Optimal design of a hydro-elastic rear axle suspension for heavy trucks

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Summary

The spring and damper parameters of a truck axle suspension are selected to yield an optimal compromise between three conflicting demands: minimisation of chassis accelerations (in order to improve driver comfort, to decrease the accelerations the cargo is exposed to and to decrease the fatigue damage caused to chassis and chassis components), minimisation of spring travel and minimisation of dynamic wheel load.

Suspension systems ideally have a behaviour that varies with the excitation frequency. However, most conventional suspension systems exhibit stiffness and damping values that do not show this behaviour. To achieve such frequency dependent stiffness and damping behaviour, a novel suspension system has been investigated. This new system relies on the application of more degrees of freedom for the suspension system. First, a systematic analysis is performed in which a single mass, linear springs and linear dampers are combined. The suspension systems which can be designed with these elements, show four types of stiffness and damping properties. Of these four frequency dependent types, one is chosen for further investigation. This suspension consists of two air volumes, acting as air springs, which are connected by a channel, filled with liquid. At the system’s eigenfrequency, the mass of the liquid resonates on the two air springs and affects the stiffness and damping properties as a function of the excitation frequency.

The performance of this suspension system, with respect to the aforementioned three design demands, has been evaluated with models of the road surface excitation, new suspension and an experimentally validated truck and semi-trailer model. Improvements found during optimisations were relatively small for driver comfort, spring travel, cargo accelerations and dynamic wheel load. Improvements found for fatigue loads, expressed in a so-called Palmgren-Miner number, were 15%. This means that, before suffering from identical fatigue, a truck with the novel suspension will have covered 15% greater distance than a similar truck equipped with a conventional suspension system.

Of the novel suspension, two prototypes were built; a so-called passive suspension and a so-called adaptive suspension. Of this second suspension, one of the two air volumes and the diameter of the liquid channel can be adjusted while driving to obtain maximal performance in variable driving conditions (different vehicle speeds, kinds of road surfaces etc.). Test-rig experiments with the suspension system itself, revealed that the desired stiffness and damping properties can be attained. Full scale truck experiments on a test-rig proved that calculations with the three-dimensional vehicle model predict the dynamic behaviour of the truck and the new suspension very well, which means that a valuable tool is available to predict the dynamic vehicle behaviour.
Samenvatting

De veer en dempingsparameters van een vrachtwagen veersysteem worden altijd zo gekozen dat er een optimaal compromis ontstaat tussen drie tegenstrijdige ontwerpeisen; een zo laag mogelijk trillingsniveau van het voertuig (beter comfort voor de chauffeur, lagere versnellingen van de belading, lagere vermoeiingsbelasting van truckonderdelen), een zo klein mogelijke inveerweg van de assen en zo laag mogelijke dynamische wielkrachten.

Een ideaal veersysteem heeft veer en demper coëfficiënten die veranderen als de excitatie-frequentie van het veersysteem verandert. Echter, de meeste conventionele veersystemen vertonen dit frequentie afhankelijke gedrag niet. Om dit gedrag te bereiken is er naar een nieuw veersysteem gezocht. Uit een systematische analyse, waarin veren, dempers en een massa op zoveel mogelijk verschillende manieren met elkaar gecombineerd werden, bleek dat er dan vier verschillende frequentie afhankelijk karakteristieken voor de stijfheid en demping mogelijk zijn. Van deze vier, is er één uitgekozen voor verder onderzoek. Deze vering bestaat uit twee luchtvolumes, die als luchtveer fungeren. Deze luchtvolumes zijn met elkaar verbonden door middel van een kanaal dat gevuld is met vloeistof. De massa van deze vloeistof resonereert op de luchtveren en beïnvloedt op deze manier de stijfheids- en dempingskarakteristiek afhankelijk van de excitatie frequentie.

Het gedrag van de nieuwe vering is geëvalueerd met behulp van modellen voor het wegdek, de nieuwe vering en een experimenteel geverifieerd drie-dimensionaal voertuigmodel. Verbeteringen door toepassing van het nieuwe veersysteem zijn klein voor comfort, veerweg, versnellingen van de belading en dynamische wielkrachten. Verbeteringen gevonden voor de vermoeiingsbelasting, uitgedrukt in een zogenaamd Palmgren-Miner getal, bedragen 15 %. Dit betekent dat, voordat er dezelfde vermoeiingsschade opgetreden is, een truck met de nieuwe vering, 1.15 maal zoveel kilometers afgelegd heeft als een truck met een conventioneel veersysteem.

Van de nieuwe vering zijn twee prototypes gebouwd. Een passieve variant, zonder instelbare parameters, en een adaptieve variant. Van dit tweede prototype, is één van de twee luchtvolumes en de diameter van het vloeistofkanaal verstelbaar om een optimaal gedrag te bewerkstelligen onder verschillende rijomstandigheden zoals bijvoorbeeld een verandering in de rijssnelheid, hoeveelheid belading en het soort wegdek waarover gereden wordt. Experimenten met beide veersystemen, lieten zien dat de verwachte stijfheids- en dempingskarakteristieken gerealiseerd kunnen worden. Experimenten met een truck en oplegger uitgerust met het conventionele veersysteem en het nieuwe veersysteem lieten zien dat het gedrag van truck en oplegger goed voorspeld wordt met het drie-dimensionale voertuig model. Dit betekent dat we een waardevol gereedschap hebben om het voertuiggedrag te beschrijven.
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Chapter 1

General introduction

Road cargo transport is growing [Statistisch Jaarboek, 1996] as shown in Figure 1.1. In general, society associates this current growth with a negative impact on their environment. To reduce this impact, cargo transport has been made more efficient over the past years. However, to make road cargo transport even more efficient, trucks have to be improved. In other words, for each kilometre driven by a truck, more cargo must be transported and the damage inflicted upon the environment during this transport in terms of emissions, road damage etc. must be reduced.

\[ \text{Figure 1.1 Road cargo transport in the Netherlands} \]

For a higher efficiency, payload volume per truck must be increased. Within the legal constraints on vehicle dimensions, greater efficiency can be achieved by reducing the required suspension working space, i.e. the space needed for the compression of the springs. This allows the trailer floor to be lowered. Another possibility is to decrease the accelerations of the cargo. This allows sensitive cargo to be transported with less packaging material to increase the amount of cargo which can be transported within a given volume. Smaller accelerations also yield better driver comfort and lower fatigue loads on truck components which enhances the truck’s reliability. Furthermore, this allows the truck to be constructed lighter which in turn, allows either heavier payload, or reduced fuel consumption (both of interest for transport companies). Next to required suspension working space and acceleration levels, dynamic wheel loads are of importance. Dynamic wheel loads are the dynamic forces acting on the road at the wheels. Smaller dynamic wheel loads not only lead to improved road holding and thus...
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safer transport, but also to reduced road damage and thus smaller government expenditures for road repairs. These three topics (suspension working space, accelerations and dynamic wheel load) are strongly influenced by the suspension system and justify the search for improved suspension systems.

The high level of technology required and the opportunities for better transport efficiency, brought several companies together in the CASCOV project (Controlled Axle Suspensions for Commercial Vehicles). This project was initiated in 1992 with funding from the European Community within the Brite/Euram II framework for a time span of four years. The project is a co-operation between DAF Trucks N.V. (truck manufacturer, the Netherlands), ContiTech Formteile GmbH (manufacturer of air springs and hydromounts, Germany), Monroe (manufacturer of shock absorbers, Belgium) and Eindhoven University of Technology (the Netherlands). Goal of the project was to develop axle suspension concepts for a heavy truck and semi-trailer combination to improve the dynamic behaviour with respect to acceleration levels, dynamic wheel loads and required suspension working space. Because previous research [Yael, 1993] revealed that the truck’s rear axle suspension most strongly affects the various dynamic aspects of the truck and semi-trailer, the project was focused on improving the rear axle suspension only. Subject of the research carried out in the project is a truck and semi-trailer combination of the kind as shown in Figure 1.2.

Current car and truck suspension systems, predominantly contain conventional springs and dampers. These suspensions have evolved to a level at which it is difficult to improve suspension behaviour without altering the suspension working principles. A great number of investigations have been carried out in the area of semi-active and active suspensions [see e.g. Yeh et al., 1994; Huisman et al., 1993; Crolla et al.,1991]. Active suspensions involve energy consumption and cause increased fuel consumption. Of the active and semi-active suspension systems, only the semi-active kind is paid attention to in the CASCOV project. Despite all valuable efforts with respect to (semi-)active suspensions, it is expected that passive suspension systems still offer room for improvement. Compared to (semi-)active suspension systems, the advantage of a passive suspension is, that expensive and complex electronic equipment is not necessary (e.g. fast switching valves) and no complicated control algorithms have to be developed. Furthermore, it is expected that, due to the absence of electronic equipment, passive suspensions are more reliable than (semi-)active suspensions. Therefore, in the research presented in this thesis, a conceptual design for a novel passive suspension for a truck’s rear axle of a truck and semi-trailer is presented.

First, in Section 1.1, the problem is described, clarifying the conflicting design demands for a suspension if acceleration levels, necessary suspension working space and dynamic wheel loads all are to be optimised. In Section 1.2, the objectives of this research are described in more detail. In Section 1.3, the research strategy is discussed. Section 1.4 contains a description of the new topics of the suspension.

1.1 Problem description

For passenger cars, a suspension system has to meet two conflicting yet important design goals [e.g. Sharp et al., 1987], i.e. minimal chassis accelerations to improve comfort, and minimal dynamic wheel loads to optimise road holding. Such a suspension system has to take the constraint of spring travel into account. As explained before, for commercial vehicles the limitation on spring travel is not just a constraint, but a design goal in itself. To reduce the required suspension working space, minimal spring travel is desired. The resulting three design goals impose conflicting demands on the settings for the stiffness and damping values. For example, larger damping will result in lower amplitudes in the region of the axle eigenfrequencies and therefore in lower dynamic wheel loads and thus in improved road holding. This is normally accompanied with a decrease of driver comfort. In general, these conflicts can be expressed graphically as shown in Figure 1.3.
Between these three design goals, a compromise has to be found. At the definition of the settings for the suspension parameters (stiffness and damping coefficient), every car manufacturer chooses this compromise according to the company's philosophy. For example, a manufacturer of sports cars will opt for suspension settings which minimise the dynamic wheel loads and will accept the lower level of comfort. A manufacturer of luxury cars will opt for suspension settings which are optimised for driver comfort knowing that this is disadvantageous for the car's road holding. Truck manufacturers also have to take into account spring travel to make the necessary suspension working space as small as possible. As shown in Figure 1.4, by the area surrounded by the solid line, this controversial compromise can be expressed in an area of equal performance. For a certain type of suspension, the compromise has to be chosen in this area of equal performance. To meet the three design demands better than is possible with a conventional suspension, the area of equal performance must be shifted to a higher level (the area surrounded by the dashed lines in Figure 1.4). This requires a new suspension system.

The research presented in this thesis, will focus on stochastic excitations. For this type of excitation, conventional suspensions show in principle the same stiffness and damping values for every excitation frequency. However, in view of the fundamental function of a suspension system, different stiffness and damping values for different frequencies are preferred. The primary task of a suspension system is to isolate the vehicle body (with passengers and cargo) from the road excitations. Ideally, a suspension would not impose any surface related forces upon the vehicle body. For a quarter car model, which in contrast to more comprehensive models can only represent the bounce modes for axle and vehicle body, the ideal suspension would possess neither stiffness nor damping. However, since the static mass of the vehicle body must be carried, a finite stiffness at a zero frequency is required. Furthermore, for low frequency excitations, which normally involve large displacements, e.g. as introduced by viaducts, the vehicle body must follow the excitation to prevent axle-body contact or even body-ground contact. Thus, also for relatively low frequencies a certain stiffness is required. For higher frequencies this stiffness ideally vanishes, since there is no need for any stiffness. Moreover, at these frequencies the stiffness introduces unnecessary forces on the vehicle body, causing undesired body motions. Due to this stiffness, the vehicle body will have body modes in this frequency range. To damp such eigenmodes and to obtain desirable comfort-, acceleration- and spring travel levels, damping must be applied. The secondary task of a suspension system is to keep the dynamic wheel load within acceptable levels. To do this, damping is required around the eigenmode of the axle (approximately 10-15 Hz). For the frequencies outside the two mentioned frequency ranges, the suspension damping only imposes unnecessary forces upon the chassis causing unnecessary motions of the vehicle body. Therefore, for these frequency ranges, the damping should ideally vanish.
The ideal stiffness and damping values are pictured in Figure 1.6 in which $k$ and $b$ represent stiffness and damping respectively.

![Figure 1.6: Stiffness and damping demands for a vehicle suspension in a quarter car model](image)

Looking at these two diagrams, it is evident that one must search for a suspension element with a frequency dependent behaviour of as well stiffness as damping. The research presented in this thesis deals with a suspension system which have stiffness and damping coefficients that vary with the excitation frequency.

### 1.3 Research objectives

The objective of the present study is to determine the maximum performance improvements - compared to a representative passive suspension without frequency dependent characteristics - obtainable with a suspension element with frequency dependent stiffness and damping properties. This element forms a part of the rear axle suspension of the truck and semi-trailer combination. To create a complete overview of the improvement in performance, the following five criteria will be addressed at:

- driver comfort,
- cargo accelerations,
- fatigue loads,
- spring travel,
- dynamic wheel loads.

Each of these objectives is optimised and the maximal performance improvement is determined under the constraint that the other objectives get not worse compared to the reference vehicle. The reference vehicle is a vehicle with the conventional suspension system. In this way, the improvements, achieved by the new suspension system, are determined.
1.5 What's new?

The concept of a frequency dependent suspension is not entirely new [e.g. Richardson, 1984]. However, the use of the inertia of a fluid oscillating in a channel to make the stiffness and damping coefficients of the suspension frequency dependent is new. This concept involves the addition of a mass effect without introducing heavy components to the sprung or unsprung weight of the vehicle. Due to the inertia of the fluid, stiffness and damping depend upon the frequencies the suspension is excited with. To our knowledge this principle has not been used before in a truck axle suspension. Furthermore, the suspension contains a number of adjustable parameters to control its dynamic behaviour for different driving conditions, like changes in vehicle speed, amount of cargo load and kind of road surface the vehicle is driven on.

Chapter 2

Suspension selection

Following a short survey of alternative suspension designs, a systematic analysis for a new suspension concept will be presented in this chapter. As a result of this analysis, four distinct suspension concepts are derived for which the dynamic behaviour is determined. The purpose of the project is to actually build, test and analyse the suspension system. Therefore, design concepts for these four suspension systems are presented. Finally, one of the suspension concepts is selected and it forms the subject of the remaining research presented in this thesis.

2.1 Alternative suspension designs

In addition to existing standard suspension systems, consisting of standard dampers and springs, many concepts for alternative suspension designs have been proposed in the past decades. These concepts can be subdivided into three classes, alternative damper designs, alternative spring designs and systems which combine these two functions in a single design.

2.1.1 Alternative damper designs

The schematic principle of a mono-tube damper is drawn in Figure 2.1.

![Schematic representation of a conventional suspension damper](image-url)
Chamber 2 in Figure 2.1). This piston contains small orifices through which the fluid may flow. To ensure the proper functioning of the damper, these orifices can be blocked by valves depending on the relative displacement of the piston relative to the cylinder. These restrictions determine the characteristics of the damper. The valves are pressed into their seats with springs (not shown in Figure 2.1). In the damper, an extra free-moving Piston 2 is placed at the bottom of the cylinder. Since in practical terms, the fluid is nearly incompressible, the air below the free-moving piston compresses or decompresses and compensates in this way for the change of volume that occurs when the piston rod moves within the cylinder.

To adjust the damping characteristics to varying driving conditions, alternative damper designs are regularly used. A change in damping characteristics is desired when the load condition or the vehicle speed changes or when the driver prefers a more comfortable or sportive ride. Adjustable dampers can be divided into three different categories. The first category makes use of adjustable valve springs [e.g. Steinbuch, 1969]. Adjustment of these springs is either done manually by the driver or by an automatic control system. For instance, in a suspension system equipped with air springs, the pressure in the air springs is adapted when the vehicle load changes. This pressure is then used to adjust the damper [e.g. Bastow, 1980]. The second category makes use of an additional bypass between Chamber 1 and Chamber 2. This bypass is placed in parallel with the normal valves and can be opened or closed using an additional valve, in this way adjusting the damper's characteristic [e.g. Reimpell et al., 1989; Richardson, 1984; Karnopp, 1974]. The control valve is adjusted electrically, manually or pneumatically. The third category makes use of a viscosity change of the damper fluid. The viscosity of the fluid is changed for instance by putting a voltage over the specific fluid [Pinkos et al., 1994; Heyer, 1988]. Due to the viscosity change, the damper characteristics change.

An interesting variant of the second category is the frequency dependent damper proposed by Richardson [1984] shown in Figure 2.2. Here, a bypass containing a valve is connected to the damper. This valve is connected to a mass which on its turn is connected to a spring. The spring in turn is connected to the sprung mass of the vehicle. For frequencies close to the sprung mass eigenfrequency, this mass resonates on the spring, thereby closing the valve. This results in higher damping in this frequency range.

Another type of damper has damping characteristics which depend on the amplitude [e.g. Lörh, 1991]. In this type, when the damper exceeds a certain stroke, a bypass is opened or closed to change the damper coefficient.

Finally, dynamic 'dampers' have to be mentioned which rely on the action of an oscillating mass. Strictly speaking, these are not dampers because an ideal dynamic damper dissipates no energy. The mass is placed on a spring which in its turn is attached to the wheel. Well known is the system used on the early Citroën 2CV. This principle is also discussed by Voy [1988]. Depending on the spring stiffness and the weight of the mass, the mass resonates at a specific frequency and counteracts the motions of the axle or vehicle's body.

2.1.2 Alternative spring designs

Alternative adjustable springs often rely on air springs which have additional volumes that can be closed or opened for specific driving conditions. Alternative designs are e.g. mentioned by Schutzner et al. [1994], describing the double air spring of Bridgestone [1992] and triple air spring of Gold [1990]. The air springs consist of two and three chambers, respectively, which are connected by small air channels. Due to the inertia of the air in the channels, the channels close for relatively high frequencies. This decreases the effective air volume which increases the stiffness at these higher frequencies.

2.1.3 Alternative suspension designs

An example of an alternative suspension design is a hydropneumatic system as, for example, used by Citroën [e.g. Bastow, 1980]. Here, an air spring and a damper are combined in a single design. The advantage of applying an air spring is the possible use of a load adjusting levelling system. By adjusting the amount of air in the spring or oil in the damper element, the
clearance between the road and vehicle body can be kept constant for different vehicle loads. Furthermore, for heavier laden vehicles, the suspension stiffness increases due to the compressed air in the air spring. Because of this increase in stiffness and higher mass, the vehicle's body eigenfrequency stays approximately the same, which makes the comfort level independent of the load condition. In a hydropneumatic suspension, a piston moves up and down in a cylinder which is filled with a fluid. On top of the cylinder an air spring is mounted as shown in Figure 2.3.

In the sphere, the fluid is separated from the gas by means of a membrane. In top of the cylinder, a restriction is placed. The air functions as a spring while the fluid flowing through the restriction functions as a damper. More recently, variations on the basic concept have been presented [Els et al, 1993; Löh, 1991].

A further development on this principle is the so-called PAWS system (P.A. Wilkinson Suspension), developed by the Motor Industry Research Association [Wilkinson, 1992]. A schematic representation of this suspension is shown in Figure 2.4.

In this system, two hydropneumatic suspensions are coupled in such a way that the damping decreases with increasing frequency. For this suspension, improved body accelerations and lower dynamic wheel loads are claimed.

For a more extensive description of a part of the mentioned systems, the reader is referred to the report by W. Smits van Oyen [1989].

2.2 Search for an alternative suspension system

The purpose of this chapter is to determine an alternative suspension system whose stiffness and damping properties correspond to the ideal properties formulated in Section 1.2. As this research only deals with stochastic road inputs, the behaviour of the suspension system is analysed in the frequency domain only. To approach this ideal suspension behaviour, the conventional suspension either has to be extended, or replaced by a completely new system. The search for a new suspension can be done in two ways. Elements with a non-linear behaviour may be used to improve the dynamic vehicle behaviour, or elements with a linear behaviour relying on dynamic effects. In this project the latter approach was taken. The approach consists of allowing the suspension system to have more than the current two degrees of freedom. Here, a system with only one additional degree of freedom has been chosen for the following reasons:

- the complexity of the system increases with increasing number of degrees of freedom,
- to limit expenditures in future designs,
- for practical reasons (e.g. required simulation times in case of more complex designs).
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Of course, this limitation may affect the improvements obtained. At this stage in the search for a new suspension system, this is considered to be acceptable. The question now to be answered is, what types of frequency dependent stiffness and damping coefficients, can be realised with a combination of linear elements (springs, dampers, masses)? Within this approach, two different suspension principles are possible.

In the first principle, the new suspension system is placed between the axle and the vehicle body as sketched in Figure 2.5.

![Figure 2.5 First principle](image)

The dynamic behaviour of the suspension system depends on the displacement of the axle relative to the vehicle body. Typical examples of this type of suspensions are springs and dampers.

In the second principle, the static mass of the vehicle is carried by a spring while dynamic 'dampers' are used to dampen the eigenmodes of the axle or the vehicle body. A dynamic damper consists of a mass placed on a spring which, in its turn, is placed on the body whose eigenmode has to be controlled (Figure 2.6). The dynamic behaviour of the suspension not only depends on the displacement of the axle relative to the vehicle body. It also depends on the movement of the vehicle body or axle.

![Figure 2.6 Second principle](image)

Suspension selection

The additional mass and the spring stiffness are chosen in such a way, that the eigenfrequency of the mass-spring combination is identical to the eigenfrequency of the body. When the system resonates at its eigenfrequency, the accompanying forces reduce the body motions. A disadvantage of dynamic dampers is that two systems are necessary to reduce both the vehicle body and axle displacements. A practical example of a dynamic damper is the Citroën 2CV system, used in the early models. The eigenmode of the wheels was reduced with a dynamic damper while the body eigenmode was damped with a friction damper.

To reduce the amplitudes of the body and axle around their eigenfrequencies, the forces generated by the resonating masses must be of the same order of magnitude as the inertia forces caused by the vehicle body or axle motions. This means that either the added masses must be of the same order of magnitude as the body or axle mass, or a smaller mass has to be subjected to higher accelerations to achieve similar dynamic forces. In the latter case, large amplitudes are required and thus a relatively large space. In the former case, the total vehicle mass will increase significantly which is considered to be unacceptable because less cargo load can then be transported within the legal constraints on vehicle weights. Due to these two reasons, the principle of adding dynamic dampers is rejected.

As a result, only systems based on the first principle are considered to be eligible for further investigation. As mentioned before, for a suspension of this type, the dynamic behaviour depends on the displacement of the axle relative to the vehicle body. Before investigating such systems in more detail, some definitions concerning the dynamic behaviour are introduced.

2.2.1 Definitions

To describe the transfer functions of the dynamic systems, first two important quantities have to be defined. These are the so-called 'dynamic stiffness' and the so-called 'dynamic damping' and are both a function of the excitation frequency. Within this thesis, a harmonic excitation $x(t)$ will be expressed in complex notation as:

$$x(t) = \text{Re}[\tilde{x} \cdot e^{j\omega t}]$$

(2.1)

In this expression, $t$ represents the time, $\tilde{x}$ represents the amplitude of the excitation signal and $j$ represents the square root of -1. Furthermore $\omega$ represents the angular frequency (being $2\pi f$, with $f$ being the frequency in Hz) and Re the real part of its argument.

If a dynamic system is excited with a displacement $x(t)$, the force $F(t)$ can be related to this displacement by calculating the transfer function $H(f)$ which describes the relation between the two quantities $\tilde{F}$ and $\tilde{x}$, with $\tilde{F}$ and $\tilde{x}$ being the amplitudes of $F(t)$ and $x(t)$, respectively. When exciting a dynamic system with two degrees of freedom (see Figure 2.7), transfer functions $\tilde{F}_1/\Delta x_1$, and $\tilde{F}_2/\Delta x_2$ can be calculated, where $\Delta x_1 = x_1 - x_2$ and $\Delta x_2 = x_2 - x_1$ are...
the relative displacements and where $F'_1$ and $F'_2$ are the amplitudes of the resulting forces. The dynamic stiffness is defined as the real part of the transfer function while dynamic damping is defined as the imaginary part of the transfer function divided by the angular frequency $2\pi f$. For the system as shown in Figure 2.7, which consists of a single spring and damper, the dynamic stiffness is for every frequency identical to the spring stiffness while the dynamic damping is identical to the damping coefficient.

$$H(f) = \frac{F'_i}{\Delta x'_i} = \frac{k + b2\pi f}{\Delta x'_i}$$

- transfer function: $H(f) = \frac{F'_i}{\Delta x'_i}$ (i=1, 2)
- dynamic stiffness: $k_{sd}(f) = \text{Re}\{H(f)\}$
- dynamic damping: $b_{sd}(f) = \frac{\text{Im}\{H(f)\}}{2\pi f}$

**Figure 2.7 Definitions of dynamic stiffness and damping**

Other definitions used in this thesis, pertain to the semi-static stiffness and damping. The semi-static stiffness is defined as the dynamic stiffness at a frequency of 0.2 Hz while the semi-static damping is defined as the dynamic damping value at a frequency of 0.2 Hz. This frequency was chosen since it is the lowest frequency used in the calculations in this thesis.

### 2.2.2 Suspension requirements

The task of a suspension was explained in Chapter 1 and the optimal dynamic stiffness and dynamic damping were derived for a quarter car model. Besides providing frequency dependent stiffness and damping characteristics, the new suspension system has to support the vehicle while it is stationary or maintain certain spring travel levels for incidental excitations. To achieve this, at this point in the research, the semi-static stiffness and damping are set to the values of the conventional suspension.

The requirements on the suspension are pictured in Figure 2.8, where $k_{sd}$ and $b_{sd}$ are the stiffness and damping of the standard vehicle. However, at this point of the research, it's worthwhile to realise, that a dynamic system as given in Figure 1.6 never can be actualised in practice. For a further explanation, the reader is referred to Appendix A.

**Figure 2.8 Ideal stiffness and damping suspension properties in a quarter car model**

#### 2.2.3 Combining springs, dampers and a mass

A number of springs, dampers and masses can be combined in various ways, resulting in a variety of dynamic systems with a variety of dynamic characteristics. Here, a systematic analysis is presented to identify dynamic stiffness and damping characteristics that can be realised. As stated before, the analysis is restricted to systems with linear springs and linear dampers and having not more than three degrees of freedom. First, the combination of only springs and dampers is discussed, afterwards, the addition of a mass is considered.

Starting with a single spring and a single damper, these components can be placed in series or in parallel. When adding a damper or spring to this system, these new components can again be placed in series or in parallel. This procedure can be repeated over and over again as shown in Figure 2.9 (next page).

Some of the systems shown in Figure 2.9 have identical basic characteristics. The properties of two dampers placed in parallel can be matched by one single damper. Also, the properties of two springs placed in parallel can be matched by one single spring. Bearing this in mind, Systems 3 and 5 are equivalent to System 1. Systems 8 and 10 are equivalent to System 2. Furthermore, Systems 8 and 10 have four degrees of freedom and are thus beyond the scope of the research presented in this thesis. The analysis of the remaining suspension systems reveals that two different types of frequency dependent curves for the dynamic stiffness and dynamic damping can be distinguished (Figure 2.10). The parallel setting of a single spring and damper (the standard suspension system, e.g. System 1 in Figure 2.9) shows a constant dynamic stiffness and dynamic damping. Suspensions that show this behaviour will further be denoted with Suspension System Group A. All other systems (systems in which a damper is placed in series with a spring, e.g. System 6 in Figure 2.9) exhibit a frequency dependent behaviour which causes the dynamic stiffness to increase and the dynamic damping to decrease, with increasing frequency. For increasing frequencies, both dynamic stiffness as well as dynamic damping assume asymptotic values [de Ruiter, 1993, (1)]. Suspensions that show this behaviour will be denoted with Suspension System Group B.
By adding a mass to these spring and damper systems, the dynamic stiffness and damping characteristics of these systems can be altered at specific frequencies. Especially at frequencies in the region of the eigenfrequency of the dynamic system, the dynamic behaviour will be affected by the so-called mass effect. For the dynamic systems considered (with a maximum of three degrees of freedom), a significant mass effect can only be achieved when the mass is located at the position shown in Figure 2.11. Locating the mass at degree of freedom $x_1$ would mean that the mass is added to the vehicle body, locating the mass at degree of freedom $x_2$ would mean that the mass is added to the axle.

To obtain a significant mass effect, the inertia forces arising from the movement of the mass must be of the same order of magnitude as the inertia forces introduced into the suspension.
due to the chassis or axle movements. To achieve this, the weight of the mass should be in the same order of magnitude as the axle or vehicle body. First, this mass addition is not desired, because it makes the vehicle heavier. Second, a mass effect is generated when the acceleration of the body relative to the axle is equal to zero, assuming that the accelerations themselves are desired which generates no forces when the relative acceleration equals zero. This can be due to the chassis or axle movements. To achieve this, the weight of the mass should be in nonzero.

In this case, large inertia forces are generated. This means that a dynamic system is desired which generates no forces when the relative acceleration equals zero. This can be accomplished using a special flywheel effect [R. Liebregts, 1990] shown in Figure 2.12.

At this stage of the design, this so-called flywheel model, is only hypothetical. The flywheel only has an inertia around its rotation axle and has no inertia in the translational direction of the suspension system. Using this system, the relative displacement \( x_1-x_2 \) is converted into a rotation \( \phi_1 \).

At the left hand side of Figure 2.12, the complete flywheel model is pictured, while the system converting the relative displacement into a rotation is pictured on the right hand side. In this figure, the four degrees of freedom \( x_1, x_2, \phi_1 \) and \( \phi_2 \) are shown. The system functions as follows. Bar \( a \) only moves in \( z \)-direction and is connected to Bar \( b \) by means of Hinge 1. Bar \( b \) is allowed to move only in the \( xz \)-plane. It is connected to Bar \( c \) by means of Hinge 2. Bar \( c \) is allowed to move only in the \( xz \)-plane and is connected rigidly and at right angles to Bar \( d \) which is placed parallel to the \( y \)-axis. The rotation of Bar \( c \) around Hinge 3 causes Bar \( d \) to rotate around the \( y \)-axis. In this way, a translation of Bar \( a \) in \( z \)-direction is converted into a rotation of Bar \( d \) around its centreline which points in the direction of the \( y \)-axis. For instance, if the displacement of Bar \( e \) is equal to zero and Bar \( a \) moves downwards, Hinge 1 will move downward also. Because Hinge 3 stays at the same position, Hinge 1 and Hinge 3 will move towards each other and Bars \( b \) and \( c \) will rotate in the \( xz \)-plane. Due to the rotation of Bar \( c \), Bar \( d \) will rotate around the \( y \)-axis. The rotational degree of freedom \( \phi_2 \) can now be expressed in terms of the translational degrees of freedom \( x_1 \) and \( x_2 \) (for small \( x_1-x_2 \)) by means of the following equation,

\[
\phi_2 = \frac{x_2 - x_1}{l}
\]

where \( l \) is the length of the flywheel arm. As a result, three degrees of freedom remain, which equals the number of degrees of freedom of the system pictured in Figure 2.11.

If \( x_2 \) is fixed for the system in Figure 2.11 and for the system with the flywheel, both systems are equivalent when identical values are chosen for the parameters \( k_1, k_2, b_1, b_2 \) and \( m \). For the flywheel model, these parameters must be multiplied by the square of the length \( l \) of the flywheel arm to obtain the proper dimensions.

Further investigation must reveal what kind of dynamic stiffness and damping characteristics can be achieved, with the flywheel system. In the analysis, the flywheel is always connected to earth via a damper. This is inherent of the design chosen for the suspension system (as will be explained in Subsection 2.3.3). Furthermore, to create the desired mass effect by means of the resonance of the flywheel, at least one spring must be attached to the flywheel. The complete flywheel suspension system must also have a proper static stiffness. All systems complying with these demands are shown in Figure 2.13 (next page).

Of these five configurations, Configurations 1 and 2 do not have a damper linking the rotational degrees of freedom \( \phi_1 \) and \( \phi_2 \). The difference between these two configurations is, that for the latter, a spring is located between the rotational degree of freedom \( \phi_2 \) and the earth. For Configuration 2, the two springs which are placed in series, satisfy the requirement concerning the static stiffness. For Configuration 1, an extra spring is required between \( x_1 \) and \( x_2 \), or the static stiffness of this configuration will vanish. Configurations 3, 4 and 5, all have a damper located between the rotational degrees of freedom \( \phi_1 \) and \( \phi_2 \). Three possible configurations remain with different spring combinations. Configurations 3 and 4 need a spring between the degrees of freedom \( x_1 \) and \( x_2 \) to achieve a static stiffness.
These five configurations yield only two different characteristics for the dynamic stiffness and damping. The first, denoted Suspension System Group C, belongs to the configurations without a damper between the rotational degrees of freedom (Configuration 1 and 2). The second, denoted Suspension System Group D, does have a damper between the rotational degrees of freedom (Configurations 3, 4 and 5). The dynamic stiffness and damping for Suspension System Groups C and D are shown in Figure 2.14. For both systems, three characteristic parts can be distinguished in the dynamic stiffness and damping curves.

1. At frequencies lower than the eigenfrequency; the flywheel rotates in phase with the excitation. The dynamic stiffness and damping are largely determined by the spring/damper elements between the rotational degrees of freedom \( \phi_1 \) and \( \phi_2 \) and the spring/damper elements between the degree of freedom \( \phi_2 \) and earth.

2. At frequencies near the eigenfrequency of the flywheel, the following occurs: Just below the eigenfrequency, the flywheel moves in phase with the excitation, increasing the excitation amplitude and thus reducing the dynamic stiffness. For frequencies just above the eigenfrequency, the flywheel rotates out of phase with the excitation which increases the dynamic stiffness.

3. At frequencies well above the eigenfrequency; the flywheel does not rotate due to its inertia. The dynamic stiffness and dynamic damping are largely determined by the spring/damper elements between the rotational degrees of freedom.

### 2.2.4 Summary of the four dynamic stiffness and damping characteristics

For dynamic systems with linear springs, linear dampers, a mass (or inertia in the form of a flywheel), and at most three degrees of freedom, four different characteristics can be distinguished. Dynamic stiffness and damping characteristics for these four systems are summarised in Figure 2.15 (next page). Also, in this figure, examples of dynamic systems yielding these stiffness and damping curves are given. Quantitative differences for relatively low and high excitation frequencies are given in Table 2.1.

<table>
<thead>
<tr>
<th>System</th>
<th>( k_{dyn} = k )</th>
<th>( b_{dyn} = b )</th>
<th>( k_{dyn} = k )</th>
<th>( b_{dyn} = b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>( k_{dyn} )</td>
<td>( b_{dyn} )</td>
<td>( k_{dyn} )</td>
<td>( b_{dyn} )</td>
</tr>
<tr>
<td>C</td>
<td>( k_{dyn} )</td>
<td>( b_{dyn} )</td>
<td>( k_{dyn} )</td>
<td>( b_{dyn} )</td>
</tr>
<tr>
<td>D</td>
<td>( k_{dyn} )</td>
<td>( b_{dyn} )</td>
<td>( k_{dyn} )</td>
<td>( b_{dyn} )</td>
</tr>
</tbody>
</table>

(Note that for Suspension System Groups B, C and D, the dynamic stiffness for very high frequencies is higher than the semi-static stiffness).
2.2.5 Discussion of the four suspension systems

Referring to Figure 2.15 and Table 2.1, Systems B, C and D will be discussed. System A is not discussed, because it is the reference suspension system.

Suspension System Group B

Suspension System Group B does not meet the required dynamic stiffness and damping curves shown in Figure 2.8. Although the large damping for low frequencies could be advantageous for comfort, accelerations and spring travel, its stiffness curve doesn't correspond at all with the desired curve. Nevertheless some preliminary research was done. This suspension system was implemented in a three-dimensional vehicle model of the truck and semi-trailer in parallel with the standard damper. Simulations were performed for several parameter values for the spring and damper characteristics. For these simulations, the vehicle was driven over a rather bad road surface with a vehicle speed of 50 km/h.

The semi-static stiffness was set at the semi-static stiffness of the standard suspension. This causes the stiffness at high frequencies to be larger than the stiffness of the standard suspension. For low frequencies, the damping of this suspension is higher than that of the standard suspension. Four different settings for \( k_1 \) (Figure 2.15), varying between the stiffness of the standard suspension and twice this stiffness, as well as four different settings for the damper were used. Parameter \( k_2 \) was then calculated to make the semi-static stiffness of the suspension identical to the semi-static stiffness of the conventional suspension using the following equation.

\[
k_{sw} = \frac{k_1 \cdot k_2}{k_1 + k_2}
\]  

(2.3)

where \( k_{sw} \) is the semi-static stiffness of the conventional suspension.

For all 16 combinations, power spectral density functions for comfort, chassis accelerations, spring travel and dynamic wheel load were calculated. The dynamic stiffness and damping characteristics for which the best results were obtained are shown in Figure 2.16.

Due to the larger damping at low frequencies, the power spectral density function for spring travel is improved. For example, the RMS value for spring travel has improved with 13.5 %. However the RMS values for the chassis accelerations increased by an average of 5%, especially in the higher frequency range. Due to the slightly higher damping around the axle's eigenfrequency, the RMS value for dynamic wheel load has improved with 3.1 %. For a truck driven over a German motorway, similar results were found.
Optimal design of a hydro-elastic rear axle suspension for heavy trucks

To illustrate the effect of this suspension, three power spectral density functions are shown. In Figure 2.17, the spectra for spring travel, the kingpin\(^1\) acceleration spectrum in z-direction, and dynamic wheel load spectrum are given. The dashed lines represent the conventional vehicle while the solid lines represent the vehicle with the new suspension system.

\[ \text{Spring travel} \]
\[ \text{Kingpin acceleration} \]
\[ \text{Dynamic wheel load} \]

\[ \text{Figure 2.17 Spectra for spring travel, kingpin acceleration and dynamic wheel load} \]

From the Figure, it can be seen that for relatively low frequencies, both spring travel and kingpin accelerations improve which is caused by the increased damping. For higher frequencies, the acceleration levels increase due to higher stiffness and damping. From these results, it was concluded that this system only shows benefits for spring travel at the expense of higher acceleration levels of the vehicle’s body.

**Suspension System Group C**

Compared to Suspension System Group B, Group C includes an additional mass which creates more freedom to manipulate the dynamic stiffness and damping curves. In contrast to Group B, Group C is capable of decreasing the stiffness over a specific frequency range. This effect can be tuned to act at different frequency ranges, while with Group B only the lower frequency range can be affected. However, a decrease of stiffness is always followed by an increase of stiffness at higher frequencies which is undesirable. Evidently, the frequency range in which the stiffness is reduced, must be selected carefully to allow the decrease of stiffness to have a significant impact on the considered spectra while allowing the increase of stiffness to have a small impact. For example, in the acceleration power spectral density function, larger stiffness results in larger power spectral density values while smaller stiffness values result in smaller power spectral density values which is shown in Figure 2.18. For the frequency range with the

\[ \text{Figure 2.18 Performance improvement and loss} \]

The high damping of Group C is expected to be advantageous for the body accelerations at low frequencies. At frequencies in the region of the axle’s eigenfrequency, it can be used to decrease the dynamic wheel load [Ruiter, 1996, (2)]. Consequently, the dip in the stiffness curve and peak in the damping curve are advantages for Suspension System Group C.

**Suspension System Group D**

Compared to Suspension System Group C, Group D has one additional design parameter. Therefore, the range of achievable dynamic stiffness and damping curves is expected to be larger. With the additional damper, it appears possible to reduce the peak in the stiffness curve of Group C. As a result, the negative influence of the stiffness peak will be less than for Group C. Also, the stiffness at high frequencies is smaller than that of Group C (Table 2.1) which is advantageous. A significant and obvious disadvantage of Group D is the higher damping at high frequencies (Table 2.1). At such high frequencies, the forces introduced into the vehicle body are mainly damping forces which will increase with increased dynamic damping. Such high damping forces result in larger accelerations and an increased noise level in the vehicle.

In principle, the benefits of Suspension System Groups C and D can be investigated with the aid of numerical simulations. However, due to the large number of parameters, a very wide range of dynamic stiffness and damping curves should be considered. Such a large number of simulations would take a lot of time. For this reason, first other important aspects will be taken into account before a final choice will be made. Apart from the theoretical

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\(^1\) the kingpin is the point were the semi-trailer is attached to the truck, this point is located close to the truck’s rear axle.
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considerations, the final choice of the suspension system also depends on practical considerations. The realisation of these systems is restricted by physical limitations (e.g. mass, number of required components, etc.) which are of great importance for the final choice.

2.3 Building the suspension in practice

Before designing one, or more, of the suspensions previously described, the practical demands with which these suspensions have to comply are discussed. Based on these demands, concept designs are presented for the systems B, C and D.

2.3.1 Design demands

The most important demands for a truck suspension are:

1. incorporation of a levelling system,
2. low additional weight,
3. small required space,
4. high reliability,
5. low additional costs,
6. simple construction,
7. exchangeability with standard suspension.

2.3.2 Practical realisation of spring, damper and mass effect

To fulfil the demand for a levelling system, a closed air volume will be used to act as a spring. For the required damping, either a standard damper or fluid flowing through restrictions in combination with the air spring can be used. The symbols used in the figures in this and the following sections, are explained in Figure 2.19 (which in fact is a hydraulic representation of Suspension System Group A). In the analysis of the concept designs, it is assumed that the air spring behaves like a linear spring and the restriction behaves like a linear damper.

As mentioned before, to achieve a significant mass effect, the inertia forces arising from the movement of the mass must be of the same order of magnitude as the inertia forces introduced into the suspension due to the movements of the chassis or axle. Such high mass forces can either be achieved by a large mass subjected to small accelerations, or a small mass subjected to large accelerations. To keep the weight down, the latter option is preferred. The corresponding accelerations must be larger compared with the accelerations of the chassis or the axle. This requires the application of a lever system, as sketched in Figure 2.20.

However, such a lever requires a relatively large space, which is in conflict with Design Demand 2. Furthermore, not only the lever but also the oscillating mass requires space. Another option would be the flywheel model itself. However, because this is only a special variant of the lever system, this option is also rejected.

To limit the required space, a so-called hydraulic lever can be used, as shown in Figure 2.21. The principle of the inertia of a fluid in a small channel which is excited by a fluid in a bigger volume is also used in the so-called hydromounts [see e.g. Freudenberg, 1988]. These hydromounts are widely in use as frequency dependent engine mounts.
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If there is a displacement of degree of freedom \( x_2 \) relative to degree of freedom \( x_1 \), fluid in the volume with cross sectional area \( A \) will be pressed into the fluid channel with the smaller cross sectional area \( a \). As a result, the accelerations of the fluid in the channel and the fluid in the volume are inversely proportional with their respective cross sections. The total mass effect of the fluid then depends on the amount of fluid in the fluid channel (depending on diameter and length of the channel) and the proportion between the two cross sections. Using this principle, designs can be generated for Systems B, C and D.

2.3.3 Design for Suspension System Group B, C and D

Suspension System Group B (Figure 2.15) can be constructed in the way shown in Figure 2.22. On the left hand side of this figure, and the other figures in this subsection, the mechanical analogon of the system is drawn, while on the right hand side of the figure, the concept design is shown. Suspension System Group B appears to be similar to the PAWS system proposed by Wilkinson [1992]. The air spring \( k_1 \) is placed in parallel with restriction \( b_1 \) which are placed in series with air spring \( k_2 \). This combination is placed in parallel to damper \( b_2 \). In this design a potential mass effect in the fluid channels is neglected. Hence, the fluid channels should be as short as possible.

Using the principle of an oscillating mass in a channel, damping always occurs when the mass is moved relative to the suspension itself.

Suspension selection

The system of Figure 2.23 is extended with a damper to yield a concept design for System D (Figure 2.24). The first element functions as a spring and damper placed in parallel. This element can move in a cylinder, acting as a piston and exciting the fluid mass in the channel.

Figure 2.22 Practical design for System B

To the concept of Figure 2.19, a mass effect is added to arrive at a practical design for System C (Figure 2.23). The channel connecting the first spring \( k_1 \) with the second spring \( k_2 \) is elongated to incorporate sufficient fluid to generate the desired mass effect.

In this suspension design, no moments are introduced into the vehicle body. Because of this, the spring in the flywheel model is attached to earth. When the suspension is moved as a whole, the fluid mass also has an inertia in vertical direction. In the flywheel model, this inertia is neglected (the suspension itself is assumed to be massless). The fluid channel acts as a restriction and extra restrictions are not necessary. For this reason, a damper is always placed between the flywheel and ground (which was an assumption for the systems in Figure 2.13).
2.4 Suspension system selection

Based on the available information, a selection has to be made between the three suspension systems. The advantages and disadvantages of the three systems are summarised in Table 2.2. A division has been made for the match of the stiffness curve, the match of the damping curve and the design complexity.

<table>
<thead>
<tr>
<th>Stiffness curve</th>
<th>Suspension System Group B</th>
<th>Suspension System Group C</th>
<th>Suspension System Group D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bad match with</td>
<td>better match with</td>
<td>best match with</td>
</tr>
<tr>
<td></td>
<td>desired curve,</td>
<td>desired curve</td>
<td>desired curve</td>
</tr>
<tr>
<td></td>
<td>negative for chassis</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>accelerations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Damping curve</th>
<th>Suspension System Group B</th>
<th>Suspension System Group C</th>
<th>Suspension System Group D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>positive for low</td>
<td>positive around body</td>
<td>negative considering</td>
</tr>
<tr>
<td></td>
<td>frequencies,</td>
<td>or axle</td>
<td>the aspect of noise in</td>
</tr>
<tr>
<td></td>
<td>especially for spring</td>
<td>eigenfrequency</td>
<td>the vehicle</td>
</tr>
<tr>
<td></td>
<td>travel</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Concept design</th>
<th>Suspension System Group B</th>
<th>Suspension System Group C</th>
<th>Suspension System Group D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low complexity of the</td>
<td>middle complexity of the</td>
<td>high complexity of the</td>
</tr>
<tr>
<td></td>
<td>concept design</td>
<td>concept design</td>
<td>concept design</td>
</tr>
</tbody>
</table>

Table 2.2 Advantages and disadvantages of the three systems

Within the CASCOV project, Suspension System Group C was selected for further investigation. The analysis in this chapter confirms this choice. Suspension System Groups B and D are rejected for the reasons given in the shaded boxes in Table 2.2. Suspension System Group C is therefore the object of the further investigations.

Chapter 3

Modelling the vehicle, the road surface and the suspension

To obtain correct parameter settings for the new suspension, numerical simulations have been performed. For these simulations, three models are necessary. To simulate the dynamic behaviour of the truck and semi-trailer accurately, a three-dimensional model of the vehicle has been used. In addition, a model describing the road irregularities which excite the truck and semi-trailer and a model to describe the dynamic behaviour of the new suspension system have been used. These three models are presented in this chapter.

3.1 Model of the truck and semi-trailer

The dynamic response of a truck and semi-trailer combination exposed to stochastic road surface excitations is evaluated in the frequency domain. In view of the objectives of the present research, the major part of the relevant eigenmodes of the truck and semi-trailer lie within the frequency range of 0-20 Hz. Consequently, the analysis is confined to this frequency range. The vehicle model is a three-dimensional model which contains only linear components. A two-dimensional model would take less calculation time, but there are two reasons why such a model is not sufficiently accurate to describe the dynamic behaviour of the truck and semi-trailer:

1. the vehicle is not symmetrical (e.g. the heavy fuel tank is attached to the right hand side of the vehicle),
2. especially for short longitudinal waves of the road surface, the difference between the excitation of the left and right wheels is not negligible.

The road irregularities used in this thesis are based on measurements [Braun et al., 1991] and will be described further on in this chapter. Important at this point is, that these measurements showed that the height of the road irregularities correspond with a Gaussian distribution. With the mean of this distribution being zero, the RMS value of the measured signal equals the standard deviation and determines the percentage of time in which the road surface irregularities remain within a specific limit. For 95.34% of the time, the road surface irregularities are smaller than twice the standard deviation [e.g. Papoulis, 1991]. For the two roads, a minor road and a motorway, and the considered vehicle speeds used in this thesis, these RMS values are respectively 2.1 cm and 0.51 cm and are relatively small compared to incidental excitations. In view of these relative small excitations, a linear model was considered sufficiently accurate to describe the dynamic behaviour. Moreover, the use of a linear model offers significant advantages for calculations in the frequency domain.
Optimal design of a hydro-elastic rear axle suspension for heavy trucks

The finite element model of the truck and semi-trailer is generated in the FEM package MSC/NASTRAN\(^2\). Subsection 3.1.1 describes the simulation model of the truck and semi-trailer and is followed by an experimental validation in Subsection 3.1.2.

3.1.1 Description of the vehicle and the three-dimensional linear vehicle model

A schematical representation of the truck and semi-trailer combination used for the simulations, is shown in Figure 3.1.

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**Figure 3.1 The truck and semi-trailer**

The truck has two axles, a front and a rear axle, and the trailer has three axles. The suspension of the truck's front axle is of the leaf spring type, while its rear axle suspension contains air springs. Because the novel suspension system is to replace the current rear axle suspension system, the rear axle and suspension will be discussed in more detail. The rear axle is of the rigid kind (see Figure 3.2) and is connected to the chassis by means of four air springs, two dampers, one anti-roll bar and three axle guidance components, two so-called lower reaction rods and one triangle. This suspension is illustrated in Figures 3.3 and 3.4.

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**Figure 3.2 The rear axle including brake drums**

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**Figure 3.3 Side view of the rear suspension**

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**Figure 3.4 Top view of the rear suspension**

The lower reaction rods are connected to the chassis and to the bottom of the axle, one on each side of the vehicle. The triangle is connected to the chassis and to the top of the axle. Due to the presence of these guidance components, the axle can only perform bounce and roll motions (neglecting the elasticities in the chassis and guidance components). The four air springs provide the mounting in vertical direction, while the anti-roll bar largely suppresses rolling of the vehicle.

The three-dimensional vehicle model, developed by Van Asperen [1991], will be discussed briefly. The main components of the truck which are modelled are:

- chassis,
- rear axle with two tyres on each side,
- rear axle suspension consisting of four air springs, two dampers, axle guidance components and an anti-roll bar,
- front axle with one tyre on each side,
- front axle suspension with two sets of leaf springs, two dampers and an anti-roll bar,
- cabin and cabin suspension with four air springs, four dampers and an anti-roll bar,
- engine, gearbox, drive train, and engine suspension on the chassis,
- spare wheel, battery pack, fuel tank and trailer coupling.

With respect to the trailer, the following components are modelled:

- chassis,
- three trailer axles with tyres, air suspension, dampers and axle guidance components,
- cargo.

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\(^2\) MSC/NASTRAN: MSC and MSC/ are registered trademarks and service marks of The MacNeal-Schwendler Corporation. NASTRAN is a registered trademark of the National Aeronautics and Space Administration.
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Not all truck components are modelled. Examples of components that have not been modelled are the exhaust system and some small air tanks, all attached to the chassis. Due to their low masses and correspondingly involved high eigenfrequencies, these components do not contribute significantly to the dynamic behaviour of the truck and semi-trailer as a whole in the relevant frequency range. However, since these components together have a significant mass, the total mass of these components is attributed to the chassis members of the model.

Figure 3.5 shows a representation of the truck and semi-trailer model (for clarity, not all modelled components are shown). The longitudinal, lateral and vertical direction will be referred to as the x-, y- and z-directions, respectively. First the truck model will be described after which a description of the semi-trailer model will be given.

Figure 3.5 The three-dimensional vehicle model

The chassis of the truck, which consists of two side members and five cross members, is modelled with flexible beams to represent the vertical, horizontal and rotational flexibility. Both rear and front axles are modelled as rigid bars incorporating the inertia properties of the axles in translational and rotational directions. The leaf springs in the front axle suspension are modelled with elastic beams, and linear viscous dampers are used to represent the real dampers. The rear axle suspension is modelled using four linear springs to represent the four air springs, and two linear viscous dampers to represent the actual dampers. The rear axle guidance components are modelled by means of rods. The front and rear anti-roll bars are modelled with beams. The vertical stiffness of the tyre is modelled as a linear spring and the lateral fixation of the tyres is modelled with a spring and damper in series.

The cab, engine, gearbox, drive train, spare wheel, battery pack and fuel tank are modelled as rigid elements with the proper inertia properties. The mountings of the spare wheel, the battery pack and the fuel tank to the chassis are modelled with beams. The cab suspension is modelled with linear springs and viscous dampers, the anti-roll bar is modelled with beams. Furthermore, the link between engine and chassis is modelled with linear springs.

Next, the trailer is discussed. When designing a truck, one is only interested in a sufficient approximation of the load input acting on the trailer at the kingpin (the kingpin is the pin of the trailer with which the trailer is attached to the truck). Therefore, the trailer model can be kept relatively simple. The chassis of the semi-trailer, which consists of two side members and seven cross members, is modelled with flexible beam elements to represent the vertical, horizontal and rotational flexibility. The three trailer axles are modelled as one single rigid bar with the combined inertia properties of the three axles. The tyres are modelled in the same way as for the truck model. The suspension is modelled with two linear springs to represent the six air springs and two linear viscous dampers to represent the six dampers. The cargo is represented by means of twelve concentrated masses distributed over the trailer model. These masses are connected to the side members using pin-ended rods which are rigid in tension and compression.

To give an impression of the size of the model, the number of times that an element is used in the model is given:

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node points</td>
<td>394</td>
</tr>
<tr>
<td>Rigid bars</td>
<td>107</td>
</tr>
<tr>
<td>Elastic beams</td>
<td>176</td>
</tr>
<tr>
<td>Rigid rods</td>
<td>41</td>
</tr>
<tr>
<td>Dampers</td>
<td>20</td>
</tr>
<tr>
<td>Springs</td>
<td>74</td>
</tr>
<tr>
<td>Concentrated masses</td>
<td>131</td>
</tr>
</tbody>
</table>

3.1.2 Validation of the vehicle model

To validate the vehicle model, calculated transfer functions have been compared with measured transfer functions. These transfer functions were measured with the aid of a test-rig [Haazen, 1995]. On this test-rig, schematically shown in Figure 3.6, the tyres of the truck are excited with shaker tables oscillating with a prescribed displacement.
During the excitation of the tyres, accelerations were measured at 28 points distributed over the truck. Measurement points were located at various chassis points, cabin points, engine points and both axles. For all points, the transfer functions between excitation and measurement point were determined.

A disadvantage of validating the truck model with this type of test-rig, is that the tyres do not rotate. Nevertheless, this method of validation also offers significant advantages:

- the excitation is exactly known which is necessary to calculate the measured transfer functions,
- the test conditions are constant and can be reproduced, since validation of the truck equipped with the new suspension system will take place in a later stadium, identical test conditions are required.

At the test-rig, the time signal of the table displacements is representative for excitations that occur when driving over a real road surface. Two levels of excitation were chosen. In contrast with the linear vehicle model, several non-linearities occur in the real truck and different transfer functions may be expected for different excitation levels. In the model, several parameter values have been linearised around conditions that occur at specific excitation levels and thus must be changed when the excitation level changes. The first of the chosen excitation levels corresponds to driving on a brick paved road, which is considered to be a rather high excitation level. The second excitation level corresponds to the truck driving on a German motorway, which is considered to be a rather low level excitation.

When driving over a road, accelerations at various points of the truck arise from the excitation of the front wheels, the rear wheels and the trailer wheels. These excitations are usually different for the right and left wheels and consist of a symmetric and an anti-symmetric part.

Therefore, transfer functions have not only been determined for symmetric excitations, but also for anti-symmetric excitations.

To meet the demands of symmetric as well as anti-symmetric excitations, four different tests were carried out at a low excitation level and a high excitation level:

- symmetric excitation of the front wheels,
- anti-symmetric excitation of the front wheels,
- symmetric excitation of the rear wheels,
- anti-symmetric excitation of the rear wheels.

In these tests, the trailer wheels were not excited. As mentioned earlier, in the future calculations, only a sufficient estimation of the kingpin input is desired, and it is assumed that the trailer model is accurate enough for this purpose.

In order to compare measured and calculated transfer functions, transfer functions were calculated with the three-dimensional model of truck and semi-trailer for the same four excitations as in the experiments and compared with these experiments [Ruiter, 1996]. A first conclusion was that the measured transfer functions for the low and high excitation level did not differ very much. The second conclusion was that for the lower frequency ranges the model was sufficiently accurate. However, for excitation frequencies in the range between 7 and 15 Hz, some mismatches between experiments and calculations were found. Such deviations were expected, because during the development of the model, uncertainties existed about the correct values of some parameters (e.g. tyre stiffness, engine mounting stiffness etc.). Based on the measured transfer functions, these parameters were adapted to achieve a better match between measurements and calculations. The aim of the adaptation process was not to adjust the model in such a way, that there were no differences between the measured and calculated transfer functions, but to achieve similar mismatches for the entire frequency range. With the new suspension, which exhibits a frequency dependent behaviour, the truck's response spectra are expected to change only in a specific frequency range. To compare a potential performance improvement in a frequency range with a potential performance loss in another range, the errors made for the two ranges should be of the same order. Therefore, the difference between the measured and calculated transfer functions should be of the same order of magnitude throughout the entire frequency range.

The following strategy was used in the adaptation process. Because the chassis is excited via the suspension which, in its turn, is excited by the axle, the axle movements should be correct in the first place. Therefore, the tyre stiffnesses, damper settings and suspension stiffnesses were adapted to achieve correct transfer functions for the front and rear axle. It appeared that, in the frequency range of 9 to 14 Hz, the tyre stiffness had a large influence on the transfer function. Higher tyre stiffnesses (+11%) resulted in a better match between measured and
Optimal design of a hydro-elastic rear axle suspension for heavy trucks

calculated transfer functions, as is illustrated in Figure 3.7. A bode plot is given for the measured and calculated transfer functions (resulting displacement divided by input-displacement) for a point on the right hand side of the front axle. In this case, the front axle was excited with a symmetric high level excitation. Before the adaptation process, the second peak in the magnitude line in the graph was shifted too far to the left, which is corrected.

![Bode plot of front axle, right hand side](image)

**Figure 3.7** Symmetric excitation of the front axle

Next, the transfer functions of other points at the chassis were considered. It appeared that the chassis stiffness and engine mounts influence the chassis accelerations in the frequency range of 7 to 13 Hz and to a lesser extent for frequencies higher than 13 Hz. Stiffer (+15 %) beam elements for the chassis members and higher stiffnesses (+21 %) for the engine mountings resulted in a better match between measured and calculated transfer functions in the mentioned frequency ranges, as illustrated in Figure 3.8. A bode plot is given for a point on the right hand side of the rear axle. Also in this case, a symmetric high level excitation of the rear axle is applied. It can be seen that after the adaptation process, calculated results correspond better with experimental results.

![Bode plot of rear axle, right hand side](image)

**Figure 3.8** Symmetric excitation of the rear axle

Next, attention was paid to the cabin movements. A bode plot of the measured and calculated transfer function in x-direction is given for the seat mounting point in the cabin in Figure 3.9. The bode plot shown, corresponds with an anti-symmetric high level excitation of the rear axle.

![Bode plot of seat mounting point](image)

**Figure 3.9** Anti-symmetric excitation of the rear axle
For the comfort point, a mismatch in the transfer functions for asymmetric excitation of the rear axle was found. In the frequency range from 12 to 16 Hz, the amplitude of the calculated transfer function for the longitudinal movement of the cabin was too large. It appeared that the rubber mountings with which the anti-roll bar is attached to the cabin were modelled too stiff. It was revealed that the cabin member to which the rubber mounting of the anti-roll bar is attached, contributes significantly to the total stiffness in longitudinal direction. Because this member is rigid in the model, the chosen stiffness for the mount was too high. By decreasing the stiffness of the rubber mount in longitudinal direction (-44%), the stiffness of the member is now incorporated in the model.

After this process, the bode plots of the calculated and measured transfer functions of all 28 points allowed a qualitative comparison. The results showed substantial improvements similar to the ones shown in the previous three figures. The vehicle model was therefore assumed to be sufficiently accurate to predict the dynamic behaviour of the real vehicle.

3.2 Model of the new suspension used in the calculations

The concept for the new suspension was shown in Chapter 2. The desired mass effect in the new suspension is generated by the accelerations of a fluid in a channel. This mass effect is modelled with the flywheel model shown in Figure 3.10. A mechanical model, instead of a hydraulic model is used, because this model enables an easy modelling in NASTRAN and it is easy to adjust the parameters.

To the flywheel model, spring $K_1$ is added between the two translational degrees of freedom $x_1$ and $x_2$. By doing so, one of the two springs, $K_2$ or $K_3$, can be removed without affecting the systems static stiffness. The introduction of the extra spring creates more design freedom in establishing the desired dynamic stiffness and damping.

The variations of dynamic stiffness and damping with frequency are shown in Figure 3.11.

![Figure 3.11 Dynamic stiffness and dynamic damping of the flywheel model](image)

At its eigenfrequency, mass $M$ will resonate on the springs. The resonating mass influences the stiffness and damping which leads to a frequency-dependent "equivalent stiffness" and to a frequency-dependent "equivalent damping". The conventional suspension - a spring placed in parallel with a damper - would show two straight lines indicated by the dotted lines in Figure 3.11.

The dynamic behaviour of the novel suspension system can be described by five characteristic quantities. These quantities are related to the five parameters $K_1$, $K_2$, $K_3$, $B$ and $M$, and are indicated in Figure 3.11. These characteristic quantities will be used in the optimisations described in Chapter 4, and are:

1. static stiffness $C_1$, the stiffness at vanishing frequency;
   \[ C_1 = K_1 + \frac{K_2 \cdot K_3}{K_2 + K_3} \]  
   \[ (3.1) \]

2. dynamic stiffness $C_2$, the stiffness at very high frequencies;
   \[ C_2 = K_1 + K_2 \]  
   \[ (3.2) \]
3. damping factor $\zeta$;

$$\zeta = \frac{B}{2 \cdot \sqrt{M \cdot (K_1 + K_3)}}$$

(3.3)

4. natural frequency or eigenfrequency $f_0$;

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_1 + K_3}{M}}$$

(3.4)

5. 'zero stiffness' frequency $f^*$ which is the frequency at which the dynamic stiffness vanishes in the case of an undamped flywheel model;

$$f^* = f_0 \cdot \sqrt{\frac{C_1}{C_2}}$$

(3.5)

3.3 The road model

The road surface height $h$, can be described by statistical models, assuming that the tracks by which the vehicle is excited, can be represented by a random process. The spectral density of such tracks can be approximated by the following model [Dodds et al., 1973];

$$G(n) = G_{(n_0)} \left[ \frac{n}{n_0} \right]^w$$

(3.6)

in which $G(n)$ represents the spectral density of the road surface height, $n$ represents the number of periodic waves per unit length of the road surface, $G_{(n_0)}$ represents the reference spectral density (for $n$ equals $n_0$) and $w$ is the so-called undulation exponent. A road surface can be characterised by the latter two parameters. Normally, the power spectral density function is presented on a logarithmic scale in which it shows a straight line. The height of this line is determined by $G_{(n_0)}$ and its slope is determined by the undulation exponent $w$. In the calculations presented in this paper, $n_0$ was set to $1 \text{ m}^{-1}$.

For the three-dimensional truck and semi-trailer model, it is necessary to make a distinction between the excitations at the excitation points. There is a difference between the tracks in lateral direction, shown in Figure 3.12, and there is a time delay for the excitation between the points located in different longitudinal directions (front, rear and trailer axle).

For this difference in lateral direction, a coherence model is required to relate the excitation of the different tracks. The coherence model used in this thesis is the model proposed by Ammon and Bormann [1991]. They describe a coherence function $\gamma^2$;

$$\gamma^2(n,b) = \left[ 1 + \left( \frac{n(2b)^a}{np} \right) \right]^{-2p}$$

(3.7)

in which $2b$ is the track width, $a$ is a non-isotropy constant (in case of isotropy $a=1$) and $p$ and $np$ are constants.

In this thesis, the parameters of the stochastic road surface were selected to represent either a German motorway [Braun et al., 1991] or a rather bad brick-paved road [Van Heck, 1994]. The parameters of the power spectral density function and coherence function for the German motorway road surface profile are given in Table 3.1.

<table>
<thead>
<tr>
<th>$G_{(n_0)}$</th>
<th>$w$</th>
<th>$np$</th>
<th>$p$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.159e-6</td>
<td>2.0</td>
<td>1.0523</td>
<td>3.737</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3.1 Parameter values of the German Motorway road surface profile

Figure 3.13 visualises both the road spectra according to Equation 3.6 and Equation 3.7.
3.4 The calculation process

With the three-dimensional vehicle model, frequency response functions $H(f)$ can be calculated for the displacement excitation of the wheel-road contact points to a set of typical points on the truck and semi-trailer. To process these results, a Matlab toolbox has been developed [Heck, 1994] called TricaT, which uses the calculated frequency response functions and road excitation as described in Section 3.3, to calculate power spectral density functions for displacements of the truck node points. This calculation process is described next.

In the frequency domain, the displacement of a point on the truck is described by means of:

$$Y = Y(f) = \sum_{i=1}^{n} H_{i}(f) X_{i}(f) = H^{T} X$$

with

$$H = \begin{bmatrix} H_{1}(f) \\ \vdots \\ H_{n}(f) \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} X_{1}(f) \\ \vdots \\ X_{n}(f) \end{bmatrix}$$

where $Y$ is the response displacement of the considered node point, $i$ is the number of the truck or semi-trailer wheel being excited and $n$ is the total number of wheels of the truck and semi-trailer. $H_{i}(f)$ is the frequency response function between wheel $i$ and the considered node point, $X_{i}$ is the excitation displacement at wheel $i$. $Y$ and $X$ are the Fourier transforms of $y(t)$ and $x(t)$, which are the response displacement and excitation displacement, both as a function of time, respectively.

The power spectral density function $G_{yy}$, for the displacement of the point can now be estimated by:

$$G_{yy} = E[Y Y^T] = H^{T} E[X X^T] H = H^{T} G_{xx} H$$

(3.9)

in which $G_{xx}$ is the excitation matrix and $E[x]$ indicates the expected value of $x$. $Y^T, X^T$ are the complex conjugates of $Y$ and $X$, respectively, while $X^T$ is the transpose of $X$, and $H^T$ is the hermetic transpose of $H$. On its diagonal, $G_{xx}$ contains the power spectral density function of the road surface excitations while the off-diagonal elements of $G_{xx}$ contain the cross power spectral densities. When the road surface is of equal quality at every point (part of assuming isotropy), the power spectral density functions are identical for every wheel. To calculate $G_{xx}$ for a specific excitation pattern, the cross spectra have to be derived from the power spectral density function of the road surface.

The wheels of the front axle, rear axle and semi-trailer axle are not located at the same longitudinal position. The resulting time delay leads to a phase delay in the frequency domain, which depends on the vehicle’s velocity. Defining the $x$-position of the front wheels as $x=0$, the time delay, $\Delta T_{i}$, and the phase delay, $\Delta \phi_{i}$, for the other excitation points are:

$$\Delta T_{i} = \frac{l_{i}}{v} \Rightarrow \Delta \phi_{i} = e^{-2\pi i \Delta T_{i}}$$

(3.10)

where $l_{i}$ is the distance in longitudinal direction between the appropriate excitation points, $v$ is the vehicle speed, $f$ is the frequency, and $n$ is the number of excitation points.

For the differences in excitation in lateral direction, the coherence function $\gamma^{2}$ for the road surface, as described in Equation 3.7, is required.

Combining these quantities results in the following expression for $G_{xx}$:

$$G_{xx_{k,l}} = \gamma^{2}_{k,l} G e^{-2\pi i k \Delta T_{l}} \quad k, l = 1 \text{ to } n$$

(3.11)

in which $G_{xx_{k,l}}$ is the element on the $k^{th}$ row and $l^{th}$ column of the matrix $G_{xx}$, while $\gamma^{2}$ is the coherence function and $G$ is the power spectral density function of the road surface.

The input of the processing routine TricaT requires a vertical displacement power spectral density function $G$ and a coherence function $\gamma^{2}$ of the road surface. With the given vehicle speed $v$, and the distances between the excitation points, and Equation 3.11, the input matrix
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$G_{xx}$ can be calculated. Using Equation 3.9, the displacement spectrum for typical response points can be calculated. These displacement spectra can easily be converted into speed or acceleration spectra by means of the following relations:

displacement spectrum: $Y(f)$  
speed spectrum: $V(f) = 2\pi f Y(f)$  
acceleration spectrum: $A(f) = -4\pi^2 f^2 Y(f)$

As a result of this chapter, using the processing routine TricaT in combination with the frequency response functions, available from the NASTRAN calculation with the three-dimensional vehicle model, power spectral density functions can be calculated for displacements, speeds and accelerations for all interesting points on the truck for different vehicle speeds and different road surfaces.

Chapter 4

Optimal parameter settings for the new suspension

The three design goals for a truck suspension design, together with the subdivision into several design criteria, were described in Chapter 1. The selection of a new suspension system was discussed in Chapter 2. In the present chapter, the settings of this new suspension will be optimised to yield the best results possible for the design criteria. A numerical optimisation technique is applied to optimise the suspension settings. First, the optimisation technique and the required input for the optimisations are described, then the results are discussed.

4.1 Description of the optimisation process

In general, parameter optimisations start with an initial set of design parameters, $x$, and an object function, $F(x)$. The object function depends on the design parameters and either its maximum or minimum must be found. However, since maximising $F(x)$ is essentially the same as minimising $-F(x)$, usually only the minimisation problem is addressed. An optimisation problem may include constraints, concerning ranges of acceptable values of the design parameters, or functions thereof. The solution of an optimisation problem is a set of admissible values of the design parameters for which the object function has an optimal value within the specified constraints. A general problem description is stated as:

Determine $x$ which minimises $F(x), \, x \in \mathbb{R}$

subject to,

$$
g_i(x) = 0 \quad i = 1, \ldots, m \quad g_i(x) \leq 0 \quad i = m+1, \ldots, n \quad (n \geq m+1) \quad x_1 \leq x \leq x_n
$$

where $F(x)$ is the object function, $x$ is the vector containing the design parameters, $x_1$ and $x_n$ are vectors containing the lower and upper constraints for $x$ and $g$ is a vector function containing the equality and inequality constraints (functions of $x$).

The optimal values for the design variables have been determined in an iterative way using an SQP, Sequential Quadratic Programming, algorithm [e.g. Gill et al., 1981, Grace, 1994]. The application of the SQP algorithm to optimise components of a truck model was extensively described by Besselink et al. [1994]. For the optimisation, starting values or initial values for
the design variables are required, which are referred to as the initial design. Based on this
initial design, the partial derivatives of the object function and the constraints are calculated.
Using this information, the optimisation algorithm determines new values for the design
variables, which are expected to decrease the object function while satisfying the constraints.
For these new values, the responses and partial derivatives are calculated again and the
iterative process is continued. The optimisation process is stopped when all constraints are
satisfied and the first order derivatives of the object function vanish. An important aspect in
optimisations is scaling of the design variables, which causes them to have the same order of
magnitude. This is also mentioned by Besselink et al. [1994] for the optimisation of truck
components. Besselink suggests a logarithmic transformation which will also be used in this
thesis. With this transformation, the number of iterations can be reduced considerably and thus
the solution of the optimisation problem is found faster. The optimisation process is shown
schematically in Figure 4.1.

Figure 4.1 The optimisation process

The object function, design variables, constraints and initial design will be discussed in more
detail in the following sections.

4.1.1 Performance criteria and definition of the object functions

As described in Chapter 1, the performance criteria for a truck and semi-trailer contain the
following aspects:

- accelerations, subdivided with respect to driver and passenger comfort, cargo accelerations
  and fatigue damage to the chassis,
- dynamic wheel load,
- spring travel.

The suspension will be optimised for each of these performance criteria separately. This, to
determine the maximal attainable improvements with the new suspension for these
performance criteria. To do this, the performance criteria must be specified in more detail and
relevant points on the truck at which these criteria are evaluated have to be selected.

Driver and passenger comfort

The driver and passenger comfort is determined by the accelerations to which their bodies are
subjected. However, the human sensitivity to accelerations depends on the excitation
frequency which means that the accelerations should be weighed as a function of this
frequency. The weighing functions for accelerations in longitudinal, lateral and vertical
direction, used in the present investigation are summarised in the ISO 2631 standard [1985].
Of the weighed signals, the RMS values were calculated and used in the criterion for comfort.

As the location for measuring the comfort, the seat mounting point on the cabin floor was
selected. Advantage of using this point, and not, e.g. a point at the belly of the driver, is that
its displacements are independent of the person sitting in the driver’s seat (the type of driver
can vary, e.g. his weight) or the type of seat that is mounted.

Summarising, the criterion for driver comfort becomes;

\[ DC = \sqrt{MS(\text{cox}) + MS(\text{coy}) + MS(\text{coz})} \]  

where \( DC \) denotes the driver comfort criterion, and \( MS(\text{cox}), MS(\text{coy}) \) and \( MS(\text{coz}) \) represent
the Mean Square values for comfort in the longitudinal, lateral and vertical directions,
respectively.

Cargo accelerations

A truck manufacturer has very little influence on the type of trailer used and the suspension of
this trailer, and can only try to reduce the accelerations, forced upon the trailer by the truck, as
much as possible. These accelerations are forced upon the trailer at the so-called kingpin
which couples the trailer to the truck. To minimise the cargo accelerations, a truck
manufacturer is mainly interested in the accelerations at this kingpin. Therefore, the power
spectral density functions of the accelerations in longitudinal, lateral and vertical directions at the kingpin are calculated. Normally, the RMS values are used as a measure to evaluate the accelerations of the cargo [Sharp et al., 1987, Hrovat, 1993]. Here, the square root of the sum of the MS values for the longitudinal, lateral and vertical direction are used as the criterion for the kingpin accelerations (under the assumption that the cargo is equally sensitive for all three directions).

Summarising, the criterion for the accelerations at the kingpin is;

\[ KA = \sqrt{MS(akx) + MS(aky) + MS(akz)} \]  

(4.3)

where \( KA \) represents the criterion for the kingpin accelerations and \( MS(akx), MS(aky) \) and \( MS(akz) \) are the Mean Square values for accelerations at the kingpin in the longitudinal, lateral and vertical direction, respectively.

Dynamic wheel load

The vertical tyre forces acting on the road, consist of a static part and a dynamic part. The dynamic part, the so-called dynamic wheel load, is of importance to road holding and road damage, since road damage is mainly caused by heavy trucks [e.g. Cebon, 1989; Goktan et al., 1995].

For a proper road holding, the following ratio should be small [Mitschke, 1984];

\[ \frac{\text{RMS}(dwl)}{F_{\text{stat}}} \]  

(4.4)

where \( \text{RMS}(dwl) \) is the Root Mean Square value for the dynamic wheel load and \( F_{\text{stat}} \) represents the static wheel load. Because the static wheel load is constant for a given vehicle with a given amount of cargo, the value of interest is the RMS value for dynamic wheel load.

The load on the road, causing road damage, can be expressed by the road stress factor \( \vartheta \) [Mitschke, 1984]. \( \vartheta \) is directly proportional to \( \eta_{\text{stat}} \), which is a coefficient related to dynamic wheel load. For homogeneously damaged roads, an average value is used for \( \eta_{\text{stat}} \) [Mitschke, 1984];

\[ \eta_{\text{stat}}^4 = 1 + 6 \left( \frac{\text{RMS}(dwl)}{F_{\text{stat}}} \right)^2 \]  

(4.5)

Again, \( \text{RMS}(dwl) \) represents the RMS value for the dynamic wheel load and \( F_{\text{stat}} \) is the static wheel load. Reducing the RMS value of the dynamic wheel loads, reduces \( \eta_{\text{stat}} \) and this reduces road damage. In view of this, the RMS value of the dynamic wheel load is a suitable criterion.
specific shape and made of a specific material, the life span, defined as the time until a fatigue fracture occurs, depends on the forces acting on it. In case of chassis components, the forces acting on the chassis, and causing stresses in the material, are inertia forces and depend on the accelerations acting on the component.

To estimate the damage content of a load with varying amplitude (in this case, the load is an acceleration signal, because the accelerations cause the forces on the construction), the relative form of the Palmgren-Miner rule [e.g. Surush, 1992] is generally used. The so-called relative Palmgren-Miner number relates the damage content of the considered load signal to a reference load signal of which the damage content is known. The rule assumes that, if for a given load signal with amplitude \( S_{a} \), the life span of a certain component is \( N \) cycles, then \( n \) (\( n < N \)) load cycles with amplitude \( S_{a} \) consume a part of \( n/N \) times the total life span of the component. From this it can be concluded that for a load with specific amplitude levels \( S_{a} \), a fracture will occur if:

\[
\sum_{i=1}^{r} \frac{n_i}{N_i} = 1
\]

(4.8)

where \( r \) represents the number of different levels of \( S_{a} \). With the Palmgren-Miner rule, the fatigue damage can be calculated for each separate load cycle. The total damage equals the sum of the separate damages. Using the Palmgren-Miner rule, the damage of a specific signal can now be expressed in the number of cycles of a specific reference signal with a constant amplitude having the same damage content. If the Palmgren-Miner numbers are determined for two vehicles with different suspension systems, e.g. a conventional suspension and a new suspension, the two suspensions can be compared with respect to the fatigue loads.

To calculate the Palmgren-Miner number, the different amplitude levels of the load signal must be categorised in ranges counting the number of cycles \( n_i \) occuring for a specific \( S_{a} \). The number of cycles for different \( S_{a} \) (\( i=1...r \)) is reflected in a so-called cumulative load-collective. Generally, the so-called rainflow-count method is used to create the cumulative load-collective out of a load signal [e.g. Surush, 1992]. A cumulative load-collective has the typical shape shown in Figure 4.2.

The horizontal axis shows the cumulative number of counts (\( n \)) for a specific amplitude range, while the vertical axis shows the amplitude range (\( S_{a} \)).

The Palmgren-Miner number is calculated from a time signal. For the calculations presented in this thesis, only an acceleration spectrum, instead of the desired time signal is available. Out of a power spectral density function, a rainflow-count must be derived. The way to do this is described in detail by Box [1995] who uses the method of Dirlik [1985] to estimate a rainflow-count, based on a power spectral density function. This method was implemented in the Matlab toolbox TricaT, described in Chapter 3.4. With the aid of this extra module, Palmgren-Miner numbers can be calculated and compared for power spectral density functions of different acceleration signals resulting from different suspension settings.

For the calculation of the fatigue damage, acceleration levels of the chassis are required. However, the calculation of the accelerations for a single point on the chassis is not sufficient. When the chosen point happens to lie in a node of a vibration mode, relatively small accelerations are calculated and the comparison of different suspension settings yields unrealistic results. The accelerations of various points distributed over the chassis are required. To determine how many points must be considered, the eigenmodes below 20 Hz have been analysed. For the vibration mode with the shortest wavelength in the xz-plane, three measurement points are sufficient (Figure 4.3). This mode corresponds to a half wavelength, and its shape can be described with three points. For the vibration mode with the shortest wavelength in the xy-plane (below 20 Hz), which is not the same vibration mode as the one in the xz-plane, four measurement points are required. However, it was decided to use the same three points as for the xz-plane. One of the nodes of the vibration mode is at the kingpin and because the measurement points for the xz-plane are all far enough away from this point, they also give an accurate approximation of the acceleration level in the xy-plane.
For these three points, accelerations are calculated in longitudinal, lateral and vertical direction. For each of these nine acceleration spectra, Palmgren-Miner numbers are calculated. To compare the new suspension with the conventional one, two criteria have been selected. The first criterion is the maximum of these Palmgren-Miner numbers. Since at the maximal value of the Palmgren-Miner number, the highest fatigue damage occurs, this number should be as low as possible in the optimisations. However, this could mean that for the other points, the Palmgren-Miner number increases, implying that, although the worst case is improved, the overall fatigue level may have become worse. Therefore, a second criterion is selected. This second criterion is the mean value of the nine Palmgren-Miner numbers.

Summarising, the criteria for fatigue damage are:

\[
PM_{\text{max}} = \max(\text{Palmgren-Miner}) \quad (4.9)
\]
\[
PM_{\text{mean}} = \text{mean}(\text{Palmgren-Miner}) \quad (4.10)
\]

where \(PM_{\text{max}}\) represents the maximal Palmgren-Miner number and \(PM_{\text{mean}}\) represents the mean of the nine Palmgren-Miner numbers.

The basic level for the dynamic stiffness is determined by the combination of the springs in the new suspension (static stiffness \(C_1\)). The basic level for the dynamic damping is determined by the damping of the new suspension in combination with the damping of the standard damper which is placed in parallel to the new suspension.

As a consequence of the chosen basic levels for the stiffness and damping characteristics of the conventional vehicle, this vehicle has specific values for the optimisation criteria defined
in the previous subsection. Also its behaviour with respect to incidental excitations is fixed by
these characteristics. Changing the stiffness and damping results in different values for the
optimisation criteria and a different behaviour with respect to incidental excitations. The goal
of the present calculations is to determine to what extent the new suspension improves the
dynamic behaviour of the vehicle. The resulting question is: "Which basic levels for the
new suspension should be chosen to allow a fair comparison with the conventional
suspension?"

With respect to the basic level of the dynamic stiffness and damping, several ways can be
followed to determine the advantages of the new suspension system over the conventional
one.

Concerning the dynamic stiffness, there are two options:

1a. the basic level of the new suspension is determined by the optimisation algorithm,
1b. the basic level of the new suspension is chosen equal to that of the conventional
   suspension.

Choosing the first option, it is possible that the static stiffness will vanish, which is not
allowed since a minimum static stiffness is always required. Also the dynamic behaviour of
the truck for incidental excitations may be influenced. Furthermore, to obtain a fair
comparison, the stiffness of the conventional suspension should also be allowed to change.
Therefore, the static stiffness of the new suspension should be chosen identical to the static
stiffness of the conventional suspension.

Concerning the dynamic damping, the optimal solution would probably be to place two of the
new suspensions in parallel. One with a damping peak around the body eigenfrequency, and
the other with the damping peak around the axle eigenfrequency (remember Figure 1.6). The
basic level of this suspension could then be set to zero. However, such a suspension has more
than three degrees of freedom which is conflicting with the design demands. Also since such a
suspension contains more components, it would be less practical to build.

As a result, the following options are considered with respect to the dynamic damping:

2a. the basic level for the damping of the new suspension is set to zero,
2b. the basic level for the damping of the new suspension is determined by the optimisation
   algorithm,
2c. the basic level for the damping of the new suspension is chosen equal to the damping of
   the conventional suspension,
2d. the semi-static damping of the new suspension is chosen equal to the semi-static
damping of the conventional suspension.

The first option is not acceptable, because the new suspension only shows one damping peak.
As a result, either the low frequency body eigenmode, or the axle eigenmode is not damped.
For the second option, the same argument applies as for the first option of the dynamic
stiffness. Comparison with a conventional vehicle is only fair if an optimisation is performed
for the conventional vehicle first, in which the conventional damping is also allowed to
change. Optimisations performed for the conventional suspension revealed that for stochastic
excitations, the optimisation algorithm makes the stiffness as low as possible, while making
the damping higher than for the conventional vehicle. This leaves the choice between the third
and fourth option. The third and fourth option offer the possibility of creating more damping
around the body eigenfrequency, or more damping around the axle eigenfrequency. The third
option implies that at frequencies lower than the suspension's eigenfrequency (the frequency
with the damping peak), the damping is higher than for the conventional vehicle. The fourth
option implies that for frequencies higher than the suspension’s eigenfrequency, the dynamic
damping is lower than for the conventional suspension. Of these two options, the latter is
preferred, lower damping for frequencies higher than the suspension’s eigenfrequency.
Especially for high frequencies, the dampers are responsible for the largest part of the forces
introduced by the suspension into the chassis, due to the high speeds and low displacements
involved at these frequencies. Higher high frequent forces give rise to an increased noise level
in the vehicle. Therefore, low damping is preferred for high frequencies, which the fourth
option provides. Summarising, the basic level for the stiffness and the basic level of the
damping of the new suspension are chosen to be identical to those of the conventional
suspension.

Next, a third constraint is added which concerns the dynamic damping at 20 Hz and which
should stay below that of the reference vehicle. This causes the dynamic damping for
frequencies above 20 Hz to be lower than the dynamic damping of the reference vehicle. The
addition of this constraint ensures that, for frequencies above 20 Hz, the forces introduced into
the chassis by the new suspension are not larger than those introduced by the conventional
suspension. This type of constraint can not be imposed on the dynamic stiffness, because at
high frequencies the stiffness will always be higher than at lower frequencies. However, this is
not considered to be a problem, since at frequencies in excess of 20 Hz, the forces introduced
into the chassis by the springs are many times smaller than those introduced by the dampers.

4.1.3 The design variables

According to Figure 3.10, the new suspension can be characterised by five design parameters;
K₁, K₂, K₃, B and M. Although it would seem appropriate to use these five parameters as the
design variables, preference is given to the use of five alternative design variables. Instead of
referring to the design parameters of the new suspension, these alternative design variables
refer to the design characteristics C₁, f₀, f' and ζ (respectively static stiffness, damping factor,
natural frequency and 'zero stiffness' frequency, see Chapter 3.2). With the aid of these design
variables, the dynamic behaviour of the new suspension can be controlled easily, while the
4.1.4 The initial design

With design variable $\text{suspension}$ should occur. Design variable example, chassis accelerations. Therefore, the object function often has several local minima fluid channel of the concept design, too much damping reduces the mass effect. Consequently, the design variable vector $x$, lower $x_i$ and upper $x_u$ bounds have been defined as;

$$x = \begin{bmatrix}
\log(C_1) \\
\log(C_{\text{min}}) \\
\log(\frac{1}{C_1}+1) \\
\log(1) \\
\log(2)
\end{bmatrix}
\begin{bmatrix}
x_i = \\
> 0 \\
> 0 \\
> 0 \\
0.01 \\
0.5
\end{bmatrix}
\begin{bmatrix}
x_u = \\
> x_i(3) \\
> x_i(4)
\end{bmatrix}
(4.11)$$

where $C_{\text{min}}$ represents the minimum static stiffness (As can be noticed, the logarithmic transformation mentioned in Chapter 4.1 is used).

With design variable $x(1)$, the static stiffness is controlled. Design variable $x(2)$ ensures that spring stiffness $K_1$ remains smaller than the static stiffness $C_1$. Otherwise, negative values of spring stiffness $K_1$ and/or negative values of spring stiffness $K_1$ may occur. Furthermore, design variables $x(3)$ and $x(4)$ control the frequency range in which the mass effect of the new suspension should occur. Design variable $x(5)$ is used to ensure that the relative damping $\zeta_1$ is adequate. Too little damping is not feasible in practice because of the inner resistance of the fluid channel of the concept design, too much damping reduces the mass effect.

4.1.4 The initial design

As a result of the large number of components mounted on a truck, there are many degrees of freedom leading to a large number of eigenfrequencies showing peaks in the spectra of, for example, chassis accelerations. Therefore, the object function often has several local minima in addition to the global minimum (Figure 4.5).

An undesirable result of using the SQP algorithm is, that when the optimisation is started from a design vector that is not close to the global minimum, the algorithm may converge to a local minimum. Therefore, it is advisable to start the optimisation from a number of different initial designs.

To reduce the number of initial designs and the calculation effort, initial designs that are already close to the optimal design should be identified. To obtain a good initial design, the method proposed by Konings [1993] is used. An extensive description of this method is given by Ruiter et al. [1996]. This method works as follows. In steps of 0.2 Hz, for specific combinations of damping and stiffness settings, the power spectral density value of the criterion to be optimised is calculated. For each frequency, out of all the combinations of the suspension settings and corresponding values for the considered criterion, that particular combination of stiffness and damping setting is selected for which the power spectral density value of the considered criterion reaches its lowest value. If this is done for each frequency and these optimal settings are plotted against the frequency, the optimal dynamic stiffness and optimal dynamic damping are known as a function of the frequency. A sensitivity analysis of these optimal settings is carried out to identify the frequency ranges in which it is important to achieve the optimal stiffness and damping coefficients. This information can then be used to tune the parameters of the new suspension in such a way that the dynamic stiffness and damping approximate the optimal characteristics as close as possible. The settings found in this tuning process are then used for the initial design.

Some disadvantages of the new suspension system were discovered during the definitions of the initial designs. First, it is not possible to approximate the optimal characteristics entirely. Second, the dynamic stiffness and damping characteristics of the new suspension are coupled which means that, if one of the optimal characteristics is approximated as close as possible, the other characteristic may deviate from its optimal course.

After defining the object function, the design variables, the constraints and the initial design, the optimisations can be started.

4.2 Performing the optimisations

Here, the performance criteria presented in Section 4.1.1 will be optimised. First, the power spectral density functions of these criteria are given for the conventional vehicle. For fatigue damage, the cumulative load collectives are shown for the chassis point with the highest Palmgren-Miner number. Next, the initial designs used for the optimisations are briefly discussed. To conclude, the results of the optimisations for all the performance criteria are presented and discussed.
4.2.1 Performance of the conventional vehicle

As stated in Chapter 3, two driving conditions were selected for optimising the suspension. First, a fully laden truck was driven over a typical minor road at a speed of 50 km/h. Second, a fully laden truck was driven over an average German motorway at a speed of 80 km/h.

The power spectral density functions for the criteria to be optimised, for a conventional truck, driven on the minor road are shown in Figure 4.6.

![Figure 4.6 Criteria for the conventional truck driven over a minor road (50 km/h)](image)

The power spectral density functions for the optimisation criteria, for a conventional truck driven on the German motorway are shown in Figure 4.7.

![Figure 4.7 Criteria for the conventional truck driven over a German motorway (80 km/h)](image)

4.2.2 Initial designs used for the optimisations

Several optimisations were performed, each with different initial designs. The initial designs were divided into three categories:

- low frequency initial designs, $f^0$ between 0 and 7 Hz,
- middle frequency initial designs, $f^0$ between 7 and 13 Hz,
- high frequency initial designs, $f^0$ between 13 and 20 Hz.

For the optimisation of the kingpin accelerations, driver comfort and dynamic wheel load, three low frequency initial designs, three middle frequency initial designs and three high frequency initial designs were used, while for the optimisation of spring travel, only three low frequency initial designs were used. This follows from an analysis of Figures 4.7 and 4.8.
which shows that spring travel only has an energy content for low frequencies, and hence spring travel improvement is only expected in this frequency range. For the optimisation of the maximum and mean Palmgren-Miner number, three middle frequency initial designs and three high frequency initial designs were used.

From the optimisations, it was concluded that, except for spring travel, only a high frequency suspension (a suspension with a high $f_0$) appeared promising. All the improvements found in the calculations with the low and middle frequency initial designs, were lower than the improvements found in the calculations started with the high frequency initial designs. Therefore, extra initial designs were defined in this high frequency range and more optimisations were carried out. In the following sections, the best results of all optimisations are presented. One section is reserved for every optimisation criterion. For every initial design, three optimisation runs are shown. During the first run, the constraints, concerning the criteria ($DC$, $KA$, $DL$, $ST$, $PM_{\text{max}}$, $PM_{\text{mean}}$) were not allowed to be violated. During the second run, the constraints were allowed to be violated by 5 % while during the third run the constraints were allowed to be violated by 15 % (denoted with $\Delta_{\text{constr}} = 0 \%$, 5 % and 15 %). Each section starts with a table in which the results for the optimisation with respect to the minor road is given. In these tables, a negative value means that the performance with respect to the corresponding criterion is improved. When the number is shaded, the corresponding criterion was an active constraint meaning that the optimisation algorithm was not able to improve the object function further, without violating that particular constraint. The table is followed by three graphs showing the dynamic stiffness, the dynamic damping, and the new spectrum for the criterion, together with the spectrum for the conventional vehicle (in the figures denoted with conv). This figure is followed by a similar table and similar graphs for the optimisation with respect to the motorway. The dynamic stiffness, dynamic damping, and the spectra shown in the graphs, correspond with the optimisations for which the constraints were not allowed to be violated. An exception was made for the optimisation of spring travel, because no improvement could be achieved without violating the constraints. The spring travel spectrum shown in the figures, belongs to the optimisations for which constraint violations of 5 % were allowed. For the optimisation of the maximum Palmgren-Miner number, the corresponding cumulative load collective is shown. For the optimisation of the mean Palmgren-Miner number, the cumulative load collective is presented which showed the largest improvement for the Palmgren-Miner number.

### Optimisation of kingpin accelerations

<table>
<thead>
<tr>
<th>$\Delta_{\text{constr}}$</th>
<th>$KA$</th>
<th>$DC$</th>
<th>$DL$</th>
<th>$ST$</th>
<th>$PM_{\text{max}}$</th>
<th>$PM_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2.9 %</td>
<td>-1.7 %</td>
<td>0.0 %</td>
<td>-0.1 %</td>
<td>-13.7 %</td>
<td>-2.2 %</td>
</tr>
<tr>
<td>5</td>
<td>-3.2 %</td>
<td>-2.2 %</td>
<td>0.8 %</td>
<td>0.0 %</td>
<td>-16.4 %</td>
<td>-5.3 %</td>
</tr>
<tr>
<td>15</td>
<td>-3.2 %</td>
<td>-2.2 %</td>
<td>0.8 %</td>
<td>0.0 %</td>
<td>-16.4 %</td>
<td>-5.3 %</td>
</tr>
</tbody>
</table>

**Table 4.1** Optimisation of kingpin accelerations for the truck driven on the minor road

![Figure 4.8 Optimisation of kingpin accelerations for the truck driven on the minor road](image)

<table>
<thead>
<tr>
<th>$\Delta_{\text{constr}}$</th>
<th>$KA$</th>
<th>$DC$</th>
<th>$DL$</th>
<th>$ST$</th>
<th>$PM_{\text{max}}$</th>
<th>$PM_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2.6 %</td>
<td>-1.4 %</td>
<td>0.0 %</td>
<td>-0.2 %</td>
<td>-15.0 %</td>
<td>0.0 %</td>
</tr>
<tr>
<td>5</td>
<td>-2.6 %</td>
<td>-1.4 %</td>
<td>0.2 %</td>
<td>-0.2 %</td>
<td>-15.5 %</td>
<td>1.2 %</td>
</tr>
<tr>
<td>15</td>
<td>-2.6 %</td>
<td>-1.4 %</td>
<td>0.2 %</td>
<td>-0.2 %</td>
<td>-15.5 %</td>
<td>1.2 %</td>
</tr>
</tbody>
</table>

**Table 4.2** Optimisation of kingpin accelerations for the truck driven on the motorway

![Figure 4.9 Optimisation of kingpin accelerations for the truck driven on the motorway](image)
4.2.4 Optimisation of driver comfort

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM$_{\text{max}}$</th>
<th>PM$_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2.9%</td>
<td>-1.8%</td>
<td>0.0%</td>
<td>-0.1%</td>
<td>-13.9%</td>
<td>0.0%</td>
</tr>
<tr>
<td>5</td>
<td>-2.8%</td>
<td>-2.6%</td>
<td>1.4%</td>
<td>0.1%</td>
<td>-19.4%</td>
<td>5.0%</td>
</tr>
<tr>
<td>15</td>
<td>-2.6%</td>
<td>-2.6%</td>
<td>1.5%</td>
<td>0.0%</td>
<td>-19.5%</td>
<td>8.4%</td>
</tr>
</tbody>
</table>

Table 4.3 Optimisation of driver comfort for the truck driven on the minor road

![Figure 4.10 Optimisation of driver comfort for the truck driven on the minor road](image1)

4.2.5 Optimisation of dynamic wheel load

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM$_{\text{max}}$</th>
<th>PM$_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.1%</td>
<td>-0.8%</td>
<td>-1.3%</td>
<td>-0.3%</td>
<td>-4.5%</td>
<td>0.0%</td>
</tr>
<tr>
<td>5</td>
<td>-0.1%</td>
<td>-0.6%</td>
<td>-2.2%</td>
<td>-0.5%</td>
<td>-1.6%</td>
<td>5.0%</td>
</tr>
<tr>
<td>15</td>
<td>2.3%</td>
<td>-0.2%</td>
<td>-3.7%</td>
<td>-1.0%</td>
<td>2.3%</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Table 4.5 Optimisation of dynamic wheel load for the truck driven on the minor road

![Figure 4.12 Optimisation of dynamic wheel load for the truck driven on the minor road](image2)

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM$_{\text{max}}$</th>
<th>PM$_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-1.5%</td>
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<td>-6.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>5</td>
<td>-0.1%</td>
<td>-1.0%</td>
<td>-2.3%</td>
<td>-0.4%</td>
<td>-4.4%</td>
<td>5.0%</td>
</tr>
<tr>
<td>15</td>
<td>1.9%</td>
<td>-0.9%</td>
<td>-3.7%</td>
<td>-0.7%</td>
<td>-1.2%</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Table 4.6 Optimisation of dynamic wheel load for the truck driven on the motorway

![Figure 4.13 Optimisation of dynamic wheel load for the truck driven on the motorway](image3)
4.2.6 Optimisation of spring travel

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM_max</th>
<th>PM_mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.6%</td>
<td>0.0%</td>
<td>0.2%</td>
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<td>0.4%</td>
<td>0.8%</td>
</tr>
<tr>
<td>5</td>
<td>0.5%</td>
<td>0.6%</td>
<td>0.7%</td>
<td>-11.6%</td>
<td>3.5%</td>
<td>5.0%</td>
</tr>
<tr>
<td>15</td>
<td>3.8%</td>
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<td>-18.6%</td>
<td>9.6%</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Table 4.7 Optimisation of spring travel for the truck driven on the minor road

Figure 4.14 Optimisation of ST for the truck driven on the minor road (Δconstr = 5 %)

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM_max</th>
<th>PM_mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>-0.1%</td>
<td>-0.2%</td>
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<td>0.1%</td>
<td>0.2%</td>
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<tr>
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<td>3.8%</td>
<td>5.0%</td>
</tr>
<tr>
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<td>6.7%</td>
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<td>3.0%</td>
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<td>9.7%</td>
<td>15.0%</td>
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Table 4.8 Optimisation of spring travel for the truck driven on the motorway

Figure 4.15 Optimisation of ST for the truck driven on the motorway (Δconstr = 5 %)

4.2.7 Optimisation of the maximum Palmgren-Miner number

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM_max</th>
<th>PM_mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>-1.8%</td>
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<td>0.0%</td>
<td>-0.1%</td>
<td>-13.9%</td>
</tr>
<tr>
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<td>-2.8%</td>
<td>-2.6%</td>
<td>1.4%</td>
<td>-0.1%</td>
<td>-19.4%</td>
<td>5.0%</td>
</tr>
<tr>
<td>15</td>
<td>-2.4%</td>
<td>-2.6%</td>
<td>1.5%</td>
<td>0.0%</td>
<td>-19.6%</td>
<td>11.8%</td>
</tr>
</tbody>
</table>

Table 4.9 Optimisation of PM_max for the truck driven on the minor road

Figure 4.16 Optimisation of PM_max for the truck driven on the minor road

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM_max</th>
<th>PM_mean</th>
</tr>
</thead>
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<td>-15.0%</td>
</tr>
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<td>-16.6%</td>
<td>5.0%</td>
</tr>
<tr>
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<td>-1.8%</td>
<td>1.9%</td>
<td>-0.1%</td>
<td>-17.3%</td>
<td>15.0%</td>
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Table 4.10 Optimisation of PM_max for the truck driven on the motorway

Figure 4.17 Optimisation of PM_max for the truck driven on the motorway
4.2.8 Optimisation of the mean Palmgren-Miner number

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM(_{\text{max}})</th>
<th>PM(_{\text{mean}})</th>
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</thead>
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<tr>
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<td>-11.3%</td>
</tr>
<tr>
<td>15</td>
<td>-1.8%</td>
<td>-1.6%</td>
<td>0.8%</td>
<td>0.0%</td>
<td>-12.4%</td>
<td>-11.3%</td>
</tr>
</tbody>
</table>

Table 4.11 Optimisation of PM\(_{\text{mean}}\) for the truck driven on the minor road

![Figure 4.18](image)

Figure 4.18 Optimisation of PM\(_{\text{mean}}\) for the truck driven on the minor road

<table>
<thead>
<tr>
<th>Δconstr</th>
<th>KA</th>
<th>DC</th>
<th>DL</th>
<th>ST</th>
<th>PM(_{\text{max}})</th>
<th>PM(_{\text{mean}})</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.9%</td>
<td>-1.0%</td>
<td>0.0%</td>
<td>-0.1%</td>
<td>-10.1%</td>
<td>-8.4%</td>
</tr>
<tr>
<td>5</td>
<td>-2.0%</td>
<td>-1.0%</td>
<td>0.2%</td>
<td>-0.1%</td>
<td>-10.4%</td>
<td>-8.6%</td>
</tr>
<tr>
<td>15</td>
<td>-2.0%</td>
<td>-1.0%</td>
<td>0.2%</td>
<td>-0.1%</td>
<td>-10.4%</td>
<td>-8.6%</td>
</tr>
</tbody>
</table>

Table 4.12 Optimisation of PM\(_{\text{mean}}\) for the truck driven on the motorway

![Figure 4.19](image)

Figure 4.19 Optimisation of PM\(_{\text{mean}}\) for the truck driven on the motorway

4.2.9 Discussion

The improvements obtained for dynamic wheel load, cargo accelerations, and driver comfort are relatively small (all < 3%). Even if the constraints are violated, the improvements still remain relatively small. This contrasts with the results of earlier investigations [de Ruiter, 1996] in which an improvement of more than 8% was found for dynamic wheel load. However, in that investigation, the constraints concerning the fatigue damage were not taken into account. The improvement of dynamic wheel load is limited by the constraint on the mean Palmgren-Miner number. Evidently, the constraint on the mean Palmgren-Miner number is a significant one. For spring travel, a considerable improvement can be achieved if the constraints may be violated. This is due to the fact that this improvement can only be achieved with a low frequency suspension which implies that the dynamic stiffness for higher frequencies is very high compared to the dynamic stiffness of the reference vehicle. This higher dynamic stiffness is the cause for the violation of the constraints. Moreover, the dynamic damping is lower at higher frequencies than for the reference vehicle which implies that the dynamic wheel load in the higher frequency ranges increases.

For fatigue damage, the improvements are satisfactorily. In view of the optimisation of the mean Palmgren-Miner number, with respect to the minor road, an improvement of 9.2% is found, while the improvement of the maximum Palmgren-Miner number is 9.6%. This means that the truck can drive 1.09 times the distance of the conventional truck before the same mean Palmgren-Miner number, and thus the same fatigue damage is experienced (assuming that the truck drives over the same route, under identical circumstances). Even at that distance, the maximum Palmgren-Miner number is lower than that of the conventional vehicle. Thus it can be concluded that the new suspension, with these settings, results in an increased life span for the vehicle of almost ten percent. The same applies when the truck is driven over the motorway. Even a larger improvement of the mean Palmgren-Miner number and the maximum Palmgren-Miner number can be obtained when the constraint concerning the dynamic damping at 20 Hz is not considered. In that case, the improvements are in the order of 20% to 25% for both the mean and maximum Palmgren-Miner number, for both roads considered. However, in this case, at frequencies in excess of 20 Hz, the dynamic damping of the new suspension is higher than the dynamic damping of the conventional suspension and calculations for these frequencies have to be performed to determine the effects of the higher damping on the criteria.

The characteristic numbers for the optimised new suspension are collected in two tables. The first table, Table 4.13, contains the characteristic numbers for the truck on the minor road, and Table 4.14 gives the optimisation results for the truck driven on the motorway.
Looking to the characteristic numbers, all eigenfrequencies (except for spring travel) are around 16 Hz. The static stiffness, which in the optimisations was allowed to exceed that of the conventional vehicle, remained at its lower bound. This corresponds to the static stiffness of the conventional vehicle.

Note, that for the minor road, the characteristic numbers for the optima of driver comfort and maximum Palmgren-Miner number are identical. This also happens for the optimisations with respect to the motorway. The characteristic numbers for the optima of cargo accelerations, driver comfort and the maximal Palmgren-Miner number are also identical.

It is noted, that the optimal settings for the new suspension parameters are not the same for the two road types and the associated vehicle speeds. For example, the optimum for the mean Palmgren-Miner number for the minor road differs from the optimum for the motorway. Consequently, when the suspension is designed for a specific road surface and vehicle speed, it will not behave optimally on other road surfaces and at other speeds. Therefore, to obtain as much benefit as possible from the new suspension, the parameters of the new suspension should be adjustable. This aspect will be elaborated in Chapter 5.

The conclusion of the present study is, that the new suspension does not show significant improvements for driver comfort, spring travel, dynamic wheel load and kingpin accelerations. However, for fatigue damage, a substantial improvement is obtained for the maximum as well as for the mean Palmgren-Miner number. This increases the life span of the vehicle. Also, when slightly violating the constraints, a considerable improvement of spring travel is attainable.

To understand why the improvements are small, the influence of the optimal dynamic stiffness and damping on the six criteria has been examined. Therefore, the optimisation criteria were calculated again. First, the optimal dynamic stiffness, combined with the dynamic damping of the conventional vehicle is used. Second, the criteria were calculated with the optimal dynamic damping of the new suspension in combination with the dynamic stiffness of the conventional vehicle.

This analysis revealed that for the maximum Palmgren-Miner number, mean Palmgren-Miner number, kingpin accelerations and driver comfort, the improvements found in the optimisations are mainly caused by the dynamic stiffness. When only the optimal dynamic damping and a constant stiffness was used, no improvement was found at all. In fact, the optimisation criteria were worse than those of the conventional vehicle.

To illustrate the effects of the dynamic stiffness and the dynamic damping separately, the spectra for the kingpin accelerations are shown in Figures 4.20 and 4.21. On the left hand side of these figures, the dynamic stiffness and dynamic damping are shown while, on the right hand side, the corresponding acceleration spectra are shown. Figure 4.20 shows the kingpin acceleration spectrum for optimal dynamic stiffness and constant damping.

<table>
<thead>
<tr>
<th>criterion</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( \zeta )</th>
<th>( f_0 ) [Hz]</th>
<th>( f^* ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>KA</td>
<td>2.60 \times 10^5</td>
<td>6.85 \times 10^4</td>
<td>2.73 \times 10^{-2}</td>
<td>16.0</td>
<td>9.9</td>
</tr>
<tr>
<td>DC</td>
<td>2.60 \times 10^5</td>
<td>7.12 \times 10^4</td>
<td>2.70 \times 10^{-2}</td>
<td>16.0</td>
<td>9.7</td>
</tr>
<tr>
<td>DL</td>
<td>2.60 \times 10^5</td>
<td>5.20 \times 10^4</td>
<td>1.13 \times 10^{-1}</td>
<td>15.4</td>
<td>10.9</td>
</tr>
<tr>
<td>ST</td>
<td>2.71 \times 10^5</td>
<td>3.42 \times 10^4</td>
<td>8.86 \times 10^{-2}</td>
<td>1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>PM_{max}</td>
<td>2.60 \times 10^5</td>
<td>7.12 \times 10^4</td>
<td>2.70 \times 10^{-2}</td>
<td>16.0</td>
<td>9.7</td>
</tr>
<tr>
<td>PM_{mean}</td>
<td>2.60 \times 10^5</td>
<td>4.97 \times 10^4</td>
<td>3.27 \times 10^{-2}</td>
<td>16.5</td>
<td>11.9</td>
</tr>
</tbody>
</table>

Table 4.13 Characteristic numbers for the truck driven on the minor road

<table>
<thead>
<tr>
<th>criterion</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( \zeta )</th>
<th>( f_0 ) [Hz]</th>
<th>( f^* ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>KA</td>
<td>2.60 \times 10^5</td>
<td>6.81 \times 10^4</td>
<td>3.35 \times 10^{-2}</td>
<td>15.8</td>
<td>9.8</td>
</tr>
<tr>
<td>DC</td>
<td>2.60 \times 10^5</td>
<td>6.81 \times 10^4</td>
<td>3.35 \times 10^{-2}</td>
<td>15.8</td>
<td>9.8</td>
</tr>
<tr>
<td>DL</td>
<td>2.60 \times 10^5</td>
<td>5.36 \times 10^4</td>
<td>1.12 \times 10^{-1}</td>
<td>15.4</td>
<td>10.7</td>
</tr>
<tr>
<td>ST</td>
<td>2.60 \times 10^5</td>
<td>3.22 \times 10^4</td>
<td>1.27 \times 10^{-1}</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>PM_{max}</td>
<td>2.60 \times 10^5</td>
<td>6.81 \times 10^4</td>
<td>3.35 \times 10^{-2}</td>
<td>15.8</td>
<td>9.8</td>
</tr>
<tr>
<td>PM_{mean}</td>
<td>2.60 \times 10^5</td>
<td>4.88 \times 10^4</td>
<td>4.12 \times 10^{-2}</td>
<td>16.3</td>
<td>11.9</td>
</tr>
</tbody>
</table>

Table 4.14 Characteristic numbers for the truck driven on the motorway

In Figure 4.20 the influence of the dynamic stiffness is clearly visible. In the frequency range in which the dynamic stiffness becomes rather small, the power spectral density values of the spectrum are smaller than the conventional or reference values. However, as soon as the dynamic stiffness is larger than the dynamic stiffness of the reference suspension, the power spectral density values for the spectrum are larger than the reference values. This is a
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A significant disadvantage of the new suspension is the useful dip in the stiffness characteristic, which is always accompanied by a peak that has a negative effect on the spectrum. For optimisation criteria in which the dynamic stiffness is more important than the dynamic damping, the new suspension only offers improvement if the stiffness dip can be placed at frequencies which have a high energy content in the acceleration spectrum. These frequencies have to be adjacent frequencies which have a smaller energy content. However, because of the flexible chassis and due to the large number of components mounted on the total system, many peaks are visible in the acceleration spectra. This means that each peak in the spectrum is followed by another peak, making it difficult to place the stiffness peak in a frequency range with a low energy content.

Figure 4.21 shows the kingpin acceleration spectrum for constant stiffness and optimal dynamic damping. The graphs show that the influence of the dynamic damping on the spectrum is very small.

For spring travel and dynamic wheel load, both especially important around the vehicle's body and axle eigenfrequency, the dynamic damping proved to be responsible for the improvements (for spring travel, again the case is considered in which the constraints were allowed to be violated with 5%). For these criteria, an even better result is obtained than in the optimisations with both varying stiffness and varying damping (dynamic wheel load 3.1% versus 1.3%, and spring travel 14.5% versus 11.6%).

Again, to illustrate the effects of the dynamic stiffness and damping separately, the spectra for dynamic wheel load are shown in Figures 4.22 and 4.23. Figure 4.22 shows the dynamic wheel load spectrum for optimal dynamic stiffness and constant damping.

When only the dynamic stiffness is varied, the dynamic wheel load increases. Applying only dynamic damping, dynamic wheel load is reduced. The same applies for spring travel. The dynamic damping has a positive effect on the spectrum over the whole frequency range. The results confirm that, around the body and axle eigenfrequencies, more damping is desired than provided by the conventional suspension system. This is possible with the new suspension, but the associated dynamic stiffness has a negative effect on the optimisation criteria. The fact that dynamic damping and dynamic stiffness can not be controlled separately is a substantial disadvantage of the new suspension system.
4.2.10 Other applications of the new suspension

As a result of the discussion in the previous subsection, possible other applications for the new suspension can be pointed out.

A major reason given for the poor results, is the complexity of the dynamic system to be suspended. This system shows a large number of peaks in the spectra of interest, meaning that the positive effect of the new suspension is always accompanied with a negative effect. If the energy content of the spectrum is lowered for a certain frequency range, the energy content will increase for the subsequent frequency range. Better results can be expected with the new suspension system if rigid components are suspended which do not exhibit this many eigenfrequencies. In this way, the new suspension could be applied to decrease a resonance peak without the large negative influence on the rest of the spectrum, occurring in our dynamic system.

In view of this, one might think of the following applications:
- cabin suspension,
- seat suspension,
- passenger car suspensions,
- train suspensions.

An example of a rigid component (rigid in the frequency range where the suspension is of importance), where these types of suspension are successfully applied, is the mounting of engines in both trucks and passenger cars.

4.3 The new suspension and the response on incidental excitations

In this section, we focus on the dynamic behaviour of the truck and semi-trailer with respect to incidental road irregularities. The new suspension is mainly intended to improve various ride aspects of the truck and semi-trailer for road excitations described by stochastic properties. This was done under the restriction that the dynamic responses to incidental excitations should not get worse compared to the reference vehicle. Incidental excitations usually involve large compression of the suspension elements, which causes a non-linear behaviour. Hence, a three-dimensional non-linear vehicle model of the truck and semi-trailer was developed. The vehicle model is a multi-body model in which the springs and dampers were treated as non-linear elements. Vehicle simulations in the time domain with both the reference vehicle and a vehicle equipped with the new suspension were performed [Stevenhagen, 1997]. In these simulations, the rear suspension compression, chassis accelerations, cargo accelerations, cabin accelerations and tyre forces were of main interest. To excite the model, a set of generic deterministic excitation profiles was used. This set contained a brick lying on the road surface, a traffic hump, the transition from normal asphalt to scraped asphalt, a well in the road of which the lid is missing, and a wave in the road surface. The exact dimensions of the excitations and the vehicle speeds with which the vehicle was driven over the profiles, are collected in Table 4.15.

<table>
<thead>
<tr>
<th>shape</th>
<th>A [m]</th>
<th>B [m]</th>
<th>v [km/h]</th>
<th>others</th>
</tr>
</thead>
<tbody>
<tr>
<td>brick</td>
<td>0.105</td>
<td>0.065</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>traffic hump</td>
<td>0.600</td>
<td>1.400</td>
<td>20</td>
<td>C=0.10</td>
</tr>
<tr>
<td>scraped road</td>
<td>0.070</td>
<td></td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>no lid on well</td>
<td>0.600</td>
<td>0.100</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>wave</td>
<td>25.00</td>
<td>0.500</td>
<td>80</td>
<td>shape: sin²</td>
</tr>
</tbody>
</table>

Simulations were performed with three variants of the new suspension designated as a so-called low frequency suspension, middle frequency suspension and high frequency suspension, respectively. The eigenfrequency \( f \) of the low frequency suspension was 1.8 Hz (damping peak around the body eigenfrequency), of the middle frequency suspension 12.5 Hz (damping peak around the axle eigenfrequency) and of the high frequency suspension 17.0 Hz. The latter as a result of the optimisations for fatigue loads as described in the Sections 4.2.7 and 4.2.8.

For all five excitations, the results revealed that only the low frequency suspension had a large influence on the dynamic responses. For this suspension, the shapes of the calculated time signals were different from the reference suspension. For the middle and high frequency suspension, only minor influences on the dynamic responses and the shape of the time signals were found. This is illustrated in Figure 4.24 which shows the time signals of the vehicle driven over the traffic hump using the reference (std) suspension and the three new suspensions (low, middle and high).
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For the five incidental excitations, the following four criteria were used to evaluate the dynamic behaviour of the vehicle:
- the maximum absolute vertical acceleration of the comfort point (max(coz)),
- the maximum absolute vertical acceleration of the kingpin (max(akz)),
- the maximum compression of the springs, as a measure for the necessary suspension working space (max(sprtr)),
- the time that the tyre force vanishes (t(tyre lift off)).

In the latter case, the tyres lose contact with the road surface which means that they are not able to transmit forces in lateral direction, which leads to inferior handling characteristics. The results of the simulations are presented in Tables 4.16 to 4.20. In these tables, a superscript is used to indicate how many times the tyre force vanishes, during a single excitation. The superscript # means that the tyre force only vanished once, the superscript ## means that the tyre force vanished twice. An index is also used for the accelerations of the comfort point and the kingpin. Sometimes, the suspension compression is so large that the axle hits the chassis, leading to very large accelerations. The number of times the axle hits the chassis is indicated as follows. Index * indicates that axle-chassis contact occurred once, index ** indicates that axle-chassis contact occurred twice.

### Table 4.16 Simulation results for the scraped road

<table>
<thead>
<tr>
<th></th>
<th>std</th>
<th>low</th>
<th>middle</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>max(coz) [m/s²]</td>
<td>9.00</td>
<td>10.4</td>
<td>9.00</td>
<td>9.01</td>
</tr>
<tr>
<td>max(akz) [m/s²]</td>
<td>5.32</td>
<td>6.66</td>
<td>6.35</td>
<td>5.51</td>
</tr>
<tr>
<td>t(tyre lift off) [s]</td>
<td>0.03*</td>
<td>0.03*</td>
<td>0.03*</td>
<td>0.03*</td>
</tr>
<tr>
<td>max(sprtr) [cm]</td>
<td>6.1</td>
<td>5.6</td>
<td>5.6</td>
<td>6.0</td>
</tr>
</tbody>
</table>

### Table 4.17 Simulation results for the well with missing lid

<table>
<thead>
<tr>
<th></th>
<th>std</th>
<th>low</th>
<th>middle</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>max(coz) [m/s²]</td>
<td>10.6</td>
<td>10.4</td>
<td>10.7</td>
<td>10.7</td>
</tr>
<tr>
<td>max(akz) [m/s²]</td>
<td>6.25</td>
<td>8.75</td>
<td>7.18</td>
<td>6.36</td>
</tr>
<tr>
<td>t(tyre lift off) [s]</td>
<td>0.07*#</td>
<td>0.06*#</td>
<td>0.04*#</td>
<td>0.06*#</td>
</tr>
<tr>
<td>max(sprtr) [cm]</td>
<td>6.9</td>
<td>6.5</td>
<td>6.3</td>
<td>6.7</td>
</tr>
</tbody>
</table>

### Table 4.18 Simulation results for the brick

<table>
<thead>
<tr>
<th></th>
<th>std</th>
<th>low</th>
<th>middle</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>max(coz) [m/s²]</td>
<td>2.56</td>
<td>2.65</td>
<td>4.02</td>
<td>4.29</td>
</tr>
<tr>
<td>max(akz) [m/s²]</td>
<td>2.51</td>
<td>1.83</td>
<td>2.27</td>
<td>2.15</td>
</tr>
<tr>
<td>t(tyre lift off) [s]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>max(sprtr) [cm]</td>
<td>0.89</td>
<td>0.14</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

### Table 4.19 Simulation results for the traffic hump

<table>
<thead>
<tr>
<th></th>
<th>std</th>
<th>low</th>
<th>middle</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>max(coz) [m/s²]</td>
<td>15.4</td>
<td>15.5</td>
<td>15.1</td>
<td>15.1</td>
</tr>
<tr>
<td>max(akz) [m/s²]</td>
<td>8.87</td>
<td>8.45</td>
<td>8.99</td>
<td>8.83</td>
</tr>
<tr>
<td>t(tyre lift off) [s]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>max(sprtr) [cm]</td>
<td>11.5</td>
<td>6.2</td>
<td>11.1</td>
<td>11.4</td>
</tr>
</tbody>
</table>

### Table 4.20 Simulation results for the wave

<table>
<thead>
<tr>
<th></th>
<th>std</th>
<th>low</th>
<th>middle</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>max(coz) [m/s²]</td>
<td>57.2**</td>
<td>34.8</td>
<td>57.0**</td>
<td>57.5**</td>
</tr>
<tr>
<td>max(akz) [m/s²]</td>
<td>43.2**</td>
<td>34.1</td>
<td>43.7**</td>
<td>43.3**</td>
</tr>
<tr>
<td>t(tyre lift off) [s]</td>
<td>0.52*#</td>
<td>0.23*#</td>
<td>0.48*#</td>
<td>0.51*#</td>
</tr>
<tr>
<td>max(sprtr) [cm]</td>
<td>13.0</td>
<td>13.7</td>
<td>12.9</td>
<td>12.8</td>
</tr>
</tbody>
</table>

Figure 4.24 Time signals calculated for the traffic hump
From the tables, the following conclusions can be drawn. The maximum accelerations of the comfort point and the kingpin, remain about the same as those of the reference vehicle. However, there are two exceptions. The first is the response for the brick excitation. The acceleration increases significantly for the middle and high frequency suspensions, although the maximal values are still rather small when compared to, for example, the wave excitation. The second is the response of the low frequency suspension on the wave excitation. The acceleration of the comfort point is much smaller than for the conventional suspension. This is caused by the fact that chassis-axle contact occurs only once instead of twice as for the reference vehicle.

The tyre lift-off time stays about the same for every excitation, except for the wave excitation. Again for the low frequency design, the tyre lift-off time is approximately split in halves and tyre lift-off only occurs once. The suspension compression stays approximately the same for all excitations, except for the traffic hump. In this latter case, the suspension compression of the low frequency suspension is considerably smaller than for the conventional suspension.

The overall conclusion, of the simulations presented in this section, is that the dynamic behaviour of the vehicle with the new suspension driven over incidental excitations has not worsened compared to the dynamic behaviour of the reference vehicle.

Chapter 5

Concept definition

In this chapter, a design concept of the new suspension is defined which should have the same dynamic stiffness and dynamic damping as the flywheel model presented in the preceding chapter. Calculations using this concept will show that the system’s settings should be adjustable to the driving conditions of the truck and semi-trailer. The changes in driving conditions that will be considered, involve changes in the vehicle’s speed, road surface, and cargo weights. Based on the original concept, an adjustable adaptive concept is presented. A method will be presented which allows the control strategy for the adaptive suspension to be derived and different driving conditions to be recognised.

5.1 Translation of the flywheel model into a concept

To arrive at a concept design for the new suspension, the simulation model used in Chapter 4 must be translated into a physically realisable concept. Therefore, this model is first translated into a hydraulic model which is based on the design of the new suspension, as presented in Chapter 2. By assuming equal transfer functions for both models, the dimensions of the real concept design can be derived.

To translate the flywheel model into the hydraulic model, the equations of motion for both models are required. The equations of motion for the flywheel model are expressed in terms of the flywheel parameters, whereas the equations of motion of the hydraulic model are expressed in terms of the hydraulic parameters (e.g. volumes, dimensions of the fluid channel). The parameters of the hydraulic model then can be determined by assuming that both models react identical to identical input signals. In the next sections, the equations of motion are derived for the flywheel model after which the equations of motion for the hydraulic model are derived.

5.1.1 Equations of motion for the flywheel model

Figure 5.1 shows the flywheel model, already described in Chapter 3.
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Figure 5.1 The flywheel model

In this figure, $F_1$ and $F_2$ are forces, $L$ is the length of the flywheel arm, $K_1$, $K_2$, and $K_3$ are spring stiffnesses, $B$ is a damping coefficient, $x_1$ and $x_2$ are displacements, $\varphi_1$ and $\varphi_2$ are rotations, and $M$ is a mass (to multiply $K_1$, $K_2$, $K_3$, $B$ and $M$ with $L^2$ yields a rotational stiffness, rotational damping and rotational inertia).

For the flywheel model, four kinematic quantities can be distinguished, $x_1$, $x_2$, $\varphi_1$ and $\varphi_2$. However, for small displacements, $\varphi_1$ can be expressed in terms of $x_1$ and $x_2$:

$$\varphi_1 = \frac{x_1 - x_2}{L}$$  \hspace{1cm} (5.1)

This implies, that the system can be described with the following three degrees of freedom, collected in the vector $q$:

$$q_F^T = [x_1 \ x_2 \ \varphi_2 L]$$  \hspace{1cm} (5.2)

where the index $F$ denotes parameters concerning the flywheel model. The last degree of freedom, $\varphi_2$, has been multiplied by $L$, so that all degrees of freedom have the same dimension.

The equations of motion of the flywheel model are:

$$M_F \ddot{q}_F + B_F \dot{q}_F + K_F q_F = F_F$$

with matrices $M_F$, $B_F$ and $K_F$ given by,

$$M_F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M \end{bmatrix}, \quad B_F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{bmatrix}, \quad K_F = \begin{bmatrix} K_1 + K_2 & -K_1 & -K_2 & -K_2 \\ -K_1 & K_1 + K_2 & K_2 & K_2 \\ -K_2 & -K_2 & K_2 & K_2 \\ -K_2 & -K_2 & -K_2 & K_3 \end{bmatrix}$$

5.1.2 Equations of motion for the hydraulic model

Figure 5.2 shows the hydraulic model of the new suspension discussed in Chapter 2, with all important design parameters.

For this model, five kinematic quantities can be distinguished, gathered in vector $q$:

$$q_H^T = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]$$  \hspace{1cm} (5.4)

where index $H$ denotes the parameters of the hydraulic model.
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If the fluid is assumed to be incompressible, the following relations can be derived;

\[ x_4 = \frac{a}{A_3} x_3 \]  
\[ x_5 = \frac{a}{A_2} x_3 \]  

As a result, three degrees of freedom remain;

\[ \begin{bmatrix} a \end{bmatrix} x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \]  

Assuming adiabatic compression of the air, and a constant effective cross sectional area of the air spring during compression, the dynamic stiffness of the two air volumes can be represented by the following formula (linearised around a specific pressure and for a specific volume) [Vo8, 1992];

\[ c_{dyn,1} = \kappa \frac{p_{1,0}}{V_{1,0}} A_1^2 \]  
\[ c_{dyn,2} = \kappa \frac{p_{2,0}}{V_{2,0}} A_2^2 \]  

where \( c_{dyn,1} \) and \( c_{dyn,2} \) are the dynamic stiffnesses for volume \( V_1 \) and \( V_2 \) respectively, \( V_{1,0} \) and \( V_{2,0} \) are the corresponding volumes for \( V_1 \) and \( V_2 \) in equilibrium conditions. \( p_{1,0} \) and \( p_{2,0} \) are the static pressures in volume \( V_1 \) and \( V_2 \) in equilibrium conditions and \( \kappa \) is the constant for adiabatic air compression (\( \kappa = 1.4 \)).

Now three quantities are defined,

1. The volumetric stiffness \( S_1 \) of chamber 1 is defined as;

\[ S_1 = \kappa \frac{p_{1,0}}{V_{1,0}} \]  

2. The volumetric stiffness \( S_2 \) of chamber 2 is defined as;

\[ S_2 = \kappa \frac{p_{2,0}}{V_{2,0}} \]

Equation 5.12 is investigated in more detail. In our suspension, the fluid flow oscillates. However, to calculate the pressure loss, normally for these oscillating flows, the formula for a steady flow through a channel is used [e.g Holzemer, 1987, Spurk et al., 1985], which reads;

\[ \Delta p = \frac{1}{2} \rho (\xi + \lambda \frac{l}{d}) x_3^2 \]  

in which \( \Delta p \) denotes the pressure loss due to entry and exit losses as well as resistance of the channel, \( \rho \) is the specific mass of the fluid, \( \xi \) represents the pressure loss coefficient of the entry and exit losses of the channel, \( \lambda \) is the loss coefficient of the channel, \( l \) is the length of the channel, and \( d \) is the diameter of the channel.

Substituting Equation 5.13 into Equation 5.12, this one can be converted into an equation for the damping \( b_{ch} \) in the channel;

\[ b_{ch} = \frac{1}{2} \rho (\xi + \lambda \frac{l}{d}) x_3 \]

This means that the damping coefficient depends on the velocity of the fluid. This will cause the system's equations of motion to be non-linear. In order to enable the comparison of the flywheel model with the hydraulic model, this non-linear term in the equation has to be linearised. This linearisation process is described in Appendix C. There it is shown how the non-linear term \( b_{ch} \) can be replaced by a linear term \( b_1 \), assuming that for a single harmonic cycle, the energy consumed by the non-linear term is identical to the energy consumed by the linear component. The linear term can then be represented by:

\[ b_1 = 12 \cdot a \cdot \frac{1}{2} \rho (\xi + \lambda \frac{l}{d}) RMS[x_3] \]

in which \( b_1 \) is the linear damping constant and \( RMS[x_3] \) is the RMS value of the fluid velocity as a function of the time.
Because the accelerations in the fluid channel are many times larger (up to 40 times) than the accelerations of the fluid in the two volumes, the inertia forces acting upon the fluid in the channel are also many times larger than the inertia forces acting upon the fluid in the two volumes. Also, since the masses of the fluid in the volumes are not incorporated in the flywheel model, they should not be incorporated in the hydraulic model. Because of this, it is assumed that only the inertia forces of the fluid in the channel are of interest. The resulting equations of motion are:

\[
M_H \ddot{q}_H + B_H \dot{q}_H + K_H q_H = F_H
\]

(5.16)

with,

\[
q_H^T = [x_1, x_2, x_3]
\]

\[
F_H^T = [F_1, F_2, 0]
\]

\[
M_H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho a l \end{bmatrix}
\]

B_H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b \end{bmatrix}

K_H = \begin{bmatrix} A_1^2 S_1 & -A_1^2 S_1 & -A_1 a S_1 \\ -A_1^2 S_1 & A_1^2 S_1 & A_1 a S_1 \\ -A_1 a S_1 & A_1 a S_1 & a^2 S_1 + a^3 S_1 \end{bmatrix}

5.1.3 Translation of the flywheel model into the hydraulic model

In order to derive the dimensions for the hydraulic model, the transfer function between \(x_1\) and \(x_2\) of the flywheel model has to be equal to the transfer function between \(x_1\) and \(x_2\) of the hydraulic model. For identical dynamic forces, the resulting displacements \(x_1\) and \(x_2\) must be identical. This is the case when the system matrices for the flywheel model equal the system matrices of the hydraulic model. This is expressed by the following equation:

\[
\begin{align*}
M_H &= M_F \\
B_H &= B_F \\
K_H &= K_F
\end{align*}
\]

(5.17)

In this equation, an extra constant \(C\) is introduced. Later, it will appear that the introduction of this constant enables the definition of a range of hydraulic designs for just a single flywheel model.

The introduction of this constant is allowed because of the following two reasons:

1. Degree of freedom \(q^L\) for the flywheel model and degree of freedom \(x_1\) for the hydraulic model, are internal degrees of freedom, and do not have to be identical. It is sufficient when they are linearly dependent; \(C x^H = x^F\). Factor \(C\) is accounted for in the system matrices and the degrees of freedom of the flywheel model.

2. The multiplication of the third rows of the system matrices with the degrees of freedom is equal to zero for both systems, therefore the last row of the system matrices for the flywheel model may also be multiplied with the constant \(C\).

The system matrices for the flywheel model can then be expressed by:

\[
q_F^T = \begin{bmatrix} x_1, x_2, \frac{\phi v L}{C} \end{bmatrix} \quad M_F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C^2 M \end{bmatrix} \quad B_F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & C^2 B \end{bmatrix}
\]

\[
K_F = \begin{bmatrix} K_1 + K_2 & -K_1 - K_2 & -C K_2 \\ -K_1 - K_2 & K_1 + K_2 & CK_2 \\ -CK_2 & CK_2 & C^2 (K_1 + K_2) \end{bmatrix}
\]

Using Equations 5.17 and 5.14, the following equations are obtained:

\[
\rho a l = C^2 M
\]

(5.18)

\[
1.2 \cdot a \cdot \frac{\dot{L}}{d} (\xi + \frac{1}{d}) R M S [\ddot{x}_3] = C^2 B
\]

(5.19)

\[
A_1^2 S_1 = K_1 + K_2
\]

(5.20)

\[
A_1 a S_1 = C K_2
\]

(5.21)

\[
a^2 S_1 + a^3 S_1 = C^2 (K_1 + K_2)
\]

(5.22)

These equations contain the following parameters of the hydraulic model, \(\xi, \lambda, \rho, \dot{L}, a, A, b, S_1, S_2, R M S [\ddot{x}_3]\) and the parameters of the flywheel model, \(C, M, B, K_1, K_2, K_F\).

In these equations, all parameters can freely be chosen, except for \(R M S [\ddot{x}_3]\) and \(\lambda\).
The value for $RMS[\dot{x}_1]$ is calculated via:

$$RMS[\dot{x}_1] = \frac{RMS[\phi_2 \cdot L]}{C}$$  \hspace{1cm} (5.23)

where $RMS[\phi_2 \cdot L]$ is calculated with the three-dimensional vehicle model of the truck and semi-trailer containing the flywheel model. The value of $RMS[\phi_2 \cdot L]$ depends on the flywheel parameters and the excitation of the vehicle.

Coefficient $\lambda$ depends on the relative roughness $k$ of the inner surface of the fluid channel, the Reynolds Number $Re$, and the diameter $d$, of the channel and can be described by the following formulas [e.g. Beitz et al., 1983]:

For laminar flows ($Re < 2320)$, the Poiseuille equation reveals;

$$\lambda = \frac{64}{Re}$$  \hspace{1cm} (5.24)

For turbulent flows ($Re > 2320$);

$$\lambda = \begin{cases} \frac{1}{(2^{6.95} \log(Re/\sqrt{2.51}))^2} & \text{if } Re < 65 \frac{d}{k} \\ \frac{1}{(2^{6.95} \log(\frac{2.51}{Re} + 0.27))^2} & \text{if } 65 \frac{d}{k} < Re < 1300 \frac{d}{k} \\ \frac{1}{(2^{6.95} \log(3.71d/k))^2} & \text{if } Re > 1300 \frac{d}{k} \end{cases}$$  \hspace{1cm} (5.25)

with,

$$Re = \frac{\rho \cdot d}{\eta}$$  \hspace{1cm} (5.28)

in which $\eta$ is the dynamic viscosity of the fluid.

5.2 From model to concept

With the two models previously described, the dimensions of the concept design for the new suspension can be determined. First a so-called passive concept design will be presented. In this context, passive means that all hydraulic parameters such as length and diameter of the fluid channel are fixed and cannot be adjusted. For the definition of this concept, a flywheel model with specific parameter values should be chosen as a baseline design. The flywheel model chosen for this purpose, is the result of a simultaneous optimisation of driver comfort in $x$-direction, dynamic wheel load, and spring travel [Konings, 1993, (1)]. The optimisation was performed for a laden truck and semi-trailer, driving at a speed of 80 km/h over an average German main road (the so-called Landesstraße [Braun et al., 1991]). For the purpose of optimising the flywheel model, a so-called Performance Index $PI$, was defined. With this Performance Index it is possible to optimise several aspects simultaneously and even weigh the different aspects. The Performance Index is defined as follows;

$$PI = \sqrt{W_1 \left( \frac{MS(\text{cox})}{MS(\text{cox})_c} \right)^2 + W_2 \left( \frac{MS(\text{dwl})}{MS(\text{dwl})_c} \right)^2 + W_3 \left( \frac{MS(\text{sprtr})}{MS(\text{sprtr})_c} \right)^2}$$  \hspace{1cm} (5.29)

where $MS(\text{cox})$, $MS(\text{dwl})$ and $MS(\text{sprtr})$ are the Mean Square values for comfort in the longitudinal direction, dynamic wheel load and spring travel, respectively. $W_1$, $W_2$, $W_3$ are weighing factors. Index $c$ denotes a truck equipped with a conventional suspension, and index $n$ denotes a truck equipped with the new suspension. In the optimisations discussed in this section, the three weighing factors were chosen equal to one. For the conventional vehicle, $PI$ then has a value of $\sqrt{3}$. In fact, in the way the Performance Index is defined, the spectra of the different aspects to be optimised are normalised first and then counted together. Of this new spectrum, the RMS value is determined.

The eigenfrequency $f_0$ of the optimised flywheel model, with $PI$ as a criterion, is 2.3 Hz. The improvement in the RMS value for comfort in longitudinal direction is 7.0 %, the RMS value for spring travel was improved by 10.1 %, while the RMS value for dynamic wheel load was improved by 0.3 %. These large improvements could not be obtained in Chapter 4. This is caused by two reasons. First, in the optimisations of $PI$, no attention was paid to the violation
of the constraints, defined in Chapter 4. Second, the truck and semi-trailer model used in the calculations was a two-dimensional vehicle model in contrast to the three-dimensional vehicle model used in the present study.

5.2.1 The passive concept
In the definition of the concept for the new suspension, a few practical problems arose which have been expressed in design demands. These design demands are:

- no major modifications to the truck are to be made, the original mounting points of the suspension may not be changed,
- the new suspension must be an optional item which can be exchanged with the standard air springs,
- the new suspension must be as small as possible,
- only little weight may be added to the vehicle.

A concept for the new suspension fulfilling these demands was defined by Van Heck [1994] and Bebermeier [1994]. This concept is pictured in Figure 5.3 showing the suspension for one side of the vehicle.

![Concept design for the new suspension shown for one vehicle side](image)

In this concept design, the original mounting points of the air springs on the axle and the chassis are maintained. The two air mounting points of the air springs are connected to Buffer 1 via an air channel. The combined volume of the air springs, air channel and the buffer corresponds to volume V₁ of the hydraulic model (Figure 5.2). The dimensions of the air channel were selected in such a way that a mass effect of the air in the channel does not occur in the frequency range of interest, i.e. 0-20 Hz (eigenfrequency of the air-mass on the two air springs; 32 Hz). Volume V₁ is connected to Buffer 2 (volume V₂) by means of the fluid channel. This fluid channel has a length l and a cross sectional area a. The amount of fluid in the buffers was selected in such a way that, at complete compression and during dynamic excitations, no air enters the liquid channel. A prototype of this concept was built and tested by Konings [1995, (2)]. Experiments with this prototype are discussed in Chapter 6.

5.2.2 The need for an adaptive suspension
With the concept design, it is possible to calculate the behaviour of the truck and semi-trailer, equipped with the new suspension, for other driving conditions than the ones it was designed for. Also, it is possible to vary the hydraulic parameters of the concept design and to determine their influence on the dynamic behaviour of the truck and semi-trailer. Such calculations were used to determine the optimal settings for the hydraulic parameters for other driving conditions.

For the new suspension, the following parameters can, in principle, be selected to be made adjustable while driving:

- the length l of the fluid channel,
- the diameter d of the fluid channel,
- the damping b in the fluid channel using a valve,
- the volume V₁,
- the volume V₂.

Nevels [1995] investigated different designs to design these adjustable parameters in practice and concluded that, due to constructive aspects, two parameters were suited for adaptivity;

- the Diameter d of the fluid channel,
- the second Volume V₂.

The effects of changes in these parameters on PI (Equation 5.29) have been calculated with the three-dimensional truck and semi-trailer model. To do this, the modified parameters of the flywheel model were calculated from the parameters of the hydraulic model.
Hence, Equations 5.18 to 5.22 were rewritten yielding:

\[ M = \frac{p_{al}}{C^2} \]  
\[ B = \frac{ba}{C^2} \]  
\[ K_1 = A_1aS_1 - \frac{A_1aS_1}{C} \]  
\[ K_2 = \frac{A_1aS_1}{C} \]  
\[ K_3 = \frac{a^3S_1 + a^3S_2 - CA_1aS_1}{C^2} \]

Unlike \( K_1, K_2, K_3 \) and \( M \), parameter \( B \) can not be calculated directly. This flywheel model parameter is a function of \( b \), which in turn is a function of \( \text{RMS}[\phi_L] \) (see Equations 5.21 and 5.22). This means that \( B \) is also a function of \( \text{RMS}[\phi_L] \) which in turn is a function of \( K_1, K_2, K_3, M \) and \( B \). Therefore, the solution of this set of equations must be determined by means of iteration.

With Equations 5.30 to 5.34, the modified parameters of the flywheel model may be calculated, corresponding with the hydraulic model in which \( d \) or \( V_2 \) has been changed. Next, the effect of this modification on the Performance Index \( PI \) was calculated for a specific driving condition.

The different driving conditions involve a change of vehicle speed, cargo weight and type of road surface. In the calculations, the basic driving condition was a fully laden vehicle, driven over a German main road at a speed of 80 km/h. The following modified driving conditions were chosen:

- vehicle speeds of 50, 60, 70 and 80 km/h,
- laden, half laden and unladen vehicle,
- the undulation exponent, \( w \), of the road surface was chosen to be half, normal and multiplied with 1.25, respectively,
- the reference spectral density, \( G_{(w)} \), of the road surface was chosen to be half, normal and multiplied with 3, respectively.

The reference spectral density \( G_{(w)} \) and undulation exponent \( w \) for the chosen German main road are \( 4.04 \times 10^{-7} \) and 2.4, respectively.
### Concept definition

#### 5.2.3 Estimation of the driving condition

Prior to adjusting the suspension for variable driving conditions, these conditions have to be measured or estimated. The vehicle speed is already measured at the vehicle (via the tachometer or via the ABS system). The amount of cargo can be estimated via the air pressure in the air bellows. Due to the automatic levelling system, the distance between axle and chassis is constant for static conditions. When the cargo weight changes, the springs are compressed or uncompressed, and the distance between axle and chassis would normally increase or decrease. The levelling system reacts by increasing or decreasing the pressure in the air bellows to achieve the normal distance between axle and chassis. The air pressure can thus be used to estimate the cargo weight.

Estimation of the type of road surface appears not so straightforward. The procedure to estimate the type of road surface is extensively described in detail by Wieringa [1995]. Only a short description is given here. A road surface can be described with a single road profile of the road \(G_{\text{rho}}\) and \(\omega\) and the correlation between the two road tracks. However, to distinguish between different road surfaces, a description of a single profile suffices. Therefore, the system to be designed has to provide a description of a single profile of a particular road surface. A method has been developed based on vehicle accelerations. The accelerations of two points on the front axle and one point on the chassis are measured, to identify the movements of the front axle system caused by the road excitation. Subsequently, the three-dimensional truck and semi-trailer model was used to convert the combination of these accelerations into the undulations of a profile of the particular road \(G_{\text{rho}}\) and \(\omega\). This knowledge can be used in the truck. If accelerations are measured on the truck, the type of road surface can be determined with the knowledge of the calculations. The main parameter in the system is the measurement time, which has to be chosen in such a way, that a compromise is achieved between response velocity and the accuracy of the estimate. A measurement time of approximately 50 seconds appeared to yield a good compromise: after about 50 seconds of driving over a specific road, the surface could be identified sufficiently accurate. Although the system was basically designed for one particular set of parameter values of truck and semi-trailer (like load condition, spring stiffnesses, damper coefficients), changes in these parameters, as may occur in practice, do not significantly influence the accuracy of the procedure.

#### 5.2.4 The adaptive concept

An adaptive version of the previously described concept was developed by Nevels [1995]. It incorporates two adaptive parameters; the Diameter \(d\) of the channel and the Volume \(V_2\). This concept, shown in Figure 5.6, is to replace the non-adaptive concept. It is connected to the two air bellows with the same air channel, as shown in Figure 5.3.
Chapter 6

Experimental verification

The functioning of the two concepts presented in Chapter 5 has been verified experimentally. The first purpose of this was to verify the dynamic behaviour of the two new suspensions before they were to be mounted on the truck. To this aim, the suspension itself was placed on a test-rig to measure its dynamic stiffness and damping, and to determine the differences between the calculated and the measured results. The second purpose of these experiments is to verify the dynamic behaviour of truck and semi-trailer, equipped with the new suspension. For this purpose, the new suspension was mounted on the truck, and transfer functions of the truck were measured on a truck test-rig.

6.1 Verification of the non-adaptive new suspension

In this section, the test procedure is explained followed by a presentation of the results. The tests were performed by Konings [1995]. Finally, the dynamic behaviour of the truck equipped with the new suspension is verified.

6.1.1 Test procedure

The dynamic stiffness and damping of the suspension has been determined using a hydraulic actuator. These measurements were carried out at ContiTech Formteile GmbH, according to standard test conditions as described in [General Motors]. The air springs of the new suspension were excited with a prescribed sinusoidal displacement while the forces to be generated for this displacement were measured. In Chapter 5, it was indicated that the damping of the hydraulic model, and consequently the dynamic stiffness and damping, depend on the RMS value of the fluid velocity in the channel. The fluid velocity, in turn, depends on the excitation amplitude. Therefore, experiments with the new suspension were carried out at different amplitudes. The amplitude \( S_m \) of the harmonic displacement was set to 0.5, 1, 2, 5, 10, 15 and 20 mm respectively. The frequencies selected for the excitation, range from 0.5 to 5.1 Hz in steps of 0.2 Hz followed by excitations from 6 to 10 Hz in steps of 1 Hz. Hysteresis loops were then determined as shown in Figure 6.1 in which \( S_m \) is the static deflection of the air spring which was set equal to the spring length when mounted on the truck. The measured static force at the proper initial air spring length then is \( F_m \).
The measured dynamic stiffness and damping are pictured in Figure 6.2, for amplitudes of 1, 2, 5 and 15 mm. Also pictured are the desired dynamic stiffness and damping of the optimal flywheel model which was used as a base for the development of the non-adaptive suspension.
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Displacement of a certain amplitude and for a specific frequency. At every time step, the damping is then determined with the current fluid speed with Equation 5.14. Performing these calculations for the whole frequency range (0-20 Hz), gives the dynamic stiffness and dynamic damping for a specific amplitude which must be repeated for different amplitudes. The amplitude dependency calculated with the described method is shown in Figure 6.3.

Evidently, the amplitude dependency near the eigenfrequency of the fluid can be accounted for. However, the deviations for frequencies lower and higher than the eigenfrequency are not predicted. In these two frequency ranges, the stiffness is too high. This is caused by the non-linear effect of the air bellows. It is known that air bellows show a higher stiffness for small amplitudes than for large amplitudes. This effect causes the deviation in stiffness over the whole frequency range, an effect which also occurs in conventional air spring suspensions.

With respect to the dynamic damping, the amplitude dependency of the damping peak can also be explained with the method described previously. Furthermore, the damping peak occurs at a frequency that is too low. However, this can be changed by adjusting the two air volumes of the buffers. Also, the damping is too high over the entire frequency range, especially for low frequencies. This is due to the structural damping in the air springs manifesting itself at low frequencies. The measured damping peak is also higher than was calculated. This is also because of energy dissipated by the air bellows which was not taken into account in the models.

Experimental verification

The mass effect of the air in the air hose between the air springs and the first buffer was also observed in the measurements. At frequencies in excess of 5 Hz, the stiffness started to decrease and at frequencies in excess of 9 Hz, the damping started to increase again. However, in the frequency range of 0 to 5 Hz, which will be considered in the following calculations, it can be concluded that this mass effect has no influence on the system's behaviour.

Conclusion of this section is that a variable dynamic stiffness and damping can be realised. The differences between the simulations and measurements can be explained and are therefore acceptable because they can be taken into account in future designs.

6.2 Experimental verification of the truck and semi-trailer equipped with the new non-adaptive suspension

To verify its dynamic behaviour in practice, the new suspension was mounted on a truck. Figure 6.4 shows the components of the new suspension, with a frame mounted on the chassis just ahead of the two rear wheels. The two buffers are mounted in the frame. The fluid channel connecting the two air volumes can be seen in front of the buffers.
The truck was placed on a test-rig at DAF Trucks NV. This test-rig is schematically shown in Figure 6.5. It consists of two excitation tables. Either the front wheels or the rear wheels can be excited with a prescribed displacement.

![Diagram of test-rig with excitation table and rear axle](image)

*Figure 6.5 The test-rig for validation of the vehicle with the new suspension*

First, the two front tyres were excited, with an anti-symmetric excitation and then with a symmetric excitation. The transfer functions were measured at typical measurement points on the truck. The same was done for the truck's rear axle. The excitation simulates the excitation level arising when the truck is driven over a minor road at a speed of 50 km/h.

The measured transfer functions were used in TricaT (Chapter 3) to estimate the response of the real vehicle on the minor road for a vehicle speed of 50 km/h (to calculate such responses, TricaT normally uses the transfer functions calculated with the three-dimensional vehicle model). The spectra calculated in this way will be denoted as the measured spectra. The measured dynamic stiffness and damping of Figure 6.2 were used in the vehicle model to calculate the response of the vehicle model for the same conditions. These spectra are denoted as the calculated spectra. Figure 6.6 shows the measured and the calculated spectra for the vehicle equipped with the conventional suspension and with the new suspension. The graphs show the acceleration of the comfort measurement point in x-direction, of a chassis point above the rear axle in z-direction and of a chassis point in the middle of the chassis in z-direction, respectively.

![Graphs showing measured and calculated spectra](image)

*Figure 6.6 Comparison of measured and calculated spectra*

The influence of the new suspension can clearly be seen in the measurements. Measurements correspond rather well with numerical simulations. In the first graph, for the frequency range 1.5-2.2 Hz, the measured signal for the conventional vehicle is larger than for the vehicle with the new suspension. The same is the case for the calculated spectra. For frequencies in the range of 2.2 to 2.7 Hz, the spectra for the conventional truck have smaller values than the spectra for the truck with the new suspension, this is also predicted with the calculations.
In the second graph, the same comments apply. In the frequency range of 1.5 to 2.2 Hz, the spectra for the new suspension have smaller values, and higher values in the frequency range of 2.2 to 2.7 Hz. Also this is predicted with the vehicle model; although, in the range of 1.5 to 2.2 Hz, the reduction in values of the calculated spectrum for the vehicle with the new suspension is slightly over estimated.

In the third graph, the influence of the new suspension can also clearly be seen. Again, it is predicted by the vehicle model. In the frequency range of 1.7 to 2.3 Hz, the measured values for the conventional vehicle are larger than for the vehicle with the new suspension. At higher frequencies, the values of the spectra are lower which is also predicted by the vehicle model.

From these measurements, it is concluded that the improvements in RMS values of the considered criteria are very small (<1%). This was also concluded in Chapter 4. A suspension tuned for low frequencies, as presented in this section, does not have much room for improvement.

6.3 Verification of the adaptive new suspension concept

This section discusses the experimental verification of the adaptive new suspension. The tests described in this chapter were performed by Nevels [1995]). To study the influence of a lower static stiffness on comfort, the partners in the CASCOV project decided to make the static stiffness of the adaptive new suspension lower than the static stiffness of the non-adaptive suspension and the conventional suspension. Although this study is not discussed in this thesis, the measurements will show the lower static stiffness. To reduce the stiffness, the air volume in the suspension system has to be increased. This means that the suspension system gets larger. To eliminate this problem, the pressure in the system was increased to 12 bars. However, at the moment the experiments were performed, the special air springs capable of handling this pressure were not available. Standard air springs, suitable for 6 bars were used. This corresponds to the pressure in the suspension system of a half laden truck and semi-trailer. Unfortunately, no appropriate experiments of the truck and semi-trailer equipped with the adaptive new suspension are available.

6.3.1 The test procedure and test-rig

The dynamic stiffness and damping of the adaptive suspension were determined on the same hydraulic actuator as used for the non-adaptive new suspension. The air springs were excited with a prescribed displacement for the same amplitudes and frequencies as described in subsection 6.1.1, and the accompanying forces were measured. The test-rig is shown in Figure 6.7, with the adaptive new suspension placed on it. The air spring which is excited, is mounted horizontally on the test-rig and can be seen in the middle of the photograph. The adaptive suspension, see the left hand side of the photograph, is connected to the air spring by the air channel. The liquid channel is not visible, because it is placed inside the container as shown in Chapter 5 (Figure 5.6).

6.3.2 Experimental results

The adaptive suspension has two adjustable parameters: the cross sectional area $a$ of the liquid channel connecting the two air volumes and the air volume $V_2$. For the adaptive suspension, the dynamic stiffness and damping were calculated in the same way as for the non-adaptive suspension. This calculation was performed for several combinations of $a$ and $V_2$. For the cross sectional area, a minimal, medium and maximal value were selected. The same subdivision was chosen for volume $V_2$. For all combinations of cross sectional area and volume, the dynamic stiffness and damping were calculated for amplitudes of 2 mm and 20 mm. The results of these calculations are shown in Figure 6.8. The graphs at the top of this figure show the effects of a variation in cross sectional area when the volume is at its minimum level. The graphs in the middle show these effects for the medium volume and finally, the graphs at the bottom show the results for the maximum volume.
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When the cross sectional area increases, the eigenfrequency increases and the dip and the peak in the stiffness increase. For an increase in Volume $V_2$, the eigenfrequency decreases, the dip as well as the peak in the stiffness characteristic increase, and the damping peak increases. This means that the height of the damping peak can be controlled with volume $V_2$, after which the eigenfrequency can be controlled by changing the cross sectional area of the liquid channel.

For the combinations used in the calculations, measurements were performed. The graphs at the top of figure 6.9 show the effects of a variation of the cross sectional area when the volume is at its minimum level. In the middle, the effects are shown when the volume is at its medium level and, finally, the graphs at the bottom show the effects for maximum volume.
The results confirm that it is possible to make the new suspension adaptive and to change the eigenfrequency, the dynamic stiffness and damping of the system by changing the cross sectional area and the volume of the second buffer.

When comparing calculated and measured results, similar differences can be observed as found earlier for the non-adaptive suspension. The amplitude dependency around the eigenfrequency is about the same and the same stiffness deviation is visible over the whole frequency range.

When the volume increases, the measured eigenfrequency remains approximately the same. This is not observed from the calculations which show an increase in eigenfrequency. This difference is explained by the design of the adaptive suspension in which a small hole (cross sectional area 1.5 mm) was drilled in top of the piston of volume $V_i$. This opening is necessary, because when the piston is used to adjust the volume, air must be able to flow between the volumes below the piston and above the piston. Initially, it was assumed that the hole was made small enough to ensure that, during dynamic excitation of the suspension, no air would flow through it. However, this was not the case, so that the calculated effect of the change of volume $V_2$ is larger than the measured effect. This also explains the differences between the measured and calculated maximum values for the dynamic damping. The air blowing through the hole, introduces extra damping. In a further prototype suspension, the hole was replaced by a valve which was opened to adjust the piston and closed during dynamic excitation of the truck. Preliminary experiments showed now, that the eigenfrequency did increase with increasing Volume $V_2$.

The expected change in the eigenfrequency as result of the change in cross sectional area, is larger than expected. This is caused by the definition of the hydraulic diameter $d_h$, for a rectangular cross section as it was used in calculations. In the definition for the hydraulic diameter, it is assumed that not all the fluid in the liquid channel takes part in the fluid motion. However, probably more liquid than expected takes part in the fluid motion, and a bigger cross sectional area should have been used in the calculations.

### 6.4 Conclusions

From the measurements performed with the two new suspensions, it can be concluded that:

- The new suspension shows the dynamic behaviour that was expected. Differences between the measured and calculated dynamic stiffness and dynamic damping can be explained and can be taken into account in future investigations.
- The dynamic behaviour of the truck and semi-trailer equipped with the new suspension can be predicted properly with calculations.

• The improvements found in the experiments confirm the conclusions drawn in Chapter 4: the obtained improvements with this suspension are not large enough to justify its series introduction.

• It is possible to design an adaptive variant of the new suspension which allows the eigenfrequency as well as the dynamic stiffness and damping to be controlled while driving.
Chapter 7

Conclusions and recommendations

7.1 Overview of the research

Choosing the spring and damping characteristics for a truck axle suspension is a compromise between conflicting design demands:

- minimisation of chassis accelerations in order to,
  - improve driver comfort,
  - reduce cargo accelerations,
  - reduce chassis fatigue damage,

- minimisation of spring travel to allow more payload volume within the legal restrictions on vehicle dimensions,

- minimisation of dynamic wheel load to improve road holding and to reduce road surface damage.

The analysis of the fundamental function of a suspension system revealed, that only for specific ranges of excitation frequencies, stiffness and damping is desired. For low frequencies (0-3 Hz), stiffness is desired to support the vehicle weight and to ensure that the vehicle body follows low frequency changes in the road surface (e.g. bridges). Due to the stiffness, the vehicle body will show a vibration eigenmode in this low frequency range. To dampen this eigenmode, damping must be applied in this frequency range. Due to the tyre stiffness, the vehicle’s axle shows a vibration eigenmode in the higher frequency range (10-15 Hz). Therefore, also at these frequencies, damping must be applied. In the medium frequency range and at frequencies in excess of the axle’s eigenfrequency, the stiffness and damping only introduce unnecessary forces into the vehicle resulting in undesired accelerations and dynamic wheel loads. Therefore, a suspension is desired which shows a stiffness and damping varying with the excitation frequency. This contrasts to the behaviour of a conventional suspension which exhibits stiffness and damping characteristics which do not vary with the excitation frequency.

7.1.1 Suspension selection

A large number of investigations, performed and aimed at improving vehicle suspensions, rely on non-linear components or (semi-)active systems. However, it is also possible to achieve desired stiffness and damping characteristics by means of linear passive components with additional degrees of freedom. Therefore, a systematic analysis has been performed to identify the suspension systems which show a frequency dependent behaviour using only linear
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springs, linear dampers and added masses. Here, for practical reasons, the number of degrees of freedom for the new suspension was restricted to three which is one degree of freedom more than a conventional suspension. The analysis revealed that within the range of all suspension systems, four typical suspensions can be distinguished. These four suspensions, show a different behaviour of the stiffness and damping characteristics. For these four suspension systems, concept designs were defined to evaluate the complexity of building such suspensions. Based on the information regarding the dynamic behaviour and the complexity of the concept designs, one of the four suspensions was selected for further investigation. This suspension consists of two air volumes (acting as air springs) connected by a channel which is filled with liquid. At the systems eigenfrequency, the liquid mass resonates on the two air springs thus influencing the effective stiffness and damping characteristics.

7.1.2 Optimisation criteria
The design demands were converted into six criteria for which the dynamic behaviour of the truck and semi-trailer was evaluated:

- for driver comfort, a combination of the RMS values of the weighed accelerations at the driver’s seat mounting point,
- for cargo accelerations, a combination of the RMS values of the accelerations at the kingpin,
- for fatigue loads, the so-called Palmgren-Miner number is calculated at nine chassis points. Of these Palmgren-Miner numbers, the maximum and the mean were considered,
- for spring travel, the RMS value of the spring travel at the rear axle.
- for dynamic wheel load, the RMS value of the dynamic wheel load of the rear tyres.

To study the performance capabilities of the new suspension with respect to these criteria, simulation models of the road surface excitation, the new suspension and the truck and semi-trailer were used. The three-dimensional vehicle model was validated by comparing its transfer functions with transfer functions measured on a test-rig. For this purpose, front and rear tyres were exposed to symmetrical and anti-symmetrical excitations. Transfer functions were determined for several points on the truck. The measured and calculated transfer functions were compared and the vehicle model was slightly modified.

As a next step, the maximum achievable improvements with the new suspension with respect to the six criteria were determined with a numerical optimisation routine. The optimisations were performed for a truck driven over a typical minor road and over an average German motorway. During the optimisation, only one of the six criteria was optimised, while the other five criteria were not allowed to become worse compared to the representative conventional vehicle. In this way, the performance capabilities of the new suspension were compared to those of the conventional suspension. Improvements found for driver comfort, spring travel, cargo accelerations and dynamic wheel load were relatively small. Improvements found for the Palmgren-Miner numbers were up to 15%, which means that a truck with the new suspension covers 15% larger distance before obtaining the same fatigue damage as the conventional truck.

Object of the research presented in this thesis, was to determine the improvements achievable with the new suspension for stochastic road excitations, under the restriction that the performance for incidental excitations should not get worse. Therefore, simulations were carried out with a multi-body vehicle model equipped with the new suspension. These simulations revealed that the dynamic responses of the vehicle with the new suspension were not significantly worse compared to the dynamic responses of the conventional suspension.

7.1.3 Experimental validation
With the help of the mechanical model, practical dimensions for the new suspension were derived after which two prototypes of the new system were developed: a passive suspension and an adaptive suspension. In the adaptive version, one of the two air volumes and the cross sectional area of the liquid channel can be adjusted while driving. In this way, maximal performance can be obtained for a range of operating conditions which include, varying vehicle speed, cargo load and road surfaces. Experiments with the two prototypes were performed on a components test-rig. These experiments revealed that the desired stiffness and damping characteristics could be reached. Then, the passive prototype was mounted on a truck and test-rig experiments proved that the calculations with the three-dimensional vehicle model predicted the dynamic behaviour of the truck equipped with the new suspension rather well.

7.2 Conclusions
From the presented study, several conclusions can be drawn.

With respect to the choice of a new suspension:
- Allowing the suspension to have one degree of freedom more than a conventional suspension, and allowing the implementation of a mass at that degree of freedom, yields four different characteristics with respect to dynamic stiffness and damping.

With respect to the vehicle model:
- The three-dimensional model of the truck and semi-trailer showed a satisfactorily correspondence.

From the optimisations of the new suspension:
- The suspension investigated in this research only shows improvements in the dynamic behaviour of the truck and semi-trailer at higher frequencies (> 15 Hz).
- Significant improvements are only found for fatigue loads.
Even when the constraints are exceeded, criteria such as comfort, cargo accelerations, spring travel, and dynamic wheel load do not show significant improvements.

Spring travel shows a substantial improvement when the constraints are violated. However, in this case these improvements might also be found with a standard suspension.

From the calculations presented in Chapter 5 and the definition of a practical concept:
- To obtain optimal performance, the new suspension should contain one or more adjustable parameters to suit the driving conditions of truck and semi-trailer.
- The different driving conditions which have been considered (change in vehicle speed, amount of cargo, type of road surface) can, in theory, be estimated during driving.

From the experiments described in Chapter 6:
- The dynamic behaviour of the new suspension can be modelled satisfactorily.
- It is possible to design an adaptive version of the new suspension in which the eigenfrequency of the mass effect and the values for the dynamic stiffness and dynamic damping can be controlled.
- The influence of the new suspension on the dynamic behaviour of truck and semi-trailer is predicted rather well with the three-dimensional vehicle model.

7.3 Recommendations

Referring to the validation of the vehicle model, the following recommendation is given:
- The vehicle model must be validated for real road surface excitations.

With respect to the optimisations, the recommendations given are:
- From the optimisations, the new suspension appears promising for fatigue loads. This is a high frequency suspension in contrast to the new suspension used in the experiments. Experiments with a high frequency suspension should be performed to validate the numerical results.
- Investigate the application of the new suspension in stiffer constructions than a truck, e.g. passenger cars or trains.

The new suspension influences both the dynamic stiffness and the dynamic damping and sometimes does this in the wrong way. The following recommendation is given:
- Determine a suspension for which these two functions, damping and stiffness are separated, but which still show a frequency dependent behaviour.

One of the criteria to be optimised was driver comfort. However, this is not only determined by the vehicle’s suspension, but also by the cabin and seat suspension. Therefore, for this specific criterion, the following is recommended:
- Investigate the application of the new suspension in the cabin and/or seat suspension.

Appendix A

Derivation of the impulse response of the desired dynamic system

The transfer function of a dynamic system is the Fourier transformation of the impulse response function (which is a function of the time $t$) of that dynamic system. The impulse response function is the response of the dynamic system to an impulse excitation at time $t_0$ (Figure A1).

![Figure A1 Impulse function $\delta(t)$ at $t=t_0$ s and impulse response](image)

This impulse excitation $\delta(t)$ has an infinite height and is infinitely small. The surface of this impulse excitation equals one. Of a physical realisable system (also called causal system), the response function is a result of the excitation. The inverse Fourier transformation of the systems transfer function results in the impulse response function $h(t)$ starting at $t=t_0$. If the calculated impulse response function shows a response for $t<t_0$, a dynamic system having this desired transfer function can not physically be realised because the response function can never be a result of the excitation. To show that the desired dynamic system with characteristics as defined in Figure 1.6, is not physically realisable, the inverse Fourier transformation of its transfer function is calculated. For this Fourier transformation, with $-\infty < f < \infty$, the transfer function must be given first for frequencies lower than zero Herz. The real part of a transfer function is always an even function ($\text{Re}(H(f))=\text{Re}(-H(f))$) and the imaginary part is always an uneven function ($\text{Im}(H(f))=\text{Im}(-H(f))$). As a result, the dynamic stiffness and dynamic damping are both even functions and are as shown in Figure A.2.
The impulse response function for $t_0=0$ s then equals:

$$h(t) = F^{-1}\{H(f)\} = \int_{-\infty}^{\infty} \left( k(f) + b(f) \cdot 2\pi f \cdot j \right) e^{2\pi ft} df$$

$$= \int_{-\infty}^{0} \left( b - b \cdot 2\pi f \cdot j \right) e^{2\pi ft} df + \int_{0}^{\infty} \left( k + b \cdot 2\pi f \cdot j \right) e^{2\pi ft} df$$

$$= -\frac{k}{2\pi} \sin(2\pi ft) + \sum_{n=1}^{\infty} \left( -1 \right)^n \frac{b}{n \pi} \sin(2\pi nft) + \left( -1 \right)^n \frac{2bft}{n} \cos(2\pi nft)$$

For a system with constant stiffness and damping over the whole frequency range, this Fourier transformation is:

$$h(t) = F^{-1}\{H(f)\} = \int_{-\infty}^{\infty} \left( k(f) + b(f) \cdot 2\pi f \cdot j \right) e^{2\pi ft} df$$

$$= \int_{-\infty}^{\infty} \left( k + b \cdot 2\pi f \cdot j \right) e^{2\pi ft} df$$

$$= \frac{k}{2\pi} \sin(2\pi ft)$$

It can be concluded that if the desired system is excited with an impulse excitation at $t_0=0$ s, the response function already reacts before this excitation is even applied. A system like this can never be realised in practice.
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\[ M = \left( \frac{K_1 + K_2}{2 \cdot \pi \cdot f^2} \right) \]  
(B.12)

\[ B = 2 \cdot \xi \cdot M \cdot (K_1 + K_2) \]  
(B.13)

from A.1, A.2, A.3 and A.9  
\[ 0 < 10^{a(2)} - 1 \leq 1 \Rightarrow 0 < K_1 < C_i \]  
(B.14)

\[ f^0 - f^* > 0 \Rightarrow f^0 > f^* \Rightarrow \left( \frac{f^0}{f^*} \right)^2 > 1 \Rightarrow C_1 > C_i \]  
(B.15)

from A.1, A.2, and A.8  
\[ C_2 = C_1 \left( \frac{f^0}{f^*} \right)^2 \]

from A.14, A.15 and A.10  
\[ 0 < K_1 < C_i, K_2 = C_1 - K_1 \]  
(B.16)

from A.14 and A.15  
\[ K_1 < C_1 \Rightarrow K_2 < C_1 < 0 \Rightarrow \frac{K_2 - C_1}{C_1 - C_2} > 0 \]  
with A.16 and A.4

\[ f^0 > f^* > 0 \Rightarrow M > 0 \]  
(B.18)

from A.2, A.12, A.16 and A.17  
\[ K_2 > 0, K_1 > 0 \]  
\[ M = \frac{K_1 + K_2}{2 \cdot \pi \cdot f^2} \]

\[ M > 0, K_1 > 0, K_2 > 0 \Rightarrow B > 0 \]  
(B.19)

\[ K_1 > 0, K_2 > 0, K_3 > 0, M > 0, B > 0 \]

Appendix C

Linearisation of the non-linear damping

Assuming that \( \dot{x}_3 \) is harmonic, \( \dot{x}_3 \) can be written as;

\[ \dot{x}_3 = v_0 \cdot \sin(\omega \cdot t) \]  
(C.1)

with \( v_0 \) being the amplitude of the speed signal, \( \omega \) the angular frequency and \( t \) being the time.

The amount of energy consumed by the non-linear component of the differential equations, must be equal to the amount of energy consumed by a linear component which must replace the non-linear component. This is expressed in the following equations.

Energy consumed by the non-linear component;

\[ dW = \frac{1}{2} \cdot \rho \cdot a \cdot \left( \xi + \frac{1}{d} \right) \cdot |\dot{x}_3|^2 \cdot dx \]  
(C.2)

where \( dW \) is the labour performed, and \( dx \) is a distance.

The energy consumed by the linear component;

\[ dW = b_1 \cdot |\dot{x}_3| \cdot dx \]  
(C.3)

where \( b_1 \) is the linear damping coefficient.

With \( dx = \dot{x}_3 \cdot dt \), taking the integral (for one cycle) for both equations and demanding that both equations are equal (same labour performed by the linear component as by the non-linear component), the following is derived;

\[ \int_0^r \frac{1}{2} \cdot \rho \cdot a \cdot \left( \xi + \frac{1}{d} \right) \cdot |\dot{x}_3| \cdot dt = \int_0^r b_1 \cdot |\dot{x}_3| \cdot dt \]  
(C.4)

With Equation B.1, Equation B.4 becomes;

\[ \frac{1}{2} \cdot \rho \cdot a \cdot \left( \xi + \frac{1}{d} \right) \cdot |v_0| \cdot \int_0^r \sin(\omega \cdot t) \cdot t \cdot dt = b_1 \cdot v_0 \cdot \int_0^r \sin(\omega \cdot t) \cdot t \cdot dt \]  
(C.5)
from this it follows that:

\[ b_i = \frac{1}{2} \rho \cdot a \cdot \left( \xi + \frac{1}{d} \right) \cdot \frac{\int_0^r \sin(\omega \cdot t) \, dt}{\int_0^r \sin(\omega \cdot t) \, dt} = \frac{1}{2} \rho \cdot a \left( \xi + \frac{l}{d} \right) \frac{8}{3\pi} \cdot v_o \]  

(C.6)

With \( v_o \) being equal to \( \text{RMS} [\dot{x}_i] \cdot \sqrt{2} \), Equation B.6 becomes:

\[ b_i = 1.2 \cdot \frac{1}{2} \rho \cdot a \left( \xi + \frac{l}{d} \right) \cdot \text{RMS} [\dot{x}_i] \]  

(C.7)

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Chapter 2

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Chapter 6


Curriculum vitae

Ard de Ruiter was born at September 3rd, 1968 in St. Odiliënberg, the Netherlands. In 1987 he started his study Mechanical Engineering at the Eindhoven University of Technology. He performed his graduation project at the Volkswagen Research Laboratory in Wolfsburg, Germany. The graduation project was about the development of a Fuzzy Logic based control algorithm for an automatic gearbox. This control algorithm was capable to detect the shifting desire of the driver and adjust the shifting behaviour of the gearbox to this desire. In 1992 he received his M.Sc. degree in Mechanical Engineering. In 1992 he started his Ph.D. project at the Laboratory for Automotive Engineering, faculty of Mechanical Engineering, Eindhoven University of Technology. The Ph.D. was about the development and optimisation of a hydro-elastic rear axle suspension with frequency dependent characteristics. The result of this work is reported in this thesis.
STELLINGEN
behorende bij het proefschrift

Design of a hydro-elastic rear axle suspension for heavy trucks

1. Een wezenlijk probleem bij het bepalen van de, met een alternatief veersysteem te behalen
winst, is het kiezen van het juiste conventionele (referentie-)veersysteem. (dit proefschrift)

2. Het minimaliseren van PSD waardes van de verschillende frequentie spectra op een
vrachtwagen middels een veersysteem-optimalisatie heeft, door de vele pieken en dalen in
de spectra, verdacht veel weg van een wandeling in de Alpen. Wanneer een hoge piek
bedwongen is, dan verrijst er weer een nieuwe. (dit proefschrift)

3. Het fileprobleem wordt een stuk kleiner, als iedereen die niet wenst te carpoolen, op de
motor naar zijn werk gaat.

4. Om een lagere milieubelasting te verkrijgen, zou vliegen alleen nog maar toegestaan mogen
worden op trajecten waarvoor andere vormen van openbaar vervoer veel meer reistijd
vergen en daardoor geen redelijk alternatief zijn.

5. Het druk hebben is een andere manier om te zeggen dat het gevraagde erg laag op het
prioriteitenlijstje staat.

6. Het kopiëren van volwassen mensen door ze te klonen druist in tegen de menselijke
evolutiedrang. De natuurlijke ontwikkeling van de mens wordt op deze manier bedreigd
door de ontwikkeling van de wetenschap.

7. Ten onrechte wordt door velen gedacht, dat een algemene invoering van de cruise control in
personen auto’s, tot een verbeterde doorstroming en rustiger verkeersbeeld zal leiden. Dit
gaat pas op als de invoering zo gebeurt dat de gereden snelheid voor alle voertuigen exact
hetzelfde is.

8. Botsveiligheid is ondergeschikt aan de voertuigdynamica, een auto is niet om mee te botsen
maar om mee te rijden.

9. Het verenigen van alle Europeanen komt neer op het vinden van een gemeenschappelijk
doel in cultuur, hobby en sport.

10. Het toppunt van genot zijn zes volledig geopende smoorkleppen van de Triumph TR6 PI uit
de CP chassis serie.

Ard de Ruiter
Eindhoven, 19 mei 1997