Solution to Problem 82-16: Malevolent traffic lights

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and satisfies

$$\min \left[0, \sum_{j=1}^N M_{ij}(x-y)_j \right] \leq (Tx - Ty)_i \leq \max \left[0, \sum_{j=1}^N M_{ij}(x-y)_j \right], \quad 1 \leq i \leq N.$$

Since M is nonnegative,

$$|(Tx - Ty)_i| \leq \sum_{j=1}^N M_{ij} |(x - y)_j|, \quad 1 \leq i \leq N.$$

Since $\text{Spr } [M] < 1$, one can then find a vector norm $\|x\|$ and associated matrix norm $\|M\| < 1$ such that $\|Tx - Ty\| \leq \|M\| \cdot \|x - y\|$ so T is a contraction operator.

A Sum of Legendre Polynomials

*Problem 83-20**, by M. L. GLASSER (Clarkson College).

Determine the sums

$$S_l = \frac{\pi}{2} \sum_{n=0}^{\infty} \{Z^n P_n(\cos \theta)\}'^l, \quad l = 1, 2, 3, \dots$$

Editorial note. The proposer has determined the sums for $l = 1$ and 2 .

SOLUTIONS

Malevolent Traffic Lights

Problem 82-16, by J. C. LAGARIAS (Bell Laboratories, Murray Hill, NJ).

Can the red-green pattern of traffic lights separate two cars, originally bumper-to-bumper, by an arbitrary distance? We suppose that:

(1) Two cars travel up a semi-infinite street with traffic lights set at one block intervals. Car 1 starts up the street at time 0 and car 2 at time $t_0 > 0$.

(2) Both cars travel at a constant speed 1 when in motion. Cars halt instantly at any intersection with a red light, and accelerate instantly to full speed when the light turns green. If the car has entered an intersection as the light turns red, it does not stop.

(3) Each light cycles periodically, alternately red and green with red time μ_j , green time λ_j and initial phase θ_j (i.e., phase at time 0) at intersection j .

Can one define triplets $(\lambda_j, \mu_j, \theta_j)$ ($j = 1, 2, \dots$) so that $\{(\lambda_j, \mu_j): j = 1, 2, \dots\}$ is a finite set and so that car 1 gets arbitrarily far ahead of car 2?

Solution by O. P. LOSSERS (Eindhoven University of Technology, Eindhoven, The Netherlands).

The answer is yes as we shall now prove. We consider two types of traffic lights (I) $\lambda = \mu = a/2$, (II) $\lambda = \mu = b/2$, where a and b are chosen in such a way that a/b is irrational. The phases of the lights are chosen in such a way that the first car never has to stop. This is easily accomplished. However, we can manipulate more with the phases. Suppose that the second car arrives at a light of type I with a delay of $ra + \theta$ seconds ($r \in \mathbb{N}$, $0 < \theta < a$). The phase is chosen in such a way that the light changes $\frac{1}{2}\theta$ seconds after car 1 passes. So the delay of car 2 increases to $(r + \frac{1}{2})a + \frac{1}{2}\theta$. If we had only used lights of type I the delay would monotonically increase to $(r + 1)a$. A similar statement is true for lights of type II. At each intersection we still have the choice of the type of light. We choose this type in such a way that the increase is maximal. Since a/b is irrational, no multiple of a is a multiple of b and the time delay will tend to ∞ .

Also solved by the proposer.

Editorial note. The proposer shows that if all the cycle times are commensurable then any two cars remain within a bounded distance of each other, no matter how the lights are specified. [C.C.R.]

A Definite Integral

Problem 82-17, by PETER HENRICI (Swiss Federal Institute of Technology, Zurich).

The integral

$$I = \int_0^1 \sin \pi x \sum_{k=1}^x (-1)^{k+1} \frac{k(k+1)}{(x+k)^3} dx$$

arose in connection with a study of indefinite numerical integration of periodic functions. Find its value.

Solution by ANDREW H. VAN TUYL (Naval Surface Weapons Center, Silver Spring, MD).

Let

$$(1) \quad f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k(k+1)}{(x+k)^3}.$$

We will consider the integral

$$(2) \quad I_m = \int_0^1 f(x) \sin (2m+1) \pi x dx,$$

$m = 0, 1, 2, \dots$, which is equal to the given integral when $m = 0$. We can verify that

$$(3) \quad f(x) = \int_0^{\infty} \frac{t^2 e^{-(x+1)t} dt}{(1+e^{-t})^3}$$

by expanding $(1+e^{-t})^{-3}$ in powers of e^{-t} in (3) and integrating term by term. From (2) and (3), we obtain

$$\begin{aligned} (4) \quad I_m &= (2m+1) \pi \int_0^{\infty} \frac{t^2 e^{-t} dt}{[(2m+1)^2 \pi^2 + t^2](1+e^{-t})^2} \\ &= \frac{(2m+1) \pi}{2} \int_0^{\infty} \left[1 - \frac{(2m+1)^2 \pi^2}{(2m+1)^2 \pi^2 + 4t^2} \right] \frac{dt}{\cosh^2 t} \\ &= \frac{(2m+1) \pi}{2} - \frac{(2m+1)^3 \pi^3}{2} J_m, \end{aligned}$$

where

$$(5) \quad J_m = \int_0^{\infty} \frac{dt}{[(2m+1)^2 \pi^2 + 4t^2] \cosh^2 t}.$$