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Entanglement between two atoms in an overdamped cavity injected with squeezed vacuum

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The generation of an entangled state for a pair of two-level atoms embedded in a bad cavity injected with a squeezed vacuum is investigated. The degree of entanglement between the two atoms strongly depends on the mean photon number and the strength of two-photon correlations of the squeezed vacuum, the cavity-induced decay rate of atoms, and the atomic separation.

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Entanglement constitutes the single most characteristic property that makes quantum mechanics distinct from any classical theory. It has been found that entanglement forms a fundamental resource for quantum-information processing. Recently, Schneider and Milburn [1] modeled the behavior of an ion trap with all ions driven simultaneously and coupled collectively to a heat bath. By use of the entanglement of formation [2], they show that the steady state of the ion trap as a many-body system driven far from equilibrium can exhibit quantum entanglement. Clark and Parkins [3] proposed a scheme employing quantum-reservoir engineering to controllably entangle the internal states of two atoms trapped in a high-finesse optical cavity. Kim *et al.* [4] studied the interaction of two two-level atoms and a single-mode thermal field, which frequently appears in problems of decoherence. It was demonstrated that such a chaotic field can entangle two atoms that are prepared initially in a separable state, which provides a degree of analytical understanding of the decoherence mechanism for a quantum system composed of a few qubits when the reservoir with which they interact retains some memory [5]. Quantum-nondemolition (QND) measurements on samples of atoms in a cavity have also been proposed as a means to entangle atoms located in a cavity [6]. Unlike most proposals for entangling atoms by cavity QED in which the cavity is required to be in a strong-coupling regime $g^2/\kappa\gamma \gg 1$, where g is the atom cavity coupling strength, κ is the cavity decay rate, and γ is the decay rate of the atoms, this limit is very hard to achieve experimentally. Sørensen and Mølmer [7] put forward a method to produce entangled spin squeezed states of a large number of atoms inside a bad optical cavity. In their scheme the created entanglement depends on the parameter $Ng^2/\kappa\gamma$, where N is the number of atoms. That is to say, a measurable entanglement can be produced even though the cavity is a bad one and the cavity-atom coupling is weak when N is large. This allows a substantial reduction in the requirements for the cavity [7,8].

On the other hand, with the generation and the detection of squeezed light increasing attention is being given to the study of the interaction of squeezed light with one or few atoms [9,10]. Georgiades *et al.* [11] have carried out the first experimental investigation of the modification of the fundamental atomic radiative processes brought about by illumination with squeezed vacuum light generated via nondegenerate parametric down conversion. The observed rate of

the two-photon transition $6S_{1/2} \rightarrow 6D_{5/2}$ for trapped atomic cesium excited by the squeezed vacuum light shows both linear and quadratic growth with light intensity for weak excitation, in contrast with the purely quadratic dependence produced by classical sources. Zhou and Swain [12] studied the two-photon excitation rate of a cascade three-level atom interacting with a resonant cavity mode coupled to a broadband squeezed vacuum through its input-output mirror. It was shown that in the bad cavity limit, the two-photon excitation rate has two components, one depending linearly and the other quadratically on the squeezed photon number, in excellent agreement with the experiment result in Ref. [11]. Banerjee [13] found that in the system of a pair of two-level atoms confined in a single-mode optical cavity driven by a squeezed vacuum, when the two atoms are close enough, the atomic squeezed state can be generated.

In this Brief Report, we study the degree of entanglement between two two-level atoms embedded in a bad cavity injected with a squeezed vacuum. It is found that if the injected field is a squeezed one, the two atoms can be entangled. The degree of entanglement between the two atoms is strongly dependent on the mean photon number and the strength of two-photon correlations of the squeezed vacuum injected into the cavity, the cavity-induced decay rate of atoms, and the distance between these two atoms.

Assuming that two identical two-level atoms are located in a single-mode cavity, the Hamiltonian of this system is

$$H = \omega a^\dagger a + \omega S_z + g(a^\dagger S_- + a S_+), \quad (1)$$

where a and $a^\dagger$ are annihilation and creation operators for the cavity field, and S_z and $S_\pm$ are the collective pseudospin operators, which are defined as $S_z = \sum_{j=1}^2 S_z^{(j)}$ and $S_\pm = \sum_{j=1}^2 S_\pm^{(j)}$. The cavity is injected by a broadband squeezed vacuum via its lossy mirror. Taking the spontaneous emission into account, the master equation for the atom-field interaction system is

$$\frac{d}{dt}\rho = -i[H, \rho] + L_a\rho + L_c\rho, \quad (2)$$

$$\begin{aligned} L_a\rho = & \gamma(2S_- \rho S_+ - S_+ S_- \rho - \rho S_+ S_-) \\ & + (\gamma_{12} - \gamma)(2S_-^{(1)} \rho S_+^{(2)} + 2S_-^{(2)} \rho S_+^{(1)} - S_+^{(1)} S_-^{(2)} \rho \\ & - S_+^{(2)} S_-^{(1)} \rho - \rho S_+^{(1)} S_-^{(2)} - \rho S_+^{(2)} S_-^{(1)}), \end{aligned} \quad (3)$$

$$\begin{aligned}
L_c \rho = & \kappa(N+1)(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) + \kappa N(2a^\dagger \rho a \\
& - a a^\dagger \rho - \rho a a^\dagger) + \kappa M e^{i\theta}(2a^\dagger \rho a^\dagger - a^{\dagger 2} \rho - \rho a^{\dagger 2}) \\
& + \kappa M e^{-i\theta}(2a \rho a - a^2 \rho - \rho a^2).
\end{aligned} \quad (4)$$

Here γ is the spontaneous emission rate of each atom resulting from the interaction of individual atoms with all the vacuum modes other than the privileged cavity mode, and γ_{12} is the collective spontaneous emission rate arising from the coupling between the atoms through the vacuum field, which depends on the atomic separation. If the distance between the atoms is much larger than the resonant wavelength, then $\gamma_{12} \approx 0$. On the contrary, if the atomic separation is much smaller than the resonant wavelength, then $\gamma_{12} \approx \gamma$ [14]. The parameter κ is the cavity decay constant. The parameter N is the mean photon number of the broadband squeezed vacuum field and M measures the strength of two-photon correlations they obey $M = \eta\sqrt{N(N+1)}$, ($0 \leq \eta \leq 1$).

Now we assume that the cavity-atom coupling is relatively weak and the cavity has a low- Q value, so that $\kappa \gg g \gg \gamma$ (the bad cavity limit), but $C_1 = g^2/\kappa\gamma$ is finite, where C_1 is the effective cooperativity parameter of a single atom familiar from optical bistability. Also, to ensure the validity of the broadband squeezing assumption, the bandwidth of squeezing would need to be large compared to κ . These conditions imply that the cavity-mode response to the reservoir is much faster than that produced by its interaction with atoms, which means that the photons emitted by the atoms cannot react back with the atoms; i.e., the Born approximation is valid. The condition $\kappa \gg g$ (g refers to the vacuum Rabi frequency) also indicates that the two-atom system has a short memory; i.e., the Markoff approximation can be used. After assuming that, at $t=0$, $\rho(0)$ can be factorized into a product of atoms and field density operators, using the Born-Markoff approximation and tracing over the field state [10,12,15], and introducing the atomic new basis as $|2\rangle = |e_1, e_2\rangle$, $|\pm\rangle = (1/\sqrt{2})(|e_1, g_2\rangle \pm |g_1, e_2\rangle)$, and $|0\rangle = |g_1, g_2\rangle$, the

FIG. 1. The concurrence C versus λ for different η ; here $N = 0.5$, $\alpha = 1$, and $\eta = 1.0$ (solid line), $\eta = 0.9$ (dashed line), and $\eta = \sqrt{1/3}$ (dotted line).

time evolution equations of density matrix elements for the atoms are written as

$$\begin{aligned}
\frac{d}{d\tau} \rho_{22} = & -(x + \ell) \rho_{22} + (x - \ell) \rho_{++} \\
& + M(e^{i\theta} \rho_{02} + e^{-i\theta} \rho_{20}), \\
\frac{d}{d\tau} \rho_{++} = & -(3x - 2\ell + 1 + \lambda\alpha) \rho_{++} + (\ell + 1 + \lambda\alpha) \rho_{22} \\
& - 2M(e^{i\theta} \rho_{02} + \rho_{20} e^{-i\theta}) - (x - \ell)(\rho_{--} - 1), \\
\frac{d}{d\tau} \rho_{20} = & M e^{i\theta} - x \rho_{20} - M e^{i\theta}(3\rho_{++} + \rho_{--}), \\
\frac{d}{d\tau} \rho_{--} = & -\lambda(1 - \alpha)(\rho_{--} - \rho_{22}),
\end{aligned}$$

$$\rho_{00} = 1 - \rho_{22} - \rho_{++} - \rho_{--}. \quad (5)$$

Here $\gamma_c = g^2/\kappa$, reflecting the atomic decay rate induced by the coupling of the cavity mode, $\tau = 2\gamma_c t$, $x = 2N + \ell$, $\ell = 1 + \lambda$, $\lambda = \kappa\gamma/g^2$, and $\alpha = \gamma_{12}/\gamma$. The above equations are valid after a short time $t \gg \kappa^{-1}$ [10,12,15]; in the following we are interested in the long-time limit—i.e., $t \gg \kappa^{-1}$. The steady-state solution of the reduced density matrix for the atoms can be expressed as

$$\begin{aligned}
\rho_{A_S} = & \rho_{22_S} |2\rangle\langle 2| + \rho_{++_S} |+\rangle\langle +| + \rho_{--_S} |-\rangle\langle -| \\
& + \rho_{00_S} |0\rangle\langle 0| + \rho_{20_S} |2\rangle\langle 0| + \rho_{02_S} |0\rangle\langle 2|.
\end{aligned} \quad (6)$$

Next we use the entanglement of formation proposed by Wootters [2] to quantify the degree of entanglement for two subsystems of $2 \otimes 2$ bipartite mixed or pure states. The underlying quantity is called concurrence. The expression relating the concurrence C to the density operator ρ_{A_S} defined by Eq. (6) can be easily written as

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \sqrt{\rho_{22_S} \rho_{00_S} + |\rho_{20_S}|^2}, 0\}, \quad (7)$$

where λ_1 is the maximal value of the three quantities ρ_{++_S} , ρ_{--_S} , and $\sqrt{\rho_{22_S} \rho_{00_S} + |\rho_{20_S}|^2}$, and λ_2 and λ_3 are the remaining two quantities. The case of $C=1$ corresponds to the existence of the maximum entanglement between the two atoms, and $C=0$ means no entanglement between the atoms. If $\sqrt{\rho_{22_S} \rho_{00_S} + |\rho_{20_S}|^2} > \rho_{++_S}$ and ρ_{--_S} , then Eq. (7) becomes

$$C = \max\{2|\rho_{20_S}| - \rho_{++_S} - \rho_{--_S}, 0\}. \quad (8)$$

However, if $\sqrt{\rho_{22_S} \rho_{00_S} + |\rho_{20_S}|^2} < \max\{\rho_{++_S}, \rho_{--_S}\}$, the concurrence C is expressed as

$$C = \max\{|\rho_{--_S} - \rho_{++_S}| - 2\sqrt{\rho_{22_S} \rho_{00_S}}, 0\}. \quad (9)$$

From Eqs. (8) and (9), we can see that there are two criteria for entanglement between the two atoms described by Eq. (6). The first one, from Eq. (8), is $|\rho_{20_S}| > 1/2(\rho_{++_S} + \rho_{--_S})$, which is decided by the two-photon atomic coherence and populations in the two intermediate states $|\pm\rangle$. And the second one, from Eq. (9), is $|\rho_{--_S} - \rho_{++_S}| > 2\sqrt{\rho_{22_S} \rho_{00_S}}$, which depends only on the populations in all four atomic

states $|2\rangle$, $|\pm\rangle$, and $|0\rangle$. By the aid of the negativity to measure the entanglement, Kim *et al.* [4] considered the time evolution of the entanglement between two identical two-level atoms interacting with a single-mode thermal field. It is found that the terms characterizing the two-photon atomic coherence in the reduced density matrix for the two atoms are absent, so the condition leading to the entanglement between the two atoms only meets the second criterion here. In the following, we will show that due to the two-photon correlation of the injected squeezed vacuum, two-photon atomic coherence is produced in the two-atom system; consequently, entanglement between the two-atom system considered here may arise from the existence of two-photon atomic coherence, which is satisfied with the first criterion and different from that in the system of two two-level atoms interacting with a single-mode thermal field [4].

First we consider the case of $\alpha=1$, corresponding to the case where the two atoms are close to each other; i.e., the atomic separation is much smaller than the resonant wavelength. Equations (5) indicate that if there is no population in the asymmetric state $|- \rangle$ initially—for example, the two atoms being initially in their excited states $|e, e\rangle$ or $|g, g\rangle$ —then this asymmetric state is decoupled from the whole system and there is no population in this state—that is, $\rho_{--s}=0$. The nonzero density matrix elements are

$$\begin{aligned}\rho_{22s} &= [x(x-\ell)^2 - 4M^2(x-2\ell)]/F_1, \\ \rho_{++s} &= [x(x^2 - \ell^2 - 4M^2)]/F_1, \\ \rho_{20s} &= e^{i\theta} 4M\ell^2/F_1,\end{aligned}\quad (10)$$

where $F_1 = x(3x^2 + \ell^2 - 12M^2)$. Combining Eqs. (10) with Eqs. (8) and (9), we can obtain the concurrence C characterizing the degree of the entanglement between the two atoms. It can easily be proved that the second criterion for the two-atom entanglement from Eq. (10) cannot be satisfied. So the entanglement between the atoms is because of the existence of the two-photon atomic coherence originating from the two-photon correlation of the injected squeezed field. Evidently, for $M=0$ ($\eta=0$), $C=0$, there is no entanglement between the two atoms at all. In this case, the two atoms are driven by a thermal field; the two atoms will be evolved into their thermal equilibrium, a maximal mixture state. Figure 1 displays C versus with λ for different η . We see that if the injected field into the cavity is an ideal squeezed vacuum ($\eta=1$) or a nonideal one ($\eta=0.9$), the two atoms can be entangled in the long-time limit, the entanglement degree C decreasing with the increase of λ . In contrast, when $\eta=\sqrt{1/3}$, corresponding to the injected field being the maximal classical correlations between pairs of photons, C increases with increasing λ . The larger the η is, the stronger the entanglement between the atoms is. Figure 2 shows the entanglement degree C versus with the mean photon number N of the injected field for different η . In these figures $N=0$ corresponds to the case of a normal vacuum entering into the cavity; no entanglement happens between the two atoms. With the increase in the value of N , C goes through a maximum and then goes to zero at a critical value $N=N_c$. The larger the

FIG. 2. C versus N for different η ; here $\lambda=0.25$, $\alpha=1$, and $\eta=1.0$ (solid line), $\eta=0.9$ (dashed line), and $\eta=\sqrt{N/(N+1)}$ (dotted line).

value of η , the larger the value of N_c . In fact, if the injected field has maximal classical two-photon correlation—i.e., $M=N$ —the critical value of N is $N_{c1} = \ell/2$, but for the injected field being in the ideal squeezed vacuum—i.e., $M=\sqrt{N(N+1)}$, $N_{c2} \approx [\ell(\ell+1) + \ell\sqrt{(\ell+1)^2 + 8(\ell-1)}]/4(\ell-1)$ —evidently N_{c2} is larger than N_{c1} coincident with the numerical result shown in Fig. 2.

Second, we study the case of $\alpha=1$ but one of the two atoms is initially prepared in its excited state $|e\rangle$ and the other one in its ground state $|g\rangle$; that is, the asymmetric state $|- \rangle$ is initially populated. Because the asymmetric state is decoupled from the whole system, the population in this state remains unchanged. Therefore the nonzero density matrix elements in the long-time limit obey $\rho_{--s2}=1/2$, $\rho_{22s2}=\rho_{22s}/2$, $\rho_{++s2}=\rho_{++s}/2$, and $\rho_{20s2}=\rho_{20s}/2$. It is easy to check that $|\rho_{20s2}| < 1/2(\rho_{++s2} + \rho_{--s2})$, which means that the first criterion for the entanglement is violated, which implies that the two-photon atomic coherence entanglement is not strong enough to create entanglement between the two atoms. However, the above equations show that the inequality $\rho_{--s2} - \rho_{++s2} > 2\sqrt{\rho_{22s2}\rho_{00s2}}$ holds for arbitrary values of N , η , and λ . That is, if the two atoms are close enough to each other and they are initially in the state $|e, g\rangle$, then these two atoms can be entangled under the interaction of vacuum field ($N=M=0$), thermal field ($M=0$ but $N \neq 0$), or squeezed vacuum ($N, M \neq 0$). The entanglement is associated with the populations in four atomic collective states, which is similar to that revealed in the system of two two-level atoms interacting with a single-mode thermal field [4]. Figure 3 plots the concurrence C versus N for different η . With the increasing of the intensity of the injected field, C decreases quickly; the stronger the two-photon correlation is, the faster the concurrence decreases.

Finally, we discuss the situation of $\alpha < 1$, which means that the separation between two atoms is not very small; then Eqs. (5) imply that $\rho_{--s} = \rho_{22s}$. Therefore, no matter whether or not the asymmetric state $|- \rangle$ is initially populated, in the long-time limit, due to the interaction of the nonclassical

FIG. 3. Same as Fig. 2 except $\eta=0$ (dashed-dotted line).

field, the asymmetric state will be equally populated as the upper level. The nonzero steady-state density matrix elements are expressed as

$$\begin{aligned}\rho_{22s3} &= \frac{1}{F_2} [x(x - \ell)^2 + 2M^2(3\ell - 2x + 1 + \lambda\alpha)], \\ \rho_{++s3} &= \frac{1}{F_2} [x(x^2 - \ell^2) - 2M^2(2x + \ell - 1 - \lambda\alpha)], \\ \rho_{20s3} &= \frac{1}{F_2} [2\ell M(\ell + 1 + \lambda\alpha)\exp(i\theta)].\end{aligned}\quad (11)$$

where $F_2 = x[4x^2 - 2\ell x + 2\ell(1 + \lambda\alpha)] + 8M^2(1 + \lambda\alpha - 2x)$. Replacing Eqs. (11) into Eqs. (8) and (9), we can obtain the expression of C . It can be found that entanglement for the two atoms only happens when the inequality $|\rho_{20s3}| > 1/2(\rho_{++s3} + \rho_{--s3})$ holds. That is, the two-photon atomic coherence plays a key role in the appearance of the entanglement. Figure 4 shows C varying with λ for different α . Comparing Fig. 1 with Fig. 4 we see that even though $\alpha=0.99$, there is a little deviation of $\alpha=1$, reflecting the strongest quantum interference effect between the

FIG. 4. C versus λ for different α ; here $N=0.5$, $\eta=1$, and $\alpha=0.99$ (solid line), $\alpha=0.5$ (dashed line), and $\alpha=0$ (dotted line).

two atoms induced by the spontaneous emission, and the degree of entanglement between the two atoms jumps down. This is because the asymmetric state $|- \rangle$ can be populated and independent of the initial atomic state because of $\alpha \neq 1$. We find that for $\lambda < 1$, implying that the incoherent decay rate of atoms due to the coupling with a squeezed vacuum is larger than the spontaneous decay rate of atoms due to a normal vacuum, C increases with the decrease of α , and in the region of $\lambda > 1$, the result is the reverse.

In conclusion, we study the degree of entanglement between two two-level atoms embedded in a bad cavity injected with a squeezed vacuum. The degree of entanglement between the two atoms strongly depends on the mean photon number of the injected squeezed vacuum, the strength of two-photon correlations of the injected squeezed vacuum, the cavity-induced decay rate of atoms, and the distance between the two atoms.

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- [1] S. Schneider and G. Milburn, Phys. Rev. A **65**, 042107 (2002).
 - [2] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
 - [3] S. G. Clark and A. S. Parkins, Phys. Rev. Lett. **90**, 047905 (2003).
 - [4] M. S. Kim, J. Lee, D. Ahn, and P. L. Knight, Phys. Rev. A **65**, 040101 (2002).
 - [5] C. Macchiavello and G. M. Palma, Phys. Rev. A **65**, 050301 (2002).
 - [6] J. Hong and H. W. Lee, Phys. Rev. Lett. **89**, 237901 (2002); A. S. Sørensen and K. Mølmer, *ibid.* **90**, 127903 (2003); L. M. Duan and J. Kimble, *ibid.* **90**, 253601 (2003).
 - [7] A. S. Sørensen and K. Mølmer, Phys. Rev. A **66**, 022314 (2002).
 - [8] L. Vernac, M. Pinard, and E. Giacobino, Phys. Rev. A **62**, 063812 (2000); A. Dantan, M. Pinard, V. Josse, and P. R. Berman, *ibid.* **67**, 045801 (2003).
 - [9] H. J. Carmichael, A. S. Lane, and D. F. Walls, Phys. Rev. Lett. **58**, 2539 (1987).
 - [10] J. I. Cirac, Phys. Rev. A **46**, 4354 (1992).
 - [11] N. Ph. Georgiades, E. S. Polzik, K. Edamatsu, H. J. Kimble, and A. S. Parkins, Phys. Rev. Lett. **75**, 3426 (1995).
 - [12] P. Zhou and S. Swain, Phys. Rev. A **54**, 2455 (1996).
 - [13] A. Banerjee, Phys. Rev. A **54**, 5327 (1996).
 - [14] Z. Ficek and R. Tanas, Phys. Rep. **372**, 369 (2002).
 - [15] G. S. Agarwal, W. Lange, and H. Walther, Phys. Rev. A **48**, 4555 (1993).