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ANALYTICITY OF SOLUTIONS TO NONLINEAR PARABOLIC EQUATIONS ON MANIFOLDS AND AN APPLICATION TO STOKES FLOW

Joachim Escher and Georg Prokert

Dedicated to Herbert Amann on the occasion of his 65th birthday

Abstract

We prove a general regularity result for fully nonlinear, possibly nonlocal parabolic Cauchy problems under the assumption of maximal regularity for the linearized problem. We apply this result to show joint spatial and temporal analyticity of the moving boundary in the problem of Stokes flow driven by surface tension.

Key words: nonlinear parabolic equation, maximal regularity, Stokes flow, surface tension

AMS subject classification: 35K55, 35Q30, 35R35, 76D07

1 Introduction

The theory of maximal regularity for (linear) parabolic evolution equations has proved itself to be a powerful tool for the treatment of nonlinear parabolic problems (see [2, 3, 18] e.a.). In a functional analytic framework of function spaces in which this theory is applicable, questions of existence, uniqueness, "linearized stability analysis", existence of invariant manifolds etc. can be treated to a great extent in a fashion similar as in the case of (finite dimensional) ordinary differential equations.

Of course, an additional issue for parabolic equations is the smoothing property of the evolution: A function satisfying a parabolic Cauchy problem will be "as smooth as the operator and the right-hand side will allow" immediately after the initial time. The classical method of proving this in the case of quasilinear equations is by "bootstrapping", i.e. proving higher smoothness of an already established solution inductively by invoking smoothness results for linear equations in every step. One encounters considerable technical difficulties, however, when one tries to apply this technique to fully nonlinear problems and to the proof of analyticity in space.

In [3, 4], Angenent introduced a different approach to the proof of the smoothing property, relying also on the maximal regularity property: By introducing two parameters representing a scaling in time and a translation in space, respectively, the desired smoothness result follows immediately from the Implicit Function theorem. More precisely, this theorem yields smooth dependence of the solution of the parameter-dependent problem on the parameters, and this can be straightforwardly translated to a smoothness result (in space and time) for the solution of the original problem. This elegant and transparent approach is immediately applicable to fully nonlinear problems and also yields analyticity of the solution. Besides the framework of maximal regularity, the crucial condition for its applicability is the equivariance of the evolution operator with respect to translations. For an application of this approach to a problem on $\mathbb{R}^n$, see [10].

In subsequent work, Angenent's idea has been generalized to apply it to broader problem classes. In [13], a localized version is given which can be used to establish regularity properties (including analyticity) for solutions of parabolic (as well as elliptic) *differential* equations on domains in $\mathbb{R}^n$. In [12], evolution equations on an arbitrary symmetric space are considered, where the underlying Lie group plays the role of the group of translations in the case of $\mathbb{R}^n$. Both in [12] and [13], the equivariance condition is relaxed to a more natural assumption which allows, for example, to treat differential operators with variable coefficients.

It is the first aim of the present paper to generalize the idea described above even further. We present a set of conditions on a family of flows on a manifold which enable us to perform this "parameter trick", and we show the existence of such a family if the underlying manifold is compact. Then a general result on the smoothing property for abstract parabolic evolution equations is shown. As our main interest is in analyticity here, the assumptions and results are formulated in the C^ω -class. It is clear, however, that the same techniques are also applicable in lower smoothness classes.

The second part of the present paper is devoted to an application of our abstract result to a moving boundary problem occurring in low Reynolds number flow. The application is nontrivial in two senses: The result we prove is new, and in the general case it cannot be obtained via previous versions of the parameter trick. We consider the deformation of a liquid drop driven by the capillary forces on its boundary. The velocity and pressure fields inside the drop are governed by the (homogeneous and incompressible) Stokes equations, and the boundary of the drop moves according to the velocity field. The influence of the capillary forces is modelled by a so-called traction boundary condition, i.e. a dynamic boundary condition relating the curvature to the stress tensor. For the precise problem formulation as well as for references to previous results on this problem, we refer to Section 5. We show that the boundary of the moving drop becomes analytic in space and time immediately. It should be pointed out that substantial parts of the analysis which is carried out here are in fact not particular to the discussed problem. Various techniques can be used also in parallel investigations of other (nonlocal) parabolic moving boundary problems.

This paper is organized as follows: In Section 2, the basic facts on continuous maximal regularity are reviewed. The abstract key result is proved in Section 3. Section 4 contains a series of technical results on the smoothness of a parameter-dependent pull-back, considered as a function of the parameter. These results are essentially parallel to the following prototype: For $u \in BUC(\mathbb{R})$, $k \in \mathbb{N}$, the mapping $[\mu \mapsto u(\cdot + \mu)]$ is in $C^k(\mathbb{R}, BUC(\mathbb{R}))$ if (and only if) $u \in BUC^k(\mathbb{R})$. The results as well as their proofs are oriented at [13]. Finally, the application of the abstract result to the Stokes flow problem is carried out in Section 5. Here, two independent results on the Stokes equation with traction boundary conditions in little Hölder spaces on a fixed, smooth domain are used. These are proved in Appendix A. Finally, we also need a generation result on little Hölder spaces for an operator which is the sum of the Dirichlet-Neumann operator for the Laplacian and a first order differential operator with smooth coefficients on the boundary of a smooth, bounded domain. This is proved in Appendix B.

2 Continuous Maximal Regularity

In this section we briefly introduce the notion of maximal regularity in the sense of Da Prato-Grisvard. For this let E_0 and E_1 be Banach spaces such that E_1 is continuously injected and dense in E_0 . Let $\mathcal{H}(E_1, E_0)$ denote the subset of all $A \in \mathcal{L}(E_1, E_0)$ such that $-A$, considered as a, in general, unbounded linear operator in E_0 , generates a strongly continuous analytic semigroup on E_0 . Let $D \subset E_1$ be open and assume that

$$P \in C^\omega(D, E_0) \quad \text{with} \quad \partial P(v) \in \mathcal{H}(E_1, E_0), \quad v \in D. \quad (2.1)$$

Given $T > 0$, set

$$\mathbb{E}_0 := C([0, T], E_0), \quad \mathbb{E}_1 := C([0, T], E_1) \cap C^1([0, T], E_0),$$

and let $\gamma : \mathbb{E}_0 \rightarrow E_0$, $u \mapsto u(0)$ denote the trace operator in $\mathbb{E}_0$. We assume that $(\mathbb{E}_0, \mathbb{E}_1)$ is a pair of **maximal regularity** for $\partial P(v)$, this means that we impose the following assumption:

$$\left(\frac{d}{dt} + \partial P(v), \gamma \right) \in \mathcal{L}_{is}(\mathbb{E}_1, \mathbb{E}_0 \times E_1), \quad v \in D. \quad (2.2)$$

We are now ready to formulate the following existence and uniqueness result:

Theorem 2.1 *Assume that (2.1) and (2.2) hold true. Then, given any $u_0 \in D$ and $f \in C(\mathbb{R}_+, E_0)$, there exist $t^+ := t^+(u_0) > 0$ and a unique maximal solution*

$$u := u(\cdot, u_0) \in C([0, t^+), D) \cap C^1([0, t^+), E_0) \quad (2.3)$$

of the initial value problem

$$\frac{d}{dt}u + P(u) = f, \quad u(0) = u_0. \quad (2.4)$$

Remarks 2.2 a) Theorem 2.1 essentially goes back to Da Prato and Grisvard [8]. For some refinements and generalizations see also [3].

b) Observe that assumption (2.2) and Theorem 2.1 coincide in the linear case, i.e., if $D = E_1$ and $P \in \mathcal{L}(E_1, E_0)$. Nevertheless, it is not at all clear whether or not property (2.2) can be verified if $E_1 \neq E_0$. In fact, it follows from a result of Baillon [7] that, in the case $E_1 \neq E_0$, property (2.3) can only be expected if E_0 contains an isomorphic copy of the sequence space c_0 . In particular, (2.3) will never be true in reflexive Banach spaces. However, in [8] the **continuous interpolation** functor $(\cdot, \cdot)_{\theta, \infty}^0$ was introduced, an interpolation method producing non-reflexive Banach spaces for which condition (2.2) can be verified.

c) Let us briefly introduce an important scale of Banach spaces, which may be realized as continuous interpolation spaces. Given $s \in \mathbb{R}$, define the **little Hölder spaces** to be

$$buc^s(\mathbb{R}^m) := \text{closure of } BUC^\infty(\mathbb{R}^m) \text{ in } B_{\infty, \infty}^s(\mathbb{R}^m),$$

where $B_{\infty, \infty}^s(\mathbb{R}^m)$ stands for the Besov spaces as defined in [23]. Note that the spaces $B_{\infty, \infty}^s(\mathbb{R}^m)$ coincide with the usual Hölder space $BUC^s(\mathbb{R}^m)$, provided $s > 0$ is not an integer, see Theorem 2.5.7 and Remark 2.2.2.3 in [23]. Then it is shown in [18], Theorem 1.2.17 that

$$(BUC(\mathbb{R}^m), BUC^n(\mathbb{R}^m))_{\theta, \infty}^0 = buc^{\theta n}(\mathbb{R}^m)$$

for all $n \in \mathbb{N}$ and $\theta \in (0, 1)$ such that $\theta n \notin \mathbb{N}$.

d) A further scale of Banach spaces for which maximal regularity can be verified are the so-called little Nikol'skii spaces. They can be realized as continuous interpolation spaces of Bessel potential spaces, cf. [8], Section 6 and [21], Section 6.

e) Assume that Σ is a smooth compact Riemannian manifold. Then $h^s(\Sigma)$ are defined to be the closure of $BUC^\infty(\Sigma)$ in $BUC^s(\Sigma)$. Again we have that

$$(BUC(\Sigma), BUC^n(\Sigma))_{\theta, \infty}^0 = h^{\theta n}(\Sigma)$$

for all $n \in \mathbb{N}$ and $\theta \in (0, 1)$ such that $\theta n \notin \mathbb{N}$, cf. the proof of Corollary 1.2.19 in [18].

f) Let Σ be as above and fix $s_0, s_1 \in (0, \infty)$, $\theta \in (0, 1)$. Setting $s_\theta := (1 - \theta)s_0 + \theta s_1$, we have

$$(h^{s_0}(\Sigma), h^{s_1}(\Sigma))_{\theta, \infty}^0 = h^{s_\theta}(\Sigma),$$

provided s_0, s_1 , and s_θ are not integers. This follows from (e), Theorem 7.4.4 in [24], and a density argument.

g) Consider again the "linear" case $D = E_1$ and $P \in \mathcal{L}(E_1, E_0)$ and suppose in addition that $f \equiv 0$. Then problem (2.4) has a unique solution in the class $\mathbb{E}_1$ for each $u_0 \in E_1$ (and any $T > 0$, of course), provided $-P$ generates a strongly

continuous semigroup, which does not need to be analytic. However, it is shown in [8] that the semigroup is automatically analytic if condition (2.2) is supposed to hold, see also Proposition III.3.1.1 in [2].

h) A well-known characterization of generators of analytic semigroups yields that $A \in \mathcal{L}(E_1, E_0)$ belongs to $\mathcal{H}(E_1, E_0)$ if and only if there are positive constants κ and ω such that $\omega \in \rho(-A)$ and

$$|\lambda| \|(\lambda + A)^{-1}\|_{\mathcal{L}(E_0)} \leq \kappa, \quad \operatorname{Re} \lambda \geq \omega.$$

i) We mention that Theorem 2.1 remains true under a much weaker regularity assumption for P . Indeed, it suffices to assume that P is continuously Fréchet differentiable. Under these regularity assumption it can also be shown that the mapping

$$\bigcup_{x \in D} [0, t^+(x)) \times \{x\} \rightarrow D, \quad (t, x) \mapsto u(t, x)$$

is a semiflow on D , provided f does not depend on t . However, since we are looking for possible smoothing properties of solutions, we presuppose analyticity of P from the very beginning in the abstract part of this paper.

j) Let Σ be as in (e) and assume that $A \in \mathcal{H}(h^{k+l+\sigma}(\Sigma), h^{k+\sigma}(\Sigma))$ for some $k, l \in \mathbb{N}$, $\sigma \in (0, 1)$. Let further $\theta \in (\sigma, 1)$ and suppose that $h^{k+l+\theta}(\Sigma)$ is the domain of the $h^{k+\theta}(\Sigma)$ -realization of A . Setting $E_0 := h^{k+\theta}(\Sigma)$ and $E_1 := h^{k+l+\theta}(\Sigma)$, it follows from Théorème 3.1 in [8] and (f) that $(\mathbb{E}_0, \mathbb{E}_1)$ is a pair of maximal regularity for A . ■

3 The smoothing property

Throughout this section we presuppose the following:

- Σ is a compact closed analytic manifold of dimension m ,
- assumptions (2.1) and (2.2) hold true,

where E_0 and E_1 are Banach spaces of functions over Σ such that

$$E_1 \hookrightarrow C^1(\Sigma), \quad E_1 \xhookrightarrow{d} E_0 \hookrightarrow C(\Sigma). \quad (A_1)$$

We fix $u_0 \in D$ and let u denote the unique maximal solution of (2.4) on $[0, t^+)$. For the sake of simplicity we assume further that $f \equiv 0$. Finally, we set

$$\hat{u}(t, p) := u(t)(p) \quad \text{for } (t, p) \in [0, t^+) \times \Sigma. \quad (3.1)$$

Our aim is to show that (2.4) enjoys a smoothing property, i.e. to show that u has regularity properties superior to those guaranteed by Theorem 2.1, see (2.3). Hence subdividing the interval of existence and using the semiflow property of u , see Remark 2.2 i), we may assume without loss of generality that $t^+ \leq 1$.

Further, we fix $T \in (0, t^+)$ and set $I := [0, T]$.

Let $\mathcal{V}^\omega(\Sigma)$ denote the vector space of all real-analytic vector fields on Σ . Our first result in this section provides a family of analytic flows $S(\mu, \cdot, \cdot)$ on Σ , where the parameter μ is taken from $\mathbb{R}^N$, such that the corresponding family of vector fields

$$\left\{ \mathbf{v}_\mu := \frac{\partial}{\partial t} S(\mu, t, \cdot)|_{t=0} \in \mathcal{V}^\omega(\Sigma); \mu \in \mathbb{R}^N \right\} \quad (3.2)$$

span the tangent space $T_p \Sigma$ for every $p \in \Sigma$ and such that $\mathbf{v}_\mu$ depends linearly on the parameter μ .

Lemma 3.1 *There exist an $N \in \mathbb{N}$ and a mapping*

$$S \in C^\omega(\mathbb{R}^N \times \mathbb{R} \times \Sigma, \Sigma) \quad (3.3)$$

with the following properties:

$$S(\mu, t, \cdot) \in \text{Diff}^\omega(\Sigma), \quad (\mu, t) \in \mathbb{R}^N \times \mathbb{R}, \quad (3.4)$$

$$\left\{ \frac{\partial}{\partial t} S(\mu, t, p)|_{t=0}; \mu \in \mathbb{R}^N \right\} = T_p \Sigma, \quad p \in \Sigma, \quad (3.5)$$

$$\left[\mu \mapsto \frac{\partial}{\partial t} S(\mu, t, \cdot)|_{t=0} \right] \in \text{Hom}(\mathbb{R}^N, \mathcal{V}^\omega(\Sigma)). \quad (3.6)$$

Proof: (i) Recall that Σ is compact and analytic. Hence Whitney's embedding theorem guarantees that there is an $N \in \mathbb{N}$ such that Σ is smoothly embedded in $\mathbb{R}^N$, i.e. the set of all smooth embeddings $\text{Emb}^\infty(\Sigma, \mathbb{R}^N)$ from Σ into $\mathbb{R}^N$ is not empty, cf. Theorem 1.3.4 in [17]. Pick $e_0 \in \text{Emb}^\infty(\Sigma, \mathbb{R}^N)$. Since $\text{Emb}^\infty(\Sigma, \mathbb{R}^N)$ is open in $C^\infty(\Sigma, \mathbb{R}^N)$, see Theorem 2.1.4 in [17], there is an open neighbourhood O of e_0 in $\text{Emb}^\infty(\Sigma, \mathbb{R}^N)$. Since we also know from Theorem 2.5.1 in [17] that $C^\omega(\Sigma, \mathbb{R}^N)$ is dense in $C^\infty(\Sigma, \mathbb{R}^N)$, we conclude that $O \cap C^\omega(\Sigma, \mathbb{R}^N) \neq \emptyset$.

(ii) Fix $e \in O \cap C^\omega(\Sigma, \mathbb{R}^N)$. Let η be the Euclidean metric on $\mathbb{R}^N$ and write $g := e^* \eta$ for the Riemannian pull-back metric on Σ . Define $\mathcal{P} : \Sigma \times \mathbb{R}^N \rightarrow TM$ by the demand that, given $p \in \Sigma$, the mapping $T_p e \circ \mathcal{P}(p, \cdot)$ is the orthogonal projection onto $T_p e[T_p \Sigma]$. In order to express $\mathcal{P}$ in local co-ordinates, let $p \in \Sigma$ be given and let $e = (e^1, \dots, e^N)$ be a real analytic local parametrization of an open neighbourhood of p in Σ . We write $\{\partial_\alpha; \alpha = 1, \dots, m\}$ for the basis of $T_p \Sigma$ induced by e . Then we have

$$\mathcal{P}(p, \mu) = (p, g^{\alpha\beta}(p) \partial_\beta e^j(p) \mu^j \partial_\alpha), \quad (3.7)$$

where $g^{\alpha\beta}$ are the local contravariant co-ordinates of the metric g . The local representation (3.7) clearly shows that $\mathcal{P}$ is an analytic bundle map over the identity. Given $\mu \in \mathbb{R}^N$, let c_μ denote the constant section $[p \mapsto (p, \mu)]$ in $\Sigma \times \mathbb{R}^N$ and define $\mathbf{v}_\mu := \mathcal{P} \circ c_\mu$ for $\mu \in \mathbb{R}^N$. Then $\mathbf{v}_\mu \in \mathcal{V}^\omega(\Sigma)$ and by construction $\{\mathbf{v}_\mu(p); \mu \in \mathbb{R}^N\} = T_p \Sigma$. Moreover, it is clear that $[\mu \mapsto \mathbf{v}_\mu] \in \text{Hom}(\mathbb{R}^N, \mathcal{V}^\omega(\Sigma))$.

(iii) Given $(\mu, p) \in \mathbb{R}^N \times \Sigma$, let $y_\mu(\cdot, p) : \mathbb{R} \rightarrow \Sigma$ denote the unique global solution to the initial value problem

$$\dot{z} = \mathbf{v}_\mu(z), \quad z(0) = p, \quad (3.8)$$

and set $S(\mu, t, p) := y_\mu(t, p)$. Recalling that $\mathbf{v}_\mu$ depends linearly on μ , we see that S satisfies (3.3) and (3.6). Due to (ii) property (3.5) holds true as well. Finally, (3.4) is a consequence of the unique solvability of (3.8). ■

Remarks 3.2 (a) Let $N_0(\Sigma)$ be the smallest integer such that (3.3)–(3.6) hold true. Then $N_0(\Sigma) \leq m + 1$, provided Σ is an embedded analytic hypersurface. Indeed, this follows from the above proof by choosing $e : \Sigma \hookrightarrow \mathbb{R}^{m+1}$ to be the natural embedding.

(b) There are situations where a parameter-dependent family of flows on Σ with the properties (3.3)–(3.6) can be obtained by a construction which is different from the one used in the above proof. Let us start with a paradigmatic example: Let $\Sigma := \mathbb{T}^m := \mathbb{R}^m / \mathbb{Z}^m$ be the m -dimensional torus. Setting $N = m$ and defining

$$S(\mu, t, p) := p + t\mu$$

satisfies (3.3)–(3.6). More generally, assume that Σ is a globally symmetric Riemannian manifold. Then there is a finite dimensional Lie group G acting analytically and transitively on Σ , and S can be constructed by

$$S(\mu, t, p) := \exp(t\mu^k X_k) \cdot p, \quad (3.9)$$

where $\{X_1, \dots, X_N\}$ is a vector space basis of the Lie algebra of G , and $\cdot$ denotes the action of G on Σ . For details and proofs we refer to [12]. Note that this construction does not rely on the compactness of Σ . ■

We now introduce

$$W_\mu(t)v := S(\mu, t, \cdot)^* v, \quad (\mu, t) \in \mathbb{R}^N \times \mathbb{R}, \quad v \in E_j, \quad j = 0, 1.$$

In general, we do not distinguish notationally between these operators acting on E_0 or on E_1 . Since $S(\mu, \cdot, \cdot)$ is a flow on Σ it follows that the mapping $[t \mapsto W_\mu(t)]$ is a representation of the group $(\mathbb{R}, +)$ in the group $(\text{Isom}(E_j), \circ)$. We next assume that the linear operators $W_\mu(t)$ are bounded on E_j and that the mapping $[t \mapsto W_\mu(t)v]$ is continuous for every $v \in E_j$, i.e. we assume that for any $\mu \in \mathbb{R}^N$:

$$[t \mapsto W_\mu(t)] \text{ is a strongly continuous group on } E_j, \quad j = 0, 1. \quad (A_2)$$

Let A_μ denote the infinitesimal generator of $\{W_\mu^0(t); t \in \mathbb{R}\}$. It is well-known that A_μ is a closed operator in E_0 , see Theorem 1.6.3 in [19]. Thus its domain $\text{dom}(A_\mu)$, endowed with the graph norm of A_μ , is a well-defined Banach space. We assume that

$$E_1 \hookrightarrow \text{dom}(A_\mu) \quad \text{for any } \mu \in \mathbb{R}^N. \quad (A_3)$$

Observe that (A₃) implies that

$$A_\mu \in \mathcal{L}(E_1, E_0) \quad \text{for any } \mu \in \mathbb{R}^N. \quad (3.10)$$

The next result expresses the action of A_μ in terms of the vector field $\mathbf{v}_\mu$:

Lemma 3.3 *Given $w \in E_1$, we have*

$$A_\mu w(p) = T_p w \mathbf{v}_\mu(p), \quad (\mu, p) \in \mathbb{R}^N \times \Sigma. \quad (3.11)$$

Proof: It follows from (3.2) and the first part of the assumption (A₁) that

$$T_p w \mathbf{v}_\mu(p) = \frac{d}{dt} w(S(\mu, t, p))|_{t=0}.$$

On the other hand, due to (A₂), we also have

$$\frac{1}{t} [W_\mu(t)w - w] = \frac{1}{t} [w(S(\mu, t, \cdot)) - w] \rightarrow A_\mu w \quad \text{in } E_0 \quad \text{as } t \rightarrow 0$$

Hence, the last part of (A₁) yields the assertion. ■

Lemma 3.3 clearly shows that $[(\mu, w) \mapsto A_\mu w]$ maps $\mathbb{R}^N \times E_1$ bilinearly into E_0 . Our next assumption ensures that this mapping is continuous.

$$[(\mu, w) \mapsto A_\mu w] \in \mathcal{L}^2(\mathbb{R}^N \times E_1, E_0). \quad (A_4)$$

We shall now formulate the crucial assumption on the nonlinear operator P . In order to do this, we set

$$S_\mu := S(\mu, 1, \cdot) \in \text{Diff}^\omega(\Sigma), \quad \mu \in \mathbb{R}^N. \quad (3.12)$$

Clearly, we have $S_\mu = \exp(\mathbf{v}_\mu)$, where $\exp : T\Sigma \rightarrow \Sigma$ stands for the usual Riemannian exponential mapping. Moreover, let S_μ^* and $S_\mu^\mu := (S_\mu)_* = (S_\mu^{-1})^*$ denote the pull-back and push-forward operators on E_j , $j = 0, 1$, respectively. This means that

$$S_\mu^* v = v \circ S_\mu \quad \text{and} \quad S_\mu^\mu v = v \circ S_\mu^{-1}, \quad v \in E_j, \quad j = 0, 1.$$

Observe that $d_0 := \text{dist}(u_0, \partial D)$ is positive and set $D_0 := \mathbb{B}_{E_1}(u_0, \frac{d_0}{2})$. Then it follows by continuity that

$$\exists r_0 > 0 : W_\mu(t)D_0 \subset D \quad \text{for all } \mu \in \mathbb{B}_{\mathbb{R}^N}(0, r_0), \quad t \in I. \quad (3.13)$$

Hence the following operator is well-defined:

$$Q(\mu, v) := S_\mu^* P(S_\mu^\mu v) \quad \text{for } (\mu, v) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, \quad (3.14)$$

so that we can formulate our crucial assumption concerning the operator P :

$$Q \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, E_0). \quad (A_5)$$

We will show in Section 5 that the operator coming from the Stokes flow with surface tension satisfies assumption (A₅). In the case of a symmetric manifold assumption (A₅) is always true, provided P is equivariant with respect to the underlying Lie-structure.

Example 3.4 Assume that Σ is a globally symmetric Riemannian manifold. Using the notation introduced in Remark 3.2 b), we say that P is **equivariant** with respect to G if there is an open neighbourhood O of the unit e in G such that $O \cdot D \subset D$ and

$$P(g \cdot v) = g \cdot P(v) \quad \text{for } (g, v) \in O \times D.$$

Here, of course, we used the notation

$$g \cdot v : \Sigma \rightarrow \mathbb{R}, \quad p \mapsto v(g \cdot p).$$

Let S be given by (3.9). Then the equivariance of P with respect to G implies that

$$Q(\mu, v) = \exp(\mu^k X_k) \cdot P(\exp(-\mu^k X_k) \cdot v) = P(v).$$

Thus, due to the assumption (2.1), hypothesis (A₅) is clearly satisfied. Concrete examples of translation and rotation equivariant operators, respectively, were discussed in detail in [10, 12]. ■

We next lift property (A₅) to the time dependent function spaces $\mathbb{E}_j$. For this let $\mathbb{D}_1 := C(I, D_0) \cap C^1(I, E_0)$, pick $(\mu, w) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \mathbb{D}_1$, and define

$$\mathbb{Q}(\mu, w)(t) := S(\mu, t, \cdot)^* P(S(\mu, t, \cdot)_* w(t)), \quad t \in I.$$

Observe that due to (3.6) we have that

$$S_{t\mu} = S(t\mu, 1, \cdot) = S(\mu, t, \cdot) \quad \text{for } (\mu, t) \in \mathbb{R}^N \times \mathbb{R}. \quad (3.15)$$

Indeed, the assertion is clear if $t = 0$. Thus fix $(\mu, t, p) \in \mathbb{R}^N \times \mathbb{R} \times \Sigma$ with $t \neq 0$ and define $y(s) := S(t\mu, s/t, p)$ for $s \in \mathbb{R}$. Then $y(0) = p$ and, using (3.6), we infer that

$$y'(s) = \frac{1}{t} \partial_2 S(t\mu, s/t, p) = \frac{1}{t} \mathbf{v}_{t\mu}(y(s)) = \mathbf{v}_\mu(y(s)). \quad (3.16)$$

Thus we have $y(s) = S(\mu, s, p)$, which gives (3.15). It follows from (3.15) that

$$\mathbb{Q}(\mu, w)(t) = Q(\mu t, w(t)), \quad (\mu, w) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \mathbb{D}_1, \quad t \in I. \quad (3.17)$$

After this preparation we can prove now the following result:

Lemma 3.5 *Assume that (A₅) holds true. Then there is an $r \in (0, r_0)$ such that*

$$\mathbb{Q} \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r) \times \mathbb{D}_1, \mathbb{E}_0).$$

Proof: For simplicity, write $B := \mathbb{B}_{\mathbb{R}^N}(0, r_0)$ and let $(\mu_0, w_0) \in B \times \mathbb{D}_1$ be given. By assumption, given any $(\lambda_0, v_0) \in B \times D_0$, there exist $M_0, r > 0$ such that

$$\|\partial^k Q(\lambda, v)[\nu, h]^k\|_{E_0} \leq M_0 r^k k! (|\nu| + \|h\|_{E_1})^k$$

for all $(\nu, h) \in \mathbb{R}^N \times E_1$ and all $(\lambda, v) \in \mathbb{B}_{B \times D_0}((\lambda_0, v_0), r)$. Observing now that $\{(\mu_0 t, w_0(t)); t \in I\}$ is a compact subset of $\mathbb{R}^N \times D_0$, we conclude that there are positive numbers M_0 and r such that

$$\|\partial^k Q(\mu_0 t, w_0(t))[\nu, h]^k\|_{E_0} \leq M_0 r^k k! (|\nu| + \|h\|_{E_1})^k \quad (3.18)$$

for all $t \in I$ and all $(\nu, h) \in \mathbb{R}^N \times E_1$. Moreover, given $t \in I$ and $(\lambda, v) \in \mathbb{B}_{B \times D_0}((\mu_0 t, w_0(t)), r)$, we have

$$Q(\lambda, v) = \sum_{k=0}^{\infty} \frac{1}{k!} \partial^k Q(\mu_0 t, w_0(t)) [\lambda - \mu_0 t, v - w_0(t)]^k. \quad (3.19)$$

Pick $\tilde{r} \in (0, r)$ and let $(\mu, w) \in \mathbb{B}_{B \times \mathbb{D}_1}((\mu_0, w_0), \tilde{r})$ be given. Since $T \leq 1$, there is a $\beta \in (0, 1)$ such that

$$|\mu t - \mu_0 t| + \|w(t) - w_0(t)\|_{E_1} \leq \beta r, \quad t \in I. \quad (3.20)$$

Further, using (3.17) and (3.19), we infer

$$\begin{aligned} \mathbb{Q}(\mu, w)(t) &= Q(\mu t, w(t)) \\ &= \sum_{k=1}^{\infty} \frac{1}{k!} \partial^k Q(\mu_0 t, w_0(t)) [(\mu - \mu_0)t, w(t) - w_0(t)]^k \end{aligned} \quad (3.21)$$

for all $t \in I$. It follows from (3.18) and (3.20) that the series in (3.21) is majorized in $\mathbb{E}_0$ by $M_0 \sum_{k=1}^{\infty} \beta^k$. Hence the Weierstrass majorant criterion and (3.21) imply that $\mathbb{Q}(\mu, w)$ is represented in $\mathbb{E}_0$ as a power series centered at (μ_0, w_0) . This completes the proof. $\blacksquare$

Remark: We have

$$\partial^k \mathbb{Q}(\mu, w)(t) = t^k \partial^k Q(\mu t, w(t)) \quad \text{for } t \in I,$$

since (3.21) is the Taylor expansion of $\mathbb{Q}$ centered at (μ_0, w_0) . $\blacksquare$

The operators W_μ will be used to transform the solution u spatially. We shall also need a dilatation in time for elements of E_0^I . For this we fix $\varepsilon_0 > 0$ such that $\lambda t \in [0, t^+)$ for all $\lambda \in (1 - \varepsilon_0, 1 + \varepsilon_0)$ and all $t \in [0, T]$. Then, given $(\lambda, \mu) \in (1 - \varepsilon_0, 1 + \varepsilon_0) \times \mathbb{R}^N$, we define $v_{\lambda, \mu} \in E_0^I$ by

$$v_{\lambda, \mu}(t) := W_\mu(t) u(\lambda t) \quad \text{for } t \in I. \quad (3.22)$$

This means that, given $(\lambda, \mu) \in (1 - \varepsilon_0, 1 + \varepsilon_0) \times \mathbb{R}^N$, we have

$$v_{\lambda, \mu}(t)(p) = \hat{u}(\lambda t, S(\mu, t, p)) \quad \text{for } (t, p) \in I \times \Sigma. \quad (3.23)$$

Our next result shows that $v_{\lambda, \mu}$, as a function of $t \in I$, solves a parameter-dependent evolution equation involving the operators $\mathbb{Q}$ and A_μ . In order to economize our notation, we assume that $\varepsilon_0 \in (0, r)$ and set

$$\Pi := \Pi(\varepsilon_0) := (1 - \varepsilon_0, 1 + \varepsilon_0) \times \mathbb{B}_{\mathbb{R}^N}(0, \varepsilon_0).$$

Lemma 3.6 *Given $(\lambda, \mu) \in \Pi$, we have*

- (i) $v_{\lambda, \mu} \in \mathbb{D}_1$.
- (ii) $v_{\lambda, \mu}$ solves the evolution equation

$$\frac{d}{dt}w + \lambda \mathbb{Q}(\mu, w) = A_\mu w, \quad w(0) = u_0. \quad (3.24)$$

Proof: (i) Given $w \in E_1$, it follows from (A₂) and (A₃) that

$$[t \mapsto W_\mu(t)w] \in C(\mathbb{R}, E_1) \cap C^1(\mathbb{R}, E_0),$$

and that

$$\frac{d}{dt}[W_\mu(t)w] = A_\mu W_\mu(t)w = W_\mu(t)A_\mu w, \quad (3.25)$$

see Theorem 1.2.4 in [19]. Hence (2.3) and (3.13) imply the assertion.

(ii) Using (3.25) and (2.4) (recall that $f = 0$), we find that

$$\begin{aligned} \frac{d}{dt}v_{\lambda, \mu} &= A_\mu W_\mu(t)u(\lambda t) + \lambda W_\mu(t) \frac{du}{dt}(\lambda t) \\ &= A_\mu v_{\lambda, \mu} - \lambda W_\mu(t)P(u(\lambda t)), \quad t \in I. \end{aligned}$$

But, by definition,

$$\begin{aligned} W_\mu(t)P(u(\lambda t)) &= S(\mu, t, \cdot)^* P(S(\mu, t, \cdot)_* S(\mu, t, \cdot)^* u(\lambda t)) \\ &= S(\mu, t, \cdot)^* P(S(\mu, t, \cdot)_* W_\mu(t)u(\lambda t)) \\ &= \mathbb{Q}(\mu, v_{\lambda, \mu}), \end{aligned}$$

which completes the proof. ■

Our next result contains the key argument to show via the Implicit Function theorem that the mapping $(\lambda, \mu) \mapsto v_{\lambda, \mu}$ is analytic.

Lemma 3.7 *Given $((\lambda, \mu), w) \in \Pi \times \mathbb{D}_1$, let*

$$F((\lambda, \mu), w) := \left(\frac{d}{dt}w + \lambda \mathbb{Q}(\mu, w) - A_\mu w, w(0) - u_0 \right).$$

Then

$$F \in C^\omega(\Pi \times \mathbb{D}_1, \mathbb{E}_0 \times E_1) \quad (3.26)$$

and

$$\partial_2 F((1, 0), w) \in \mathcal{L}_{is}(\mathbb{E}_1, \mathbb{E}_0 \times E_1), \quad w \in \mathbb{D}_1, \quad (3.27)$$

where $\partial_2 F$ denotes the Fréchet derivative of F with respect to $w \in \mathbb{D}_1$.

Proof: (i) Recall that γ stands for the trace operator $[w \mapsto w(0)]$. Hence it is clear that

$$\left(\frac{d}{dt}, \gamma\right) \in \mathcal{L}(\mathbb{E}_1, \mathbb{E}_0 \times E_1).$$

Moreover, it follows from assumption (A₄) that

$$[(\mu, w) \mapsto A_\mu w] \in \mathcal{L}^2(\Pi \times \mathbb{E}_1, \mathbb{E}_0).$$

Therefore (3.26) follows from Lemma 3.5.

(ii) Let $(w, h) \in \mathbb{D}_1 \times \mathbb{E}_1$ be given. Then we have

$$\partial_2 F((1, 0), w)h = \frac{d}{d\varepsilon} F((1, 0), w + \varepsilon h)|_{\varepsilon=0} = \left(\frac{d}{dt}h + \partial P(w)h, h(0)\right).$$

Combining (2.2) with Remark III 3.4.2(c) in [2] we conclude that, given $(f, \varphi) \in \mathbb{E}_0 \times E_1$, there is a unique solution $h \in \mathbb{E}_1$ to the inhomogeneous evolution equation

$$\frac{d}{dt}h + \partial P(w(t))h = f(t), \quad h(0) = \varphi.$$

(3.27) is now a consequence of the open mapping theorem. ■

We are now prepared to prove that the mapping $[(\lambda, \mu) \mapsto v_{\lambda, \mu}]$ is analytic.

Proposition 3.8 *There is an $\varepsilon_0 > 0$ such that $[(\lambda, \mu) \mapsto v_{\lambda, \mu}] \in C^\omega(\Pi(\varepsilon_0), \mathbb{D}_1)$.*

Proof: Let F be defined as in Lemma 3.7 and observe that $F((\lambda, \mu), w) = 0$ if only if $w \in \mathbb{D}_1$ is a solution to

$$\frac{d}{dt}w + \lambda \mathbb{Q}(\mu, w) = A_\mu w, \quad w(0) = u_0.$$

Thus the assertion follows from Lemma 3.6, Lemma 3.7, and the Implicit Function theorem in Banach spaces. ■

It remains to translate the above proposition into the desired analyticity of $\hat{u}$, see (3.1).

Theorem 3.9 *Assume that (A₁)–(A₅) hold true. Then $\hat{u} \in C^\omega((0, t^+) \times \Sigma)$.*

Proof: Let $(t_0, p_0) \in (0, T] \times \Sigma$ be given. It follows from Lemma 3.1 that there are $b_1, \dots, b_m \in \mathbb{R}^N$ with $\|b_k\| = 1$ for $k = 1, \dots, m$ such that $(v_{b_1}, \dots, v_{b_m})$ is a basis of $T_{p_0}\Sigma$. In the following we write $\hat{\mu}\mathcal{B} := \sum_{k=1}^m \mu^k b_k$ for $(\mu^1, \dots, \mu^m) \in \mathbb{R}^m$. Moreover, given $\varepsilon \in (0, \varepsilon_0)$, we define

$$\varphi : \Pi(\varepsilon) \rightarrow (0, t^+) \times \Sigma, \quad (\lambda, (\mu^1, \dots, \mu^m)) \mapsto (\lambda t_0, S(\hat{\mu}\mathcal{B}, t_0, p_0)).$$

Due to Lemma 3.1 we have that $\varphi \in C^\omega(\Pi(\varepsilon), (0, t^+) \times \Sigma)$ and (3.15) implies that

$$T_{(1,0)}\varphi(\xi, (\eta_1, \dots, \eta_m)) = t_0 \left(\xi, \sum_{k=1}^m \eta^k v_{b_k}(p_0) \right) \in \mathbb{R} \times T_{p_0}\Sigma$$

for all $(\xi, (\eta^1 \dots, \eta^m)) \in \mathbb{R} \times \mathbb{R}^m$, hence $T_{(1,0)}\varphi$ is bijective. Thus the Inverse Function theorem ensures that φ is an analytic parametrization of an open neighbourhood O of (t_0, p_0) in $(0, t^+) \times \Sigma$, provided $\varepsilon > 0$ is chosen small enough. Observe further that from assumption (A_1) we know that $\mathbb{D}_1 \subset C(I, C(\Sigma))$. Thus the evaluation mapping

$$\mathbb{D}_1 \rightarrow \mathbb{R}, \quad w \mapsto w(t_0)(p_0)$$

is well-defined and clearly analytic. Combining this with Proposition 3.8 we get

$$[(\lambda, \hat{\mu}) \mapsto v_{\lambda, \hat{\mu}\mathcal{B}}(t_0)(p_0)] \in C^\omega(\hat{\Pi}(\varepsilon), \mathbb{R}).$$

But $\varphi^*\hat{u}(\lambda, \hat{\mu}) = v_{\lambda, \hat{\mu}\mathcal{B}}(t_0)(p_0)$ for $(\lambda, \hat{\mu}) \in \hat{\Pi}(\varepsilon)$, see (3.23). This shows that $\hat{u} \in C^\omega(O, \mathbb{R})$ and completes the proof. $\blacksquare$

Remark: Let $k \in \mathbb{N} \cup \{\infty\}$ with $k \geq 1$ be given and assume that the hypotheses (2.1) and (A_5) are replaced by those formulated in the corresponding C^k -category. Then the above proof shows that $\hat{u} \in C^k((0, t^+) \times \Sigma)$. $\blacksquare$

4 Parameter-dependent pull-back

To apply the abstract result of the previous section, a crucial point is to check assumption (A_5) . For a simple but illuminating example we refer to Remark 3.7 b) in [12]. It is the purpose of this section to prove a rather general analyticity result for parameter-dependent pull-backs which is a key tool for proving (A_5) not only in our application given below but for large classes of concrete evolution equations. Both our approach and our result are partial generalizations of those in [13], Sections 2 and 3.

We start by considering a local case. Let X be a domain in $\mathbb{R}^m$, $x_0 \in X$ and $\varepsilon_0 > 0$ such that $\overline{\mathbb{B}}(x_0, 4\varepsilon_0) \subset X$, and $\chi \in \mathcal{D}(\mathbb{B}(x_0, 2\varepsilon_0), \mathbb{R})$ such that $\chi \equiv 1$ on $\mathbb{B}(x_0, \varepsilon_0)$ and $0 \leq \chi \leq 1$. Furthermore, let

$$\mathbf{w} = [\mu \mapsto \mathbf{w}_\mu] \in \text{Hom}(\mathbb{R}^N, \mathcal{V}^\omega(X))$$

be given, i.e. $\mathbf{w}$ is a linear map from $\mathbb{R}^N$ into the space of real-analytic vector fields on X . For $x \in \mathbb{B}(x_0, 3\varepsilon_0)$, consider the initial value problem

$$\dot{z} = \mathbf{w}_\mu(z), \quad z(0) = x. \tag{4.1}$$

If $r_0 > 0$ is sufficiently small and $|\mu| < r_0$, then

$$z(1) =: T_\mu(x) \tag{4.2}$$

exists and defines a mapping

$$T = [(\mu, x) \mapsto T_\mu(x)] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \mathbb{B}(x_0, 3\varepsilon_0), \mathbb{B}(x_0, 4\varepsilon_0)).$$

Note that $T_\mu = \exp(\mathbf{w}_\mu)$; in particular, $T_0 = \text{Id}_{\mathbb{B}(x_0, 3\varepsilon_0)}$ and, as $\mathbf{w}$ is linear,

$$D_\mu T(0, x)[\eta] = \mathbf{w}_\eta(x), \quad \eta \in \mathbb{R}^N, \quad x \in \mathbb{B}(x_0, 3\varepsilon_0),$$

cf. (3.16). As T is real-analytic, it extends to a holomorphic map

$$T^{\mathbb{C}} \in C^{\omega}(V, \mathbb{C}^m)$$

where V is some open neighbourhood of $\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \mathbb{B}(x_0, 3\varepsilon_0)$ in $\mathbb{C}^N \times \mathbb{C}^m$. A standard compactness argument shows that by diminishing r_0 we can ensure

$$\mathbb{B}_{\mathbb{C}^N}(0, r_0) \times \mathbb{B}(x_0, 2\varepsilon_0) \subset V$$

and

$$T^{\mathbb{C}}(\mathbb{B}_{\mathbb{C}^N}(0, r_0) \times \mathbb{B}(x_0, 2\varepsilon_0)) \subset \mathbb{B}_{\mathbb{C}^m}(x_0, 3\varepsilon_0).$$

Hence, given $x \in X$, $\mu \in \mathbb{B}_{\mathbb{C}^N}(0, r_0)$, we can define

$$\theta_{\mu}(x) := x + \chi(x)(T_{\mu}^{\mathbb{C}}(x) - x). \quad (4.3)$$

Lemma 4.1 (*Properties of θ_{μ}*)

There is an $r_0 > 0$ such that

- (i) $[(\mu, x) \mapsto \theta_{\mu}(x)]$ is Lipschitz on $\mathbb{B}_{\mathbb{C}^N}(0, r_0) \times X$,
- (ii) $\theta_{\mu} \in \text{Diff}^{\infty}(X)$, $\mu \in \mathbb{B}_{\mathbb{R}^N}(0, r_0)$.

Proof: Note that $[(\mu, x) \mapsto \theta_{\mu}(x)]$ is smooth and

$$\begin{aligned} D_{\mu}\theta_{\mu}(x) &= \chi(x)D_{\mu}T_{\mu}^{\mathbb{C}}(x), \\ D_x\theta_{\mu}(x) &= \text{Id}_{\mathbb{R}^m} + \chi(x)(D_xT_{\mu}^{\mathbb{C}} - \text{Id}_{\mathbb{R}^m}) + \nabla\chi(x) \otimes (T_{\mu}(x) - x). \end{aligned}$$

These derivatives are uniformly bounded for small $|\mu|$, and (i) follows. Moreover, $D_x\theta_{\mu} = \text{Id}_{\mathbb{R}^m} + R_{\mu}$ where R_{μ} vanishes due to $D_xT_0 = T_0 = \text{Id}_{\mathbb{R}^m}$. Using the uniform boundedness of $D_{\mu}T_{\mu}$ and $D_xD_{\mu}T_{\mu}$ we get

$$\sup_{x \in X} \|R_{\mu}(x)\| \leq \frac{1}{2}, \quad \mu \in \mathbb{B}_{\mathbb{R}^N}(0, r_0)$$

for r_0 sufficiently small. Now (ii) can be proved in the same way as Proposition 2.2. in [13]. ■

Let $U \subset X$ be a bounded open set with smooth boundary such that $\overline{\mathbb{B}}(x_0, 4\varepsilon_0) \subset U$. Fix $l \in \mathbb{N}$ and $\sigma \in (0, 1)$, and let r_0 be as above. Clearly $\theta_{\mu} \in \text{Diff}^{\infty}(U)$.

Lemma 4.2 (*Properties of θ_{μ}^**)

- (i) $\theta_{\mu}^* \in \mathcal{L}_{is}(h^{l+\sigma}(U))$, and there is an $M > 0$ such that

$$\|\theta_{\mu}^*\|_{\mathcal{L}(h^{l+\sigma}(U))} \leq M, \quad \mu \in \mathbb{B}_{\mathbb{R}^N}(0, r_0).$$

- (ii) For any $u \in h^{l+\sigma}(U)$,

$$[\mu \mapsto \theta_{\mu}^*u] \in C(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{l+\sigma}(U)).$$

(iii) For any $u \in h^{l+1+\sigma}(U)$,

$$[\mu \mapsto \theta_\mu^* u] \in C^1(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{l+\sigma}(U)),$$

and the partial derivatives are given by

$$\partial_{\mu_j}(\theta_\mu^* u) = \sum_{i=1}^m \chi(\theta_\mu^* \partial_i u) \partial_{\mu_j} T_\mu^i, \quad j = 1, \dots, N. \quad (4.4)$$

Proof: The assertion (i) can be proved literally as the corresponding result in Proposition 2.4 (a) in [13]. To show (ii), we first prove

$$[\mu \mapsto \theta_\mu^* u] \in C(\mathbb{B}_{\mathbb{R}^N}(0, r_0), BUC(U)) \quad (4.5)$$

for $u \in BUC(U)$. For given $\varepsilon > 0$, there is a $\delta > 0$ such that $x, y \in U$, $|x - y| < \delta$ implies $|u(x) - u(y)| < \varepsilon$. Due to Lemma 4.1 (i) there is a $\tilde{\delta} > 0$ such that $\mu, \mu_0 \in \mathbb{B}_{\mathbb{R}^N}(0, r_0)$, $|\mu - \mu_0| < \tilde{\delta}$ implies $|\theta_\mu(x) - \theta_{\mu_0}(x)| < \delta$ and thus $|\theta_\mu^* u(x) - \theta_{\mu_0}^* u(x)| < \varepsilon$. Hence (4.5) is proved. Now (ii) can be proved in the same way as the corresponding result in Proposition 2.4 (b) in [13].

Fixing $u \in h^{l+1+\sigma}(U)$ and setting $g(\mu) := \theta_\mu^* u$, we get for $\mu \in \mathbb{B}_{\mathbb{R}^N}(0, r_0)$, $|h|$ sufficiently small, and $j = 1, \dots, N$:

$$\begin{aligned} & \frac{1}{h}(g(\mu + he_j) - g(\mu)) - \sum_{i=1}^m \chi(\theta_\mu^* \partial_i u) \partial_{\mu_j} T_\mu^i \\ &= \sum_{i=1}^m \int_0^1 (\chi(\theta_{\mu+\tau he_j}^* \partial_i u) \partial_{\mu_j} T_{\mu+\tau he_j}^i - \chi(\theta_\mu^* \partial_i u) \partial_{\mu_j} T_\mu^i) d\tau. \end{aligned}$$

It follows from the continuity results of (ii) that the integrals on the right approach 0 in $h^{l+\theta}(U)$ as $h \rightarrow 0$. This proves (4.4). It follows from (ii) that the partial derivatives are continuous, hence the differentiability result of (iii) follows. $\blacksquare$

Lemma 4.3 (*Higher regularity*)

(i) Suppose $k \in \mathbb{N}^* \cup \{\infty\}$, $a \in C^{l+k}(X)$. Then

$$[\mu \mapsto \chi^{|\alpha|} \theta_\mu^* \partial^\alpha a] \in C(\mathbb{B}_{\mathbb{R}^N}(0, r_0), BUC^l(U)), \quad 0 < |\alpha| \leq k.$$

(ii) Suppose $k \in \mathbb{N} \cup \{\infty, \omega\}$, $a \in C^{l+k}(X) \cap BUC^l(U)$. If $k = \omega$ suppose additionally

$$a \text{ has a holomorphic extension whose domain of definition contains } B_{\mathbb{C}^m}(x_0, 3\varepsilon_0). \quad (4.6)$$

Then

$$[\mu \mapsto \theta_\mu^* a] \in C^k(\mathbb{B}_{\mathbb{R}^N}(0, r_0), BUC^l(U)),$$

and

$$\begin{aligned} & \partial_\mu^\alpha (\theta_\mu^* a) \\ &= \sum_{k=1}^{|\alpha|} \sum_{\beta_1 + \dots + \beta_k = \alpha} C_{\beta_1 \dots \beta_k} \chi^k (\theta_\mu^* \partial_{j_1 \dots j_k} a) \partial_\mu^{\beta_1} T_\mu^{j_1} \dots \partial_\mu^{\beta_k} T_\mu^{j_k}, \quad |\alpha| \leq k. \end{aligned}$$

(iii) Suppose $k \in \mathbb{N} \cup \{\infty, \omega\}$, $a \in C^{l+1+k}(X) \cap h^{l+\theta}(U)$. If $k = \omega$ suppose additionally that (4.6) holds. Then

$$[\mu \mapsto \theta_\mu^* a] \in C^k(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{l+\theta}(U)).$$

Remarks:

- (a) For any fixed a , condition (4.6) can be satisfied by choosing ε_0 small enough.
- (b) The sum in (4.7) has to be taken over the set of all (non-ordered) k -element sets of multiindices $\{\beta_1, \dots, \beta_k\}$ such that $\sum \beta_i = \alpha$.

Proof: A simple modification of the proof of Lemma 3.1 in [13] yields (i). Using this result, (ii) can be proved using the same arguments as in the proof of Proposition 3.2 in [13], although the explicit expressions for the partial derivatives are more complicated in our situation. Finally, (iii) follows in the same way as the corresponding result in Theorem 3.3 in [13]. ■

Now we are prepared to prove a corresponding global result for pull-back on manifolds. As our main concern is analyticity, we restrict ourselves to the analytic case here. It will become clear, however, that corresponding results also hold for C^∞ -smoothness and for functions and manifolds with finite smoothness. As usual, the main tools are localization and compactness.

Let $l \in \mathbb{N}$, $\theta \in (0, 1)$ be fixed and let v_μ and S_μ be defined as in (3.2) and (3.12).

Lemma 4.4 (*Pull-back on manifolds*)

Let M and O be open sets in Σ with $M \subset\subset O$, and let $a \in C^\omega(O)$. There is an $r_0 > 0$ such that

$$[\mu \mapsto (a \circ S_\mu)|_M] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{l+\theta}(M)).$$

Proof: Let $\{(\tilde{\phi}_j, \tilde{W}_j) \mid j \in \mathcal{J}\}$ be a real-analytic atlas for Σ . Set $W_j := \tilde{W}_j \cap O$, $\phi_j := \tilde{\phi}_j|_{W_j}$ and note that $\{(\phi_j, W_j) \mid j \in \mathcal{J}\}$ is a real-analytic atlas for O . Given $\xi \in \overline{M}$, there is a $j = j_\xi \in \mathcal{J}$ and an $\varepsilon = \varepsilon_\xi > 0$ such that $\mathbb{B}_{\mathbb{R}^m}(\phi_j(\xi), 5\varepsilon) \subset \phi_j(W_j)$ and $a \circ \phi_j^{-1}$ has a holomorphic extension whose domain of definition contains $\mathbb{B}_{\mathbb{C}^m}(\phi_j(\xi), 3\varepsilon)$. Moreover, there is an open neighbourhood $V = V_\xi$ of ξ in O such that $\phi_j(V) \subset \mathbb{B}_{\mathbb{R}^m}(\phi_j(\xi), \frac{\varepsilon}{2})$. From the open covering $\{V_\xi \mid \xi \in \overline{M}\}$ of $\overline{M}$ we choose a finite subcover $\{V_{\xi_1}, \dots, V_{\xi_K}\}$. To simplify notation, we write $j_k := j_{\xi_k}$, $V_k := V_{\xi_k}$, $\varepsilon_k := \varepsilon_{\xi_k}$ for $k = 1, \dots, K$. Moreover, we set $x_k := \phi_{j_k}(\xi_k)$, $U_k := \mathbb{B}_{\mathbb{R}^m}(x_k, 4\varepsilon_{\xi_k})$, $X_k := \phi_{j_k}(W_{j_k})$, and $w_\mu^k := \phi_{j_k*}(v_\mu|_{U_k})$.

In this setting, we construct T_μ^k and θ_μ^k from $x_k, \varepsilon_k, \mathbf{w}_\mu^k$ in the same way as T_μ and θ_μ have been constructed from $x_0, \varepsilon_0, \mathbf{w}_\mu$ in (4.1)–(4.3). There is an $s_k > 0$ such that $|\mu| < s_k$ implies $T_\mu^k(\mathbb{B}_{\mathbb{R}^m}(x_k, \frac{\varepsilon_k}{2})) \subset \mathbb{B}_{\mathbb{R}^m}(x_k, \varepsilon_k)$. This ensures that

$$\theta_\mu^k = T_\mu^k = \phi_{j_k} \circ (S_\mu|_{\phi_{j_k}^{-1}(\mathbb{B}_{\mathbb{R}^m}(x_k, \frac{\varepsilon_k}{2}))}) \circ \phi_{j_k}^{-1}, \quad |\mu| < s_k, \quad (4.7)$$

on $\mathbb{B}_{\mathbb{R}^m}(x_k, \frac{\varepsilon_k}{2})$. Moreover, let $\pi_k \in C_0^\infty(X_k)$ be such that $\pi_k \equiv 1$ on U_k . For $b \in h^{l+\theta}(U_k)$, define $\mathcal{T}_k(b) := ((\pi_k b) \circ \phi_{j_k})|_M$ and note that

$$\mathcal{T}_k \in \mathcal{L}(h^{l+\theta}(U_k), h^{l+\theta}(M)). \quad (4.8)$$

We have $a \circ \phi_{j_k}^{-1} \in C^\omega(X_k) \cap h^{k,\theta}(U_k)$ and therefore, by Lemma 4.3 (iii),

$$[\mu \mapsto \theta_\mu^{k*}(a \circ \phi_{j_k}^{-1})] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_k), h^{l+\theta}(U_k)) \quad (4.9)$$

for some $r_k > 0$. Set $r_0 := \min\{r_k, s_k \mid k = 1, \dots, K\}$.

Let $\{\psi_0, \psi_1, \dots, \psi_K\}$ be a partition of unity for Σ subordinate to the open covering $\{\Sigma \setminus \overline{M}, V_1, \dots, V_K\}$. In particular, $\sum_{k=1}^K \psi_k \equiv 1$ on M . Using (4.7) and (4.9) we get by multiplication with $\psi_k \circ \phi_{j_k}^{-1}$

$$[\mu \mapsto (\psi_k(a \circ S_\mu)) \circ \phi_{j_k}^{-1}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{l+\theta}(U_k)), \quad (4.10)$$

$k = 1, \dots, K$. Finally,

$$\mathcal{T}_k((\psi_k(a \circ S_\mu)) \circ \phi_{j_k}^{-1}) = \psi_k(a \circ S_\mu)|_M$$

and (4.8) yield

$$[\mu \mapsto (\psi_k(a \circ S_\mu))|_M] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{l+\theta}(M)),$$

and the lemma follows by summation over k . ■

Remarks:

- (a) As Σ is compact, the case $M = O = \Sigma$ is covered in the above lemma.
- (b) The lemma continues to hold if Σ is noncompact. In particular, we can set $\Sigma = \mathbb{R}^m$.

5 Stokes flow driven by surface tension

In the remaining part of this paper we discuss an application of the abstract result to a parabolic moving boundary problem from fluid mechanics. This application is nontrivial in the sense that the evolution equation describing the problem is nonlocal and the underlying manifold is not assumed to be diffeomorphic to a symmetric space, hence the results of [13] and [12] are not applicable here.

Our problem models the creeping flow of a capillary liquid drop and has the following exact description (in normalized form): Given a (bounded) initial

domain $\Omega^* \subset \mathbb{R}^{m+1}$, we are looking for a family of domains $\Omega(t) \subset \mathbb{R}^{m+1}$, parametrized by time $t \geq 0$ such that $\Omega(0) = \Omega^*$, and $t \mapsto \Gamma(t) := \partial\Omega(t)$ is a moving hypersurface such that

$$V_n(t) = U(\cdot, t)|_{\Gamma(t)} \cdot \nu(t). \quad (5.1)$$

Here $V_n(t)$ denotes the normal velocity of $\Gamma(t)$, $\nu(t)$ is the outer unit normal vector field on $\Gamma(t)$ and $U(\cdot, t) \in C^2(\Omega(t), \mathbb{R}^{m+1})$ is the velocity field of Stokes flow driven by surface tension on $\Omega(t)$, i.e. the first component in the solution $(U(\cdot, t), \Pi(\cdot, t))$ of

$$\left. \begin{aligned} -\Delta U(\cdot, t) + \nabla \Pi(\cdot, t) &= 0 && \text{in } \Omega(t), \\ \operatorname{div} U(\cdot, t) &= 0 && \text{in } \Omega(t), \\ \mathcal{T}(U(\cdot, t), \Pi(\cdot, t))\nu(t) &= \kappa\nu(t) && \text{on } \Gamma(t), \end{aligned} \right\} \quad (5.2)$$

$t \geq 0$. Here $\kappa(t)$ denotes the (m -fold) mean curvature of $\Gamma(t)$ with the sign chosen such that $\kappa(t)$ is negative if $\Omega(t)$ is convex, and $\mathcal{T}(U, \Pi)$ denotes the stress tensor whose cartesian coordinates are

$$\mathcal{T}_{ij}(U, \Pi) := \partial_i U_j + \partial_j U_i - \delta_{ij} \Pi.$$

To enforce uniqueness, (5.2) has to be augmented by the auxiliary conditions

$$\begin{aligned} \int_{\Omega(t)} U(x, t) dx &= 0, \\ \int_{\Omega(t)} (\partial_i U_j - \partial_j U_i)(x, t) dx &= 0, \quad 1 \leq i, j \leq m+1, \end{aligned} \quad (5.3)$$

$t \geq 0$. (The solvability of (5.2), (5.3) for fixed t in appropriate function spaces will be addressed below.)

Short-time existence and uniqueness results for (5.1)–(5.3) holding in arbitrary spatial dimensions are given in [16] and [22], in [16] it is also shown that the free boundary is C^∞ -smooth for all positive times for which the solution exists, even if $\Gamma(0)$ has finite smoothness only. For $m = 1$, see also [5, 6, 14, 20]. In these papers, analyticity of the free boundary is proved under the assumption that $\Gamma(0)$ is analytic. Our method enables us to prove analyticity of $\Gamma(t)$ for $t > 0$ jointly in space and time under the assumption $\Gamma(0) \in h^{3+\theta}$.

To reformulate our moving boundary problem as an evolution equation on a fixed manifold, let Σ be compact closed embedded real-analytic hypersurface in $\mathbb{R}^{m+1}$ near $\Gamma(0)$ which bounds the bounded domain Ω . We fix $\theta \in (0, 1)$. Given $k \in \mathbb{N}$, the norm of the space $h^{k+\theta}(\Sigma)$ will be denoted by $\|\cdot\|_{k+\theta}$. Let n denote the normal field on Σ (exterior to Ω), let $\mathcal{U} \subset \mathbb{R}^{m+1}$ be an open neighbourhood of Σ and let $\delta > 0$ be such that $X \in \operatorname{Diff}^\omega(\Sigma \times (-\delta, \delta), \mathcal{U})$ with

$$X(\xi, \rho) = \xi + \rho n(\xi).$$

Let $X^{-1} := (\Xi, d)$. Then $\nabla d(x) = n(\Xi(x))$.

Suppose $r \in h^{2+\theta}(\Sigma)$ is sufficiently small. Then we define

$$\Gamma_r := \{X(\xi, r(\xi)) \mid \xi \in \Sigma\}.$$

This is an embedded hypersurface in $\mathbb{R}^{m+1}$ which bounds a bounded domain Ω_r . To describe the moving boundary we use a mapping $[t \mapsto r(t)]$, $t \in [0, T]$, taking small values in $h^{2+\theta}(\Sigma)$, such that $\Gamma(t) = \Gamma_{r(t)}$, and derive a (nonlocal) evolution equation for r . To simplify notation, we will write $r(t, \xi)$ instead of $r(t)(\xi)$ for $t \in [0, T]$, $\xi \in \Sigma$.

Let $[t \mapsto x(t) := X(\xi(t), r(t, \xi(t)))]$ be the path of a "particle" on the moving boundary in $\mathbb{R}^{m+1}$. Then $\dot{x}(t) = U(x(t), t)$. This implies (5.1), and we have, in local coordinates,

$$\begin{aligned} \dot{\xi}^\alpha(t) &= \partial_i \Xi^\alpha(X(\xi, r(t, \xi))) U^i(X(\xi, r(t, \xi)), t), \\ \partial_t r(t, \xi) + \partial_\alpha r \dot{\xi}^\alpha(t) &= n_i(\xi) U^i(X(\xi, r(t, \xi)), t), \end{aligned}$$

Hence, suppressing the time argument and setting $\psi_r := \Xi(\cdot, r(\cdot))$ we get

$$\partial_t r = -P(r) = (n_i - \partial_\alpha r((\partial_i \Xi^\alpha) \circ \psi_r))(U^i \circ \psi_r). \quad (5.4)$$

This is the nonlinear evolution equation to which we will apply our abstract result.

Our next aim is to show assumption (A₅) for the operator P . Pick a mapping S having properties (3.3)–(3.6) and define S_μ according to (3.12). Let $\tilde{D}$ be an open neighbourhood of 0 in $h^{2+\theta}(\Sigma)$ which is small enough to ensure that P is well defined on $\tilde{D}$. Set $D := \tilde{D} \cap h^{3+\theta}(\Sigma)$. There are neighbourhoods $\tilde{D}_0$ and D_0 of 0 in $h^{2+\theta}(\Sigma)$ and $h^{3+\theta}(\Sigma)$, respectively, and an $r_0 > 0$ such that $D_0 \subset \tilde{D}_0$, $S_\mu^* v \in \tilde{D}$ for all $(\mu, v) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0$, and $S_\mu^* w \in D$ for all $(\mu, w) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0$. (In the sequel, r_0 , $\tilde{D}_0$, and D_0 will be diminished when necessary without explicit mentioning.)

For $(\mu, w) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0$, let $Q(\mu, w)$ be defined by (3.14) with P from (5.4). Then, setting $r := S_\mu^* w$, using that this implies

$$\partial_\alpha w = S_\mu^* \partial_\beta r \partial_\alpha S_\mu^\beta,$$

and introducing

$$\phi_{\mu, w} := \psi_r \circ S_\mu = X(S_\mu(\cdot, w(\cdot))) = S_\mu + w(n \circ S_\mu),$$

we obtain

$$\begin{aligned} Q(\mu, w) &= P(r) \circ S_\mu = ((\partial_\alpha r \circ S_\mu)(\partial_i \Xi^\alpha \circ \phi_{\mu, w}) - n_i \circ S_\mu) U^i \circ \phi_{\mu, w} \\ &= (\partial_\beta w ([\partial_\gamma S_\mu^\delta]^{-1})_\alpha^\beta (\partial_i \Xi^\alpha \circ \phi_{\mu, w}) - n_i \circ S_\mu) U^i \circ \phi_{\mu, w}. \end{aligned} \quad (5.5)$$

Due to (3.3) and Lemma 4.4 we have

$$[\mu \mapsto n \circ S_\mu] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0), (h^{3+\theta}(\Sigma))^{m+1}). \quad (5.6)$$

From this and the fact that $h^{2+\theta}(\Sigma)$ is a Banach algebra it follows that

$$[(\mu, w) \mapsto \phi_{\mu, w}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0, (h^{2+\theta}(\Sigma))^{m+1}). \quad (5.7)$$

Let $\mathcal{E} \in \mathcal{L}(h^{2+\theta}(\Sigma), h^{2+\theta}(\Omega))$ be an arbitrary but fixed right inverse of the trace operator and define

$$\Phi_{\mu, w} := \text{Id}_\Omega + \mathcal{E}(\phi_{\mu, w} - \text{Id}_\Sigma).$$

From (5.7) it follows that

$$[(\mu, w) \mapsto \Phi_{\mu, w}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0, (h^{2+\theta}(\Omega))^{m+1}). \quad (5.8)$$

Moreover, $\Phi_{\mu, w}$ is near the identity for $(\mu, w) \in \mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0$, and therefore $\Phi_{\mu, w} \in \text{Diff}^{2+\theta}(\Omega, \Omega_r)$. To carry out the transformation of (5.2), (5.3) to a fixed domain, we set

$$\begin{aligned} u(\mu, w) &:= U \circ \Phi_{\mu, w}, \\ p(\mu, w) &:= \Pi \circ \Phi_{\mu, w} \end{aligned} \quad (5.9)$$

and introduce the Banach spaces

$$\begin{aligned} V &:= \{(c^{ij}) \mid i, j = 1, \dots, m+1, c^{ij} \in \mathbb{R}, c^{ij} = -c^{ji}\}, \\ X^\theta &:= (h^{2+\theta}(\Omega))^{m+1} \times h^{1+\theta}(\Omega) \times (\mathbb{R}^{m+1} \times V), \\ Y^\theta &:= (h^\theta(\Omega))^{m+1} \times h^{1+\theta}(\Omega) \times (h^{1+\theta}(\Sigma))^{m+1} \times \mathbb{R}^{m+1} \times V, \end{aligned}$$

and the operator

$$L_{\mu, w} \in \mathcal{L}(X^\theta, Y^\theta)$$

defined by

$$L_{\mu, w}(u, p, \lambda) := \begin{bmatrix} (-a_i^j \partial_j (a_i^k \partial_k u^i) + a_i^j \partial_j p + \lambda_1^i) e_i \\ a_i^j \partial_j u^i \\ (\text{Tr}_\Sigma(a_i^k \partial_k u^j + a_j^k \partial_k u^i - p \delta_{ij}) + \lambda_2^{ij}) \nu^j e_i \\ \int_\Omega \sqrt{g} u \, dx \\ \int_\Omega \sqrt{g} (a_i^k \partial_k u^j - a_j^k \partial_k u^i) \, dx \end{bmatrix},$$

where

$$\begin{aligned} a_i^j &:= [D\Phi_{\mu, w}^{-1}]_{ij}, & g &:= \det(D\Phi_{\mu, w}^T(D\Phi_{\mu, w})), \\ \nu &:= \frac{(D\Phi_{\mu, w}^{-1})^T n}{|(D\Phi_{\mu, w}^{-1})^T n|}, & \tilde{g}_{\alpha\beta} &:= \partial_\alpha \phi_{\mu, w} \partial_\beta \phi_{\mu, w}, \\ \tilde{G} &:= [\tilde{g}_{\alpha\beta}], & \tilde{g} &:= \det \tilde{G}, \\ \tilde{g}^{\alpha\beta} &:= [\tilde{G}^{-1}]^{\alpha\beta}, & f^i &:= \frac{1}{\sqrt{g}} \partial_\alpha (\sqrt{g} \tilde{g}^{\alpha\beta} \partial_\beta \phi_{\mu, w}^i), \end{aligned}$$

$i, j = 1, \dots, m+1$, $\alpha, \beta = 1, \dots, m$. Here $|\cdot|$ denotes the euclidean norm in $\mathbb{R}^{m+1}$, and the indices α, β refer to local coordinates. (We will skip the necessary localizations as they are standard. Moreover, the dependence of the above functions on (μ, w) is suppressed in the notation for the sake of brevity.)

Pulling back our boundary value problem from $\Omega(t) = \Omega_r$ to Ω via $\Phi_{\mu,w}$ shows that (5.2), (5.3) is equivalent to

$$L_{\mu,w}(u(\mu,w), p(\mu,w), 0) = (0, 0, f, 0, 0),$$

$f := f^i e_i$. Here we have used the well-known fact that on an embedded Riemannian hypersurface Γ with the metric induced by the ambient space and normal vector n , the curvature vector κn is given by

$$\kappa n = \Delta_\Gamma x \quad (5.10)$$

where Δ_Γ is the Laplace-Beltrami operator on Γ and x denotes the embedding of Γ into the ambient space. The space V is a finite dimensional space of auxiliary parameters which are necessary to enforce bijectivity of $L_{\mu,w}$.

Lemma 5.1 *We have*

$$\begin{aligned} a_i^j, g, \sqrt{g} &\in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0, h^{1+\theta}(\Omega)), \\ \nu &\in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0, (h^{1+\theta}(\Sigma))^{m+1}), \\ \tilde{g}_{\alpha\beta}, \tilde{g}, \sqrt{\tilde{g}} \tilde{g}^{\alpha\beta} &\in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, h^{2+\theta}(\Sigma)), \\ f &\in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, (h^{1+\theta}(\Sigma))^{m+1}). \end{aligned}$$

Proof: From (5.8) we get

$$[(\mu, w) \mapsto D\Phi_{(\mu,w)}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, h^{1+\theta}(\Omega)^{(m+1) \times (m+1)}). \quad (5.11)$$

The arguments given in the sequel are based on the fact that the spaces $h^{k+\theta}(\Omega)$, $h^{k+\theta}(\Sigma)$ are Banach algebras with respect to pointwise multiplication for $k \in \mathbb{N}$. This will be used without explicit mentioning. The assertion on g is implied by (5.11), and the assertion on $\sqrt{g}$ follows from this and the fact that the square root is an analytic map on the open set

$$h_+^{1+\theta}(\Omega) := \{v \in h^{1+\theta}(\Omega) \mid v > 0\}$$

This can be proved, for example, using the analytic version of the Implicit Function Theorem. As $D\Phi_{0,0} = I$, we get the assertion on a_i^j from the analyticity of matrix inversion. Noting additionally that $(D\Phi_{\mu,w})^{-1})^T n$ does not vanish, we obtain the assertion on ν^i from the analyticity of the map $z \mapsto \frac{1}{|z|}$ from

$$\{v \in h^{1+\theta}(\Sigma) \mid v(x) \neq 0 \forall x \in \Sigma\}$$

to $h^{1+\theta}(\Sigma)$.

Using (5.6), we get, parallel to (5.7),

$$[(\mu, w) \mapsto \phi_{\mu,w}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, (h^{3+\theta}(\Sigma))^{m+1}),$$

and the remaining assertions of the lemma follow from this by arguments analogous to the ones given above. $\blacksquare$

Lemma 5.2 *We have*

$$[(\mu, w) \mapsto u(\mu, w)] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, (h^{2+\theta}(\Omega))^{m+1}).$$

Proof: Note at first that

$$L_{\mu, w}(u(\mu, w), p(\mu, w), \lambda) = (0, 0, f, 0, 0),$$

implies $\lambda = 0$. This follows from the special structure of the right hand side and can be proved easily using the weak formulation of (5.2). The details are given in [16], Lemma 1(ii), for $(\mu, w) = (0, 0)$, the general case is analogous.

It easily follows from Lemma 5.1 that

$$[(\mu, w) \mapsto L_{\mu, w}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0, \mathcal{L}(X^\theta, Y^\theta)). \quad (5.12)$$

It is proved in Lemma A.1 of Appendix A that $L_{0,0} \in \mathcal{L}_{is}(X^\theta, Y^\theta)$. As the set $\mathcal{L}_{is}(X^\theta, Y^\theta)$ is open in $\mathcal{L}(X^\theta, Y^\theta)$, this implies

$$[(\mu, w) \mapsto L_{\mu, w}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times \tilde{D}_0, \mathcal{L}_{is}(X^\theta, Y^\theta)).$$

Hence

$$(u(\mu, w), p(\mu, w), 0) = L_{\mu, w}^{-1}(0, 0, f, 0, 0),$$

and the lemma follows from the analyticity of the map $[L \mapsto L^{-1}]$ and Lemma 5.1. $\blacksquare$

Lemma 5.3 *Assumption (A₅) is valid for the operator P defined by (5.4), i.e. we have*

$$Q \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, h^{2+\theta}(\Sigma)).$$

Proof: (i) Let $\{(\zeta_j, W_j)\}$ be a real-analytic atlas for Σ . Due to the compactness of Σ , there is an open covering $\{V_1, \dots, V_K\}$ of Σ such that $S_\mu(V_k) \subset W_{j_k}$ for $|\mu| < r_0$. Let $\{\pi_k | k = 1, \dots, K\}$ be a partition of unity subordinate to this covering. From (5.5) and (5.9) we get

$$Q(\mu, w) = \sum_{k=1}^K \pi_k (\partial_\beta w [\partial_\gamma S_\mu^\sigma]^{-1} {}^\beta_\alpha ((\partial_i \Xi^\alpha) \circ \phi_{\mu, w}) - n_i \circ S_\mu) u^i(\mu, w), \quad (5.13)$$

where $\alpha, \dots, \sigma$ refer to local coordinates on W_{j_k} and the dependence of these coordinates on k is suppressed in the notation.

(ii) Fix $k \in \{1, \dots, K\}$ and consider the corresponding local coordinates. Clearly

$$[w \mapsto \partial_\beta w] \in \mathcal{L}(h^{3+\theta}(\Sigma), h^{2+\theta}(V_k)). \quad (5.14)$$

(iii) Taking into account that $S_\mu^\sigma = (\zeta_{j_k}^\sigma \circ S_\mu)$ it follows from Lemma 4.4 that

$$[\mu \mapsto \partial_\gamma S_\mu^\sigma] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0), h^{3+\theta}(V_k)). \quad (5.15)$$

Recall that $\partial_\gamma S_0^\sigma = \delta_\gamma^\sigma$. Hence the matrix $[\partial_\gamma S_\mu^\sigma]$ is invertible in the Banach algebra $(h^{3+\theta}(V_k))^{m \times m}$ for small μ , and by the analyticity of matrix inversion,

$$[\mu \mapsto [\partial_\gamma S_\mu^\sigma]^{-1}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0), (h^{3+\theta}(V_k))^{m \times m}). \quad (5.16)$$

(iv) From $\Xi \circ \psi_r = \text{Id}_\Sigma$ it follows that $\Xi \circ \phi_{\mu,w} = S_\mu$. Differentiating this identity with respect to the spatial coordinates and with respect to w yields, in local coordinates,

$$\begin{aligned} \partial_\beta \phi_{\mu,w}^i (\partial_i \Xi^\alpha) \circ \phi_{\mu,w} &= \partial_\beta S_\mu^\alpha, \quad \beta = 1, \dots, m, \\ (n^i \circ S_\mu) (\partial_i \Xi^\alpha) \circ \phi_{\mu,w} &= 0 \end{aligned}$$

where $\alpha = 1, \dots, m$. For any fixed α , these equations constitute a regular linear system of equations for $(\partial_i \Xi^\alpha) \circ \phi_{\mu,w}$, $i = 1 \dots m+1$, provided (μ, w) is small. Again, by the analyticity of matrix inversion and (5.15),

$$[(\mu, w) \mapsto (\partial_i \Xi^\alpha) \circ \phi_{\mu,w}] \in C^\omega(\mathbb{B}_{\mathbb{R}^N}(0, r_0) \times D_0, h^{3+\theta}(\Sigma)). \quad (5.17)$$

The assertion of the lemma follows now from (5.6), Lemma 5.2, (5.13), (5.14), (5.16), and (5.17). $\blacksquare$

Due to

$$P(r) = Q(0, r) = (\partial_\alpha r (\partial_i \Xi^\alpha \circ \phi_{0,r} - n_i) u^i(0, r),$$

Lemma 5.3 implies $P \in C^\omega(D_0, h^{2+\theta}(\Sigma))$. Our next aim is to determine the Fréchet derivative $\partial P(0)$ and to show that it generates an analytic semigroup on $h^{2+\theta}(\Sigma)$. From Lemma 5.2 and (5.17) we get

$$\begin{aligned} [r \mapsto \partial_i \Xi^\alpha \circ \phi_{0,r}] &\in C^\omega(D_0, h^{3+\theta}(\Sigma)), \\ u(0, \cdot) &\in C^\omega(D_0, (h^{2+\theta}(\Omega))^{m+1}). \end{aligned}$$

Suppressing the fixed argument $\mu = 0$ and writing Π_1 for the canonical projection from X_θ onto $(h^{2+\theta}(\Omega))^{m+1}$, we get from this

$$\begin{aligned} \partial P(0)[h] &= -n_i \partial u^i(0)[h] + \partial_\alpha h \partial_i \Xi^\alpha u^i(0) \\ &= -n \cdot \Pi_1(L_0^{-1}(0, 0, \partial f(0)[h], 0, 0) - L_0^{-1} \partial L_0[h](u(0), p(0), 0)) \\ &\quad + \partial_\alpha h \partial_i \Xi^\alpha u^i(0). \end{aligned} \quad (5.18)$$

It remains to calculate $\partial f(0)[h]$. Using (5.10) and the well-known formula

$$\kappa = \tilde{g}^{\alpha\beta} \partial_{\alpha\beta} \phi_r^k \nu^k$$

we get

$$f = \tilde{g}^{\alpha\beta} \partial_{\alpha\beta} \phi_r^k \nu^k \nu.$$

The Fréchet derivatives of ν^k , $\tilde{g}$, and $\tilde{g}^{\alpha\beta}$ at $r = 0$ are given simply by first-order differential operators with smooth coefficients, hence

$$\partial f(0)[h] = \tilde{g}^{\alpha\beta}(0) \partial_{\alpha\beta} h^h n^k n + R_1 h = \Delta_\Sigma(h^k n^k) n + R_2 h$$

where R_1, R_2 are given by first-order differential operators with smooth coefficients and the $\tilde{g}^{\alpha\beta}(0)$ are the contravariant coefficients of the metric on Σ induced from the ambient space.

Consequently, from this and (5.18) we have

$$\partial P(0) = A + B + R_3 \quad (5.19)$$

where

$$\begin{aligned} A &= n \cdot \Pi_1 L_0^{-1}(0, 0, -\Delta_\Sigma(h \cdot n)n, 0, 0), \\ R_3 &= n \cdot \Pi_1 L_0^{-1}((0, 0, -R_2 h, 0, 0) + \partial L_0[h](u(0), p(0), 0)), \end{aligned}$$

and B is a first-order differential operator with smooth coefficients. From (5.12) and Lemma A.1 it follows that

$$R_3 \in \mathcal{L}(h^{2+\theta}(\Sigma)). \quad (5.20)$$

The operator A involves the solution of a BVP for the Stokes equations. Instead of proving a generation result for $A + B$ directly, we will identify A as a small perturbation of the well-known Dirichlet-Neumann operator for the Laplacian for which generation results are more easily available. To be precise, recall the standard result on the Dirichlet problem for the Laplacian, cf. Theorem B.1,

$$(\Delta, \text{Tr}_\Sigma) \in \mathcal{L}_{is}(h^{3+\theta}(\Omega), h^{1+\theta}(\Omega) \times h^{3+\theta}(\Sigma))$$

and define

$$A_0 := \partial_n(\Delta, \text{Tr}_\Sigma)^{-1}(0, \cdot).$$

Lemma 5.4 *We have*

$$A - \frac{1}{2}A_0 \in \mathcal{L}(h^{3+\theta}(\Sigma)).$$

Proof: For $h \in h^{3+\theta}(\Sigma)$, let ψ be defined by

$$\psi := (\Delta, \text{Tr}_\Sigma)^{-1}(0, h \cdot n).$$

Then

$$L_0(\nabla \psi, 0, 0) = (0, 0, 2(\partial_n^2 \psi + \mathcal{P}(\partial_{ij} \psi n_j e_i)), \int_\Omega \nabla \psi \, dx, 0)$$

where $\mathcal{P}$ denotes the orthogonal projection from $\mathbb{R}^{m+1}$ onto $T\Sigma$ again. Defining (v, q, λ) by

$$L_0(v, q, \lambda) = (0, 0, -2\Delta_\Sigma(h \cdot n)n, 0, 0)$$

and using the identity

$$(\Delta \psi)|_\Sigma = \Delta_\Sigma(\psi|_\Sigma) + \partial_n^2 \psi + \kappa \partial_n \psi$$

we get from Lemmas A.1 and A.2

$$\begin{aligned}
& \|(A_0 - 2A)h\|_{3+\theta} = \|(\nabla\psi - v) \cdot n\|_{3+\theta} \\
&= \|n \cdot \Pi_1 L_0^{-1}(0, 0, 2(\Delta_\Sigma(\psi|_\Sigma) + \partial_n^2 \psi + \mathcal{P}(\partial_{ij}\psi n_j e_i)), \int_\Omega \nabla\psi \, dx, 0)\|_{3+\theta} \\
&\leq 2\|n \cdot \Pi_1 L_0^{-1}(0, 0, \mathcal{P}(\partial_{ij}\psi n_j e_i), 0, 0)\|_{3+\theta} \\
&\quad + C\|(0, 0, -2\kappa\partial_n\psi, \int_\Sigma \psi \cdot n \, dx, 0)\|_{Y_{1+\theta}} \\
&\leq C\|\psi\|_{3+\theta} \leq C\|h\|_{3+\theta}.
\end{aligned}$$

Together with the fact that both A and A_0 map smooth functions to smooth functions, this implies the lemma. $\blacksquare$

Now we are able to prove that P satisfies the crucial assumption on the parabolic character of the evolution problem.

Lemma 5.5 *The operator P defined by (5.4) satisfies*

$$\partial P(r) \in \mathcal{H}(h^{3+\theta}(\Sigma), h^{2+\theta}(\Sigma)), \quad r \in D_0.$$

Proof: As $\mathcal{H}(h^{3+\theta}(\Sigma), h^{2+\theta}(\Sigma))$ is open in $\mathcal{L}(h^{3+\theta}(\Sigma), h^{2+\theta}(\Sigma))$, it is sufficient to show $\partial P(0) \in \mathcal{H}(h^{3+\theta}(\Sigma), h^{2+\theta}(\Sigma))$. We have, according to (5.19),

$$\partial P(0) = \frac{1}{2}(A_0 + 2B) + (A - \frac{1}{2}A_0) + R_3.$$

It is shown in Appendix B that $A_0 + 2B \in \mathcal{H}(h^{3+\theta}(\Sigma), h^{2+\theta}(\Sigma))$. Fix $\theta' \in (0, \theta)$, and $\varepsilon > 0$. Using Lemma 5.4 with θ replaced by θ' and the fact that the little Hölder spaces are stable under continuous interpolation, cf. Remark 2.2 f), we get

$$\|(A - \frac{1}{2}A_0)h\|_{2+\theta} \leq \|(A - \frac{1}{2}A_0)h\|_{3+\theta'} \leq \|h\|_{3+\theta'} \leq \varepsilon\|h\|_{3+\theta} + C_\varepsilon\|h\|_{2+\theta}$$

for any $h \in h^{3+\theta}(\Sigma)$ with C_ε independent of h . Together with (5.20), this implies the lemma by a well-known perturbation result on generators of analytic semigroups. $\blacksquare$

Remark: Actually, we even have $A - \frac{1}{2}A_0 \in \mathcal{L}(h^{2+\theta}(\Sigma))$ and hence it is possible to have $\varepsilon = 0$ in the last inequality. A proof for this, however, would be more technical, and we do not need this result for our present purposes.

Setting

$$\hat{r}(p, t) := (r(t))(p), \quad p \in \Sigma,$$

we can prove now our final result on Stokes flow driven by surface tension:

Theorem 5.6 *For arbitrary $r_0 \in D_0$, there is a $t^+ = t^+(r_0) > 0$ such that the initial value problem*

$$\left. \begin{aligned} \partial_t r + P(r) &= 0, \\ r(0) &= r_0 \end{aligned} \right\} \quad (5.21)$$

has a unique maximal solution

$$r = r(\cdot, r_0) \in C([0, t^+), D_0) \cap C^1([0, t^+), h^{2+\theta}(\Sigma))$$

and

$$\hat{r}|_{\Sigma \times (0, t^+)} \in C^\omega(\Sigma \times (0, t^+)).$$

Proof: Existence and uniqueness of the solution to (5.21) follow from Lemma 5.5, Remark 2.2 j), and Theorem 2.1. It is easy to check that Assumptions (A₁)–(A₄) hold for the function spaces $E_0 = h^{2+\theta}(\Sigma)$ and $E_1 = h^{3+\theta}(\Sigma)$ and any family S of mappings satisfying (3.3)–(3.6). Finally, Lemma 5.3 shows the validity of Assumption (A₅). Thus, our analyticity result follows from Theorem 3.9. $\blacksquare$

Appendices

In the following we prove two results for the BVP (5.2), (5.3) on a fixed domain and a perturbation result for the Dirichlet-Neumann operator which were used in Section 5, but may be of interest in their own right.

Throughout these appendices, let Ω denote a bounded domain in $\mathbb{R}^{m+1}$ with smooth boundary Σ . Given $s \geq 0$, we write $\|\cdot\|_{s,\Omega}$ and $\|\cdot\|_s$ for the norm of the spaces $h^s(\Omega)$ and $h^s(\Sigma)$, respectively. Furthermore, we fix $\theta \in (0, 1)$.

A The Stokes problem with traction boundary conditions in little Hölder spaces

In this appendix we first establish a regularity result in little Hölder spaces in a form suitable for our purposes. Secondly, a sharper estimate is proved which can be interpreted in the following way: Note that (5.2) is a Neumann problem as the traction boundary condition (5.2)₃ is the natural boundary condition for the Stokes operator. The estimate we are going to prove is a consequence of the fact that the "Neumann-to-Dirichlet operator" belonging to the Stokes equations has diagonal structure in highest order with respect to the decomposition into tangential and normal components at the boundary. This means, in particular, that the operator mapping the tangential component of the Neumann data to the normal component of the velocity field at the boundary is of order -2 while the full operator has order -1 only. We will prove this using an auxiliary BVP for the Laplacian.

Recall that we have set

$$\begin{aligned} V &:= \{(c^{ij}) \mid i, j = 1, \dots, m+1, c^{ij} \in \mathbb{R}, c^{ij} = -c^{ji}\}, \\ X^\theta &:= (h^{2+\theta}(\Omega))^{m+1} \times h^{1+\theta}(\Omega) \times (\mathbb{R}^{m+1} \times V), \\ Y^\theta &:= (h^\theta(\Omega))^{m+1} \times h^{1+\theta}(\Omega) \times (h^{1+\theta}(\Sigma))^{m+1} \times \mathbb{R}^{m+1} \times V. \end{aligned}$$

We further define $L := L_{0,0} \in \mathcal{L}(X_\theta, Y_\theta)$ by

$$L(u, p, \lambda) = \begin{bmatrix} -\Delta u + \nabla p + \lambda_1 \\ \operatorname{div} u \\ \mathcal{T}(u, p)n + \lambda_2^{ij} n_j e_i \\ \int_\Omega u \, dx \\ \int_\Omega (\partial_i u^j - \partial_j u^i) \, dx \end{bmatrix}.$$

Lemma A.1 *We have*

$$L \in \mathcal{L}_{is}(X_\theta, Y_\theta).$$

Proof: Let

$$\begin{aligned} V_R &= \{v \in (C^\infty(\Omega))^{m+1} \mid v_i(x) = s_{ij}x_j + c_i, \\ &\quad s_{ij}, c_i \in \mathbb{R}, \, s_{ij} = -s_{ji}, \, i, j = 1, \dots, m+1\} \end{aligned}$$

be the space of velocity fields belonging to the rigid body motions. Define the mappings ϕ_1 and ϕ_2 , valued in $\mathbb{R}^{m+1}$ and V , respectively, by

$$\begin{aligned} \phi_1 : \quad v &\mapsto \int_\Omega v \, dx, \\ \phi_2^{ij} : \quad v &\mapsto \frac{1}{2} \int_\Sigma (v^i n_j - v^j n_i) \, d\Sigma = \frac{1}{2} \int_\Omega (\partial_j v^i - \partial_i v^j) \, dx \end{aligned}$$

and note that (ϕ_1, ϕ_2) is bijective from V_R onto $\mathbb{R}^{m+1} \times V$. Choose $w_j, z_{ij} \in V_R$ such that

$$\phi_1^j(w_k) = \delta_{jk}, \quad \phi_2^{ij}(z_{kl}) = \delta_{ik}\delta_{jl}, \quad \phi_1(z_{kl}) = 0, \quad \phi_2(w_k) = 0.$$

Assume $L(u, p, \lambda) = (f, g, h, M_1, M_2)$, $\lambda = (\lambda_1, \lambda_2)$. The integral identity

$$\begin{aligned} &\frac{1}{2} \int_\Omega (\partial_i u^j + \partial_j u^i)(\partial_i v^j + \partial_j v^i) \, dx - \int_\Omega p \operatorname{div} v \, dx \\ &= \int_\Omega (-\Delta u + \nabla p - \nabla \operatorname{div} u) \cdot v \, dx + \int_\Sigma \mathcal{T}(u, p)n \cdot v \, d\Sigma, \end{aligned} \quad (\text{A.1})$$

valid for arbitrary $u, v \in (h^{2+\theta}(\Omega))^{m+1}$, $p \in h^{1+\theta}(\Omega)$ then implies

$$\begin{aligned} \lambda_1^j &= \int_\Omega (f - \nabla g) \cdot w_j \, dx + \int_\Sigma h \cdot w_j \, d\Sigma, \\ \lambda_2^{ij} &= \int_\Omega (f - \nabla g) \cdot z_{ij} \, dx + \int_\Sigma h \cdot z_{ij} \, d\Sigma. \end{aligned}$$

Consequently,

$$|\lambda| \leq C(\|f\|_{\theta, \Omega} + \|g\|_{1+\theta, \Omega} + \|h\|_{1+\theta}). \quad (\text{A.2})$$

In particular, $L(u, p, \lambda) = 0$ implies $\lambda = 0$ and by (A.1) with $v = u$ also $\partial_i u^j + \partial_j u^i = 0$, i.e. $u \in V_R$. This yields $u = 0$ because of $\int_\Omega u \, dx = 0$, $\int_\Omega (\partial_i u^j - \partial_j u^i) \, dx = 0$. Consequently, also $p = 0$. Hence, L is injective.

To show surjectivity, fix $(f, g, h, M_1, M_2) \in Y_\theta$ and approximate f , g , and h by sequences of smooth functions $\{f_k\}$, $\{g_k\}$, and $\{h_k\}$, respectively, in the norms of the corresponding spaces. It follows from [16], Lemma 2(i), and the Sobolev imbedding theorem that there are $(u_k, p_k, \lambda_k) \in (C^\infty(\Omega))^{m+1} \times C^\infty(\Omega) \times (\mathbb{R}^{m+1} \times V)$ such that

$$L(u_k, p_k, \lambda_k) = (f_k, g_k, h_k, M_1, M_2). \quad (\text{A.3})$$

We recall that the Stokes equations form an elliptic system in the sense of Douglis-Nirenberg and the boundary operator $(u, p) \mapsto \mathcal{T}(u, p)n$ satisfies the complementing boundary condition. Consequently, we have an a priori estimate (cf. [1])

$$\begin{aligned} & \|u_k - u_l\|_{2+\theta, \Omega} + \|p_k - p_l\|_{1+\theta, \Omega} \\ \leq & C(\|f_k - f_l - ((\lambda_k)_1 - (\lambda_l)_1)\|_{\theta, \Omega} + \|g_k - g_l\|_{1+\theta, \Omega} \\ & + \|h_k - h_l - ((\lambda_k)_2^{ij} - (\lambda_l)_2^{ij})n_j e_i\|_{1+\theta} \\ & + \|u_k - u_l\|_{L^2(\Omega)^{m+1}} + \|p_k - p_l\|_{L^2(\Omega)}). \end{aligned}$$

From a discussion of the weak formulation of (A.3) it follows that

$$\begin{aligned} & \|u_k - u_l\|_{H^1(\Omega)^{m+1}} + \|p_k - p_l\|_{L^2(\Omega)} \\ \leq & C(\|f_k - f_l\|_{((H^1(\Omega))^{m+1})'} + \|g_k - g_l\|_{L^2(\Omega)} + \|h_k - h_l\|_{H^{\frac{1}{2}}(\Omega)^{m+1}}). \end{aligned}$$

Together with an estimate for $\lambda_k - \lambda_l$ parallel to (A.2) this yields

$$\begin{aligned} & \|u_k - u_l\|_{2+\theta, \Omega} + \|p_k - p_l\|_{1+\theta, \Omega} \\ \leq & C(\|f_k - f_l\|_{\theta, \Omega} + \|g_k - g_l\|_{1+\theta, \Omega} + \|h_k - h_l\|_{1+\theta}), \end{aligned}$$

and the surjectivity follows by a standard approximation argument. ■

Remark: A very similar result is stated in [22], Proposition 2.

Lemma A.2 Suppose $h \in (h^{1+\theta}(\Sigma))^{m+1}$ with $h \cdot n = 0$ and

$$(u, p, \lambda) = L^{-1}(0, 0, h, 0, 0). \quad (\text{A.4})$$

Then $u \cdot n \in h^{3+\theta}(\Sigma)$, and there is a constant C independent of h such that

$$\|u \cdot n\|_{3+\theta} \leq C\|h\|_{1+\theta}.$$

Proof: In the sequel, we will write $\partial_n = n_i \partial_i$ for the normal derivative at the boundary. Fix a function $d \in BUC^\infty(\Omega)$ with the properties

$$d|_\Sigma = 0, \quad (\nabla d)|_\Sigma = n, \quad \partial_n(\nabla d) = 0.$$

Such a function can easily be constructed by using the signed distance function near Σ and cutting it off at large distance from Σ . Extend the normal vector field into Ω by setting $n = \nabla d$ there.

Note that

$$\Delta p = \operatorname{div}(\Delta u - \lambda_1) = 0 \quad \text{in } \Omega, \quad (\text{A.5})$$

and, due to the assumptions on h and the extension of n ,

$$\begin{aligned} \mathcal{T}(u, p)n \cdot n &= 2n_i n_j \partial_i u_j - p = 2n_j \partial_n u_j - p \\ &= 2\partial_n(u \cdot n) - p = -\lambda_2^{ij} n_i n_j \quad \text{on } \Sigma. \end{aligned} \quad (\text{A.6})$$

Set now

$$\psi := u \cdot n - d \frac{p}{2}.$$

Then, because of (A.4) and (A.5),

$$\begin{aligned} \Delta \psi &= (\nabla p - \lambda_1) \cdot n + 2\partial_i u_j \partial_i n_j + u \cdot \Delta n - \frac{p}{2} \Delta d - \nabla d \nabla p \\ &= -\lambda_1 \cdot n + 2\partial_i u_j \partial_i n_j + u \cdot \Delta n - \frac{p}{2} \Delta d \end{aligned}$$

and on Σ , due to (A.6),

$$\partial_n \psi = \partial_n(u \cdot n) - \frac{p}{2} = -\frac{1}{2} \lambda_2^{ij} n_i n_j.$$

A standard a priori estimate for the Laplacian with Neumann boundary data in Hölder spaces and Lemma A.1 yield

$$\begin{aligned} \|u \cdot n\|_{3+\theta} &\leq \|\psi\|_{3+\theta, \Omega} \leq C(\|\Delta \psi\|_{1+\theta, \Omega} + \|\partial_n \psi\|_{2+\theta} + \|\psi\|_{1+\theta, \Omega}) \\ &\leq C(\|u\|_{2+\theta, \Omega} + \|p\|_{1+\theta, \Omega} + |\lambda|_{\mathbb{R}^{m+1} \times V}) \leq C\|h\|_{1+\theta}. \end{aligned}$$

■

B A perturbation result for the Dirichlet-Neumann operator

It is known that the negative Dirichlet-Neumann operator is an elliptic pseudo-differential operator of first order which generates a strongly continuous analytic semigroup on suitable Hölder spaces. We show in this appendix that this generation property is stable under additive perturbations by first order differential operators. Note that we do not assume any condition on the L_∞ -bound of the coefficients of the perturbation. This means that the general perturbation results for generators of analytic semigroups are not applicable in our setting.

We assume that we are given

$$a^{jk}, a^j, a_0 \in C^\infty(\overline{\Omega}), \quad 1 \leq j, k \leq m+1, \quad b_0 \in C^\infty(\Sigma),$$

and that there is a positive constant c such that

$$a^{jk}(x) \xi_j \xi_k \geq c|\xi|^2, \quad x \in \overline{\Omega}, \quad \xi \in \mathbb{R}^{m+1}.$$

Given $u \in h^{2+\theta}(\Omega)$, let

$$\mathcal{A}u := -a^{jk}\partial_j\partial_k u + a^j\partial_j u + a_0 u, \quad \mathcal{B}u := a^{jk}(\text{Tr}_\Sigma\partial_j u)\nu_k + b_0\text{Tr}_\Sigma u,$$

where $\nu = (\nu_1, \dots, \nu_{m+1})$ stands for the outer unit normal on Σ and where Tr_Σ denotes the trace operator with respect to Σ , i.e. $\text{Tr}_\Sigma u = u|_\Sigma$ is the restriction of $u \in C^1(\overline{\Omega})$ to Σ . For simplicity we assume that $a_0 \geq 0$. It will become clear later that this is without loss of generality for our purposes, see Theorem B.1. Let $(f, g) \in h^\theta(\Omega) \times h^{2+\theta}(\Sigma)$ be given. Then it is well-known that there exists a unique classical solution the following BVP

$$\mathcal{A}u = f \quad \text{in } \Omega, \quad \text{Tr}_\Sigma u = g \quad \text{on } \Sigma. \quad (\text{B.1})$$

It can be shown that this solution belongs to $h^{2+\theta}(\Omega)$, see Theorem B.1 below. Let

$$\mathcal{T} := (\mathcal{A}, \text{Tr}_\Sigma)^{-1} | (\{0\} \times h^{2+\theta}(\Sigma)) \in \mathcal{L}(h^{2+\theta}(\Sigma), h^{2+\theta}(\Omega))$$

denote the corresponding solution operator with $f = 0$. Then we call $\mathcal{BT}$ the Dirichlet-Neumann operator on Σ . It is not difficult to see that

$$\mathcal{BT} \in \mathcal{L}(h^{2+\theta}(\Sigma), h^{1+\theta}(\Sigma)).$$

Let now X be a smooth vector field on Σ and define $\mathcal{V} \in \mathcal{L}(h^{2+\theta}(\Sigma), h^{1+\theta}(\Sigma))$ by

$$\mathcal{V}h := dh(X) \quad \text{for } h \in h^{2+\theta}(\Sigma).$$

Observe that

$$\mathcal{V}h = \mathcal{L}_X h = Xh = (\nabla_\Sigma h|X),$$

where $\mathcal{L}_X$ stands for the Lie derivative of h with respect to X .

We next fix $l \in \mathbb{N}$ with $l \geq 1$ and set

$$E_0 := h^{l+\theta}(\Sigma), \quad E_1 := h^{l+1+\theta}(\Sigma).$$

Let $L : \text{dom}(L) \subset h^{1+\theta}(\Sigma) \rightarrow h^{1+\theta}(\Sigma)$ be a linear operator. We define the E_0 -realization L_{E_0} of L to be the linear operator in E_0 , given by $\text{dom}(L_{E_0}) := \{u \in \text{dom}(L) \cap E_0; Lu \in E_0\}$ and $L_{E_0}u = Lu$ for $u \in \text{dom}(L_{E_0})$. Using this notation we define

$$A := E_0 - \text{realization of } \mathcal{BT}, \quad V := E_0 - \text{realization of } \mathcal{V}.$$

In order to provide a detailed study of the operator A we need the following adaption of the well-known Agmon-Douglis-Nirenberg a priori estimate:

Theorem B.1 *There are $\lambda_0 \geq 0$ and $\mu_0 > 0$ such that, given $\lambda \geq \lambda_0$ and $\mu \geq \mu_0$, we have*

$$\begin{aligned} (\lambda + \mathcal{A}, \text{Tr}_\Sigma) &\in \mathcal{L}_{is}(h^{l+1+\theta}(\Omega), h^{l-1+\theta}(\Omega) \times h^{l+1+\theta}(\Sigma)), \\ (\lambda + \mathcal{A}, \mu\text{Tr}_\Sigma + \mathcal{B}) &\in \mathcal{L}_{is}(h^{l+1+\theta}(\Omega), h^{l-1+\theta}(\Omega) \times h^{l+\theta}(\Sigma)). \end{aligned}$$

Moreover, given $\lambda_1 \geq \lambda_0$, there is a positive constant C such that

$$\|u\|_{l+1+\theta, \Omega} \leq C(\|(\lambda + \mathcal{A})u\|_{l-1+\theta, \Omega} + \|\text{Tr}_\Sigma u\|_{l+1+\theta})$$

for all $u \in h^{l+1+\theta}(\Omega)$ and all $\lambda \in [0, \lambda_1]$.

Theorem B.1 was shown in [9] in the case $l = 1$ and for a particular BVP $(\mathcal{A}, \mathcal{B})$ on an unbounded domain. The proof there is based on the classical Agmon-Douglis-Nirenberg estimates in Hölder spaces, the maximum principle, and the continuity method. A careful inspection of that proof shows that it carries over to our situation here. Some arguments are even easier, since we consider here a BVP with smooth coefficients on a bounded domain.

Based on Theorem B.1 we can show the following mapping properties of A :

Lemma B.2 (i) $E_1 = \text{dom}(A)$, $E_1 \subset \text{dom}(V)$, and $A, V \in \mathcal{L}(E_1, E_0)$.

(ii) $\mu + A \in \mathcal{L}_{is}(E_1, E_0)$, provided $\mu \geq \mu_0$.

Proof: (i) It follows from Theorem B.1 that $E_1 = h^{l+1+\theta}(\Omega)$ belongs to the domain of A and that $A \in \mathcal{L}(E_1, E_0)$. To verify that $\text{dom}(A)$ is contained in E_1 , let $g \in \text{dom}(A)$ be given. By construction we have that $h := \mu_0 g + \mathcal{B}\mathcal{T}g \in E_0$ and that $v := \mathcal{T}g$ is a solution to

$$\mathcal{A}v = 0 \quad \text{in } \Omega, \quad (\mu_0 \text{Tr}_\Sigma + \mathcal{B})v = \mu_0 g + \mathcal{B}\mathcal{T}g = h \quad \text{on } \Sigma.$$

Hence Theorem B.1 implies that $v \in h^{l+1+\theta}(\Omega)$ and therefore $g = v|_\Sigma \in h^{l+1+\theta}(\Sigma) = E_1$. The assertion for V is clear.

(ii) Let $\mu \geq \mu_0$ and assume that $g \in E_1$ satisfies $(\mu + A)g = 0$. Writing $v := \mathcal{T}g$, we have

$$\mathcal{A}v = 0 \quad \text{in } \Omega, \quad (\mu \text{Tr}_\Sigma + \mathcal{B})v = \mu g + \mathcal{B}\mathcal{T}g = 0 \quad \text{on } \Sigma.$$

Thus Theorem B.1 implies that $g = 0$. Let now $f \in E_0$ be given. Then Theorem B.1 ensures that there is a $v \in h^{l+1+\theta}(\Omega)$ such that

$$\mathcal{A}v = 0 \quad \text{in } \Omega, \quad (\mu \text{Tr}_\Sigma + \mathcal{B})v = f \quad \text{on } \Sigma.$$

This yields

$$\mathcal{T}\text{Tr}_\Sigma v = (\mathcal{A}, \text{Tr}_\Sigma)^{-1}(0, \text{Tr}_\Sigma v) = (\mathcal{A}, \text{Tr}_\Sigma)^{-1}(\mathcal{A}, \text{Tr}_\Sigma)v = v.$$

Putting now $g := \text{Tr}_\Sigma v$, we conclude that $g \in E_1$ and that

$$(\mu + A)g = (\mu + \mathcal{B}\mathcal{T})g = (\mu \text{Tr}_\Sigma + \mathcal{B})v = f.$$

Hence (i) and the open mapping theorem imply the assertion. This completes the proof. $\blacksquare$

Remark: Assume that $X(p) \neq 0$ for all $p \in \Sigma$. Then one can show that $\text{dom}(V) = E_1$ if and only if $m = 1$. We omit a proof of this assertion, since we

shall not need this property of V in the following.

Let now (U, φ) be a local chart of $\overline{\Omega}$ such that $U \cap \Sigma \neq \emptyset$. We may assume that the corresponding parameter domain $\varphi(U)$ is given by $B := \mathbb{B}_{m+1} \cap \mathbb{H}_+^{m+1}$, where $\mathbb{H}_+^{m+1} := \{x \in \mathbb{R}^{m+1}, x^{m+1} \geq 0\}$ denotes the "positive half space" in $\mathbb{B}^{m+1}$ with respect the $(m+1)$ -th co-ordinate. The local representations of $(\mathcal{A}, \mathcal{B}, \mathcal{V})$ are denoted by $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}}, \tilde{\mathcal{V}})$. This means that

$$\tilde{\mathcal{A}}\varphi_* = \varphi_*\mathcal{A}, \quad \tilde{\mathcal{B}}\varphi_* = \varphi_*\mathcal{B}, \quad \tilde{\mathcal{V}}\varphi_* = \varphi_*\mathcal{V}.$$

It is well-known that there are

$$\begin{aligned} \tilde{a}^{jk}, \tilde{a}^j, \tilde{a}_0, \tilde{b}^j &\in BUC^\infty(B), \quad 1 \leq j, k \leq m+1, \\ \tilde{b}_0, \tilde{c}^\alpha &\in BUC^\infty(B \cap (\mathbb{R}^m \times \{0\})), \quad 1 \leq \alpha \leq m, \end{aligned}$$

such that

$$[\tilde{a}^{jk}(x)] \text{ is symmetric and positive definite, } \tilde{b}^n(x) > 0, \quad (\text{B.2})$$

for all $x \in B$, and such that

$$\tilde{\mathcal{A}} = -\tilde{a}^{jk}\partial_j\partial_k + \tilde{a}^j\partial_j + \tilde{a}_0, \quad \tilde{\mathcal{B}} = -\tilde{b}^j\partial_j + \tilde{b}_0, \quad \tilde{\mathcal{V}} = \tilde{c}^\alpha\partial_\alpha.$$

Let now

$$a_0^{jk} := \tilde{a}^{jk}(0), \quad b_0^j := \tilde{b}^j(0), \quad c_0^\alpha := \tilde{c}^\alpha(0), \quad 1 \leq j, k \leq m+1, \quad 1 \leq \alpha \leq m.$$

Furthermore, we introduce

$$\vec{a} := (a_0^{1(m+1)}, \dots, a_0^{m(m+1)}), \quad e_0 := a_0^{(m+1)(m+1)}, \quad d_0(\xi) := a_0^{\alpha\beta}\xi_\alpha\xi_\beta,$$

where $\xi \in \mathbb{R}^m$, and define the following parameter-dependent quadratic polynomial:

$$q_\xi(z) := 1 + d_0(\xi) + 2i(\vec{a}|\xi)z - e_0z^2, \quad z \in \mathbb{C}.$$

It follows from the fact that the matrix $[a_0^{jk}]$ is positive definite that, given $\xi \in \mathbb{R}^m$, there exists exactly one root $\lambda(\xi)$ of $q_\xi(\cdot)$ with positive real part, which is given by

$$\lambda(\xi) = \frac{i(\vec{a}|\xi)}{e_0} + \frac{1}{e_0} \sqrt{e_0(1 + d_0(\xi)) - (\vec{a}|\xi)^2}.$$

This root is now used to introduce the following Fourier multiplication operator, acting on functions defined on $\mathbb{R}^m$. Given $g \in h^{l+1+\theta}(\mathbb{R}^m)$, let

$$\mathcal{T}_0 g(x, y) := [\mathcal{F}^{-1} e^{-\lambda(\cdot)y} \mathcal{F} g](x), \quad (x, y) \in \mathbb{R}^m \times \mathbb{R}_+,$$

where $\mathcal{F}$ denotes the Fourier transform on $\mathbb{R}^m$. Finally, we define the following operators

$$\mathcal{G}_t := \mathcal{B}_0 \mathcal{T}_0 + t \mathcal{V}_0, \quad t \in [0, 1], \quad (\text{B.3})$$

where $\mathcal{B}_0 := -b_0^j \partial_j$ and $\mathcal{V}_0 := c_0^\alpha \partial_\alpha$, and the function

$$g_t(\xi) := b_0^{m+1} \lambda(\xi) - i(\vec{b} + t\vec{c}|\xi), \quad \xi \in \mathbb{R}^m, \quad t \in [0, 1],$$

where $\vec{b} := (b_0^1, \dots, b_0^m)$ and $\vec{c} := (c_0^1, \dots, c_0^m)$. Then one shows as in Lemma 4.1 in [11] that $\mathcal{G}_t$ is a Fourier multiplication operator with symbol g_t , i.e. we have

$$\mathcal{G}_t = \mathcal{F}^{-1} g_t \mathcal{F}, \quad t \in [0, 1]. \quad (\text{B.4})$$

Based on (B.4), we now obtain as in [11, Theorem 4.2]:

Theorem B.3 $\mathcal{G}_t \in \mathcal{H}(h^{l+1+\theta}(\mathbb{R}^m), h^{l+\theta}(\mathbb{R}^m)), \quad t \in [0, 1].$

Setting $G_t := A + tV$ for $t \in [0, 1]$, we are prepared to prove for the following generation result:

Theorem B.4 $G_t \in \mathcal{H}(E_1, E_0), \quad t \in [0, 1].$

Proof: (i) We first introduce an equivalent norm on $h^{l+k+\theta}(\Sigma)$, $k \in \{0, 1\}$, which will be useful in what follows. In order to do so, we note that there exists a finite family of charts $\{(U_r, \varphi_r); 1 \leq r \leq N\}$ such that $\{U_r \cap \Sigma, \varphi_r|_{(U_r \cap \Sigma)}; 1 \leq r \leq N\}$ is an atlas for Σ . Further, there exist test functions $\psi_r \in \mathcal{D}(U_r)$ such that $\{\psi_r; 1 \leq r \leq N\}$ is a partition of unity on a compact neighbourhood of Σ in $\cup_{r=1}^N U_r$. Finally, in the following we write $\|\cdot\|_{s, \mathbb{R}^m}$ for the norm in $h^s(\mathbb{R}^m)$, $s > 0$. It is known that

$$[h \mapsto \max_{1 \leq r \leq N} \|(\varphi_r)_*(\psi_r h)\|_{l+k+\theta, \mathbb{R}^m}] \quad (\text{B.5})$$

defines an equivalent norm on $h^{l+k+\theta}(\Sigma)$ for $k \in \{0, 1\}$, see [23].

(ii) Given $r \in \{1, \dots, N\}$, we obtain a family of operators $\{\mathcal{G}_t^r; t \in [0, 1]\}$ of the form (B.3), using the above construction on the chart (U_r, φ_r) . In the following we suppress the index $r \in \{1, \dots, N\}$. Clearly, we can and will choose the constants in the estimates below to be independent of r . Invoking Theorem B.3, we know that there are positive constants C_1 and λ_1 such that

$$\|g\|_{l+1+\theta, \mathbb{R}^m} + |\lambda| \|g\|_{l+\theta, \mathbb{R}^m} \leq C_1 \|(\lambda + \mathcal{G}_t)g\|_{l+\theta, \mathbb{R}^m} \quad (\text{B.6})$$

for all $g \in h^{l+1+\theta}(\mathbb{R}^m)$, $\lambda \in [\text{Re } z \geq \lambda_1]$, and $t \in [0, 1]$.

(iii) Fix $\sigma \in (0, \theta)$. Shrinking the diameter of the chart domain U , we may assume that there exists a positive constant C_2 such that

$$\|\varphi_*(\psi G_t h) - \mathcal{G}_t \varphi_*(\psi h)\|_{l+\theta, \mathbb{R}^m} \leq \frac{1}{2C_1} \|\varphi_*(\psi h)\|_{l+1+\theta, \mathbb{R}^m} + C_2 \|h\|_{l+1+\sigma} \quad (\text{B.7})$$

for all $h \in h^{l+1+\theta}(\Sigma)$. The proof of (B.7) is based on the fact that $\mathcal{T}_0$ can be interpreted as the solution operator of the elliptic BVP on the half space $\mathbb{H}_+^{m+1}$, which arises from the original BVP given by $(\mathcal{A}, \mathcal{B})$ after flattening the

boundary, freezing the coefficients, and considering top order terms only. For the details we refer to [11, Lemma 5.1]. Combining (B.6) and (B.7), we get

$$\begin{aligned} & \|\varphi_*(\psi h)\|_{l+1+\theta, \mathbb{R}^m} + |\lambda| \|\varphi_*(\psi h)\|_{l+\theta, \mathbb{R}^m} \\ & \leq 2C_1 \{\|\varphi_*(\psi(\lambda + G_t)h)\|_{l+\theta, \mathbb{R}^m} + C_2 \|h\|_{l+1+\sigma}\} \end{aligned} \quad (\text{B.8})$$

for all $h \in h^{l+1+\theta}(\Sigma)$, $\lambda \in [\operatorname{Re} z \geq \lambda_1]$, and $t \in [0, 1]$. Using the norm given in (B.5), we find a positive constant C such that

$$\|h\|_{l+1+\theta} + |\lambda| \|h\|_{l+\theta} \leq \frac{C}{2} \|(\lambda + G_t)h\|_{l+\theta} + C \|h\|_{l+1+\sigma} \quad (\text{B.9})$$

for all $h \in h^{l+1+\theta}(\Sigma)$, $\lambda \in [\operatorname{Re} z \geq \lambda_1]$, and $t \in [0, 1]$.

(iv) We next absorb the second term on the right-hand side of (B.9) into the left-hand side of (B.9). For this recall that

$$h^{l+1+\sigma}(\Sigma) = (h^{l+\theta}(\Sigma), h^{l+1+\theta}(\Sigma))_{1-\theta+\sigma, \infty}^0,$$

see Remark 2.2 f). Using the corresponding interpolation inequality, it follows from Young's inequality that there is a positive constant C_3 such that

$$\|h\|_{l+1+\sigma} \leq \frac{1}{2C} \|h\|_{l+1+\theta} + C_3 \|h\|_{l+\theta}, \quad h \in h^{l+1+\theta}(\Sigma). \quad (\text{B.10})$$

Combining (B.9) and (B.10), we find

$$\|h\|_{l+1+\theta} + |\lambda| \|h\|_{l+\theta} \leq C \|(\lambda + G_t)h\|_{l+\theta} \quad (\text{B.11})$$

for all $h \in h^{l+1+\theta}(\Sigma)$, $\lambda \in [\operatorname{Re} z \geq \lambda_*]$, and $t \in [0, 1]$, where $\lambda_* := 2 \max\{\lambda_1, CC_3\}$.

(v) It follows from (B.11), Lemma B.4(ii), and a well-known homotopy argument, cf. [15, Theorem 5.2], that there is a $\lambda_0 > 0$ such that

$$\lambda_0 + G_t \in \mathcal{L}_{is}(h^{l+1+\theta}(\Sigma), h^{l+\theta}(\Sigma)), \quad t \in [0, 1]. \quad (\text{B.12})$$

Now the assertion follows from (B.11), (B.12) and Remark 2.2 h). ■

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