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Channel Modeling and Target Design for Two-Dimensional Optical Storage Systems

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In this paper, we first show that the symbol response of the two-dimensional (2-D) optical storage (TwoDOS) channel can be computed by a one-dimensional (1-D) Hankel transform, instead of the 2-D Fourier transform. This results in a computationally efficient approach for generating readback signals in the presence of pit-size variations. We also show how to design a 2-D minimum mean-square error (MMSE) equalizer for channels with these distortions. Second, we present a novel way to jointly design equalizer and target under linear constraints on the target, by transforming the 2-D target design problem into a 1-D form. Using a 2-D Viterbi detector, we investigated different target constraints. The results show that the newly proposed “2-D monic constraint” is a reasonable target constraint for a TwoDOS system.

Index Terms—Hankel transform, monic constraint, target design, two-dimensional optical storage.

I. INTRODUCTION

IN RESPONSE to the continuously increasing demand for storage capacity, optical recording research has undergone three generations [namely, compact disc (CD; 650 MB), digital versatile disc (DVD; 4.7 GB), blu-ray disc (BD; 25 GB)] of optical media, with BD being the most recent [1]–[4]. Even though each generation offers significant improvement in storage capacity, the growth rate of recording density in optical storage lags behind that of magnetic storage, largely due to the slow pace at which the wavelength of laser diode has improved. The relatively slow pace of physical improvements in optical storage motivates the use of advanced signal processing techniques to achieve further increase in recording density. One promising example of this is the recently introduced two-dimensional optical storage (TwoDOS) approach [5]. Compared with the conventional one-dimensional (1-D) optical storage, this technology noticeably reduces the track pitch and thus makes it possible to record at much higher track density that is comparable to the linear density. This higher track density is realized by grouping a number of adjacent tracks with shorter track pitch between them, and using a larger track pitch as the group boundaries. Further, TwoDOS maximizes the capacity of the disc by adopting hexagonal bit cells instead of traditional square/rectangular cells (which was used in [6], [7]). Moreover, a higher data rate can be achieved with the use of parallel writing and read-out. Even though the presence of a guard-band between groups prevents the interference from adjacent groups, the use of shorter track pitch between the tracks within the group results in severe two-dimensional (2-D) intersymbol interference (ISI) from bits across as well as along the tracks. Consequently, it becomes very important to develop powerful 2-D signal processing and coding techniques to ensure reliable data recovery. In this paper, we address the issues of channel modeling, and the design of equalizer and partial response (PR) target for a 2-D Viterbi detector (VD) based receiver.

The linear 2-D channel model for TwoDOS was first introduced in [8] by taking the 2-D inverse Fourier transform of the 2-D modulation transfer function (MTF), resulting in 2-D impulse response of the channel. However, this channel model may not be accurate enough unless the density is sufficiently high since this model does not account for the presence of the linear pulse modulator in the write circuit. In this paper, we present the linear channel model which incorporates the linear pulse modulator. More importantly, we show that by exploiting the radial symmetry property of the symbol response, the 2-D symbol response can be efficiently calculated by a 1-D Hankel transform approach [9]. Though our linear channel model is constructed solely from a signal processing point of view, it reflects physical reality since it is consistent with the linear part of the physical channel model constructed by Coene [10] based on scalar diffraction theory. We also develop the 2-D symbol response model in the presence of pit-size variations, and show that the Hankel transform approach can be used to develop a computationally simple approach for generating readback signals in the presence of pit-size variations.

Because of the presence of serious 2-D ISI in TwoDOS readback signals, the 2-D VD has been used for bit detection [11]. As the complexity of 2-D VD grows exponentially with both channel length and number of tracks per group, 2-D PR equalization has been used to shorten the channel memory [12]. In our work, instead of choosing a fixed target to design the equalizer, we optimize both the equalizer and target responses based on minimum mean-square error (MMSE) approach to improve the performance of 2-D VD. However, to avoid the trivial solution, we need to impose a suitable constraint on the 2-D PR target. Since doing constrained minimization in 2-D is quite difficult, we propose a novel method which converts the 2-D target design problem into a 1-D problem. We also develop an appropriate target constraint that results in good bit detection performance from 2-D VD.

Section II describes the TwoDOS channel model and the Hankel transform approach to the computation of symbol response. In Section III, the 2-D MMSE equalizer design approach is presented. In Section IV, we investigate the theoretical

performance of 2-D VD in the presence of pit-size variations. A novel 2-D target design method is proposed in Section V. The investigation of different target constraints along with performance comparison through bit-error-rate (BER) simulations are presented in Section VI. Finally, Section VII concludes the paper.

II. CHANNEL MODEL

TwoDOS uses a multispot light beam for parallel read-out. For linear read-out, each spot can be characterized by the MTF given by the Braat–Hopkins formula [14]

$$\mathbf{F}_{\text{sf}}(\phi, \rho) = \begin{cases} \frac{2}{\pi} \left[\arccos\left(\frac{\rho}{\rho_c}\right) - \left(\frac{\rho}{\rho_c}\right) \sqrt{1 - \left(\frac{\rho}{\rho_c}\right)^2} \right], & \text{if } \rho \leq \rho_c \\ 0, & \text{else} \end{cases} \quad (1)$$

where ρ is the spatial angular frequency, ρ_c is the angular cutoff frequency, and ϕ is the azimuth angle in the 2-D spatial frequency plane. The 2-D linear pulse modulator corresponds to the pit region used in the writing process (i.e., pit hole), and is defined as

$$\mathbf{C}_s(\theta, r) = \begin{cases} 1, & \text{when } r \leq R \\ 0, & \text{else} \end{cases} \quad (2)$$

where r and θ are radial and angular coordinates, respectively, in spatial domain and R is the radius of the pit hole.

We use polar coordinates in deriving the channel symbol response. Let $\mathbf{C}_{\text{sf}}(\phi, \rho)$ represent the linear pulse modulator in spatial frequency domain. Then, the symbol response in spatial frequency domain is given by

$$\mathbf{H}_{\text{sf}}(\phi, \rho) = \mathbf{C}_{\text{sf}}(\phi, \rho) \mathbf{F}_{\text{sf}}(\phi, \rho). \quad (3)$$

By taking 2-D Fourier transform on both sides of (2), we obtain

$$\mathbf{C}_{\text{sf}}(\phi, \rho) = 2\pi R \frac{J_1(\rho R)}{\rho} \quad (4)$$

where $J_1(x)$ is the Bessel function of the first kind and first order. Then the symbol response in spatial domain, $\mathbf{H}_s(\theta, r)$, can be obtained by taking the 2-D inverse Fourier transform of $\mathbf{H}_{\text{sf}}(\phi, \rho)$, i.e.,

$$\mathbf{H}_s(\theta, r) = \frac{1}{(2\pi)^2} \iint \rho \mathbf{H}_{\text{sf}}(\phi, \rho) e^{[j\rho r \cos(\theta - \phi)]} d\phi d\rho. \quad (5)$$

It should be noted that the above symbol response considers only linear ISI and it is developed purely from a signal processing point of view. In the latter part of this section, we will modify this to include nonlinear interferences also. A rigorous derivation of the channel model based on physical generation of the signal, including linear and nonlinear ISI, can be found in [10]. However, as we show below, our model in (3) is consistent with the model in [10] when only linear ISI is considered.

Based on the general formulation in [10], we can write the linear symbol response $H_{\text{sc}}(x, y)$ in spatial domain as ((x, y) being Cartesian coordinates)

$$H_{\text{sc}}(x, y) = \iint |p(\vec{x}, \vec{y})|^2 C_s(\vec{x} - x, \vec{y} - y) d\vec{x} d\vec{y} \quad (6)$$

where $p(\vec{x}, \vec{y})$ is the spot function and $C_s(\vec{x} - x, \vec{y} - y)$ is the window function centered at (x, y) . Because $C_s(x, y)$ is circularly symmetric in TwoDOS, $C_s(\vec{x} - x, \vec{y} - y) = C_s(x - \vec{x}, y - \vec{y})$. Therefore, (6) implies that $H_{\text{sc}}(x, y)$ is the 2-D convolution between $|p(x, y)|^2$ and $C_s(x, y)$. Since $\mathbf{F}_{\text{sf}}(\phi, \rho)$ is the 2-D Fourier transform of $|p(x, y)|^2$, the signal processing based model given in (3) is consistent with the physical model given in (6).

While studying TwoDOS channels with time-varying characteristics (e.g., channels with pit-size variations), it may be necessary to recompute the channel symbol response very often. Therefore, in such situations, directly using the 2-D Fourier transform approach may not be desirable in view of its computational complexity. We now show that the 2-D Fourier transform can be replaced by Hankel transform in TwoDOS, thus resulting in significant computational saving in the calculation of $\mathbf{H}_s(\theta, r)$. From (1) and (4) we find that since $\mathbf{F}_{\text{sf}}(\phi, \rho)$ and $\mathbf{C}_{\text{sf}}(\phi, \rho)$ are both radially symmetric, the symbol response in spatial frequency domain $\mathbf{H}_{\text{sf}}(\phi, \rho)$ is also radially symmetric, i.e., $\mathbf{H}_{\text{sf}}(\phi, \rho) = \hat{\mathbf{H}}_{\text{sf}}(\rho)$. Then, (5) can be simplified as

$$\mathbf{H}_s(\theta, r) = \frac{1}{2\pi} \int_0^{\rho_c} \rho \hat{\mathbf{H}}_{\text{sf}}(\rho) J_0(\rho r) d\rho \quad (7)$$

where $J_0(x)$ is the Bessel function of the first kind and zero order, and is independent of θ . Thus, $\mathbf{H}_s(\theta, r)$ is also radially symmetric, i.e., $\mathbf{H}_s(\theta, r) = \hat{\mathbf{H}}_s(r)$. Equation (7) shows that $\mathbf{H}_s(\theta, r)$ is the inverse Hankel transform of $\hat{\mathbf{H}}_{\text{sf}}(\rho)$ [9]. Thus, by taking advantage of the circular symmetry of the TwoDOS channel, the 2-D Fourier transform can be reduced to a 1-D Hankel transform. This is important since, as we show below, the Hankel transform approach is a computationally efficient means to compute $\mathbf{H}_s(\theta, r)$, compared with 2-D Fourier transform. The fast Hankel transform can be computed with a complexity of about $O(N_s \log_2 N_s)$, where N_s denotes the number of data points in each dimension [15], [16]. Compared to this, the complexities of traditional 2-D discrete Fourier transform (DFT) and 2-D fast Fourier transform (FFT) are $O(N_s^4)$ and $O(N_s^2 \log_2 N_s)$, respectively [17]. Thus, the fast Hankel means of computing the channel symbol response is indeed a computationally efficient approach when the radial symmetry property is maintained in TwoDOS systems.

Note that $\hat{\mathbf{H}}_s(r)$ characterizes a single spot only. The channel response of the 2-D system is given by $\mathbf{H} = [\mathbf{H}_0, \mathbf{H}_1, \dots, \mathbf{H}_{N_h-1}]^T$, where N_h is the channel length, $\mathbf{H}_k$ is a $N_r \times N_r$ matrix, and N_r denotes the number of tracks forming one group. Fig. 1 shows the arrangement of bit cells in TwoDOS recording system. The dashed slanted rectangular boxes indicate the array of bit cells onto which the parallel read-out laser beam is focused at each time instant.

Fig. 1. Arrangement of bit cells in TwoDOS system. Bit cells with “circles” inside indicate “+1” (i.e., pits) and the bit cells without circles indicate “-1” data bits.

Considering the hexagonal structure of each bit cell, the (i, j) th element of $\mathbf{H}_k$ can be obtained as

$$H_k(i, j) = \tilde{\mathbf{H}}_s \left(\sqrt{x_{ij,k}^2 + y_{ij,k}^2} T \right), \quad i, j = 1, 2, \dots, N_r \quad (8)$$

where $x_{ij,k} = (i - j) \cos 60^\circ + t_k$, $y_{ij,k} = (j - i) \sin 60^\circ$, $t_k = (k - (N_h + 1)/(2))T$ is the spatial difference along the track (for odd N_h), and T is the center-to-center distance between adjacent bits. In fact, $H_k(i, j)$ can be considered as the interference of j th track to i th track. The readback signal resulting from the parallel read-out at time index “ n ” is given by

$$\mathbf{z}(n) = \sum_{k=0}^{N_h-1} \mathbf{H}_k \mathbf{a}(n-k) + \boldsymbol{\theta}(n) \quad (9)$$

where $\mathbf{z}(n) = [z_1(n), z_2(n), \dots, z_{N_r}(n)]^T$, $z_k(n)$ denotes the readback signal component from the k th track, $\mathbf{a}(n) = [a_1(n), a_2(n), \dots, a_{N_r}(n)]^T$, $a_k(n) \in \{-1, 1\}$ denotes the data bit written on the k th track, $\boldsymbol{\theta}(n) = [\theta_1(n), \theta_2(n), \dots, \theta_{N_r}(n)]^T$ and $\theta_k(n)$ denotes the noise picked up from the k th track, for $k = 1, 2, \dots, N_r$.

It has been shown that the optimum radius of the pit hole is $R = (1/2)\sqrt{(\sqrt{3}/\pi)T}$ [11]. In practice, however, it is difficult to accurately control the size of the recorded domain. For example, the domain bloom, which is due to under- or over-etching during the mastering process, causes pits to be systematically larger or smaller and it is one of the major write imperfections. In this case, (4) becomes

$$\mathbf{C}_{\text{sf}}(\phi, \rho) = 2\pi(1 + \Delta_b)R \frac{J_1(\rho(1 + \Delta_b)R)}{\rho} \quad (10)$$

where Δ_b reflects the degree of bloom. A positive value of Δ_b means that the recorded pits are systematically larger than the nominal pits and vice versa when Δ_b is negative. The channel symbol response $\mathbf{H}_k$ ($k = 0, 1, \dots, N_h - 1$) should be computed by taking this into account. Unlike the 1-D system, where bloom manifests only when pit-to-land or land-to-pit transitions occur, the transitions in TwoDOS occur whenever bits corresponding to “+1” are written. This is because every “+1” is recorded as a pit of radius R smaller than $T/2$ and thus there are two transition edges between consecutive pits, instead of none in the 1-D system.

Fig. 2. Discrete-time channel model of TwoDOS system with PR equalization and Viterbi detector.

In this section, it has been implicitly assumed that the recorded domains has infinitely steep vertical edges. In reality, however, the edges of the recorded domains are not always sharply defined and tend to exhibit random pit-size variations. Accounting for this random pit-size variation in (10), we get

$$\mathbf{C}_{\text{sf}}(\phi, \rho) = 2\pi(1 + \Delta_b + \Delta_t)R \frac{J_1(\rho(1 + \Delta_b + \Delta_t)R)}{\rho} \quad (11)$$

where the normalized parameter Δ_t is a zero-mean random variable that is statistically independent of the input data vector $\mathbf{a}(n)$. Due to the same reason as explained above, this random pit-size variation occurs whenever the input bit is “+1.” Thus, the random variable $\Delta_i = \Delta_b + \Delta_t$ may be used to represent the effective pit-size variation. The bloom component Δ_b , which remains constant for all pits, is the mean of Δ_i , and the component Δ_t varies randomly for each pit.

In the presence of pit-size variations, it is easy to see that the channel becomes a time-varying linear system since the symbol response tends to differ from one bit to another. As a result, the readback signal takes the form (see Fig. 2)

$$\mathbf{z}(n) = \sum_{k=0}^{N_h-1} \mathbf{H}_{k,n} \mathbf{a}(n-k) + \boldsymbol{\theta}(n) \quad (12)$$

where the channel matrices $\mathbf{H}_{k,n}$ are now also dependent on the time reference “ n .” The elements $H_{k,n}(i, j)$ ($i, j = 1, 2, \dots, N_r$) of $\mathbf{H}_{k,n}$ can be computed by (8), (7), and (3) after replacing the $\mathbf{C}_{\text{sf}}(\phi, \rho)$ in (3) with the time-varying form given in (10) or (11). Thus, to compute $\mathbf{z}(n)$ for each “ n ,” we need to recompute $\mathbf{H}_{k,n}$ for all k , which is a very computationally demanding task. A simple table lookup can be used to save computational complexity by storing precomputed channel responses for quantized values of pit sizes. On the other hand, since the time-varying symbol response still satisfies the radial symmetry condition in spatial frequency domain, we can use the fast Hankel transform approach to compute the individual symbol responses. This approach results in significant savings in computational complexity and memory requirements compared with the 2-D Fourier transform approach and the table lookup approach, respectively. This is very important for doing simulation studies over a very large length of data bits.

III. 2-D MMSE EQUALIZER DESIGN

Fig. 2 shows the discrete-time channel model of TwoDOS system with PR equalization and VD. In the figure, $\mathbf{G}_k$ ($k = 0, 1, \dots, N_g - 1$) and $\mathbf{W}_k$ ($k = 0, 1, \dots, N_w - 1$) represent

$N_r \times N_r$ coefficient matrices of PR target and equalizer, respectively, m_0 and Δ denote the delays (in number of bits) from channel input to equalizer output and detector output, respectively. In this section, we assume that the channel symbol response $\mathbf{H}_k$ ($k = 0, 1, \dots, N_h - 1$) is linear and time-invariant, and the PR target $\mathbf{G}_k$ is known.

The 2-D MMSE equalizer design for TwoDOS was first proposed in [13]. However, the derivation in [13] assumes the input data $\mathbf{a}(n)$ as uncoded and the channel noise $\boldsymbol{\theta}(n)$ as 2-D white. In this section, we present a somewhat generalized approach, which can also be used when the channel contains pit-size variations.

From Fig. 2, we get the error vector $\boldsymbol{\varepsilon}(n)$ as

$$\begin{aligned} \boldsymbol{\varepsilon}(n) &= \mathbf{x}(n) - \mathbf{d}(n) \\ &= \sum_{k=0}^{N_w-1} \mathbf{W}_k \mathbf{z}(n-k) - \sum_{k=0}^{N_g-1} \mathbf{G}_k \mathbf{a}(n-k) \\ &= \mathbf{W}^T \mathbf{z}(n) - \mathbf{G}^T \mathbf{a}(n-m_0) \end{aligned} \quad (13)$$

where $\mathbf{z}(n) = [\mathbf{z}^T(n), \mathbf{z}^T(n-1), \dots, \mathbf{z}^T(n-N_w+1)]^T$, $\mathbf{a}(n) = [\mathbf{a}^T(n), \mathbf{a}^T(n-1), \dots, \mathbf{a}^T(n-N_g+1)]^T$, $\mathbf{W} = [\mathbf{W}_0, \mathbf{W}_1, \dots, \mathbf{W}_{N_w-1}]^T$, and $\mathbf{G} = [\mathbf{G}_0, \mathbf{G}_1, \dots, \mathbf{G}_{N_g-1}]^T$. Then, the total mean square error (MSE) at the equalizer output can be expressed as

$$\begin{aligned} J(\mathbf{W}) &= E[\boldsymbol{\varepsilon}^T(n)\boldsymbol{\varepsilon}(n)] \\ &= \text{trace}(\mathbf{W}^T \mathbf{R}_z \mathbf{W} - 2\mathbf{W}^T \mathbf{R}_{za} \mathbf{G} + \mathbf{G}^T \mathbf{R}_{aa} \mathbf{G}) \end{aligned} \quad (14)$$

where $\mathbf{R}_z = E[\mathbf{z}(n)\mathbf{z}^T(n)]$, $\mathbf{R}_{za} = E[\mathbf{z}(n)\mathbf{a}^T(n-m_0)]$, and $\mathbf{R}_{aa} = E[\mathbf{a}(n-m_0)\mathbf{a}^T(n-m_0)]$. By minimizing the MSE, the optimum taps of the equalizer are obtained as

$$\mathbf{W}_{\text{opt}} = \mathbf{R}_z^{-1} \mathbf{R}_{za} \mathbf{G} \quad (15)$$

and the MMSE becomes

$$\begin{aligned} J_{\min}(\mathbf{W}_{\text{opt}}) &= \text{trace} \left[\mathbf{G}^T \mathbf{R}_{aa} \mathbf{G} - (\mathbf{R}_{za} \mathbf{G})^T \mathbf{R}_z^{-1} (\mathbf{R}_{za} \mathbf{G}) \right]. \end{aligned} \quad (16)$$

We now derive the expressions for $\mathbf{R}_z$, $\mathbf{R}_{za}$, and $\mathbf{R}_{aa}$. Here, $\mathbf{R}_z$ is a $N_w N_r \times N_w N_r$ autocorrelation matrix of the equalizer input and its (k, l) th submatrix ($k, l = 0, 1, \dots, N_w - 1$) is given by $\mathbf{R}_{kl}^z = E[\mathbf{z}(n-k)\mathbf{z}^T(n-l)]$. Since $\mathbf{a}(n)$ and $\boldsymbol{\theta}(n)$ are mutually uncorrelated and are of zero mean, using (9) we get

$$\mathbf{R}_{kl}^z = \mathbf{H}^T \mathbf{R}_{kl}^a \mathbf{H} + \mathbf{R}_{kl}^\theta \quad (17)$$

where $\mathbf{R}_{kl}^a = E[\mathbf{a}(n-k)\mathbf{a}^T(n-l)]$ is a $N_h N_r \times N_h N_r$ autocorrelation matrix of the input data, $\mathbf{a}(n) = [\mathbf{a}^T(n), \mathbf{a}^T(n-1), \dots, \mathbf{a}^T(n-N_h+1)]^T$, and $\mathbf{R}_{kl}^\theta = E[\boldsymbol{\theta}(n-k)\boldsymbol{\theta}^T(n-l)]$ is a $N_r \times N_r$ autocorrelation matrix of the noise. The (k', l') th submatrix of $\mathbf{R}_{kl}^a$ is given by

$$\mathbf{R}_{kl,k'l'}^a = E[\mathbf{a}(n-k-k')\mathbf{a}^T(n-l-l')] \quad (18)$$

for $k', l' = 0, 1, \dots, N_h - 1$, and (i, j) th elements of $\mathbf{R}_{kl,k'l'}^a$ and $\mathbf{R}_{kl}^\theta$ are given by

$$\mathbf{R}_{kl,k'l'}^a(i, j) = E[a_i(n-k-k')a_j(n-l-l')] \quad (19)$$

$$\mathbf{R}_{kl}^\theta(i, j) = E[\theta_i(n-k)\theta_j(n-l)] \quad (20)$$

respectively, for $i, j = 1, 2, \dots, N_r$.

The matrix $\mathbf{R}_{za}$ is a $N_w N_r \times N_g N_r$ cross-correlation matrix between the channel output and input data, and its (k, l) th submatrix ($k = 0, 1, \dots, N_w - 1; l = 0, 1, \dots, N_g - 1$) is given by $\mathbf{R}_{kl}^{za} = E[\mathbf{z}(n-k)\mathbf{a}^T(n-m_0-l)]$. Then, as before, we get

$$\mathbf{R}_{kl}^{za} = \mathbf{H}^T \tilde{\mathbf{R}}_{kl}^a \quad (21)$$

where $\tilde{\mathbf{R}}_{kl}^a = E[\mathbf{a}(n-k)\mathbf{a}^T(n-m_0-l)]$ is a $N_h N_r \times N_r$ autocorrelation matrix of the input data. Its (k') th $N_r \times N_r$ submatrix is given by

$$\tilde{\mathbf{R}}_{kl,k'}^a = E[\mathbf{a}(n-k-k')\mathbf{a}^T(n-m_0-l)] \quad (22)$$

for $k' = 0, 1, \dots, N_h - 1$, and (i, j) th element of $\tilde{\mathbf{R}}_{kl,k'}^a$ is given by

$$\tilde{\mathbf{R}}_{kl,k'}^a(i, j) = E[a_i(n-k-k')a_j(n-m_0-l)] \quad (23)$$

for $i, j = 1, 2, \dots, N_r$.

Finally, $\mathbf{R}_{aa}$ is a $N_g N_r \times N_g N_r$ autocorrelation matrix of input data. As before, its (k, l) th submatrix ($k, l = 0, 1, \dots, N_g - 1$) is given by $\mathbf{R}_{kl}^{aa} = E[\mathbf{a}(n-m_0-k)\mathbf{a}^T(n-m_0-l)]$, and (i, j) th element of $\mathbf{R}_{kl}^{aa}$ is given by

$$\mathbf{R}_{kl}^{aa}(i, j) = E[a_i(n-m_0-k)a_j(n-m_0-l)] \quad (24)$$

for $i, j = 1, 2, \dots, N_r$.

Special Cases:

- 1) **Input data and noise are 2-D wide-sense stationary.**

In this case, (19), (20), (23), and (24) can be simplified as

$$\mathbf{R}_{kl,k'l'}^a(i, j) = r_a(l-k+l'-k')(i-j) \quad (25)$$

$$\mathbf{R}_{kl}^\theta(i, j) = r_\theta(l-k)(i-j) \quad (26)$$

$$\tilde{\mathbf{R}}_{kl,k'}^a(i, j) = r_a(l-k-k'+m_0)(i-j) \quad (27)$$

$$\mathbf{R}_{kl}^{aa}(i, j) = r_a(l-k)(i-j) \quad (28)$$

respectively, where $r_a(k)(i)$ and $r_\theta(k)(i)$ are defined as $E[a_{i+j}(n+k)a_j(n)]$ and $E[\theta_{i+j}(n+k)\theta_j(n)]$, respectively, for any n and j .

- 2) **The noise is 2-D white with variance σ^2 and the input data bits are uncoded.**

This is the simplest case. In this case, (19), (20), (23), and (24) can be simplified as

$$\mathbf{R}_{kl,k'l'}^a(i, j) = \delta_{l-k+l'-k'}\delta_{i-j} \quad (29)$$

$$\mathbf{R}_{kl}^\theta(i, j) = \sigma^2\delta_{l-k}\delta_{i-j} \quad (30)$$

$$\tilde{\mathbf{R}}_{kl,k'}^a(i, j) = \delta_{l-k-k'+m_0}\delta_{i-j} \quad (31)$$

$$\mathbf{R}_{kl}^{aa}(i, j) = \delta_{l-k}\delta_{i-j} \quad (32)$$

respectively, where $\delta_k = 1$ when $k = 0$ and $\delta_k = 0$ when $k \neq 0$. Substituting (29) and (30) in (17), we get

$$\underline{\mathbf{R}}_{kl}^z = \begin{cases} (\mathbf{H} * \mathbf{H}_-^T)_{l-k} + \sigma^2 \mathbf{I}_{N_r}, & \text{if } k = l \\ (\mathbf{H} * \mathbf{H}_-^T)_{l-k}, & \text{else} \end{cases} \quad (33)$$

where $\mathbf{H}_-$ is the mirror filter of $\mathbf{H}$, i.e., $(\mathbf{H}_-)_k = \mathbf{H}_{-k}$, $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix and “*” refers to 2-D convolution given by $(\mathbf{A} * \mathbf{B})_k = \sum_n \mathbf{A}_n \mathbf{B}_{k-n}$. Using (21), we get the k th submatrix of $\underline{\mathbf{R}}_{za} \underline{\mathbf{G}}$ as

$$(\underline{\mathbf{R}}_{za} \underline{\mathbf{G}})_k = \sum_{k'=0}^{N_h-1} \sum_{l=0}^{N_g-1} \mathbf{H}_{k'} \tilde{\mathbf{R}}_{kl,k'}^a \mathbf{G}_l^T. \quad (34)$$

Substituting (31) in (34) and letting $l = k + k' - m_0$, we obtain

$$\begin{aligned} \underline{\mathbf{R}}_{za} \underline{\mathbf{G}} &= \sum_{k'=0}^{N_h-1} \begin{bmatrix} \mathbf{H}_{k'} \mathbf{G}_{k'-m_0}^T \\ \mathbf{H}_{k'} \mathbf{G}_{k'-m_0+1}^T \\ \vdots \\ \mathbf{H}_{k'} \mathbf{G}_{k'-m_0+N_w-1}^T \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{H}_- * \mathbf{G}^T)_{-m_0} \\ (\mathbf{H}_- * \mathbf{G}^T)_{1-m_0} \\ \vdots \\ (\mathbf{H}_- * \mathbf{G}^T)_{N_w-1-m_0} \end{bmatrix}. \end{aligned} \quad (35)$$

We can compute the optimum equalizer taps by using (33) and (35) in (15). Since $\underline{\mathbf{R}}_{aa}$ is a $N_g N_r \times N_g N_r$ identity matrix in this simplest case, by using (16) we get the corresponding MMSE as

$$\begin{aligned} J_{\min}(\mathbf{W}) \\ = \text{trace} \left[\underline{\mathbf{G}}^T \underline{\mathbf{G}} - (\underline{\mathbf{R}}_{za} \underline{\mathbf{G}})^T \underline{\mathbf{R}}_z^{-T} (\underline{\mathbf{R}}_{za} \underline{\mathbf{G}}) \right]. \end{aligned} \quad (36)$$

The expressions (33)–(36) corresponding to the simplest case can be found in [13].

3) Channel contains pit-size variations.

The Fourier transform of the channel symbol response in the presence of pit-size variations is given by [using (11) and (3)]

$$\begin{aligned} \mathbf{H}_{sf,t}(\phi, \rho) \\ = 2\pi(1 + \Delta_i) R \frac{J_1(\rho(1 + \Delta_i)R)}{\rho} \mathbf{F}_{sf}(\phi, \rho) \end{aligned} \quad (37)$$

where $\Delta_i = \Delta_b + \Delta_t$ with Δ_b and Δ_t are the normalized domain bloom and random pit-size variation for the bit under consideration, respectively. If Δ_b and Δ_t are sufficiently small and the recording density is sufficiently high, the above transfer function can be approximated as [18]

$$\begin{aligned} \mathbf{H}_{sf,t}(\phi, \rho) &\approx 2\pi(1 + \Delta_i) R \frac{J_1(\rho R)}{\rho} \mathbf{F}_{sf}(\phi, \rho) \\ &= (1 + \Delta_i) \mathbf{H}_{sf}(\phi, \rho) \end{aligned} \quad (38)$$

where $\mathbf{H}_{sf}(\phi, \rho)$ is the time-invariant response given in (3). Using this approximation, we can write (12) as

$$\mathbf{z}(n) = \sum_{k=0}^{N_h-1} \mathbf{H}_k \tilde{\mathbf{a}}(n-k) + \boldsymbol{\theta}(n) \quad (39)$$

where $\mathbf{H}_k(k = 0, 1, \dots, N_h - 1)$ corresponds to the time-invariant symbol response matrices and $\tilde{\mathbf{a}}(n) = [\tilde{a}_1(n), \tilde{a}_2(n), \dots, \tilde{a}_{N_r}(n)]^T$ is the modified input data vector where

$$\tilde{a}_i(n) = \begin{cases} (1 + \Delta_i(n))a_i(n), & \text{if } a_i(n) = +1 \\ a_i(n), & \text{if } a_i(n) = -1 \end{cases} \quad (40)$$

for $i = 1, 2, \dots, N_r$, where $\Delta_i(n)$ corresponds to the effective pit-size variation experienced by the bit $a_i(n)$.

It is now easy to see that Case 1 discussed above (i.e., wide-sense stationary data and noise) can be used to design the optimum equalizer for the case with random pit-size variation since the modified data $\tilde{a}_k(n)$ and white electronics noise $\theta_k(n)$ are mutually uncorrelated and wide-sense stationary. In this case, (19), (20), (23), and (24) can be simplified as

$$\underline{\mathbf{R}}_{kl,k'l'}^a(i, j) = r_{\tilde{a}}(l - k + l' - k')(i - j) \quad (41)$$

$$\mathbf{R}_{kl}^{\theta}(i, j) = r_{\theta}(l - k)(i - j) \quad (42)$$

$$\tilde{\mathbf{R}}_{kl,k'}^a(i, j) = r_{\tilde{a}}(l - k - k' + m_0)(i - j) \quad (43)$$

$$\underline{\mathbf{R}}_{kl}^{aa}(i, j) = r_{\tilde{a}}(l - k)(i - j) \quad (44)$$

respectively, where $r_{\tilde{a}}(k)(i)$ and $r_{\theta}(k)(i)$ are defined as $E[\tilde{a}_{i+j}(n+k)\tilde{a}_j(n)]$ and $E[\theta_{i+j}(n+k)\theta_j(n)]$, respectively, for any n and j . Let $\Delta_j^t(n)$ represent the random pit-size variation experienced by the bit $a_j(n)$. Noting that $\Delta_j^t(n)$ is statistically independent of $a_j(n)$ and $\mathbf{a}(n)$ is of zero mean, it can be shown that

$$\begin{aligned} r_{\tilde{a}}(k)(i) &= \frac{[1 + r_a(k)(i)][\Delta_b^2 + r_{\Delta_t}(k)(i)]}{4} \\ &\quad + (1 + \Delta_b)r_a(k)(i) \end{aligned} \quad (45)$$

where $r_{\Delta_t}(k)(i)$ is defined as $E[\Delta_{i+j}^t(n+k)\Delta_j^t(n)]$, for any n and j . If $\Delta_b = 0$, the noise is 2-D white with variance σ^2 and the input data bits are uncoded, then the expressions of correlations given in (19), (20), (23), and (24) can be simplified for this case as

$$\underline{\mathbf{R}}_{kl,k'l'}^a(i, j) = (1 + \sigma_t^2/2) \delta_{l-k+l'-k'} \delta_{i-j} \quad (46)$$

$$\mathbf{R}_{kl}^{\theta}(i, j) = \sigma^2 \delta_{l-k} \delta_{i-j} \quad (47)$$

$$\tilde{\mathbf{R}}_{kl,k'}^a(i, j) = (1 + \sigma_t^2/2) \delta_{l-k-k'+m_0} \delta_{i-j} \quad (48)$$

$$\underline{\mathbf{R}}_{kl}^{aa}(i, j) = (1 + \sigma_t^2/2) \delta_{l-k} \delta_{i-j} \quad (49)$$

respectively, where σ_t^2 is the variance of random pit-size variation Δ_t . The factor $1/2$ accounts for the probability of pit bit “+1” where random pit-size variation may occur.

IV. PERFORMANCE OF VITERBI DETECTOR FOR TWODOS

To deal with the serious 2-D ISI present in TwoDOS read-back signal and to ensure reliable data recovery, powerful detection approaches such as the Viterbi algorithm become necessary [11]. The BER performance of 2-D VD may be used as a criterion to evaluate different PR targets. Even though a BER expression for 2-D VD has been reported in [13], the variables used in the expression correspond to 1-D VD. In this section,

we briefly give the derivation of BER of 2-D VD using the 2-D definitions and notations of error event, target, etc.

As a starting point for determining the BER, we define a 2-D error event vector $\underline{\mathbf{e}}$. Let $\hat{\mathbf{a}}(n)$ denote the detected data vector corresponding to $\mathbf{a}(n)$. Further, define $\underline{\mathbf{a}} = [\mathbf{a}^T(n), \mathbf{a}^T(n+1), \dots, \mathbf{a}^T(n+N_e-1)]^T$, $\hat{\underline{\mathbf{a}}} = [\hat{\mathbf{a}}^T(n), \hat{\mathbf{a}}^T(n+1), \dots, \hat{\mathbf{a}}^T(n+N_e-1)]^T$, $\underline{\mathbf{e}} = (\underline{\mathbf{a}} - \hat{\underline{\mathbf{a}}})/2 = [\mathbf{e}^T(n), \mathbf{e}^T(n+1), \dots, \mathbf{e}^T(n+N_e-1)]^T$ with $\mathbf{e}(n) = (\mathbf{a}(n) - \hat{\mathbf{a}}(n))/2$. Then, $\underline{\mathbf{e}}$ is a 2-D error event of length N_e if:

- 1) $\mathbf{e}(n) \neq 0$ and $\mathbf{e}(n+N_e-1) \neq 0$;
- 2) the length of strings of zero vectors in $\underline{\mathbf{e}}$ does not exceed N_f ;
- 3) $\hat{\mathbf{a}}(n+i) = \mathbf{a}(n+i)$ for $-N_f \leq i < 0$ and $N_e \leq i \leq N_e + N_f - 1$.

Here, N_f is called the “error-free interval” and $N_f \geq N_g$. Let $W(\underline{\mathbf{e}})$ be the number of error bits in the error event $\underline{\mathbf{e}}$. Then, the BER of 2-D VD is given by

$$P_e = \sum_{\underline{\mathbf{e}} \in E_s} \sum_{\underline{\mathbf{a}} \in S_e} W(\underline{\mathbf{e}}) P(\underline{\mathbf{a}}) P(\text{error} | \underline{\mathbf{a}}) \quad (50)$$

where E_s is the set of all possible error events, S_e is the set of all possible data patterns that support the error event $\underline{\mathbf{e}}$, and $P(\text{error} | \underline{\mathbf{a}})$ is the conditional probability that the VD detects $\hat{\underline{\mathbf{a}}}$ as the recorded data when the true recorded data is $\underline{\mathbf{a}}$.

According to the principle of VD, $P(\text{error} | \underline{\mathbf{a}})$ can be upper-bounded by the probability that the metric corresponding to path $\hat{\underline{\mathbf{a}}}$ is smaller than that corresponding to path $\underline{\mathbf{a}}$. That is

$$\sum_i [\mathbf{x}(n+i) - (\mathbf{G} * \hat{\mathbf{a}})_{n+i}]^T [\mathbf{x}(n+i) - (\mathbf{G} * \hat{\mathbf{a}})_{n+i}] < \sum_i [\mathbf{x}(n+i) - (\mathbf{G} * \mathbf{a})_{n+i}]^T [\mathbf{x}(n+i) - (\mathbf{G} * \mathbf{a})_{n+i}] \quad (51)$$

where $\mathbf{x}(n)$ is the equalizer output which can be expressed as (see Fig. 2)

$$\begin{aligned} \mathbf{x}(n) &= \sum_i \mathbf{W}_i \mathbf{z}(n-i) \\ &= [(\mathbf{G} + \mathbf{M}) * \mathbf{a}]_n + (\mathbf{W} * \boldsymbol{\theta})_n \end{aligned} \quad (52)$$

where $\mathbf{M}_0, \mathbf{M}_1, \dots, \mathbf{M}_{N_h+N_w-2}$ represent the taps of the 2-D residual ISI channel, i.e., $\mathbf{M}_n = (\mathbf{W} * \mathbf{H})_n - \mathbf{G}_n$, and $\boldsymbol{\theta}(n)$ is a 2-D white Gaussian noise vector with variance σ^2 . Using (52), we can express (51) as

$$d^2(\underline{\mathbf{e}}) + \sum_{i=0}^{N_e+N_g-2} \tilde{\mathbf{e}}^T(n+i) [(\mathbf{M} * \mathbf{a})_{n+i} + (\mathbf{W} * \boldsymbol{\theta})_{n+i}] < 0 \quad (53)$$

where $d^2(\underline{\mathbf{e}}) = \sum_{i=0}^{N_e+N_g-2} [(\mathbf{G} * \mathbf{e})_{n+i}^T (\mathbf{G} * \mathbf{e})_{n+i}]$ and $\tilde{\mathbf{e}}(n) = (\mathbf{G} * \mathbf{e})_n$. For given $\underline{\mathbf{a}}$ and $\underline{\mathbf{e}}$, the left-hand side of the inequality in (53) is a Gaussian random variable with mean

$$m_u = d^2(\underline{\mathbf{e}}) + \sum_{i=-(N_h+N_w-2)}^{N_e+N_g-2} (\mathbf{M}_i^T * \tilde{\mathbf{e}})_{n+i}^T \mathbf{a}(n+i) \quad (54)$$

and variance

$$\sigma_u^2 = \sigma^2 \sum_{i=-(N_w-1)}^{N_e+N_g-2} (\mathbf{W}_i^T * \tilde{\mathbf{e}})_{n+i}^T (\mathbf{W}_i^T * \tilde{\mathbf{e}})_{n+i}. \quad (55)$$

Thus, we get

$$P(\text{error} | \underline{\mathbf{a}}) \leq Q\left(\frac{m_u}{\sigma_u}\right) \quad (56)$$

where $Q(\alpha) = (1/\sqrt{2\pi}) \int_{\alpha}^{\infty} e^{-x^2/2} dx$ is the tail probability of Gaussian distribution. By substituting (56) into (50), we get the BER of 2-D VD as

$$P_e \leq \sum_{\underline{\mathbf{e}} \in E_s} \sum_{\underline{\mathbf{a}} \in S_e} W(\underline{\mathbf{e}}) P(\underline{\mathbf{a}}) Q\left(\frac{m_u}{\sigma_u}\right). \quad (57)$$

Based on the BER expression given above, we may define an effective detection signal-to-noise ratio (SNR_{eff}) for a given error event $\underline{\mathbf{e}}$ and target $\underline{\mathbf{G}}$ as

$$\text{SNR}_{\text{eff}} = \left(\frac{d^2(\underline{\mathbf{e}}) + \sum_n (\mathbf{M}_n^T * \tilde{\mathbf{e}})_n^T \mathbf{a}(n)}{\sigma \sqrt{\sum_n (\mathbf{W}_n^T * \tilde{\mathbf{e}})_n^T (\mathbf{W}_n^T * \tilde{\mathbf{e}})_n}} \right)^2. \quad (58)$$

The error event $\underline{\mathbf{e}}_d$ that minimizes SNR_{eff} for a given target $\underline{\mathbf{G}}$ is called the dominant error event since this error event has the highest probability of occurrence. Consequently, we could use SNR_{eff} as a criterion to determine the target since it is closely related to BER and can be computed very easily compared to BER. The optimum target should maximize the SNR_{eff} corresponding to the dominant error event. In other words, the design of optimum target is a max-min optimization problem

$$\underline{\mathbf{G}}_{\text{opt}} = \arg\{\max_{\underline{\mathbf{G}}} \min_{\underline{\mathbf{e}}} \text{SNR}_{\text{eff}}\}. \quad (59)$$

As seen from (58), the presence of the residual ISI makes this optimization data-dependent in nature. Therefore, the usual practice is to assume that the residual is very small and to add it as part of the Gaussian noise $(\mathbf{W} * \boldsymbol{\theta})_n$. Further, the effect of noise correlation on the detection performance can be inferred by examining the denominator of SNR_{eff} .

For the case when the channel contains pit-size variations, (53) is revised as

$$\begin{aligned} &\sum_{i=0}^{N_e+N_g-2} \tilde{\mathbf{e}}^T(n+i) [(\mathbf{M} * \mathbf{a})_{n+i} + (\mathbf{W} * \boldsymbol{\theta})_{n+i}] \\ &< -d^2(\underline{\mathbf{e}}) - \sum_{i=0}^{N_e+N_g-2} \tilde{\mathbf{e}}^T(n+i) [\tilde{\mathbf{G}} * (\tilde{\mathbf{a}} - \mathbf{a})]_{n+i} \end{aligned} \quad (60)$$

where $\tilde{\mathbf{a}}(n)$ is the modified input data vector defined in (40) and $\tilde{\mathbf{G}}_n = \mathbf{G}_n + \mathbf{M}_n$. Under the assumption that random pit-size variation and thermal noise $\boldsymbol{\theta}_n$ are uncorrelated and jointly Gaussian distributed, the left-hand side of the inequality in (60) is still Gaussian with mean

$$\begin{aligned} m_{ur} &= d^2(\underline{\mathbf{e}}) + \sum_i (\mathbf{M}_i^T * \tilde{\mathbf{e}})_{n+i}^T \mathbf{a}(n+i) \\ &\quad + \sum_i \Delta_b (\tilde{\mathbf{G}}_i^T * \tilde{\mathbf{e}})_{n+i}^T \mathbf{u}(n+i) \end{aligned} \quad (61)$$

and variance

$$\sigma_{ur}^2 = \sigma^2 \sum_i (\mathbf{W}_-^T * \tilde{\mathbf{e}})_{n+i}^T (\mathbf{W}_-^T * \tilde{\mathbf{e}})_{n+i} + \sigma_t^2 \sum_i [\tilde{\mathbf{p}}(n+i)]^T [\tilde{\mathbf{p}}(n+i)]. \quad (62)$$

where $\tilde{\mathbf{p}}(n) = (\tilde{\mathbf{G}}_-^T * \tilde{\mathbf{e}})_n \cdot \mathbf{u}(n)$, Δ_b reflects the degree of domain bloom, σ_t^2 is the variance of random pit-size variation, $\mathbf{u}(n) = (1 + \mathbf{a}(n))/2$, and “ $\cdot$ ” indicates element-wise multiplication of vectors. Then, (58) becomes

$$\text{SNR}_{\text{eff}} = \left(\frac{m_{ur}}{\sigma_{ur}} \right)^2. \quad (63)$$

From (63), it can be seen that the presence of random pit-size variation will degrade the performance since it increases the denominator of (63). However, its destructive effect may be reduced if we consider it in the target design that will be covered in the next section.

V. TARGET DESIGN FOR TWODOS

Several techniques have been reported in the literature for designing the PR target for 1-D data storage systems [19]–[21]. For the sake of convenience, the cost function used for the design is mostly the MSE (at equalizer output) rather than the SNR_{eff} given in (58). The design of equalizer and target by minimizing MSE subject to monic constraint (i.e., first tap of the target is set to unity) has been reported to result in near-optimum performance in 1-D system [20]. The use of a constraint is necessary to avoid the trivial all-zero solutions for equalizer and target. In this section, we investigate the type of target constraint required to achieve good performance from 2-D VD. Since the PR target is in the form of a sequence of matrices in the 2-D case (as compared to a vector target in 1-D case), solving the constrained minimization is quite difficult. Therefore, we introduce a novel method which transforms the 2-D target design problem from 2-D into 1-D form. Using this method, targets with different constraints can be designed with relatively less effort as in the 1-D case.

Without any loss of generality, we define a vector $\mathbf{g}$ which transforms the target $\underline{\mathbf{G}} = [\mathbf{G}_0, \mathbf{G}_1, \dots, \mathbf{G}_{N_g-1}]^T$ into a 1-D vector target $\mathbf{g} = [g_0, g_1, \dots, g_{N_v-1}]^T$ where g_i ($i = 0, 1, \dots, N_v - 1$) are the distinct nonzero elements in $\underline{\mathbf{G}}$. It should be noted that imposing some symmetry constraints on the taps of the target can decrease the number of parameters to optimize and thus reduce the complexity of target design process. Replacing $\underline{\mathbf{G}}$ with $\mathbf{g}$ in (14), the MSE for the 2-D case becomes

$$J(\underline{\mathbf{W}}) = \text{trace}[\underline{\mathbf{W}}^T \underline{\mathbf{R}}_z \underline{\mathbf{W}} - 2(\underline{\mathbf{W}}^T)'(\underline{\mathbf{R}}_{za})' \mathbf{g} + \mathbf{g}^T (\underline{\mathbf{R}}_{aa})' \mathbf{g}] \quad (64)$$

where $(\underline{\mathbf{R}}_{za})'$, $(\underline{\mathbf{W}}^T)'$, and $(\underline{\mathbf{R}}_{aa})'$ represent rearranged forms of $\underline{\mathbf{R}}_{za}$, $\underline{\mathbf{W}}^T$, and $\underline{\mathbf{R}}_{aa}$, respectively. More specifically, $(\underline{\mathbf{W}}^T)'$ is a vector given by

$$(\underline{\mathbf{W}}^T)' = [\text{1st row of } \underline{\mathbf{W}}^T \quad \dots \quad (N_r)\text{th row of } \underline{\mathbf{W}}^T]. \quad (65)$$

Then, we can obtain $(\underline{\mathbf{R}}_{za})'$ as

$$[(\underline{\mathbf{R}}_{za})']^T = \begin{bmatrix} \begin{bmatrix} \text{1st column of } \underline{\mathbf{R}}'_{za,g_0} \\ \vdots \\ N_r\text{th column of } \underline{\mathbf{R}}'_{za,g_0} \end{bmatrix}^T \\ \vdots \\ \begin{bmatrix} \text{1st column of } \underline{\mathbf{R}}'_{za,g_{N_v-1}} \\ \vdots \\ N_r\text{th column of } \underline{\mathbf{R}}'_{za,g_{N_v-1}} \end{bmatrix}^T \end{bmatrix}^T \quad (66)$$

where $\text{trace}(\underline{\mathbf{W}}^T \underline{\mathbf{R}}_{za} \underline{\mathbf{G}}) = (\underline{\mathbf{W}}^T)'(\underline{\mathbf{R}}_{za})' \mathbf{g}$ and $\underline{\mathbf{R}}'_{za,g_k}$ is the matrix associated with the coefficient g_k such that

$$\underline{\mathbf{R}}_{za} \underline{\mathbf{G}} = \underline{\mathbf{R}}'_{za,g_0} g_0 + \underline{\mathbf{R}}'_{za,g_1} g_1 + \dots + \underline{\mathbf{R}}'_{za,g_{N_v-1}} g_{N_v-1}. \quad (67)$$

Similarly, $(\underline{\mathbf{R}}_{aa})'$ is a $N_v \times N_v$ matrix that can be obtained such that $\text{trace}(\underline{\mathbf{G}}^T \underline{\mathbf{R}}_{aa} \underline{\mathbf{G}}) = \mathbf{g}^T (\underline{\mathbf{R}}_{aa})' \mathbf{g}$.

To obtain the solutions subject to different constraints, we formulate a cost function using Lagrange multiplier method [23]:

$$J(\underline{\mathbf{W}}, \mathbf{g}) = \text{trace}[\underline{\mathbf{W}}^T \underline{\mathbf{R}}_z \underline{\mathbf{W}} - 2(\underline{\mathbf{W}}^T)'(\underline{\mathbf{R}}_{za})' \mathbf{g} + \mathbf{g}^T (\underline{\mathbf{R}}_{aa})' \mathbf{g}] - \lambda c(\mathbf{g}) \quad (68)$$

where $c(\mathbf{g})$ represents the constraint function and λ is the Lagrange multiplier. Taking the gradient of $J(\underline{\mathbf{W}}, \mathbf{g})$ with respect to $\mathbf{g}$, we get

$$\nabla_{\mathbf{g}} J(\underline{\mathbf{W}}, \mathbf{g}) = -2[(\underline{\mathbf{R}}_{za})']^T [(\underline{\mathbf{W}}^T)]^T + 2(\underline{\mathbf{R}}_{aa})' \mathbf{g} - \lambda \nabla_{\mathbf{g}} c(\mathbf{g}). \quad (69)$$

From (15) and (67), the optimum equalizer matrix can be expressed as

$$\underline{\mathbf{W}}_{\text{opt}} = \underline{\mathbf{R}}_z^{-1} \left(\underline{\mathbf{R}}'_{za,g_0} g_0 + \underline{\mathbf{R}}'_{za,g_1} g_1 + \dots + \underline{\mathbf{R}}'_{za,g_{N_v-1}} g_{N_v-1} \right). \quad (70)$$

Then the transpose of the rearranged transposed optimum equalizer matrix can be written as

$$\left[(\underline{\mathbf{W}}_{\text{opt}}^T)' \right]^T = (\underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za})' \mathbf{g} \quad (71)$$

where $(\underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za})'$ can be obtained as

$$\left[(\underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za})' \right]^T = \begin{bmatrix} \begin{bmatrix} \text{1st column of } \underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za,g_0} \\ \vdots \\ N_r\text{th column of } \underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za,g_0} \end{bmatrix}^T \\ \vdots \\ \begin{bmatrix} \text{1st column of } \underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za,g_{N_v-1}} \\ \vdots \\ N_r\text{th column of } \underline{\mathbf{R}}_z^{-1} \underline{\mathbf{R}}'_{za,g_{N_v-1}} \end{bmatrix}^T \end{bmatrix}^T. \quad (72)$$

Substituting (71) in (69), we get

$$\nabla_{\mathbf{g}} J(\mathbf{W}, \mathbf{g}) = -2[(\mathbf{R}_{za})']^T (\mathbf{R}_z^{-1} \mathbf{R}'_{za})' \mathbf{g} + 2(\mathbf{R}_{aa})' \mathbf{g} - \lambda \nabla_{\mathbf{g}} c(\mathbf{g}). \quad (73)$$

Thus, the crux of this novel method is appropriate rearrangement of the matrices. An example is given in [22] to illustrate the method for $N_g = 3$ and $N_r = 3$.

In view of the hexagonal bit-cells and the radially symmetric symbol response, we may choose a target that is also symmetric, i.e., we assume that all the values of the cross interference to the nearest spot are the same. Further, we assume the interference from beyond adjacent bits to be zero. Then, the target can be written as ($N_g = 3, N_r = 5$)

$$\begin{aligned} \mathbf{G}_0 &= \begin{bmatrix} g_0 & 0 & 0 & 0 & 0 \\ g_0 & g_0 & 0 & 0 & 0 \\ 0 & g_0 & g_0 & 0 & 0 \\ 0 & 0 & g_0 & g_0 & 0 \\ 0 & 0 & 0 & g_0 & g_0 \end{bmatrix} \\ \mathbf{G}_1 &= \begin{bmatrix} g_1 & g_0 & 0 & 0 & 0 \\ g_0 & g_1 & g_0 & 0 & 0 \\ 0 & g_0 & g_1 & g_0 & 0 \\ 0 & 0 & g_0 & g_1 & g_0 \\ 0 & 0 & 0 & g_0 & g_1 \end{bmatrix} \\ \mathbf{G}_2 &= \begin{bmatrix} g_0 & g_0 & 0 & 0 & 0 \\ 0 & g_0 & g_0 & 0 & 0 \\ 0 & 0 & g_0 & g_0 & 0 \\ 0 & 0 & 0 & g_0 & g_0 \\ 0 & 0 & 0 & 0 & g_0 \end{bmatrix} \end{aligned} \quad (74)$$

and the 1-D form of the target matrix becomes $\mathbf{g} = [g_0 \ g_1]^T$, where g_0 is value of the cross interference to the nearest spot, and g_1 is value of the self interference of the central spot. Even though we are considering symmetric targets here for illustration, the target design approach presented here is applicable for any general target.

We design the targets for three different constraints. The first constraint used is $g_0 = 1$, which we call “1-D monic constraint.” The constraint is widely used in 1-D systems and is known to have good noise-whitening properties. Secondly, we use the energy constraint. The rationale behind this constraint is as follows. For a single-bit error event $\mathbf{I}_e = [1 \ 0 \ \dots \ 0]^T$ of length $N_e = 1$, neglecting noise colorization and channel mis-equalization, the resulting minimum SNR_{eff} is given by

$$\text{SNR}_{\text{eff}} = \frac{\sum_k (\mathbf{G}_k \mathbf{I}_e)^T (\mathbf{G}_k \mathbf{I}_e)}{\sigma_{\min}^2} = \frac{4g_0^2 + g_1^2}{\sigma_{\min}^2} \quad (75)$$

where $\sigma_{\min}^2$ is the MMSE at the equalizer output. The “energy constraint” mentioned above refers to setting $4g_0^2 + g_1^2 = 1$ so that minimizing the MSE results in maximizing the SNR_{eff} . A similar argument for choosing a similar energy constraint in 1-D data storage system was made in [21]. The third constraint used is $g_1 = 1$, which we call “2-D monic constraint.” The rationale behind this target constraint is as follows. Let the target response for a single spot in the spatial frequency domain be radially symmetric, i.e., $\mathbf{G}_{\text{sf}}(\phi, \rho) = \tilde{\mathbf{G}}_{\text{sf}}(\rho)$. Then for the

noise to be 2-D white, $|\tilde{\mathbf{G}}_{\text{sf}}(\rho)|^2$ should be ideally of the form $\mu |\tilde{\mathbf{H}}_{\text{sf}}(\rho)|^2$, where μ is some constant of proportionality. Of all the targets with the same magnitude characteristics of $|\tilde{\mathbf{G}}_{\text{sf}}(\rho)|$, the one with the energy optimally concentrated near g_1 has the largest possible amplitude g_1 . Equivalently, for a constrained value $g_1 = 1$, the target with the energy optimally concentrated near g_1 will yield the smallest possible μ and therefore the smallest possible noise enhancement. Since we put symmetric constraint that appears in (74), the target has its energy almost concentrated near g_1 . Consequently, making the target with the energy optimally concentrated near g_1 will not burden the finite length equalizer much. So setting $g_1 = 1$ is a reasonable target constraint.

For the 1-D monic constraint, the constraint function is $c(\mathbf{g}) = 2(\mathbf{i}^T \mathbf{g} - 1)$, where $\mathbf{i} = [1 \ 0]^T$. Then, the optimum target can be obtained as [using (73)]

$$\mathbf{g} = \lambda \left[(\mathbf{R}_{aa})' - [(\mathbf{R}_{za})']^T (\mathbf{R}_z^{-1} \mathbf{R}'_{za})' \right]^{-1} \mathbf{i} \quad (76)$$

where

$$\lambda = \frac{1}{\mathbf{i}^T \left[(\mathbf{R}_{aa})' - [(\mathbf{R}_{za})']^T (\mathbf{R}_z^{-1} \mathbf{R}'_{za})' \right]^{-1} \mathbf{i}}. \quad (77)$$

And it can be shown that MMSE is equal to the Lagrange multiplier λ .

For the energy constraint, we have $c(\mathbf{g}) = \mathbf{g}^T (\mathbf{R}_{\text{ec}})' \mathbf{g} - 1$, where $(\mathbf{R}_{\text{ec}})'$ is such that $\sum_k (\mathbf{G}_k \mathbf{I}_e)^T (\mathbf{G}_k \mathbf{I}_e) = \mathbf{g}^T (\mathbf{R}_{\text{ec}})' \mathbf{g}$. Using this in (73) and setting the gradients zero, we obtain

$$\begin{aligned} & \{[(\mathbf{R}_{\text{ec}})']^{-1/2} (\mathbf{R}_{aza})' [(\mathbf{R}_{\text{ec}})']^{-1/2}\} [(\mathbf{R}_{\text{ec}})']^{1/2} \mathbf{g} \\ &= \lambda [(\mathbf{R}_{\text{ec}})']^{1/2} \mathbf{g} \end{aligned} \quad (78)$$

where $[(\mathbf{R}_{\text{ec}})']^{1/2}$ is such that $\{[(\mathbf{R}_{\text{ec}})']^{1/2}\}^T [(\mathbf{R}_{\text{ec}})']^{1/2} = (\mathbf{R}_{\text{ec}})'$ and $(\mathbf{R}_{aza})' = [(\mathbf{R}_{aa})' - [(\mathbf{R}_{za})']^T (\mathbf{R}_z^{-1} \mathbf{R}'_{za})']$. It can be shown that the MMSE is given by the minimum eigenvalue of the matrix $\{[(\mathbf{R}_{\text{ec}})']^{-1/2} (\mathbf{R}_{aza})' [(\mathbf{R}_{\text{ec}})']^{-1/2}\}$ and $[(\mathbf{R}_{\text{ec}})']^{1/2} \mathbf{g}$ is the corresponding eigenvector.

For the 2-D monic constraint, the solutions are identical to the 1-D monic constraint case except for $\mathbf{i} = [0 \ 1]^T$.

VI. SIMULATION RESULTS

Computer simulations are carried out for the TwoDOS system shown in Fig. 2 with channel length $N_h = 41$, equalizer length $N_w = 31$, target length $N_g = 3$, and $N_r = 5$ tracks per group. The target matrix $\mathbf{G}$ is set up using the symmetric constraint, and the coefficients are given by (74). The normalized optical cutoff frequency is set to $1/\sqrt{3}$ to avoid aliasing [5], [11]. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = 10 \log_{10} \left(\frac{\sum_{x,y} \check{H}_s^2(x,y)}{\sigma^2} \right) \quad (79)$$

where $\sum_{x,y} \check{H}_s^2(x,y)$ is the energy for a single spot with $\check{H}_s(x,y)$ being the symbol response in Cartesian coordinates, and σ^2 is the variance of the additive white

TABLE I
NOISE CORRELATION AT EQUALIZER OUTPUT FOR DIFFERENT TARGETS

SNR	Spatial difference along the track	0	1	2	3	4
32dB	Noise correlation for fixed PR target	1	0.2811	-0.1390	0.0548	-0.0083
	Noise correlation for 1D monic constrained target	1	0.2903	-0.1446	0.0492	-0.0038
	Noise correlation for 2D monic constrained target	1	0.2320	-0.1059	0.0714	-0.0266
	Noise correlation for energy constrained target	1	0.2607	-0.1259	0.0639	-0.0169
34dB	Noise correlation for fixed PR target	1	0.2595	-0.1365	0.0655	-0.0061
	Noise correlation for 1D monic constrained target	1	0.2545	-0.1306	0.0684	-0.0100
	Noise correlation for 2D monic constrained target	1	0.2094	-0.0811	0.0773	-0.0329
	Noise correlation for energy constrained target	1	0.2312	-0.1043	0.0761	-0.0239

Fig. 3. BER performance of 2-D VD for different target constraints.

Gaussian noise (AWGN) on each individual track (i.e., $E[\theta_i^2(n)] = \sigma^2, i = 1, 2, \dots, N_r$). In addition to AWGN, we also consider pit size variations for performance evaluation. We use four different PR targets for performance comparison: fixed PR target ($g_0 = 1, g_1 = 2$), 1-D monic constrained target ($g_0 = 1$), energy constrained target ($4g_0^2 + g_1^2 = 1$), and 2-D monic constrained target ($g_1 = 1$). BER is used as the performance measure. The equalizer and target are designed with the approaches described in Sections III and V for each SNR.

Fig. 3 shows the performance of 2-D VD for the four different targets. Observe that the 1-D monic constrained target performs worse than all the other targets when SNR is smaller than 33 dB, whereas the fixed PR [1 2] target becomes the worst for SNR > 33 dB. At high SNRs, residual ISI dominates the MSE. Hence, jointly designing the target and equalizer, rather than fixed target approach, is preferable at higher SNRs since noise is not significant at these SNRs. For this reason, for SNR > 33 dB, the 1-D monic constrained target outperforms the fixed PR target. Fig. 3 also shows that the energy constraint is a reasonable target constraint. More importantly, the target with 2-D monic constraint performs best among the four targets at all the SNRs, thus indicating that concentrating the energy around g_1 is a good choice for symmetric targets [e.g., see (74)].

To further understand the performance trends observed in Fig. 3, we examine the noise correlation at the VD input for the

TABLE II
NORMALIZED g_1 WITH RESPECT TO g_0 FOR DIFFERENT TARGET CONSTRAINTS

SNR	32	33	34	35
Fixed PR Target	2	2	2	2
1D monic constrained target	1.9468	1.9901	2.0310	2.0696
2D monic constrained target	2.2784	2.2828	2.2901	2.2997
Energy constrained target	2.1150	2.1411	2.1668	2.1919

different target designs, since the performance of VD is known to be influenced significantly by noise correlation. Moreover, the 2-D equalization could result in significant noise correlation. This problem becomes even more severe for shorter targets (e.g., $N_g = 3$). Furthermore, we know that the MMSE design approach focuses on minimizing the total noise power, and not noise correlation. The normalized noise correlation along the third track is shown in Table I for the four different targets at SNR = 32 dB and SNR = 34 dB. Similar trends in the correlation are observed for other tracks as well as in the cross-track direction. Of all the targets, we see that the 2-D monic constrained target results in least noise correlation at both SNRs, which is consistent with its best BER performance among the four targets. Further, comparing the noise correlation produced by the fixed PR target with the 1-D monic constrained target at SNR = 32 dB and SNR = 34 dB, we can roughly understand why the BER curves for these two targets cross each other around SNR = 33 dB in Fig. 3. Finally, the relative noise correlation obtained for the energy constraint target with respect to the other targets is consistent with the BER performance observed in Fig. 3.

Table II shows the values of g_1 (normalized by g_0) for the four targets at different SNRs. Comparing Table II with Fig. 3, it can be concluded that the target that has the smallest normalized g_1 results in the worst BER performance and vice versa. This observation reconfirms that the 2-D monic constraint is a reasonable target constraint since it causes the energy to be concentrated near g_1 , thus resulting in the largest normalized g_1 compared with the other targets. The 1-D monic constraint can be considered as causing the target with the energy to be concentrated near g_0 and as a result it minimizes the normalized g_1 . For this reason, the 1-D monic constraint is not a good target constraint for 2-D channels.

We did further investigations to see if the BER performance can be further improved (beyond that in Fig. 3) by removing the symmetry constraint on the target. This addresses the situa-

Fig. 4. BER performance for different target constraints with -3% domain bloom.

Fig. 5. BER performance for different target constraints with 3% domain bloom.

tion where the channel response is not symmetric. Since the 2-D monic constraint resulted in the best BER performance in Fig. 3, we designed asymmetric targets under the constraint $g_1 = 1$. We considered two kinds of asymmetry. First, we broke the assumption that the values of cross interference to the adjacent bits are constant. Therefore, we used six different values to represent the values of cross interference from the six adjacent bits. Secondly, we broke the assumption that the target responses for different tracks are the same. Therefore, we used 26 different values to represent the values of cross interferences from the adjacent bits and for five tracks. Simulations showed that the BER performances (not shown here) of these two asymmetric targets are similar to that of the 2-D monic constraint target in Fig. 3. This further reconfirms the effectiveness of the simple 2-D monic constraint.

Figs. 4 and 5 compare the performance of different targets as a function of SNR in the presence of domain bloom. The equal-

Fig. 6. BER performance for different target constraints in the presence of random pit-size variation when $\text{SNR} = 31$ dB. The pit-size variation is normalized with respect to the bit duration T .

izers and targets used are the same as that in Fig. 3 and information of domain bloom is not used in the detector. This is a practical consideration since in many cases we may not know the degree of domain bloom. As illustrated, the 2-D monic constrained target still outperforms all the other targets for both the under- and over-etching cases. Further, the BER curves in these figures indicate that the influence of the value of normalized g_1 on BER in the presence of domain bloom remain the same as that in the absence of bloom, i.e., larger g_1 results in better performance and vice versa. Finally, the amount of degradation in the presence of domain bloom is so significant that effective compensation techniques will be necessary to guarantee reliable data recovery in the presence of domain bloom.

Fig. 6 shows the performance of the different targets in the presence of random pit-size variation (with $\Delta_b = 0$) whose probability density function is Gaussian with zero mean and variance σ_t^2 , where σ_t is specified as a percentage of T . In this case, the equalizer and target are designed using the modified correlation matrices given by (46)–(49). Observe that the 2-D monic constrained target still outperforms other targets. In conclusion, the 2-D monic constraint is a suitable target constraint in TwoDOS with or without pit-size variations.

VII. CONCLUSION

In this paper, Hankel transform is used to construct the symbol response of TwoDOS in the presence and absence of pit-size variations. Based on this channel, we introduce a novel method which transforms the 2-D target design problem from 2-D into 1-D form. We also propose a novel 2-D monic constraint for 2-D partial response targets. By evaluating the BER performance of 2-D Viterbi detector for targets designed using different constraints, the proposed 2-D monic constraint is shown to be a reasonable target constraint for TwoDOS systems. The noise correlation resulting from the different target constraints is examined to support the observed BER performances.

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