Relations between local and effective air-side mass and heat transfer coefficients in fluidized bed drying

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RELATIONS BETWEEN LOCAL AND EFFECTIVE AIR-SIDE MASS AND HEAT TRANSFER COEFFICIENTS IN FLUIDIZED BED DRYING

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ABSTRACT

Effective mass and heat transfer coefficients are defined, describing the bed-averaged fluxes in terms of effective driving forces, based on the difference between interfacial and inlet values of humidity and temperature. Relations are developed between these effective coefficients and the local film coefficients for the case of well-mixed solids and plug flow of air. In these relations the effects of net mass transfer on mass and heat transfer coefficients and the variation over the bed height are included. Approximate simple relationships are shown to give an accurate description of the humidity and temperature profiles.

INTRODUCTION

The fluidized bed drying of particulates such as food extrudates can often be described by the relatively simple model of ideally mixed solids through which the air flows in plug flow. In case of batch fluidized bed drying a model consists of a description of the internal moisture transport in the particles, of the air side mass and heat transfer, coupled by the appropriate balances and thermodynamic and psychrometric equations.

Depending on the material to be dried the mechanisms of internal moisture transport can be different. Zimmermann & Bauer (1) used a shrinking-core approach for drying of bioproduct pellets, while Lievense et al (2) used an effective diffusion coefficient depending on water content and temperature for a similar system while drying in excess air. Other authors (3,4) use the approach of a characteristic drying curve in combination with balances over the drying air. Strumillo et al. (5) used an equilibrium model for the internal particle moisture content with the surroundings in their simulation of continuous fluidized bed drying.

In the continuous air phase the local mass and heat transfer fluxes vary over the height of the bed and thus the effect of the net water flux on the local mass and heat transfer rates also varies over the bed height. Zimmermann & Bauer (1) accounted for these variations by integrating the external flux equations over the height together with the internal kinetic equations, as Strumillo et al (5) did for the equilibrium model.

In our own work (6) on the development of a more rigorous treatment of systems with internal diffusion limitation one of the difficulties encountered is the multi-distributed nature of the moisture- and heat transfer. This would imply that the internal diffusion equation has to be solved for particles at various heights in the bed with height-dependent driving forces, alternated with the integration of the mass and heat balances over the drying air as it passes through the bed. Although this is possible, it takes a lot of computational effort. In order to simplify the modelling the mass and heat transfer around the particles can be described by time-averaged fluxes (6), which for a well-mixed particle bed are equal to bed-volume averaged fluxes, thus for a given set of

conditions the diffusion equation is solved once with bed-averaged boundary conditions.

Assuming constant transfer coefficients independent of the net mass transfer one can follow the classical approximation of Kast et al. (7), from which follows for the profiles along the dimensionless bed volume and the effective transfer coefficients :

$$\frac{H_i - H_b}{H_i - H_{in}} = \exp(-N_{tm}\xi) \quad k_{eff} \approx k_f \frac{1}{N_{tm}} [1 - \exp(-N_{tm})] \quad (1a)$$

$$\frac{T_b - T_i}{T_{in} - T_i} = \exp(-N_{th}\xi) \quad \alpha_{eff} \approx \alpha_f \frac{1}{N_{th}} [1 - \exp(-N_{th})] \quad (1b)$$

in which the effective coefficients are based on the driving force at the bed inlet. For a more accurate formulation of such averaged boundary conditions, also the bed-averaged effects of the net mass transfer on the mass and heat transfer should be available.

In the following relationships including these effects will be developed for these integrated air-side mass and heat transfer coefficients, and compared to numerical solutions. These relations are independent of the internal moisture transport kinetics and of the dependence of the film coefficients on bed parameters and velocity and consequently are of a general nature. *The only basic assumption is the "well-mixed-bed" hypothesis that, viewed from the air, the particle temperature and interfacial humidity may be considered constant over the bed.*

MASS TRANSFER

In Fig1. a schematic view is given of the film as envisioned at a particle interface, and a diagram of the fluidized bed with some relevant notations.

As a starting point we define an *effective mass transfer coefficient* by means of the following equation :

$$k_{eff} \equiv \langle j_{w,i} \rangle / [C_{tf} M_a (H_i - H_{in})] \quad (2)$$

in which $\langle j_{w,i} \rangle$ is the bed-averaged water flux ($\text{kg}/\text{m}^2\text{s}$), C_{tf} is the molar density of the gas at film conditions (8), M_a is the molecular mass of the dry air, and H stands for the absolute humidity expressed as $\text{kg water}/\text{kg dry air}$. The subscript "i" refers to the interface, "in" to the conditions of the inlet air to the bed.

FIGURE 1. Diagram of film near interface (left) and fluid-bed air flow (right). Dotted lines in film indicate linear driving force models.

Eq.(1) is understood here to include the effect of net mass transfer as well. The product $C_{tf}Ma$ is for low humidities approximately equal to the film density ρ_f .

Local mass transfer

For the local film coefficient for mass transfer we use that following from Bird et al.(8) for mass transfer through a stagnant layer :

$$N_{w,i} = k_f C_{tf} \ln \left[\frac{1-x_b}{1-x_i} \right] \quad (3)$$

in which k_f is the local film mass transfer coefficient, and $N_{w,i}$ is the molar water flux.

In making balances over the drying air it is very convenient to work on dry air basis, using the absolute humidity H rather than mole fractions.

The relation between H and x reads :

$$H = \frac{x}{1-x} \frac{M_w}{M_a} \quad (4a)$$

For convenience we will use in the following the *molar absolute humidity*, defined as moles H_2O /mole dry air :

$$U = \frac{x}{1-x} = \frac{M_a}{M_w} H \quad (4b)$$

Going over to the mass flux on weight basis we obtain :

$$j_{w,i} = k_f C_{tf} M_w \ln \left[\frac{1+U_i}{1+U_b} \right] \quad (5)$$

Let now a *modified mass transfer coefficient* k_f° be defined by the easy-to-handle equation :

$$j_{w,i} = k_f^\circ C_{tf} M_a (H_i - H_b) = k_f^\circ C_{tf} M_w (U_i - U_b) \quad (6)$$

and let us write :

$$k_f^\circ = \beta_m k_f \quad (7)$$

with β_m the conversion factor for the case of net mass transfer :

$$\beta_m = \frac{1}{U_i - U_b} \ln \left[\frac{1+U_i}{1+U_b} \right] \quad (8)$$

Integration over the bed

For the case of well-mixed solids and air flowing in plug flow through the bed we have the balance :

$$V \frac{dH_b}{dz} = a_p (1 - \varepsilon_b) A_b j_{w,i} \quad (9)$$

Substitution of Eq.(5) and integration gives :

$$N_{tm} \xi = \int_{U_{in}}^U \frac{dU_b}{\ln \left[\frac{1+U_i}{1+U_b} \right]} \quad (10)$$

in which $\xi = z/L$ is the dimensionless height in the bed and N_{tm} is the number of mass transfer units :

$$N_{tm} = \frac{k_f C_{tf} M_a a_p (1 - \varepsilon_b) A_b L}{V} \quad (11)$$

In Fig.2. the local value of β_m , as obtained by numerical integration of Eq. (9) for H_b over the bed, is presented as it varies over the relative height in the bed, for two values of the interfacial humidity H_i . It should be noted that $H_i = 0.1$ is a very high value, not likely to be encountered in practice; it is used here however to provide a severe test for the methods developed. It is clear that β_m varies considerably; at high values of N_{tm}

FIGURE 2. Mass transfer conversion factor β_m as it varies over the bed height, for two values of the interfacial humidity H_i . The inlet air humidity H_{in} has been taken equal to 0.

we have a rapid change near the bottom plate and an asymptotic approach to a constant value. The difference of this asymptotic value from unity stems from the definition of k_f° as based on humidity differences. For very low values of N_{tm} , corresponding to virtually single-particle behaviour in excess air, we have hardly any variation of β_m .

Since Eq. (10) cannot be solved in closed form, we search for an approximate form by defining:

$$N_{tm}^\circ = \langle \beta_m \rangle N_{tm} \quad (11)$$

in which N_{tm}° is obtained by inserting Eq.(6) for the flux into Eq. (9), and integrating over the bed height for an effective value of $k_f^\circ C_{ff}$ assumed constant:

$$N_{tm}^\circ \xi \approx -\ln \left[\frac{U_i - U_b}{U_i - U_{in}} \right] \quad (13)$$

or

$$U_b \approx U_i - (U_i - U_{in}) \exp(-N_{tm}^\circ \xi) \quad (14)$$

In order to obtain $\langle \beta_m \rangle$ we expand Eq.(8) in series :

$$\begin{aligned} \beta_m &= \frac{1}{U_i - U_b} [U_i - U_b - (U_i^2 - U_b^2)/2 + (U_i^3 - U_b^3)/3 \dots] \\ &\approx 1 - \frac{1}{2}(U_i + U_b) + \frac{1}{3}(U_i^2 + U_i U_b + U_b^2) \end{aligned} \quad (15)$$

and for the height averaged value $\langle \beta_m \rangle$ we substitute the height-averaged value of the bulk molar humidity $\langle U_b \rangle$:

$$\begin{aligned} \langle \beta_m \rangle &\approx 1 - \frac{1}{2}(U_i + \langle U_b \rangle) + \frac{1}{3}(U_i^2 + U_i \langle U_b \rangle + \langle U_b \rangle^2) \\ &= 1 - \frac{1}{2}(U_i + \langle U_b \rangle) + \frac{1}{3}(U_i - \langle U_b \rangle)^2 + U_i \langle U_b \rangle \end{aligned} \quad (16)$$

Eq. (14) gives for the height-averaged value $\langle U_b \rangle$:

$$\begin{aligned} \langle U_b \rangle &\approx U_i - (U_i - U_{in}) \frac{1}{N_{tm}^\circ} [1 - \exp(-N_{tm}^\circ)] \\ &\approx U_i - (U_i - U_{in}) \frac{1}{N_{tm}} [1 - \exp(-N_{tm})] \end{aligned} \quad (17)$$

The simplification in the second part of (17) is allowed for the use in deriving the conversion factor $\langle \beta_m \rangle$; the effect of the difference between N_{tm}° and N_{tm} is a higher-order conversion on $\langle \beta_m \rangle$. Substitution in (16) then gives after some rearrangement :

$$\begin{aligned} \langle \beta_m \rangle &\approx 1 - U_i + U_i^2 + \frac{(\frac{1}{2} + \frac{1}{3} U_i)(U_i - U_{in})}{N_{tm}} [1 - \exp(-N_{tm})] \\ &\quad + \frac{(U_i - U_{in})^2}{N_{tm}^2} [1 - \exp(-N_{tm})]^2 \end{aligned} \quad (18)$$

Numerical evaluation showed that (18) gives a slight overestimation of about 1-2 %; a somewhat simpler but more accurate equation is obtained if most of the higher order terms are omitted, and the U_i^2 -term is halved :

$$\langle \beta_m \rangle \approx 1 - U_i + \frac{1}{2} U_i^2 + \frac{1}{2} \frac{(U_i - U_{in})}{N_{tm}} [1 - \exp(-N_{tm})] \quad (19)$$

Results for approximated humidity profiles and conversion factors

For the case of $H_i = 0.1$ the humidity profiles over the bed as follow from numerical integration of Eq.(10) are presented in Fig. 3. Since the

FIGURE 3. Humidity profiles over the bed height, as obtained from numerical integration, for $H_i = 0.1$ and $H_{in} = 0$, for different numbers of mass transfer units.

differences between these profiles and those from the approximation (14) are very small, the latter profiles are not drawn, but in Fig. 4 the relative differences between the approximation and the numerical solution are given. The differences are small; using Eq. (19) for $\langle \beta_m \rangle$ leads for high N_{tm} to an error of maximal 2 % in the humidity profile, while for lower N_{tm} the error is smaller, as can be seen from the left picture calculated for $H_i = 0.1$ and $H_{in} = 0$. Interesting is the effect of the inlet air humidity; in the right picture the deviations are shown for $H_{in} = 0.05$, from which follows that the deviation for the classical approach increases, while that for the present approach is considerably smaller. This reflects the non-linear effect of H on the local mass transfer rate, as follows from Eq.(8), which is incorporated in the present method but not in the classical relations. These situations can be met in practice when direct combustion

FIGURE 4. Deviation in % between approximated and exact humidity profiles as obtained from Eq. (14) (drawn lines), and from simple exponential approximation Eq.(1a) neglecting net mass transfer correction (dotted lines). Interfacial humidity $H_i = 0.1$; inlet air humidity $H_{in} = 0$ (left figure) and $H_{in} = 0.05$ (right figure). Parameter N_{tm} .

gases are used in the drying air. Investigation of results for lower H_i showed that the errors decrease for both approximations, as may be expected because of the lower water flux.

The comparison between the approximated values of $\langle \beta_m \rangle$ and the results from numerical integration are summarized in Table 1, together with the heat transfer results. Inspection shows that the error is lower than 1%.

Combination of Eq. (1) and (6) leads to the bed-averaged mass transfer coefficient :

$$k_{\text{eff}} = \frac{k_f^{\circ}}{N_{tm}} [1 - \exp(-N_{tm})] \quad (20)$$

$$k_f^{\circ} = \langle \beta_m \rangle > k_f$$

HEAT TRANSFER

Local heat transfer

First we will derive the effect of mass transfer on the heat flux in the film, along the lines of Bird et al. (8). For the heat flux at a given

TABLE 1. COMPARISON OF APPROXIMATE AND EXACT
CONVERSION FACTORS FOR MASS AND HEAT TRANSFER
COEFFICIENTS

N_{tm}	H_{in}	H_i	$<\beta_m>_{ex}$	$<\beta_m>_{app}$	$\Delta(\%)$	$\Phi_{h,ex}$	$\Phi_{h,app}$	$\Delta(\%)$
0.1	0	0.05	0.9601	0.9611	0.11	0.9092	0.9126	0.37
	0	0.1	0.9241	0.9286	0.50	0.8740	0.8807	0.77
	0.05	0.1	0.8914	0.8904	-0.11	0.9162	0.9164	0.02
1	0	0.05	0.9486	0.9483	-0.04	0.5940	0.5942	0.03
	0	0.1	0.9034	0.9030	-0.05	0.5744	0.5780	0.62
	0.05	0.1	0.8820	0.8776	-0.50	0.6166	0.6145	-0.35
5	0	0.05	0.9330	0.9308	-0.23	0.1856	0.1849	-0.39
	0	0.1	0.8753	0.8681	-0.82	0.1841	0.1821	-1.11
	0.05	0.1	0.8684	0.8601	-0.95	0.2011	0.1993	-0.91
50	0	0.05	0.9264	0.9236	-0.30	0.0187	0.0186	-0.43
	0	0.1	0.8629	0.8538	-1.06	0.0186	0.0184	-1.18
	0.05	0.1	0.8622	0.8530	-1.07	0.0203	0.0201	-0.94

coordinate y in the film we have :

$$j_q = -\lambda \frac{dT}{dy} + j_{w,i} h_v = -\lambda \frac{dT}{dy} + j_{w,i} (c_{pv}T + \Delta h_{vo}) \quad (21)$$

in which the reference for the enthalpy is taken that of liquid water at 0 °C.

Integration between $y = 0$, at the interface, and $y = \delta$, at the bulk gas phase, gives :

$$\frac{1}{j_{w,i} c_{pv}} \ln \left[\frac{j_{w,i} (c_{pv}T_b + \Delta h_{vo}) - j_q}{j_{w,i} (c_{pv}T_i + \Delta h_{vo}) - j_q} \right] = \int_0^\delta \frac{dy}{\lambda} = \frac{1}{\alpha_f} \quad (22)$$

in which α_f is the *film heat transfer coefficient* excluding the effect of mass transfer. Rearrangement leads to :

$$\begin{aligned} j_q &= j_{w,i} (c_{pv}T_i + \Delta h_{vo}) + \frac{\gamma}{\exp(\gamma) - 1} \alpha_f (T_i - T_b) \\ &= j_{w,i} (c_{pv}T_i + \Delta h_{vo}) + \alpha^\circ (T_i - T_b) \end{aligned} \quad (23)$$

in which

$$\gamma = \frac{j_{w,i} c_{pv}}{\alpha_f} \quad \text{and} \quad \alpha^\circ = \beta_h \alpha_f \quad \text{with} \quad \beta_h = \frac{\gamma}{\exp(\gamma) - 1} \quad (24)$$

Integration over the bed

For the heat flux averaged over the bed we have :

$$\langle j_q \rangle = \int_0^1 j_q d\xi = \langle j_{w,i} \rangle (c_{pv}T_i + \Delta h_{vo}) + \alpha_f \int_0^1 \beta_h (T_i - T_b) d\xi \quad (25)$$

We now define an *effective bed-average heat transfer coefficient* by :

$$\langle j_q \rangle = \langle j_{w,i} \rangle (c_{pv}T_i + \Delta h_{vo}) + \alpha_{eff} (T_i - T_{in}) \quad (26)$$

For the relation between the effective and the film coefficient we write :

$$\alpha_{eff} = \phi_h \alpha_f$$

$$\phi_h = \frac{1}{T_i - T_{in}} \int_0^1 \beta_h (T_i - T_b) d\xi \quad (27)$$

Now let us consider the enthalpy balance over a horizontal slice through the fluid bed, assuming again plug flow for the air :

$$\begin{aligned} V \frac{di}{dz} &= A_b (1 - \varepsilon_b) a_p j_q \\ &= A_b (1 - \varepsilon_b) a_p \left[j_{w,i} (c_{pv}T_i + \Delta h_{vo}) + \frac{\gamma}{\exp(\gamma) - 1} \alpha_f (T_i - T_b) \right] \end{aligned} \quad (28)$$

in which i is the enthalpy of the air, expressed as kJ/kg dry air, given by :

$$i = (c_{pa} + c_{pv}H_b)T_b + H_b \Delta h_{vo} \quad (29)$$

which leads to :

$$\begin{aligned} V \left[(c_{pv}T_b + \Delta h_{vo}) \frac{dT_b}{dz} + (c_{pa} + c_{pv}H_b) \frac{dT_b}{dz} \right] &= \\ A_b (1 - \varepsilon_p) a_p \left[j_{w,i} (c_{pv}T_i + \Delta h_{vo}) + \frac{\gamma}{\exp(\gamma) - 1} \alpha_f (T_i - T_b) \right] \end{aligned} \quad (30)$$

Substitution of Eq. (8) and rearrangement leads to :

$$\begin{aligned} V (c_{pa} + c_{pv}H_b) \frac{dT_b}{dz} &= A_b (1 - \varepsilon_b) a_p \left[j_{w,i} c_{pv} + \frac{\gamma}{\exp(\gamma) - 1} \alpha_f \right] (T_i - T_b) \\ &= A_b (1 - \varepsilon_b) a_p \alpha_f \frac{\gamma \exp(\gamma)}{\exp(\gamma) - 1} (T_i - T_b) \end{aligned} \quad (31)$$

Defining the *number of heat transfer units* N_{th} :

$$N_{th} = \frac{A_b (1 - \varepsilon_b) a_p \alpha_f L}{V c_{pa}} \quad (32)$$

we can integrate (31) to find :

$$-\ln \left(\frac{T_b - T_i}{T_{in} - T_i} \right) = N_{th} \int_0^{\xi} \frac{c_{pa}}{c_{pa} + c_{pv}H_b} \frac{\gamma \exp(\gamma)}{\exp(\gamma) - 1} d\xi' \quad (33)$$

with ξ' as a dummy integration variable.

Eq. (33) can be rearranged to an expression for the temperature profile :

$$T_r = \left(\frac{T_b - T_i}{T_{in} - T_i} \right) = \exp[-N_{th} f_h(\xi)] \quad (34)$$

in which :

$$f_h(\xi) = \int_0^\xi g_h(\xi') d\xi' \quad (35)$$

$$g_h(\xi) = \frac{c_{pa}}{c_{pa} + c_{pv} H_b} \frac{\gamma \exp(\gamma)}{\exp(\gamma) - 1}$$

From the definition of α_{eff} by Eq. (27) we have then for the conversion factor :

$$\phi_h = \int_0^1 \beta_h \exp[-N_{th} f(\xi)] d\xi = \int_0^1 \beta_h T_r d\xi \quad (36)$$

Derivation of approximate relation for conversion factor ϕ_h

In the following we will derive approximate expressions for the temperature profile and the conversion factor ϕ_h . This is done by a series of successive approximations and linearizations of the various parameters. We start with an approximation for the γ -term from series development for small values of γ :

$$\frac{\gamma \exp(\gamma)}{\exp(\gamma) - 1} \approx 1 + \frac{\gamma}{2} \quad (37)$$

We now use the expressions (6) and (7) for the flux, combined with (14) for the humidity profile to obtain for γ :

$$\gamma = \frac{j_{w,i} c_{pv}}{\alpha_f} = \frac{k_f \beta_m C_{tf} M_a (H_i - H_{in}) \exp(-N_{tm}^\circ \xi) c_{pv}}{\alpha_f} \quad (38)$$

For the relation between the film coefficients we use the Chilton-Colburn analogy :

$$\frac{k_f}{\alpha} = \frac{1}{\rho_f c_{pf}} Le^{a_1} \quad \text{with : } Le = \frac{Sc}{Pr} \quad \text{and } a_1 \approx 0.33 \quad (39)$$

For the film thermal capacity we have :

$$\rho_f c_{pf} = C_{tf} (x_a M_a c_{pa} + x_w M_w c_{pw}) = C_{tf} M_a c_{pa} \frac{1 + m_1 H_f}{1 + m_2 H_f} \quad (40)$$

with : $m_1 = c_{pv}/c_{pa}$ and $m_2 = M_a/M_w$.

Thus we obtain approximately for γ :

$$\gamma \approx \text{Le}^{a_1} \beta_m m_1 (H_i - H_{in}) \exp(-N_{tm}^\circ \xi) \frac{1 + m_2 H_f}{1 + m_1 H_f} \quad (41)$$

$$\approx \text{Le}^{a_1} \beta_m m_1 (H_i - H_{in}) \exp(-N_{tm}^\circ \xi) [1 + (m_2 - m_1) H_f]$$

For the film humidity we obtain :

$$H_f = \frac{1}{2} (H_i + H_b) = \frac{1}{2} (H_i + H_{in}) - \frac{1}{2} (H_i - H_{in}) \exp(-N_{tm}^\circ \xi) \quad (42)$$

For the mass transfer conversion factor β_m we have from (15) in first approximation :

$$\beta_m = 1 - \frac{1}{2} (U_i + U_b) = 1 - \frac{1}{2} m_2 (H_i + H_b) \quad (43)$$

Combination of the foregoing equations while leaving out higher order terms in H leads to an approximation for $g_h(\xi)$:

$$g_h(\xi) \approx \frac{1 + \frac{1}{2} \text{Le}^{a_1} m_1 (H_i - H_{in}) \exp(-N_{tm}^\circ \xi)}{1 + m_1 [H_i - (H_i - H_{in}) \exp(-N_{tm}^\circ \xi)]} \quad (44)$$

Integration while neglecting the changes in N_{tm}° gives for $f_h(\xi)$:

$$f_h(\xi) = \frac{\xi}{1 + m_1 H_i} + \frac{1}{N_{tm}^\circ} \left(\frac{1}{1 + m_1 H_i} + \frac{1}{2} \text{Le}^{a_1} \right) \ln [1 + m_1 (H_i - H_{in}) \{1 - \exp(-N_{tm}^\circ \xi)\}] \quad (45)$$

For the dimensionless temperature profile we obtain :

$$T_r = \exp[-N_{th} f_h(\xi)]$$

$$= \exp \left[-\frac{N_{th} \xi}{1 + m_1 H_i} \left[1 + m_1 (H_i - H_{in}) \{1 - \exp(-N_{tm}^\circ \xi)\} \right] - \frac{N_{th}}{N_{tm}^\circ} \left(\frac{1}{1 + m_1 H_i} + \frac{1}{2} \text{Le}^{a_1} \right) \right] \quad (46)$$

The exponent in (46) can be expanded by :

$$\frac{N_{th}}{N_{tm}^\circ} = \frac{1}{\beta_m} \text{Le}^{-a_1} [1 + (m_1 - m_2) H_f] \quad (47)$$

Using the approximation for small x :

$$(1 + ax)^b \approx 1 + abx$$

and again leaving out the higher order terms in H , we obtain for the approximate temperature profile :

$$T_r \approx \exp \left[-\frac{N_{th} \xi}{1 + m_1 H_i} \left[1 - m_1 \left(\text{Le}^{-a_1} + \frac{1}{2} \right) (H_i - H_{in}) \{1 - \exp(-N_{tm}^\circ \xi)\} \right] \right] \quad (48)$$

For the further development towards the integrated conversion factor φ_h we write:

$$\beta_h = \frac{\gamma}{\exp(\gamma) - 1} \approx 1 - \frac{\gamma}{2} \quad (49)$$

$$\approx 1 - \frac{1}{2} \text{Le}^{a_1} m_1 (H_i - H_{in}) \exp(-N_{th}^\circ \xi) [1 - m_1 H_i + (m_2 - m_1) H_f]$$

and obtain for the integrand in Eq. (36):

$$\beta_h T_r \approx \exp(-d_3 N_{th} \xi) \times$$

$$\left[(1 - d_4 (H_i - H_{in})) + (H_i - H_{in}) \left[-d_1 d_2 + d_4 + d_1 d_2 d_4 (H_i - H_{in}) \exp(-N_{th}^\circ \xi) \right] \right] \quad (50)$$

with:

$$d_1 = \frac{1}{2} \text{Le}^{a_1} m_1$$

$$d_2 = 1 - m_1 H_i + (m_2 - m_1) H_f \approx 1 + (m_2 - 2m_1) H_i$$

$$d_3 = \frac{1}{1 + m_1 H_i} \approx 1 - m_1 H_i \quad (51)$$

$$d_4 = m_1 (\text{Le}^{-a_1} + \frac{1}{2})$$

Using Eq. (47) integration of (50), neglecting 3^d-order terms then leads to the expression for φ_h :

$$\varphi_h \approx \frac{1}{d_3 N_{th}} [1 - \exp(-d_3 N_{th})] [1 - d_4 (H_i - H_{in})] +$$

$$\frac{1}{d_5 N_{th}} [1 - \exp(-d_5 N_{th})] [d_6 (H_i - H_{in}) + d_7 (H_i - H_{in})^2] \quad (52)$$

with:

$$d_5 = d_3 + d_2 \text{Le}^{a_1}$$

$$d_6 = -d_1 d_2 + d_4 \quad (53)$$

$$d_7 = \frac{1}{2} m_1^2 (1 + \text{Le}^{a_1})$$

In the Appendix the constants and relations are presented in numerical form.

For clarity the definition of φ_h is repeated here:

$$\alpha_{eff} = \varphi_h \alpha_f \quad (54)$$

From numerical study follows that also within 1.5 % accuracy the following simpler relation for the conversion factor holds:

$$\varphi_h \approx \frac{1}{d_3 N_{th}} [1 - \exp(-d_3 N_{th})] \left[1 - \frac{d_4}{2} (H_i - H_{in}) \right] \quad (55)$$

and so with either of Eqs. (52) or (55) the conversion from film to effective heat transfer can be made by means of Eq. (54).

Results for approximate temperature profiles and conversion factors

In Fig. 5 the temperature profiles are presented as calculated numerically. In Fig. 6 the difference between the results of the numerical integration and the approximate solution (48) are given for $H_i = 0.1$ and $H_{in} = 0$ in the left picture. We see that the maximum deviations are in the order of 1 % for both approximations. At higher inlet humidities however the deviations of the present work are much lower. For lower drying rates the deviations in both approximations are smaller. Since the absolute humidity influences the thermal capacity of the film this affects the value of γ , and the relation between k_f and α_f ; this is included in the present approximations, but not in the classical approach.

In Table 1 the results of the approximation are again compared with the results of the numerical integration. It is seen that the errors are smaller than 1.2 %, and the largest deviations occur at high driving forces. At higher absolute humidities, up to $H_i = 0.2$, this accuracy is still well within 1.5 % for inlet humidities $H_{in} > 0.1$.

TEMPERATURE-DEPENDENCE OF THE CONVERSION FACTORS

In the foregoing the effects of the evaporation rate on the effective mass and heat transfer coefficients was treated. In evaluating the coefficients however the film temperature must be used; since the bulk temperature varies over the bed one would also expect a variation in k_f and α_f . For the mass transfer film coefficient many correlations are available, which may be well approximated by a dependence like :

$Sh \sim Re^b Sc^c$, with $b \approx 0.5$, and $c \approx 0.33$. Thus we find for k_f :

$$k_f \sim D_f^{1-b_2} \rho_f^{b_1-b_2} \mu_f^{b_2-b_1} = D_f^{0.67} \rho_f^{0.17} \mu_f^{-0.17}$$

For the temperature dependence on the absolute temperature T we may approximate (8) :

$$D \sim T^{1.5}$$

$$\rho \sim T^{-1}$$

$$\mu \sim T^{0.5}$$

FIGURE 5. Dimensionless temperature profiles as obtained by numerical integration, for different numbers of mass transfer units. Interfacial humidity is taken equal to $H_i = 0.1$, inlet air humidity equal to $H_{in} = 0$.

FIGURE 6. Difference in % in temperature profile from Eq.(48) (drawn lines) and exact solution, and from simple exponential Eq. (1b) (dotted lines). Interfacial humidity $H_i = 0.1$; inlet air humidity $H_{in} = 0$ (left figure) and $H_{in} = 0.05$ (right figure). Parameter N_{tm} .

leading to $k \sim T^1$. Since the total molar density equals :

$C_{tf} = p_{tot}/(RT)$, we have for the product $(k_f C_{tf})$, as it appears in the flux and in the number of mass transfer units, virtually no dependence on T .

For the heat transfer coefficient α_f we have :

$Nu \sim Re^b Pr^c$, with $b \approx 0.5$, and $c \approx 0.33$. Thus we find for α_f :

$$\alpha_f \sim \lambda_f^{1-b_2} \rho_f^{b_1-b_2} \mu_f^{b_2-b_1} = \lambda_f^{0.67} \rho_f^{0.17} \mu_f^{-0.17}$$

Since $\lambda \sim \sqrt{T}$ we would have a dependence of α of $T^{0.08}$. In a typical situation for the drying of salt (9) with an inlet temperature of 150 °C and a bed temperature of 46 °C, the film temperature would vary from 98 °C to 46 °C, provided we have enough transfer units. If we would evaluate α at the average of both, 72 °C, the maximum deviation in α would be :

$\Delta\alpha/\alpha = 0.08 \Delta T/T = 6 \times 10^{-3}$, or 0.6 %. For the conversion factor ϕ_h this error is within the error of the other approximations.

CONCLUSIONS

Approximate relationships have been derived for the bed-integrated effective mass and heat transfer coefficients in which the effect of the net mass transfer and its variation over the bed height have been taken into account. Comparison with numerical solution over a large range of conditions shows a high accuracy even at high humidities and water fluxes. Since the developed equations do neither depend on the product internal mass transfer mechanism nor on the correlation between continuous phase transfer coefficients and bed parameters, they can be generally applied for a very large field of modelling.

As intermediate results also the approximate humidity and temperature profiles have been derived, which are very accurate over a wide range of conditions. Especially at higher humidity levels the accuracy is considerably better than that by the classical approach.

APPENDIX : PHYSICAL DATA AND NUMERICAL VALUES

Physical data :

$$c_{pa} = 1.00 \text{ kJ/kg } ^\circ\text{C}$$

$$c_{pv} = 1.87 \text{ kJ/kg } ^\circ\text{C}$$

Approximate relationships and constants

$$a_1 = 0.33$$

$$m_1 = 1.87$$

$$c_{pw} = 4.18 \text{ kJ/kg } ^\circ\text{C}$$

$$m_2 = 1.60$$

$$\Delta h_{vo} = 2504 \text{ kJ/kg}$$

$$d_1 = 0.88$$

$$M_a = 28.8 \text{ kg/kmole}$$

$$d_2 = 1 - 2.14 H_i$$

$$M_w = 18 \text{ kg/kmole}$$

$$d_3 = 1/(1 + 1.84 H_i)$$

$$Sc = 0.588$$

$$d_4 = 2.92$$

$$Pr = 0.705$$

$$d_5 = 1.94 - 3.88 H_i$$

$$Le = 0.834 \text{ (@ } 50^\circ\text{C and}$$

$$d_6 = 2.04 + 1.88 H_i$$

$$H = 0.015 \text{ kg/kg, (2)})$$

$$d_7 = 2.57$$

$$\langle \beta_m \rangle \approx 1 - 1.60 H_i + 1.28 H_i^2 + 0.80 \frac{H_i - H_{in}}{N_{tm}} [1 - \exp(-N_{tm})]$$

$$\frac{N_{th}}{N_{tm}} \approx 1.06 [1 + 0.135(H_i + H_{in})] \text{ for } N_{tm} > 3$$

$$T_r \approx \exp\left(-\frac{N_{th}}{1 + 1.87 H_i}\right) [1 - 2.69(H_i - H_{in}) \{1 - \exp(-N_{tm})\}]$$

SYMBOLS

a_p specific area of particles [m^2/m^3 particle]

A_p bed cross-sectional area [m^2]

b, c exponents [-]

C_t molar density [kmole/m^3]

c_p specific heat [$\text{J}/\text{kg } ^\circ\text{C}$]

d constants in Equations (51) and (52)

f_h defined in Equation (34)

g_h defined in Equation (35)

H absolute humidity [kg/kg dry air]

h enthalpy [J/kg]

Δh_{vo} heat of evaporation at 0°C [J/kg]

i enthalpy on dry air basis [J/kg dry air]

j_w mass flux of water [$\text{kg}/\text{m}^2 \text{ s}$]

j_q heat flux [W/m^2]

k mass transfer coefficient [m/s]

L bed height [m]

Le Lewis nr.

M molecular mass [kg/kmole]

m constants, defined in Equation (40)

N_t	nr. of transfer units [-]
N_w	molar water flux [kmole/m ² s]
Pr	Prandtl number [-]
p	pressure [Pa]
R	gas constant [J/kmole K]
Re	Reynolds number [-]
Sc	Schmidt number [-]
T	temperature [°C]
T	absolute temperature [K]
U	molar absolute humidity [kmole/kmole dry air]
V	mass flow rate of dry air [kg/s]
x	mole fraction [-]
y	distance coordinate in film [m]
z	distance coordinate in bed [m]
α	heat transfer coefficient [W/m ² °C]
β_h	conversion factor for heat transfer coefficient [-]
β_m	conversion factor for mass transfer coefficient [-]
γ	defined in Equation (24) [-]
Δ	deviation [%]
δ	film thickness [m]
ϵ_b	bed porosity [-]
λ	thermal conductivity [W/m K]
μ	dynamic viscosity [Pa s]
ρ	density [kg/m ³]
ξ	relative bed height [-]
< >	averaged over bed

Subscripts, superscripts

1,2..	index
a	air
b	bulk gas phase
eff	effective, related to driving force from interfacial and inlet conditions
f	film
h	heat
i	interfacial
m	mass
r	reduced

tot	total
v	vapour
w	water, liquid water in c_p
o	included the effect of net mass transfer

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