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1/f noise in homogeneous and inhomogeneous media

L.K.J. Vandamme and Gy. Trefán

Abstract: Some experimental techniques for low-frequency resistance noise measurements are discussed. The criterion for using a low-noise current amplifier instead of voltage amplifier is given. A distinction is made between contact and bulk resistance contributions to the observed 1/f noise. The merits and difficulties of the application of the empirical relation for the 1/f noise in homogeneous and inhomogeneous media are addressed. The criterion that 1/f noise in homogeneous samples can only be detected for a number of free carriers \( N < 10^{14} \) is calculated. The authors explain why the enhanced 1/f noise, due to poor crystal quality, current crowding at contacts or at grain boundaries, and at inhomogeneous internal interfaces can be used as a diagnostic tool for quality and reliability assessment of electronic devices.

List of symbols

\( x \) Hooge's 1/f noise parameter (relative 1/f noise in the conductance normalised for carrier number and frequency) [dimensionless]

\( C_{1/f} \) relative 1/f noise for ohmic samples normalized at 1 Hz for bias condition and frequency (at 1 Hz) [dimensionless]

\( C_n \) 1/f noise coefficient for unit area \( (C_n = C_{1/f,A}) \) [cm\(^2\)]

\( D, D(t) \) diffusion coefficient, its fluctuations [cm\(^2\) s\(^{-1}\)]

\( f_c, f_{ch}, f_{cv} \) corner frequency of thermal and 1/f noise, \( f_c \) for current fluctuations, \( f_{ch} \) for voltage fluctuations [Hz]

\( g_{m0}, g_{m0}(t) \) transconductance, its fluctuations [mA/V]

\( G, G(t) \) conductance, its fluctuations [\( \Omega^{-1} \)]

\( J, \dot{J} \) current density, adjacent current density [A/cm\(^2\) in 3-dimensional case, A/cm in 2-dimensional case]

\( k \) Boltzmann constant [Joule/K] or [eV/K]

\( \mu, \mu_1, \mu_\text{mob} \) mobility, \( \mu_\text{mob} \) due to lattice scattering, due to impurity scattering [cm\(^2\)/V s]

\( N, N_\text{eff} \) number, effective number of charge carriers [dimensionless]

\( R, \dot{R}, \delta R(t) \) resistance, its average, its fluctuations [\( \Omega \)]

\( R_{eq} \) equivalent noise resistance of an amplifier \( 4kTR_0 = S_{1/f}:R_0 \) [\( \Omega \)]

\( R_{||}, R_{\perp} \) input resistance of voltage, current amplifier [\( \Omega \)]

\( p, \delta p(t) \) resistivity, its fluctuations [\( \Omega \) cm]

\( S_{1/f}, S_\alpha, S_v \) spectrum of resistance, current, voltage fluctuations [\( \Omega^2/Hz, \Omega^2/Hz, V^2/Hz \)]

\( S_{1/f}, S_v \) spectrum of the equivalent noise at the input of the amplifier [\( V^2/Hz \)]

\( S_{V_{th}, S_{1/f}} \) spectrum of thermal noise voltage, current [\( V^2/Hz, A^2/Hz \)]

\( \sigma, \delta \sigma(t) \) conductivity, its fluctuations [\( \Omega^{-1} \) cm\(^{-1}\)]

\( T, \delta T(t) \) temperature on the absolute scale, its fluctuations [K]

\( \tau \) collision time of charge carriers [s]

\( \nu_\text{th} \) thermal velocity of charge carriers [cm/s]

1 Introduction

Although we do not know much about the origin of 1/f noise, it can be used as a complementary diagnostic tool for the quality assessment of a semiconductor material or device [1]. 1/f Noise is named after its typical inverse proportionality between spectral values and frequency; it is a fluctuation in the conductivity. One school of thought says it is due to number fluctuations of the charge carriers and the other school of thought considers it due to mobility fluctuations. Our point of view is biased towards a bulk origin where mobility fluctuations are the origin of the conductance fluctuations. The 1/f noise is often described in terms of Hooge's empirical relation, where the 1/f noise parameter \( x \) plays an important role [2-7]. Other groups often interested only in low-frequency noise in MOS transistors consider 1/f noise to be a surface phenomenon, where the conductance fluctuation stems from trapping and detrapping of free carriers close to the silicon/silicon dioxide interface. The 1/f noise-dependence of the spectrum is calculated from a superposition of Lorentzians [8-10]. Weisssmann, among others, attributes the 1/f noise to defect motion in the bulk of the material [11]. In 1976 Voss and Clarke presented an explanation for the 1/f-like noise based on temperature fluctuations [12]. Klingenpenning showed that temperature fluctuations do not explain real 1/f noise in semiconductors over a large range of frequencies [13]. No serious models exist for 1/f noise in metals that invoke number fluctuations. The proposed physical origins for the mobility fluctuations in metals are different and depend on the school of thought see for example [14]. A fundamental quantum-mechanical explanation of the noise was proposed by Handel [15]. Rejections of this theory are given in [16-19].

This paper is an introduction to the topic and is partially based on lecture notes about 1/f noise given at the University of Montpellier since 1987, at Eindhoven.
University of Technology, at the University of Bordeaux in 1998, at the University of Uppsala in 1999, and at EPFL Lausanne in 2000 [20, 21].

2 Calculated resistance spectra \( S_R \) from \( S_V \) or \( S_I \)

2.1 Two-point measurements

2.1.1 Measuring resistance fluctuations from current or voltage fluctuations: Consider a sample with fluctuating resistance \( R = R + \delta R(t) \) with two contacts only in a circuit (Fig. 1) and biased by a noise-free battery \( U \) and a series resistance \( R_V \) that has thermal noise only. What is measured as \( S_V \) or \( S_I \) often is different from an open-circuit voltage noise obtained from a perfect current source or a short-circuit current noise obtained from a perfect voltage source, respectively.

In Fig. 1, we observe the voltage noise \( S_V \) by an ideal voltage amplifier \( B \) and the current noise \( S_I \) by an ideal current amplifier \( A \) in series with the sample. For the input resistance of the voltage and current amplifiers holds \( R_{inB} \gg R_V/(R+R_V) \) and \( R_{inA} \ll R+R_V \), respectively. The current fluctuations \( \delta I \) caused by sample resistance fluctuations \( \delta R \) are computed from Ohm’s law \( I = U/(R+R_V) \) by logarithmic differentiation. Similarly, the voltage fluctuations \( \delta V \), due to the sample resistance fluctuations, are obtained by logarithmic differentiation of \( V = IR = UR/(R+R_V) \). For \( \delta I \) and \( \delta IV \) we obtain:

\[
\frac{\delta I}{I} = -\frac{\delta R}{R+R_V} \tag{1a}
\]

and

\[
\frac{\delta V}{V} = \frac{\delta R}{R} \left(\frac{R_V}{R+R_V}\right) \tag{1b}
\]

\[\text{Fig. 1 Bias circuit for measuring resistance fluctuations of } R \text{ with ideal noise-free voltage amplifier } B \text{ and ideal noise-free current amplifier } A \]

\[R = R + \delta R\]

Eqs. 1a and 1b show that the current and voltage fluctuations are anticorrelated because they are fully correlated but with a different sign. An increase in resistance \( R \) leads to a decrease in current \( I \) with an increase in voltage drop \( V \). These relations hold for quasistatic changes in \( R \) around its average value \( R \). After band-pass filtering, squaring and averaging, it also holds for the spectral values of \( \delta R(t) \), \( \delta I(t) \) and \( \delta V(t) \). Considering fluctuations due to resistance fluctuations only and ignoring the thermal noise and proper noise of low-noise amplifiers we obtain the following from eqs. 1a and 1b:

\[
S_I = S_R \left(\frac{R}{R+R_V}\right)^2 \tag{2a}
\]

and

\[
S_V = \frac{S_R}{R^2} \frac{R_V}{(R+R_V)^2} \tag{2b}
\]

The correction factors due to the bias conditions are always smaller than one. On top of the fluctuations caused by resistance fluctuations in \( R \), we will observe the thermal noise and contribution of the noise of the amplifiers. If \( R_V \) tends to zero, this highest value for \( S_I \) is observed and \( S_V \) tends to zero as can be seen from eqns. 2a and 2b. This is the so-called short-circuit limit and only current fluctuations are observed. Only then do we find relative current fluctuations equal to the relative resistance fluctuations,

\[
\frac{S_I}{I^2} = \frac{S_R}{R^2} \tag{3}
\]

For \( R_V \gg R \), keeping \( I = U/(R+R_V) \) constant, \( S_I \) tends to zero as can be seen from eqn. 2a and we are in the so-called open-circuit limit and only voltage fluctuations are observed and only then we find

\[
\frac{S_V}{V^2} = \frac{S_R}{R^2} \tag{4}
\]

The maximum value for \( S_V \) for a fixed maximum value of battery voltage is calculated from \( dS_V/dR_V = 0 \) with \( S_V = \frac{U^2R_S}{R_V} \left(\frac{R}{R+R_V}\right) \). The highest value for \( S_V \) will be observed for \( R_V = R \). The maximum is given by

\[
S_{V\text{max}} = \frac{I^2}{4} \frac{S_R}{R^2} = \left(\frac{U}{R}\right)^2 \frac{S_R}{16} = \frac{1}{4} \frac{V^2}{R^2} S_R \tag{5}
\]

If no low-noise current amplifier is available, current fluctuations can be measured with a voltage amplifier just like a current can be measured with a voltmeter across a small, precise series resistance. For \( R_V \ll R \) with \( S_I = S_V/R_V \), from eqn. 2b \( S_I/V^2 = S_R/R^2 (R_V/R)^2 \) holds, and it follows that \( S_I/V^2 = S_R/(R+R_V)^2 \), where \( S_I \) is measured either across \( R_V \) or \( R \) which will give the same result, although the DC voltage drop across \( R_V \) and \( R \) can be quite different.

2.1.2 Determining the corner frequency below which 1/f noise merges: The thermal noise of the bias resistor \( R_V \) and the sample \( R \) will always be the dominant noise at higher frequencies. Hence, without taking into account the noise properties of the amplifiers and the influence of the impedances \( R_{inB} \) and \( R_{inA} \) we find

\[
S_{V \text{th}} = 4kTR_V \frac{R}{R+R_V} \tag{6a}
\]

and

\[
S_I = 4kT \frac{R}{R+R_V} \tag{6b}
\]

The so-called corner frequency \( f_c \) or \( f_{cl} \) is the frequency at which the 1/f noise \( S_{V \text{th}} \) or \( S_I \) and the white noise \( S_{V \text{th}} \) or \( S_{V \text{th}} \) contributions are equal, respectively. The 1/f noise of the resistor can be written as

\[
S_R = C_{1/f} \frac{R}{f} \tag{7}
\]

with \( C_{1/f} \) a dimensionless constant often between \( 10^{-10} < C_{1/f} < 10^{-8} \). The 1/f noise in samples with \( C_{1/f} < 10^{-16} \) is hardly observable due to the competition of thermal noise even at extreme bias conditions and samples with \( C_{1/f} \geq 10^{-4} \) shows other types of noise (random telegraph signal noise, RTS) on top of the 1/f noise. Then the corner frequency \( f_c \) from eqns. 2b, 6a and 7 for the voltage
amplifier is as follows:

\[ f_{cl} = \frac{U^2 Rv RCV_{eff}}{(R + Rv)^3} \frac{4kT}{U^2} \]  

(8)

For \( f_{cl} \) of the ideal current amplifier an expression holds which is different from eqn. 8. It is calculated from eqns. 6b and 7:

\[ f_{cl} = \frac{U^3 R^2 C_{eff}}{(Rv + R)^3} \frac{4kT}{U^2} \]  

(9)

As can be seen from eqns. 8 and 9, both \( f_{cl} \) and \( f_{cl} \) and hence the frequency range in which to study the resistance fluctuations shrinks with increasing values of \( R \) and \( Rv \). The maximum value for \( f_{cl} \) applying a low noise voltage amplifier and a fixed battery voltage \( U \) is calculated from \( df_{cl} \)/\( dRv = 0 \) and leads to the condition \( Rv = R/2 \). This condition is slightly different from the condition \( Rv = R \) for a maximum in the value of \( f_{cl} \). With only a current amplifier in Fig. 1, \( Rv = 0 \) is chosen, and then follows that the ideal current amplifier is always a better choice than an ideal voltage amplifier to study conductance noise because

\[ f_{cl} \frac{4kT}{U^2 C_{eff}} = \frac{Rv R}{(Rv + R)^3} < f_{cl} \frac{4kT}{U^2 C_{eff}} = \frac{1}{R} \]  

(10)

Ignoring the fact that current amplifiers and voltage amplifiers are nonideal leads to the wrong conclusion that current amplifiers always result in corner frequencies at least 27/4 times higher than voltage amplifiers. Current amplifiers are in general the best choice when the sample resistance is of the same order of magnitude or larger than the input impedance of the voltage amplifier \((R > Rb)\).

2.2 Four-point measurements, how to suppress contact noise

The noise in the conductivity is often contaminated by an extra noise contribution at the contacts. The contact noise contribution can be reduced [22] in a sample provided with a pair of current contacts \((D1, D2)\) and a pair of voltage contacts \((Q1, Q2)\) (see Fig. 2). If the input resistance of the amplifier \( R > R(Rv + r + r) / (R + Rv + r + r) \) then the following equations hold:

\[ S_{cl} = \frac{R^2 S_k}{(1 + R(Rv + r + r))} \]  

\[ + (1 + (Rv + r + r)/R) \frac{S_{cl}}{R} \]  

\[ + (1 + (Rv + r + r)/R) \frac{S_{cl}}{R^2} \]  

(11a)

\[ S_{cl} = \frac{S_k}{R^2} \left( \frac{Rv + r + r + r}{Rv + r + r + r + R} \right)^2 \]  

\[ + \left( \frac{S_{cl}}{R} \right) \left( \frac{Rv + r + r + r}{Rv + r + r + r + R} \right)^2 \]  

(11b)

\( S_k \) represents the resistance fluctuations between the sensing contacts \( Q1, Q2 \) and \( S_{cl} \) and \( S_{cl} \) are the uncorrelated contact noise between the driver and sensor contacts \( D1Q1 \) and \( D2Q2 \) respectively. Normally we wish to suppress the influence of \( S_{cl} \) and \( S_{cl} \). Keeping the same current \( f \) by increasing \( U \) and \( Rv \) the contact contributions can be reduced (see eqns. 11a and 11b). By contrast, for \( Rv \to 0 \), mainly contact noise is observed as can be seen from eqns. 11a and 11b.

However, contact noise reduction or assessment is not that trivial, as mentioned above. If the samples have only one pair of contacts, as in FETs or MOSFETs, there is only one way to distinguish between contact noise and bulk noise: by applying an L-array [23, 24]. An L-array is a set of samples submitted to homogeneous fields all having the same width but different lengths \( L \) between contacts. Then it is assumed in the analysis that the resistance \( R \) and the noise resistance \( S_{cl} \) scale with \( L \) while the contact contribution is independent of \( L \).

For a single sample with one pair of current driver contact \( D1, D2 \) and one pair of sensor contacts \( Q1, Q2 \), with the variable \( Rv \)-method we can detect whether or not the contact noise contributions are dominant on the bulk contribution. By changing \( U \) and \( Rv \) together and keeping about an average current through the sample constant, we suppress the noise contribution of the current carrying contacts as can be seen from eqn. 11b. If the noise is dominated by the part in between the sensor contacts, \( S_{cl} \) should not change in this experiment \((f, constant; U \) and \( Rv \) increasing). However, this does not mean that a noise contribution of the sensor contacts \( Q1 \) and \( Q2 \) is excluded! Although the sensor contacts \( Q1, Q2 \) do not pass a current out of the sample through the amplifier in Fig. 2 there can be a current through this contact entering and leaving at the rims and not going out of the contacts. This contact noise is not suppressed by a four-point probe analysis with the variable \( Rv \)-method. Therefore, the sensor contacts must also be of high quality, low noise and must not be silver-paint contacts. The best way to avoid contact noise contributions from \( D1, D2, Q1, Q2 \) is to put \( Q1, Q2 \) in a field and current density free area of the sample and not in a current path. A cross-shaped sample as shown in [25] is a satisfactory compromise between a good suppression of the possible contact noise at the current driving electrodes \( D1, D2 \) and sensors \( Q1, Q2 \) and an increase of thermal noise at the sensors by contact arms which are too long. This will be explained in Section 5.3 in terms of the dot product of current density and adjoint current densities.

3 1/f noise

The physical origin(s) of 1/f noise are still unknown.

3.1 What do we know about 1/f noise?

(a) It is a fluctuation in the conductivity \((\delta \sigma)\) already present in thermo-equilibrium: even for \( f = 0 \), \( S_{cl} \propto 1/f \) if it already exists [4]. Passing a noise-free DC or AC current from a current source through the sample translates the resistance fluctuations into voltage fluctuations as follows:

\[ I^2 S_{R} = S_{cl} \propto 1/f \]  

(12)
For AC current excitations with a carrier frequency \(\omega_c\), considering a value of \(\delta R\) at \(\omega_c\) in the resistance fluctuations we find the following for the voltage fluctuations:

\[
\delta V(t) = 1 \sin(\omega_c t) \cdot \delta R(t) \cdot \cos(\omega_c t)
\]

\[
= \frac{(1/2)}{T} \delta R(t) \cdot \left[ \sin(\omega_c + \omega_1) t + \sin(\omega_c - \omega_1) t \right]
\]

By applying an AC current, a modulation of the carrier by \(\delta R(t)\) is observed in voltage fluctuations. This is the origin of low-frequency phase noise very close to the carrier frequency in amplifiers and oscillators. Around the carrier frequency the so-called 1/f noise appears [4], as indicated by the sum and the difference in frequency from eqn. 13. This 1/f noise is nowadays often the strongest contribution to the so-called phase noise at frequencies beyond 1 GHz close to the carrier frequency.

(b) The conductivity fluctuations cannot be caused by temperature fluctuations \(\delta T\) [4, 26]. This is because samples with a negligibly small temperature coefficient have the same 1/f noise as samples with a normal temperature coefficient, while the temperature coefficient enters the temperature-induced 1/f noise should be observed. The experimental results on constantan and manganin with 0 \(\Omega\) show the same 1/f noise as in other metals with a normal temperature coefficient. In 1969 \(\Omega\) only can be calculated in Section 4, eqn. 33. This is due to the competition of thermal noise and the 1/f noise limited by the maximum power that can be dissipated in the sample. Metal thin-film samples with \(N < 10^9\) often are homogeneous on a microscopic scale. This results in a noise increase due to current crowding on a microscopic scale. In eqn. 33. \(N\) should be replaced by a lower effective value \(N_{eff}\) as will be explained in the section on current crowding (Section 5). This value \(N_{eff}\) only can be calculated if the electric field is known even on a microscopic scale. If not, \(N\) is not the case 1/f noise can better be expressed in Cu/g values or \(\omega_m = area \times C_{Cu}\) instead of \(ax\) values. Normally \(ax\) values observed in bulk n- and p-type Si at 77 K and 300 K for \(n = 3 \times 10^{15} \text{cm}^{-3}\), \(p = 6 \times 10^{15} \text{cm}^{-3}\) are \(ax = 10^{-6}\) [33]. Similar \(ax\) values have been observed from 1/f noise in silicon diodes, bipolar transistors and MOS transistors although the latters are typical surface devices [1, 10]. The parameter \(ax\) is the contribution of one electron to the relative noise at 1 Hz assuming that the \(N\) electrons are uncorrelated fluctuators.

The usefulness of eqn. 16 lies in the fact that a comparison in 1/f noise in \(ax\) values is made independent of the bias frequency and size \((N)\) of the device. The misuse of eqn. 16 to calculate \(ax\) values from experimental results by overlooking the nonuniform current densities on a microscopic scale always leads to overestimation of the apparent \(ax\) values, as shown by Vandamme et al. [1, 10, 30, 34, 36], so they represent no arguments against the validity of the empirical relation (eqn. 16). \(ax\) values from well-defined structures are in the range \(10^{-6}\) to \(10^{-9}\). Values lower than \(10^{-2}\) often are apparent \(ax\) values and stem from calculations overlooking current crowding on a microscopic scale. Values lower than \(10^{-2}\) sometimes occur from analysis of results in bipolar transistors in hot electron regime applying too simple models.

### 3.2 Meaning of 1/N dependence in the empirical relation

Let us try the mobility fluctuation noise source \(\delta \mu \) in a homogeneous sample with \(N\) free carriers to explain the resistance fluctuations. The conductance \(G\), its average value \(\bar{G}\) and the variance \(\delta G^2\) are proportional to \(G \propto \sum N, \mu_1, \mu_2 \propto G \propto N\mu_1\), thus \(\delta G^2 \propto N\delta \mu^2\), because the fluctuations in \(\mu_1\) and \(\mu_2\) are uncorrelated, so \(\delta \mu_1 \delta \mu_2 = 0\) for \(i \neq j\), respectively. Thus, the relative noise with \(N\) independent noise fluctuators is calculated as follows:

\[
\frac{\delta G^2}{G^2} = \frac{N_\mu \delta \mu^2}{N^2} = \frac{1}{N} \left( \frac{\delta \mu}{\mu} \right)^2
\]

Hence, the 1/N dependence for the relative noise is not a proof for number fluctuations \(\delta N\). However, in n-channel MOS transistors it is believed that 1/f noise stems from number fluctuations, (see for example [10] and [37]). The variance \(\delta N^2\) is due to traps at the interface (with \(N_t\) traps occupied by electrons and \(P_t\) empty traps). Biasing the MOS transistor above threshold voltage generally results in \(N_t\) and \(P_t\) smaller than \(N\) and the variance equals \(\delta N^2 \approx \min(N_t, P_t)\), \(N\) can be changed with the gate voltage and then \(1/N^2\) dependence holds for the relative noise due to number fluctuations. Expressing the noise in terms of the empirical relation then results in \(x \propto 1/V_{th}\) with \(V_{th} = |V_G - V_t|\) the effective gate voltage:

\[
\frac{\delta G^2}{G^2} = \frac{\delta N^2}{N^2} \propto 1/N^2
\]
In general for number fluctuations in semiconductors the Fermi statistics holds. For the generation–recombination noise between a single trap level and conduction or valence band, we find

\[
\frac{1}{\delta N^2} = \frac{1}{N_t} + \frac{1}{N} + \frac{1}{P_t}
\]

Eqn. 19 shows the strong sensitivity of generation–recombination noise on the position of the Fermi level \(E_F\) relative to the trap level. For either \(N_t \to 0\) (empty trap levels well above Fermi level) or \(P_t \to 0\) (full trap levels far below Fermi level), the variance \(\delta N^2 \to 0\) and the \(\delta N\)-noise disappears. If Noise, on the other hand, is not sensitive at all to the position of the Fermi level as long as \(N\) remains constant. This is an indication, among others, that \(1/f\) noise is not a \(\delta N\) noise (see next Section).

### 3.3 Some \(1/f\) noise problems

(a) The spectral density \(S_V\) is divergent for \(f \to 0\). The experimental verification may need an infinite measuring time. The lowest frequency where the \(1/f\) dependence was still observed was \(10^{-8}\) Hz. The variance of the \(1/f\) noise increases logarithmically with the measuring time \(t\) and is logarithmically divergent for \(f \to 0\) and \(f \to \infty\). For pure \(1/f\) noise the following holds for the variance \(\delta^2\) in a bandwidth \(f_0 - f_1\) spanned by high limit frequency \(f_0\) and low limit frequency \(f_1\) for \(S_V = Af_f^2\):

\[
\delta^2 = \int_{f_1}^{f_0} S_V df_f = \int_{f_1}^{f_0} (A/f_f) df_f
\]

\[
= A \ln(f_0/f_1) = A \ln f_0 + \ln t
\]

With the help of this relation, the \(1/f\) nature of a noise signal in the microhertz range was verified by measuring the variance as a function of measuring time e.g. \(t \leq 3 \times 10^5\) s. At \(f_1 = 3 \times 10^{-6}\) Hz the spectral density of the noise signal was still found to be inversely proportional to the frequency [38]. The problem of the logarithmically divergent variance can be solved by introducing two corner frequencies: \(f_c\) where \(S_V\) levels off for \(f < f_c\), and a cut-off frequency \(f_{ch}\) where \(S_V \propto 1/f^2\) with \(\gamma > 1\) for \(f > f_{ch}\). However, \(f_c\) and \(f_{ch}\) have never been observed experimentally. The \(1/f\) noise component normally disappears in the white noise thermal voltage, Hall voltage, hot carriers and \(1/f\) noise in thin samples with a different mixture of lattice and surface scattering. This can be found in [4].

### 4 Criterion for observing \(1/f\) noise in homogeneous resistors

\(1/f\) noise voltage exceeds the thermal noise and background noise of the amplifier only above a certain minimum power dissipated in the sample. In thin metal and semiconductor films, power dissipation is limited by a maximum allowable temperature increase of the sample. The \(1/f\) noise voltage in homogeneous samples submitted to homogeneous fields exceeds the thermal noise and amplifier noise if the following holds (from eqn. 16):

\[
\frac{\alpha V^2}{jN} > 4kT(R + R_{eq}) = 4kT \left( \frac{L^2}{qjIN} + R_{eq} \right)
\]

where \(V\) is the average voltage drop across the resistor \(R, L\) is the distance between the electrodes, and \(N\) the number of carriers. The equivalent noise resistance \(R_{eq}\) of an amplifier is often given by a frequency independent part \(R_0\) and a \(1/f\) noise contribution below a corner frequency \(f_c\). The noise is represented at the input of an amplifier scattering, \(\mu_{amp}\), that

\[
\frac{1}{\mu} = \frac{1}{\mu_{amp}} + \frac{1}{\mu}
\]

From the hypothesis that \(1/f\) noise only is in lattice scattering, we calculate the following for the fluctuations \(\delta\mu\) (\(\delta\mu_{amp} = 0\) and in spectral values \(S_{\delta\mu}/V^2\) in terms of \(x\) values respectively from eqns. 21 and 16:

\[
\frac{\delta\mu_0}{\mu} = \frac{\delta\mu_{amp}}{\mu_{amp}^2} \Rightarrow \frac{\delta\mu}{\mu} = \left( \frac{\mu}{\mu} \right)^2 \frac{\delta\mu}{\mu_{amp}}
\]

\[
S_{\delta\mu} = \left( \frac{\mu}{\mu} \right) \frac{\delta\mu_{amp}}{\mu_{amp}^2} = \left( \frac{\mu}{\mu} \right)^2 \alpha f \frac{N}{N} = \frac{\alpha_{measured}}{N_f}
\]

Eqs. 23 and 24 have been verified experimentally [3] down to a reduction factor value \((\mu/\mu) = 10^{-3}\). \(\delta N\) cannot explain these experimental results. The \(x\) value for pure lattice scattering depends on crystal lattice quality and can be as low as \(3 \times 10^{-6}\) in silicon [1, 30, 33]. More recent experimental results on thin-film MBE grown III–V compounds are in agreement with this model [39, 40]. This reduction in \(x\) with a reduction in \(\mu\) has been observed for surface scattering too [41]. Under hot electron conditions \(\mu\) is also decreased and \(x\) is also reduced [42].

A consequence of mobility fluctuations is a fluctuation in the diffusion coefficient. The Einstein relation \(D/\mu = kT/\eta\) holds and hence we can expect \(1/f\) fluctuations in the diffusion coefficient too, as given by the following:

\[
\frac{\delta\mu}{\mu} = \frac{\delta D}{D}
\]

This explains why diodes and bipolar transistors, where the current transport is governed by diffusion, also exhibit \(1/f\) noise. In MOS transistors the transconductance \(g_{m0}\) is proportional to mobility and hence will fluctuate as \(\delta g_{m0}/g_{m0} = \delta\mu/\mu\). Other experimental evidence for mobility fluctuation stems from \(1/f\) noise in thermal voltage, Hall voltage, hot electrons and \(1/f\) noise in thin samples with a different mixture of lattice and surface scattering. This can be found in [4].
by $S_{ne} = 4kT\rho_{eq}$ with

$$R_{eq} = R_e (1 + f_c/f)$$

(27)

$f_c$ is the corner frequency where the 1/f noise and white noise contribution of the amplifier are equal. Two versions of eqn. 26 are considered under the conditions $R_{eq} \ll R$, and $R = L'/(a \mu N)$ for homogeneous samples submitted to homogeneous fields. One condition is given in terms of a critical electric field criterion $E = V/L'$, which is appropriate for dielectric types of materials, and the other condition is in terms of a power density criterion $\rho F$, which is more appropriate for low ohmic materials like metal and semiconductor films:

$$E^2 > 4kTf / \omega_{eq} \mu$$  \(\text{or}  \ E > \sqrt{4kTf / \omega_{eq} \mu}$$

(28)

$$\rho F > 4kT f n / \alpha$$

(29)

with $\rho$, $J$, and $n$ the resistivity, the current density and the free carrier concentration respectively. Eqs. 28 and 29 are independent of sample dimensions and $N$. Due to the low value of mobility in dielectric types of materials and low $\alpha$ values, the 1/f noise is hard to detect for $f > 10^7$ Hz and for an electrical field below breakdown, as can be seen from eqn. 28. For thin metal layers and semiconductors, eqn. 29 is more appropriate. Hence, the 1/f noise can only be expected below a critical frequency $f_c$,

$$f < f_c = \frac{\alpha \rho F}{4kT}$$

(30)

Again, the critical frequency $f_c$ is independent of the size of the sample if $R > R_{eq}$. For thin metal films there is an absolute maximum value for the dissipated power density $P_d = \rho F = 10^9 W/cm^3$ while avoiding damage. From eqn. 30 it follows that at room temperature metals with an $\alpha$ value of $10^{-1}$ will display 1/f noise only for $f < 250$ Hz.

With a resistivity of about $\rho = 2 \times 10^{-6} \Omega \ cm$, a limiting value for the current density of $J = 7 \times 10^6 A/cm^3$ is found. At such current densities, an onset of electromigration goes hand in hand with current-induced resistance fluctuations. Resistance against time starts to show spikes and a drift in the resistance value. This results in a strong degradation and a typical 1/f current-induced noise, and finally damage [1].

The corner frequency for semiconductors, biased at maximum current density $J_{max} = qn_{max}$, is calculated as follows:

$$f < f_c = \frac{3c}{4\tau}$$

(31)

with $\tau$ the collision time. Here we assume that the maximum value of $\rho F = 3kTn/a$. For a semiconductor with an $\alpha$ value of $4 \times 10^{-3}$ and $\tau = 10^{-12}$ s, the corner frequency can be as high as $3 \times 10^3$ Hz. Here, the problems of an excessive temperature increase in the semiconductor for $n > 10^{15}$ cm$^{-3}$ is ignored. The background noise of the amplifier in eqns. 28–31 is ignored compared to the thermal noise of the sample. This erroneously suggests that the number of carriers $N$ and the distance between electrodes $D$ does not play a role in the conditions for minimum electric field $E$ or power density $\rho F$, which is necessary to see 1/f noise above the thermal noise. For inhomogeneous samples (carbon resistors and grain-boundary structures) the empirical relation for 1/f noise given in eqn. 16 cannot be used and the relative 1/f noise must be expressed in the dimensionless parameter $C_{1/f}$ as defined in eqn. 16. The corner frequency where the 1/f noise equals the thermal noise in eqn. 30 then becomes in terms of $C_{1/f}$:

$$f_c = \frac{J^2 R C_{1/f}}{4kT}$$

(32)

The 1/f noise parameter $C_{1/f}$ is inversely proportional to the volume of the sample. On the one hand it can be seen from eqn. 32 that for $R > R_{eq}$ the length of the sample has no influence on $f_c$. On the other hand, decreasing the cross-section by a factor $x$ increases $f_c$ by a factor $x^2$ for the same $I$.

For homogeneous samples with a resistance $R$ larger than $R_{eq}$, the rule of thumb for detecting conductance fluctuations above thermal noise under reasonable bias conditions is $N < 10^{16}$. This criterion is calculated from eqn. 29 by multiplying both sides of the equation with the volume of the sample which results in

$$N < \frac{P}{4kTf}$$

(33)

where $P$ is the power dissipation in the sample and $f$ is the frequency at which 1/f noise must be detectable. For real laboratory conditions with $P = 200 mW$, $x = 10^{-3}$, $f = 10^7$ Hz, and $4kT = 1.6 \times 10^{-20}$ Joule, the relation $N < 10^{16}$ holds.

5 Conduction fluctuations and current crowding

5.1 Sensitivity coefficient in a distributed system

The voltage fluctuation across a pair of contacts (terminals), where a constant current $I$ is applied in an electrical network due to a resistance fluctuation can be calculated from a sensitivity coefficient $\delta V / \delta R$. From the Tellegen theorem [43] it follows that

$$\delta V = \frac{\sigma}{I}$$

(34)

with $I$ the current through the fluctuating resistor. The noise in a resistive network due to a fluctuating resistance $R$ where a current $i$ flows is given by

$$\delta V^2 = \left( \frac{\sigma}{I} \right)^2 \delta R^2$$

(35)

This is independent of the physical origin of the resistance fluctuations. Either a temperature-induced resistance fluctuation, a 1/f or a generation–recombination noise induced resistance fluctuation can be calculated from eqn. 35. A distributed system can be considered as a limit case of a network, a local change in the resistivity $\rho = 1/\sigma$ provokes a voltage fluctuation if a constant current source is applied [44]. The resistance $R$ and conductance $G$ between two electrodes is easily calculated using the total dissipated energy in the conductor. When a constant current $I$ is passed through the electrodes, the power relation leads to:

$$R = \frac{1}{I^2} \int_\Omega \sigma E^2 d\Omega$$

(36a)

$$\Rightarrow G = \frac{1}{V^2} \int_\Omega \delta V / \delta R$$

(36b)

The total dissipated power $P^R$ is the sum over the whole sample volume $\Omega$ of the power density multiplied by subvolumes $d\Omega$. The general eqns. 36a and 36b reduce to the well-known equation $R = \rho L / A = 1/G$ for a homogeneous sample with $\rho$ and $\sigma$ uniform and subjected to uniform electric field $E = (J \mu)$. The sensitivity coefficient $\delta V / \delta R$ is taken from the derivative of $V = (1/2) \int_{\Omega} \sigma E^2 d\Omega$, in the sub-area $\Omega$, where the resistivity increase $\delta \rho$ occurs

$$\frac{\delta V}{\delta \rho} = \frac{1}{I} \int_{\Omega} J^2 d\Omega$$

(37)
After multiplying eqn. 36b with $V$, it follows that $I = \frac{1}{\sqrt{V}} \int_{\Omega} \rho E^2 d\Omega$ and the sensitivity coefficient $d\rho/d\Omega$ for a change $d\rho$ in an area $\Omega$ becomes under constant voltage bias (short-circuited current noise)

$$\frac{dI}{d\sigma} = \frac{1}{V} \int_{\Omega} E^2 d\Omega \quad (38)$$

For a homogeneous increase in resistivity $\delta \rho$ over the complete volume $\Omega$ the integral must be taken over $\Omega$. The voltage or current fluctuation is given by the product of the sensitivity coefficient and $\delta \rho$ or $d\sigma$. From eqns. 37 and 38 it follows that

$$\delta V = \frac{1}{I} \int_{\Omega} \delta \rho J^2 d\Omega \quad (39a)$$

or

$$\delta I = \frac{1}{V} \int_{\Omega} \delta \sigma E^2 d\Omega \quad (39b)$$

If the $1/f$ noise source is spatially uncorrelated and distributed homogeneously over the sample, the empirical relation can be applied to a sub-volume where $(d\rho)^2 = \rho^2/\eta d\Omega$. This results in a noise voltage spectral density $45$:

$$S_V = \frac{1}{f^2} \int_{\Omega} \frac{\alpha \rho^2 J^4}{n} d\Omega \quad (40)$$

For homogeneous samples, $\alpha$, $\rho$, and the concentration of free carriers $n$ can be in front of the integral. The relative $1/f$ noise is inversely proportional to $N$ in homogeneous samples (see eqn. 16). For inhomogeneous current densities $N$ in eqn. 16 has to be replaced by a reduced effective number $N_{eff}$. From eqn. 36a it follows that

$$V^2 = \frac{1}{f^2} \left[ \int_{\Omega} \rho J^2 d\Omega \right]^2 = \frac{\rho^2}{f^2} \left[ \int_{\Omega} J^2 d\Omega \right]^2 \quad (41)$$

In the latter expression of eqn. 41, the resistivity is assumed to be uniform and hence in front of the integral. Then the relative noise $S_V/V^2$ becomes, from eqns. 40 and 41:

$$\frac{S_V}{V^2} = \frac{\alpha}{f^2} \left[ \frac{\int_{\Omega} J^2 d\Omega}{\int_{\Omega} J^2 d\Omega} \right]^2 \equiv \frac{\alpha}{n N_{eff} f} \quad (42)$$

with the effective number of carriers $N_{eff} = nN_{eff}$. The effective volume seems to be concentrated at the spots of the highest current density and can be much smaller than the complete sample volume $\Omega$. From the definition for $\Omega_{eff}$ in eqn. 42 we find

$$\Omega_{eff} = \left[ \frac{\int_{\Omega} J^2 d\Omega}{\int_{\Omega} J^2 d\Omega} \right] < \Omega \quad (43)$$

$\Omega_{eff}$ is $\Omega$ only for uniform current density ($J$ constant). When the resistivity is homogeneous and the current density is not, an effective volume and number of carriers should be introduced as given in eqn. 43 to calculate the $1/f$ noise or an $x$ value instead of eqn. 16.

5.2 Remarks about $1/f$ noise in inhomogeneous media

(a) Deviations from homogeneity in electric field and current density in samples with homogeneous properties in $\rho$, $n$ and $x$ result in an increase in excess noise. Examples:

- Electrical contacts, vias and conductive adhesives
- Granular thin layer with thickness $10 \text{ nm} < t < 100 \text{nm}

(b) Interfaces are notorious for the excess noise, because local value of $\alpha$ or $n$ can be much higher than outside the interface. Due to a poor local crystal quality or a native oxide, $\sigma$ can be higher than outside the interface. Due to depletion at the crystal boundaries $n_i$ can be lower thus increasing $1/f$ noise. This is understood without considering trapping as the origin of the $1/f$ noise.

(c) The main reasons why $1/f$ noise is a diagnostic tool for reliability and quality assessment of electronic devices are: (i) the strong sensitivity to current crowding and (ii) the strong sensitivity to the ultra-thin interfaces with high local values of $\alpha$ or $n$ perpendicular to the current flow and (ii) poor crystal quality shows up more pronounced in high $x$ values than in low mobilities [5, 46]. The above-mentioned trends in eqn. 40 about current crowding and interfaces do not invoke trapping ($\delta N$-fluctuations) as an explanation for the increase in noise.

(d) Apparent $x$ values from misusing the empirical relation are often too high. Eqn. 16 is not suited for inhomogeneous materials. For inhomogeneous current densities the volume of the conducting material has to be replaced by an effective volume as shown in eqn. 43; and hence an effective number $N_{eff}$ has to be calculated before $x$ can be calculated. Thick film resistors (TFRs) are said to form an inhomogeneous resistive network of conducting grains in an insulating matrix and the $1/f$ noise can better not be represented by an $x$ value.

To illustrate the point that overlooking the current crowding problems can lead to apparent high $x$ values, we propose a simple thought experiment [35]. A cube with a pair of opposite contacts with area $l^2$ at a distance $l$ is considered see Fig. 3 [36]. The volume $V$ has a uniform current and a uniform noise source characterised by a parameter $x$. In between the contacts we reduce the conducting zone to a circular spot with radius $a$. This reduced contact zone induces a constriction current flow and higher measured noise [45]. Assuming hemispherical equipotential surfaces around the contact spot, the resistance is higher than $R = \rho/l$ and becomes

$$R = \frac{\rho}{\pi a} \quad (44)$$

![Fig. 3 Schematic representation of the homogeneous cubic volume with a reduced circular contact zone with diameter $2a$](image)
where \( a \) is the radius of the spot contact and \( \rho \) the resistivity of the material. The calculated effective volume \( V_{\text{eff}} \) is [26, 45]

\[ V_{\text{eff}} = 20\pi a^3 \]  

(45)

This effective volume is introduced in the Hooge empirical relation.

\[ \frac{S_y}{R^2} = \frac{\alpha}{n\Omega_{\text{eff}}} f = \frac{\alpha}{n20\pi^3 a^3 f} \]  

(46)

Overlooking the problem of current crowding and applying the empirical relation for homogeneous media with \( \Omega = \Omega_{\text{eff}} \) instead of \( \Omega_{\text{eff}} \) given by eqn. 45 results in overestimated apparent \( \alpha \) values, which have nothing to do with the characteristic \( 1/f \) noise parameter of the material. The apparent noise parameter \( \alpha_{\text{app}} \) can be calculated from the experimentally observed relative noise and the number of carriers \( N \). The total number of free carriers is not always known and for homogeneous samples submitted to a homogeneous field it holds that \( N = \frac{F}{\mu q} \). By overlooking the current crowding problems an apparent \( \alpha_{\text{app}} \) is then calculated:

\[ \alpha_{\text{app}} = \frac{S_y}{R^2} = \frac{\alpha}{n\Omega_{\text{eff}}} f = \frac{\alpha}{n20\pi^3 a^3 f} \]  

(47)

so from the erroneous estimation of \( N = \frac{F}{\mu q} \mu R \) we call it \( \alpha_{\text{app}} \):

\[ \alpha_{\text{app}} = \frac{\alpha}{20a^2} \]  

(48)

This example shows that an inhomogeneous field can lead to a calculated noise parameter \( \alpha_{\text{app}} \) higher than the real noise parameter \( \alpha \) characteristic of the material. If one were to calculate \( \alpha \) ignoring current crowding from relation (eqn. 1) by using

\[ N = n\Omega = nV \]  

(49)

with \( \Omega \) the volume of the conducting material, one will find another apparent \( \alpha \) value given by \( \alpha_{\text{app}} \):

\[ \alpha_{\text{app}} = \frac{V_{y}}{I^2} N = \frac{\alpha}{n20\pi^3 a^3 nV} = \frac{\alpha V}{20\pi^3 a^3} \]  

(50)

Eqs. 48 and 50 in this approach for \( a/l < 1/20 \) lead to apparent \( \alpha \) values of at least a factor 15 higher than the real values describing the material properties. In general, the trimming of a resistor increases the noise owing to a reduction in effective volume by the enhanced current crowding (see eqn. 43). If the trimming process does not damage the material around the trim-cut, there is no reason for an increase in \( a \).

5.3 Conductance fluctuations in four-probe arrangements

Conductance fluctuations are easily probed by passing a constant current through a pair of driver electrodes and measuring the voltage fluctuations across a separate pair of sensor electrodes. The excess noise voltage \( S_y \) across the sensors is still proportional to the square of the current through the drivers. In the special case of perfect sample-contact alignment, the average voltage across the sensor electrodes remains zero as in a perfect Hall voltage plate without applied magnetic field. The observed noise is then the so-called transverse noise and is not proportional to the square of the voltage across the sensors (because in the ideal case \( V'_{\text{HOOG}} = 0 \)). Vannamme and van Bokhoven [44] have given a general relation for the noise voltage between arbitrarily shaped and placed sensors when a constant current is applied to arbitrarily shaped and placed drivers:

\[ S_y = \frac{1}{I^2} f \frac{\alpha^2}{n\Omega_{\text{app}}} f |J \cdot J|^2 d\Omega \]  

(51)

The integral must be taken over the whole sample except the electrodes. Eqn. 51 is quite similar to eqn. 40 except that the fourth power of the modulus of \( J \) is replaced by \( |J \cdot J|^2 \), which is the square of the scalar product of the current density caused by the current \( I \) through the drivers and adjacent current density \( J \). In a thought experiment, the adjacent current is the current that flows when the current source has been switched from the drivers to the sensors. This general relation was inspired by the sensitivity theorem in electrical networks based on the Tellegen theorem [43].

Areas with conductivity fluctuations where the \( J \) and \( J \) are perpendicular do not contribute to the observed voltage noise at the sensor contacts. For a given current \( I \) at the driver contacts we always find the voltage fluctuations at the current contacts \( S_{y_2} \) higher than at the sensor contacts \( S_{y_1} \). For a suppression of \( 1/f \) noise of poor contacts, the pair of sensor electrodes must be in an area of low current density and for all contacts hold \( J \cdot J \rightleftharpoons 0 \) [25, 44].

6 Discussion: merits and problems of the empirical relation

One of the merits of applying the empirical relation is to check the bulk and contact contribution to the \( 1/f \) noise and to check whether or not the noise source is distributed homogeneously in the volume of the sample (\( 1/\Omega \) or more precisely \( 1/N \) dependence). For example, in the smallest samples a possible deviation can be observed between experimental results and the simple empirical relation (eqn. 16). This points to inhomogeneities due to too small a thickness of layers, which can hardly be detected from a dependence of resistance on the cross-section. Inhomogeneities turn the comparison of the experimentally observed \( 1/f \) noise with the empirical relation into a diagnostic tool for quality assessment. Applying the empirical relation to samples with noise-free contacts can be seen as a crystal quality test. High values of \( \alpha \) together with low mobilities are strong indicators of poor crystal quality [46] or grain boundaries leading to current crowding on a microscopic scale [35]. Nowadays, it is rather common to characterise new crystal growth technologies and crystal quality of, for example, GaN or carbon fibres by \( 1/f \) noise in terms of \( \alpha \) values [47-52]. On the other hand, low \( \alpha \) values and a high mobility point to a mature technology for the material, especially for polycrystalline material and polycrystalline thin-film transistors. In a good quality crystal material grown by for example MBE, a reduction in \( \alpha \) often goes hand in hand with a reduction in the mobility due to scattering mechanisms additional to the lattice scattering [2, 39, 40].

In summary, these are the most important points:

- The empirical relation is a useful tool in prediction the \( 1/f \) noise for downscaled devices like in (MOS transistors) [10, 24].
- \( \alpha \) values in Si can easily be as low as \( 10^{-6} \) in bulk material and for MOS transistors this enables us to predict corner frequencies between \( 1/f \) noise and the thermal noise of devices.
- \( \alpha \) values above \( 10^{-3} \) point to poor quality material or current crowding on a microscopic scale and often must be considered as apparent \( \alpha \) values [30, 35, 36].
● $z$ value provides a useful measure about the strength of the 1/f noise in a material independent of the volume or geometry of the sample.

One of the problems in using the empirical relation for the 1/f noise is the fact that a relation is used where $z$ is a dimensionless parameter only for pure 1/f noise, while on the other hand the observed low-frequency noise often has a frequency dependence, such as $S_R \propto f^{-\gamma}$ with $0.8 < \gamma \leq 1.2$. Here, we would like to stress that sometimes $y$ values are presented out of the above region. This is often due to overlooking the fact that an observed spectrum consist of more different components or the frequency index is taken from too short a frequency range, where the background noise is not or badly corrected. A second origin for low $y$ values is a misinterpretation of a plateau value of a Lorentzian in competition with 1/f noise. This always results in $y$ values $\leq 0.8$. High $y$ values (i.e. $\gamma \geq 1.2$) often stem from the superpositions of genuine noise and temperature drifts in the sample due to high bias conditions, or a competition between the 1/f and 1/f$^2$ part of a Lorentzian contribution. Apart from the two artefacts $\gamma \leq 0.8$ and $\gamma \geq 1.2$, there are theoretical and practical problems to catch such experimental results with one single dimensionless $z$ value. The practical problem is not too serious and it does not influence the precision of the $z$ value. Now let us calculate the errors. Consider a measured, real spectrum $S = A(f) f^\gamma$ within the limits $f_1$ and $f_2$ whose central frequency on the logarithmic scale is $f_0 = (f_1 f_2)^{1/2}$. The variance $\sigma^2_{\text{real}}$, in the band width $f_2 - f_1$ for $y \neq 1$ then becomes:

$$\sigma^2_{\text{real}} = A f_0 \int_{f_1}^{f_2} \frac{df}{f} \left[ f^{\gamma-1} - f_1^{\gamma-1} \right]$$

To apply the empirical eqn. 16 and so obtain an $z$ value interpretation, the measured spectrum $S(f) = A f f^\gamma$ is approached by a 1/f spectrum $S_{1/f} = A f_0 f$. This approximation assumes a spectrum whose value at the middle frequency $f_0$ coincides with the measured spectrum, i.e. $A = S(f_0) = S_{1/f}(f_0)$. The erroneous variance $\sigma^2_{1/f}$ in this approximation then becomes:

$$\sigma^2_{1/f} = A f_0 \int_{f_1}^{f_2} \frac{df}{f} = A f_0 \ln \frac{f_2}{f_1}$$

The error in the approximation expressed by the ratio variances $\sigma^2_{\text{real}}/\sigma^2_{1/f}$ becomes:

$$\frac{\sigma^2_{\text{real}}}{\sigma^2_{1/f}} = \frac{f_0^{\gamma-1} \left[ f_1^{\gamma-1} - f_2^{\gamma-1} \right]}{(1 - \gamma) \ln f_2/f_1}$$

The ratio between the spectral values $S_{\text{real}}$ and $S_{1/f}$ at 1 Hz becomes:

$$\frac{S_{\text{real}}}{S_{1/f}} = \frac{A(f_0)}{A(f_0)} \left( \frac{f_0}{f_1} \right)^{\gamma-1}$$

The results are shown for different $y$ values and strengths of the low-frequency noise (or bandwidths) in Table 1. In the thought experiment we choose $f_1 = 1$ Hz, $f_2 = 10^3$ Hz, $10^6$ Hz and $10^9$ Hz, respectively. Although the errors in spectral values at 1 Hz can be large (a factor 1/4 4), the errors in the variances are rather low considering the scattering of noise results.

From experimental results it is observed that the relative 1/f noise at a fixed frequency can scatter from one device to another by a factor of three at least, so a systematic error introduced by characterising the spectrum with a frequency index $0.8 \leq \gamma \leq 1.2$ by a genuine 1/f noise introduces only an error of 30% at most, which is acceptable. For the above mentioned reasons a must not be expressed with a high precision.

7 Conclusions

We derived the criteria to observe 1/f conductance noise. The total number of free carriers $N < 10^{14}$. We calculate that for samples with a resistance value of the order of the input impedance of the voltage amplifier a low-noise current amplifier is a better choice for measuring resistance fluctuations under constant voltage bias than under constant current.

The application of the empirical relation for low-frequency noise with $S_R \propto f^{-\gamma}$ with $0.8 < \gamma < 1.2$ is acceptable considering the precision in variance over a limited bandwidth. The application of the empirical relation is very useful for distinguishing between bulk and contact contributions to the 1/f noise. Current crowding on a microscopic scale in inhomogeneous materials always leads to apparent high $z$ values. Therefore 1/f noise can be used as a diagnostic tool for the quality assessment of materials and devices.

The empirical relation does not suggest a physical model to explain the 1/f dependence of the spectrum. This can be considered a weakness and a drawback or a merit because there is a stimulation to find a physical origin in terms of mobility fluctuations. The McWhirter model claims a solution for the shape of the spectrum but is not general at all and does not consider the bulk effect.

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9 References


Table 1: Systematic errors due to a 1/f noise approach for a 1/f noise

<table>
<thead>
<tr>
<th>$f_0$ (Hz)</th>
<th>$f_1$ (Hz)</th>
<th>$10^2 f_0$</th>
<th>$10^3 f_0$</th>
<th>$10^4 f_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.8$</td>
<td>$0.63$</td>
<td>$1.036$</td>
<td>$0.4$</td>
<td>$1.15$</td>
</tr>
<tr>
<td>$\gamma = 1.2$</td>
<td>$1.58$</td>
<td>$1.036$</td>
<td>$2.5$</td>
<td>$1.15$</td>
</tr>
</tbody>
</table>

The data table shows the pairs $(S_{\text{real}}/S_{1/f})$ at 1 Hz $\sigma^2_{\text{real}}/\sigma^2_{1/f}$ for several $f_0/f_0$ with $f_0 = (f_1 f_2)^{1/2}$ as calculated from eqns. 55 and 54, respectively, $f_0 = 1$ Hz


