Sparse identification for model predictive control to support long-term voltage stability

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Abstract

Along with the increased installation of distributed energy resources (DER), long-term voltage stability can be improved if proper coordination is developed between DERs and grid controllers, for example, Load Tap Changer (LTC). In most of the proposed methods, the steady-state voltage-sensitivity analysis has been implemented to predict the voltage state, which is the state’s dynamic evolution in abnormal operations of the grid with high shares of DERs. This paper presents a system identification-based model predictive control (MPC), which can coordinate DERs and LTC to restore the voltage to the pre-fault condition after emergencies. First, the online voltage evolution is predicted based on the sparse identification of the nonlinear dynamics (SINDY) technique. Then, the SINDY-based voltage prediction is combined with an adaptive MPC in the centralized controller. In addition, the nonlinear of DERs have been modeled to avoid the MPC-based coordination jeopardizing the local constraints of DERs. The proposed method has been tested in a modified CIGRE benchmark network. Simulation results show that the dynamic voltage is effectively estimated by the SINDY method. Furthermore, the developed MPC model smoothly supports faster voltage recovering time with the least number of control actions of LTCs.

1 INTRODUCTION

The rapid increase of distributed energy resources (DER) poses challenges as well as opportunities to support grid stability. The impact of DERs on long-term voltage stability via coordination at the grid edges, that is, the coupling points between transmission-distribution (T-D) systems and DERs, has been investigated by ENTSO, ISGAN, NERC [1–3]. The report emphasized the importance of considering the interaction between DERs and the existing grid controllers, for example, load tap changer (LTC).

The coordination between DERs and LTC has been investigated in [4, 5] to address the instability issue at the primary side of the high-voltage/medium-voltage (HV/MV) transformer. The main cause of this issue is the presence of voltage-dependent load which creates an adverse effect on the primary side of the transformer while LTC keeps the voltage at the secondary side within the bound [6]. In [4], three control strategies are developed to control DERs and LTC to increase the power transfer into an affected area, thus increasing the voltage stability margin. Further, the voltage instability mechanism in the transmission system due to control actions in the distribution system is explained in [5]. The authors proposed synchronization of LTC actions and DERs to support long-term voltage stability. However, the sequential control of DERs and LTC leads to unnecessary LTC control actions which reduce the lifespan of the transformer, as well as increase the recovering time of voltage.

To address the above limitations, extensive studies focusing on optimal voltage control have been carried out. In [7], voltage-constrained centralized management was developed based on a physics-based model in form of a sensitive matrix. Similarly, the advantage model predictive control (MPC) was developed in reference [8], a centralized MPC was presented to regulate the distribution network voltage using the steady-state voltage-sensitivity analysis, which is extracted from an offline power flow calculation. Furthermore, the concept of distributed MPC was developed in [9] for long-term voltage coordination in multi-area power systems. The authors assumed that all system buses are observed locally via the phasor measurement units.
(PMUs). Then, the distributed MPC is operated with neighbor-to-neighbor communications. The optimal voltage coordination issues have been stacked in different ways in [7–9]. However, the voltage response is either merely analyzed based on steady-state voltage-sensitivity analysis, which is not applicable for time-variant DERs and system loads, or assumed available with PMUs installation, which is insufficient in practicality.

To tackle the problem of physics-based voltage-sensitivity analysis, data-driven models are promising solutions to improve grid distribution system monitoring and control. In [10], an artificial neural network (ANN) was used to approximate the voltage-sensitivity matrix, which was then used for voltage prediction in the centralized MPC model. Recently, a data-based learning and control method was proposed in [11] using a feed-forward back propagation neural network for long-term voltage instability problems. A database for optimal control actions is then trained offline by mapping the relationship system dynamics and saving control actions. Although data-driven approaches can achieve good performance in voltage coordination. However, these methods are purely using data to map the dynamic relationship between the input measurement and output variable in the dynamic power system, which requires large volumes of training data to learn models leading to long training time and over-fitting issues.

To solve the mentioned issues of the physics-based models and the data-based models, the physics-aware (physics-informed) data-based models were proposed in recent publications. The neural network models introduced in [12–14] used a combination of collected data from the systems as well as the knowledge of physics systems to prevent over-fitting issues, reduce the training time and improve the accuracy of data-based models. Over the last years, different physics-aware data-based system identification techniques for analyzing complex dynamical systems, which are strongly related to the concept of Koopman operator theory, have been proposed, such as dynamic mode decomposition (DMD) [15, 16], extended DMD (EDMD) [17, 18], and sparse identification of nonlinear dynamics (SINDY) [19]. There are several applications of the Koopman theory in power systems [20–24]. In [20] the properties of the point spectrum of the Koopman operator are used to detect a loss of power system stability without system modeling. Similarly, in [21] the Koopman operator is combined with perturbation theory to analyze the impact of multi-mode interactions on the nonlinear response of nonlinear dynamical systems. The authors in [22, 23] proposed a robust Koopman operator-based Kalman filter for dynamic power system estimation, showing the model is completely independent of the network model and more computationally efficient than the extended Kalman filter. Recently, a data-driven approach for short-term load forecasting is proposed in [24] using DMD showing the advantage of load prediction in different seasons as well as the less computational burden in the training phase.

Among these methods, SINDY models have shown a superior performance when dealing with highly nonlinear, high-dimensional, multi-scale dynamical systems with limited training data. In addition, the intrinsic sparse, parsimonious characteristics of SINDY models allow them to real-time identify system dynamics with the fewest system knowledge, and significantly less training data and computational burden, in comparison with other methods [25]. Motivated by these advantages of SINDY models, this work adopts a SINDY with control (SINDYc) based identification integrated into MPC to support long-term voltage stability. In the proposed method, the online voltage evolution is predicted based on the SINDYc technique. By doing so, the voltage evolution model is identified by locally recorded measurements. Then, it is used as the predictor for the MPC model, where the optimal control action for LTC and DERs are processed. The primary goal of the proposed method is to effectively restore the voltage at the grid edges back to the pre-fault condition after a disturbance with a minimum number of LTC control actions.

The key contributions of this paper are listed below:

- Sparse identification to capture non-linearity voltage evolution from large disturbances is proposed. The method is a combination of physics-based (i.e. built in the library of dynamic terms) and data-based (i.e. collected measurement data), which can be used to replace the static voltage-sensitivity analysis for voltage prediction.
- The voltage prediction model is then combined with MPC to optimally coordinate the LTC and DERs power. The proposed control can support the system in emergency conditions with an optimal control horizon, which can reach faster voltage recovery time with the least number of LTC control actions.

This paper is organized as follows: the long-term voltage instability model and instability problem are formulated in Section 2. Then, the proposed sparse identification-based MPC is presented in Section 3. The adopted MPC coordination in different case studies is presented in Section 4. Finally, the paper is concluded in Section 5.

2 LONG-TERM VOLTAGE STABILITY MODEL AND INSTABILITY PROBLEM

In this section, the power system modeling for long-term voltage stability and the problem formulation are discussed. The system overview and involved devices in long-term control actions such as over-excitation limiters (OXL), DERs, and dynamic loads are explained in Section 2.1. Then, the long-term voltage stability issue is analyzed in Section 2.2 using the modified CIGRE medium voltage benchmark distribution system.

2.1 System modelling for long-term voltage stability

Generally, a dynamic model of the power system, which is typically used in dynamic studies, includes a set of nonlinear differential-algebraic equations (DAEs) equations, and is
expressed as follows [6]:

\[
\begin{align*}
0 &= g(x, y, z, u), \\
\dot{x} &= f(x, y, z, u), \\
\dot{z}(k+1) &= h(x, y, z, u(k)), \\
\end{align*}
\]

(1a)

Here, Equation (1a) accounts for an instantaneous response of the network, including the electrical transmission system and other passive devices. Equation (1b) captures the short-term system dynamics associated with dedicated components such as synchronous generators and their regulators, induction motors, and other dynamical components. Whereas, Equations (1c) and (1d) represent the long-term system dynamics that occur from a few seconds up to several minutes following the short-term dynamics, associated with continuous and discrete-time control actions, respectively. The long-term dynamics involve actions of slow-acting devices, including generator current limiters, LTC, and controlled loads.

In these above equations, \(x\) is the vector of short-term state variables (e.g. the electromagnetic field voltage, phase angle, and velocity of a synchronous generator), and \(y\) is the vector of bus voltages. Whereas, \(z\) and \(u\) are the vectors of continuous and discrete-time long-term state variables, respectively. It should be noted that this work focuses only on long-term voltage stability. Therefore, the short-term voltage dynamics are neglected, that is, Equation (1b) is replaced by the following new equation:

\[
0 = f(x, y, z, u).
\]

(2)

In the next subsections, the devices that involve in the long-term voltage stability, that is, OXL of synchronous generators, controlled loads, and DERs, are modeled in the form of the above-mentioned dynamic model to comprehensively investigate their contribution to long-term voltage stability.

### 2.1.1 Over-excitation limiter

An excitation system is used to control a synchronous machine’s voltage and reactive power flow by controlling the direct current (DC) at synchronous machine field winding. This field current is adjusted automatically to maintain the terminal voltage following the reference. From the power system stability point of view, the excitation system is needed to control voltage and enhancement transient stability effectively. However, if the terminal voltage cannot reach the reference value after a fixed delay, a signal will be sent to limit the field current. As a result, the voltage violation will occur, affecting the long-term voltage stability.

Figure 1 shows the OXL circuit, which was designed to protect the generator from overheating due to high field current. In other words, the OXL allows the synchronous generator to operate under a high field current for a defined delay time period, then reduces to a rated value. There are two types of delay with fixed time and with inverse time. The dynamic model of the OXL circuit takes on the discrete-time form:

\[
\begin{align*}
x_2 &= \begin{cases} 
S_1 (E_q - I_{ld}), & \text{if } E_q \geq I_{ld}, \\
S_2 (E_q - I_{ld}), & \text{otherwise}, \\
0, & \text{if } x_2 = K_2 \text{ and } x_2 \geq 0, \\
0, & \text{otherwise}. 
\end{cases} \\
\dot{x}_2 &= \begin{cases} 
0, & \text{if } x_2 = -K_1 \text{ and } x_2 < 0, \\
K_3 x_3, & \text{otherwise}, 
\end{cases} \\
x_{oXL} &= \begin{cases} 
0, & \text{if } x_{oXL} = 0 \text{ and } x_3 < 0, \\
K_3 x_3, & \text{otherwise}, 
\end{cases}
\end{align*}
\]

(3)

(4)

(5)

where \(S_1\) and \(S_2\) are the positive numbers which defined the slope of the block (1) in the Figure 1. Then, the first OXL state equation \(\dot{x}_2\) after the non-windup limited integrator is as in Equation (4). Finally, Equation (5) presents second OXL state equation, which is related to block (4).

#### 2.1.2 Load tap changer

The HV/MV transformer is considered as an interface between the T-D network. Normally, it is installed with the LTC which operates automatically to maintain voltage of the secondary side of the transformer within a predefined limit. While a disturbance occurs that cause a voltage drop, the LTC adjusts transformer’s tap-setting to bring back the secondary voltage to its pre-disturbance level. The operation of taps can be summarized as follows [5]:

\[
T_{k+1} = \begin{cases} 
T_k + \Delta T, & \text{if } V^{M/M'} > V_0^{M/M'} + d \text{ and } T_k < T_{\text{max}}, \\
T_k - \Delta T, & \text{if } V^{M/M'} < V_0^{M/M'} - d \text{ and } T_k > T_{\text{min}}, \\
T_k, & \text{otherwise}, 
\end{cases}
\]

(6)

where \(T_k\) and \(T_{k+1}\) are the current and next tap position; \(\Delta T\) is the size of each tap step; and the \(T_{\text{min}}, T_{\text{max}}\) are the minimum and maximum tap limit, respectively.

The tap is activated depending on the measured secondary voltage \(V^{M/M'}\). The tap position \(T_{k+1}\) will be increased or decreased if \(V^{M/M'}\) is out of a deadband \(V_0^{M/M'} \pm d\), where
$V_0^\text{MV}$ is the secondary voltage reference and $d$ is the dead-band limit. Furthermore, tap movement needs a fixed delay time (normally, from 5 to 8 s) to reach a new position due to mechanical requirements.

### 2.1.3 Load model

The attempt to restore power consumption of a controlled load is one of the reasons for long-term voltage instability [6, 26]. Therefore, it is important to consider the dynamic model of these loads in long-term voltage study. Regarding load voltage characteristics, there are two types of load models such as exponential and polynomial load models. In this work, the exponential load is used, which is expressed as follows:

\[
P = P_0 \left( \frac{V}{V_0} \right)^\alpha, \tag{7}
\]

\[
Q = Q_0 \left( \frac{V}{V_0} \right)^\beta, \tag{8}
\]

where $V_0$, $P_0$, and $Q_0$ are the rated terminal voltage, active power, and reactive power, respectively. The $\alpha$ and $\beta$ values define the load types, and are set to 1 and 2 for constant current and constant impedance, respectively.

### 2.1.4 DERs with power electronic based interface

Figure 2 shows the detailed configuration of a DER with a multi-loop controller. The DER is modelled as a voltage source connected with the distribution system at the PCC point through an LCL filter and a step-up transformer. In this work, the DERs are operated in the grid following mode [27], which follow the $P^\ast$, $Q^\ast$ setpoints from the upper control layer. The outer power control loop compares the power setpoints with the power outputs of the DER to determine the setpoints for the inner current control loop in the $dq$ frame, as shown below:

\[
\begin{bmatrix}
P^\ast \\
Q^\ast
\end{bmatrix} =
\begin{bmatrix}
P^d \\
Q^d
\end{bmatrix} =
\begin{bmatrix}
3 & 0 \\
0 & 2
\end{bmatrix}
\begin{bmatrix}
\frac{P}{V_d} \\
\frac{Q}{V_q}
\end{bmatrix},
\tag{9}
\]

where $V_d$ and $V_q$ are the inverter terminal voltages in the $dq$ frame. The PI controller minimizes the error between input current setpoints (obtained from Equation 9) and measured inverter currents (i.e., filter inductor currents) and generates the PWM modulation used to control the inverter.

### 2.2 Long-term voltage instability problem

In the recent published paper [28], the above discussed devices are included and the long-term voltage stability issues have been investigated. A three-phase fault is considered at one of the two parallel transmission lines between external bus and Bus 0, as depicted in Figure 3. This transmission line trips at time $t = 0$ to isolate the fault. After a short-term period with dynamics assumed to be stable, the system enters a long-term period where voltage stability is of concern. A simulation for this study of long-term voltage stability involves slow-acting equipment (e.g., LTC, OXL). In this simulation, the voltage measurement at HV bus (i.e., Bus 0) and at MV bus (i.e., Bus 1) are shown in Figure 4. After a short-term dynamic period, the voltages at HV and MV buses are stable at $V_{HV} = 0.973 \text{ p.u.}$, and $V_{MV} = 0.928 \text{ p.u.}$, respectively. Thus, the LTC controller is activated to increase the...
Model predictive control based voltage stability

In this section, the proposed sparse identification for MPC to support long-term voltage stability is presented. As discussed, this new method is proposed to replace the traditional voltage-sensitivity analysis in terms of dynamic voltage prediction.

First, Section 3.1 recalls the concept of MPC including future state prediction, optimization and moving horizon. Then, the traditional MPC based on voltage-sensitivity analysis is analyzed and discussed. The proposed coordinated control architecture is shown in Figure 5. The DERs are operated in the grid following mode, which follows the \( P, Q \) control signals from the upper control layer. The local controller of each DER and transformer exchanges information (e.g., measured active, reactive power, and tap position) with the central controller through a communication network. The calculated optimal control signals are sent back from the centralized controller to the local controllers.

Second, the sparse identification based MPC is proposed in Section 3.2, where the sparse identification of nonlinear dynamics without control (i.e., SINDY) and with control (i.e., SINDYc) methods are explained step by step.

### 3.1 Model predictive control based voltage control

MPC is an advanced control method that uses a discrete-time model of a system to predict the future behavior of the desired control variables and compute a set of future control actions by optimizing an objective function with predefined constraints. The MPC can solve single or multiple objectives together with discrete and continuous control variables. The basic concept of MPC is described in Figure 6. At current time step \( k \), the MPC will receive all available measurements and the state variables (e.g., voltage) will be predicted from time step \( k \) to \( k + N_p \) (\( N_p \) is the prediction horizon) based on the system model. Then a new set of optimal control actions \( \Delta u \) from time step \( k \) to \( k + N_c \) will be determined by solving the optimization problem with input constraints. It should be noted that only the first element \( \Delta u(k) \) is applied. The above process is receding until the end of the simulation.

It is obvious that the computational complexity of MPC at each step increases proportionally with the fixed number of the prediction horizon. Therefore, this work adopts a simple adaptive prediction horizon strategy to relieve the computational burden while sustaining the sub-optimal performance of the proposed MPC. In Figure 7, the proposed strategy shrinks the prediction horizon based on the level of the voltage error \( \Delta V_k = V_k - V_{k-1} \), and is explained as follows:

- It is assumed that at the time of step \( k \), the proposed MPC is running with the prediction horizon \( N_p \). At step \( k + N_p \), the voltage error is changed and reaches a new...
subject to:
\[
\begin{align*}
\alpha_{\text{min}} & \leq \alpha(k + i) \leq \alpha_{\text{max}}, \\
\triangle \alpha_{\text{min}} & \leq \triangle \alpha(k + i) \leq \triangle \alpha_{\text{max}}, \\
0.9 \mu & \leq \Delta V_{k}^{\text{MV}} \leq 1.1 \mu,
\end{align*}
\] (11)

for \( i = 0, 1, \ldots, N_{c} - 1 \), where \( \Delta V_{k} \) is the voltage measurement at the time step \( k \). \( \Delta V_{k}^{\text{HV}} = V_{k}^{\text{HV}} - V_{0}^{\text{HV}} \) is the variation at time step \( k \) of the primary voltage \( (V_{k}^{\text{HV}}) \) of HV-MV transformer bus from pre-fault value \( (V_{0}^{\text{HV}}) \), and \( V_{k}^{\text{MV}} \) is secondary voltage of HV-MV transformer bus at time step \( k \). \( \Delta \alpha(k) = [\Delta P_{\text{DER}}(k), \Delta Q_{\text{DER}}(k), \Delta V_{k}^T(k)]^T \) are the change of control variables at time step \( k \) compare to \( k - 1 \), where \( P_{\text{DER}} \) and \( Q_{\text{DER}} \) are the active and reactive power outputs of \( \text{DER} \) with \( j = 1, 2, 3, \ldots \). The transpose of a matrix is denoted by \( [\cdot]^T \).

In Equation (10), \( R_{p} \) and \( R_{T} \) are weight matrices for voltage regulation and LTC actions, which was used to determine the most important objective. In this work, the voltage regulation is assigned with a higher weight, which means the proposed MPC prioritizes the voltage regulation back to the reference value while minimizing the number of LTC actions.

In literature, a voltage prediction model for MPC can be expressed as follow:

\[
V_{k+1}^{\text{HV}} = V_{k}^{\text{HV}} + \frac{\partial V_{k}^{\text{HV}}}{\partial P_{\text{DER}}(k)} \Delta P_{\text{DER}}(k) + \frac{\partial V_{k}^{\text{HV}}}{\partial Q_{\text{DER}}(k)} \Delta Q_{\text{DER}}(k) + \frac{\partial V_{k}^{\text{HV}}}{\partial V_{k}^T} \Delta V_{k}^T,
\] (12)

where \( V_{k}^{\text{HV}} \) is the voltage measurement at the time step \( k \). \( \frac{\partial V_{k}^{\text{HV}}}{\partial V_{k}^T} \) is the voltage sensitivity matrix with respect to an LTC position. It can be calculated by changing an LTC position and obtaining the ratio of voltage variations. \( \frac{\partial V_{k}^{\text{HV}}}{\partial P_{\text{DER}}(k)} \) and \( \frac{\partial V_{k}^{\text{HV}}}{\partial Q_{\text{DER}}(k)} \) are the voltage sensitivity matrices corresponding to the change of the active and reactive power, respectively. These terms can be obtained from the inverse of the Jacobian matrix \( J \), as follow:

\[
\begin{bmatrix}
\Delta \alpha_1 \\
\vdots \\
\Delta \alpha_{N_{p}} \\
\Delta V_1 \\
\vdots \\
\Delta V_{N_{p}}
\end{bmatrix} = J^{-1}
\begin{bmatrix}
\Delta P_1 \\
\vdots \\
\Delta P_{N_{p}} \\
\Delta Q_1 \\
\vdots \\
\Delta Q_{N_{p}}
\end{bmatrix},
\] (13)

where \( \Delta \alpha_i \) and \( \Delta V_i \) are the absolute change in voltage angle and voltage magnitude at bus \( i \) which corresponds to the change in active \( \Delta P \) and reactive power \( \Delta Q \).
It is noted that the sensitivity matrix (i.e. inverse of the Jacobian matrix), \( f^{-1} \), is obtained from an equilibrium operation point, that is, at the steady state, which is insufficient to consider with dynamic voltage control. Furthermore, an accurate sensitivity matrix should be calculated based on real-time measurements. However, this information is not always available in practice [29]. Thus, most of the research used a single snapshot of the system based on an offline study to obtain the sensitivity matrix [7–9].

In this work, an advanced data-based system identification based on a SINDY technique is used to obtain the dynamic voltage response model to overcome the aforementioned issues. The details of this technique and its application to the proposed MPC are discussed in the next subsection.

### 3.2 SINDY-based long-term voltage prediction

In this subsection, first, the basis of the system identification method based on the SINDY is presented. Then, the use of this SINDY technique as a predictor for the future voltage dynamics in the proposed MPC is explained.

#### 3.2.1 Sparse identification of nonlinear dynamics

From the available dynamic data set of the systems, the SINDY identifies possibly fully non-linear dynamical terms. Giving the dynamic model as follows:

\[
X = \Xi \Theta^T(X), \tag{14}
\]

where \( X \) is \((M_t \times M_x)\) dimensional matrix of \( M_x \) state variables, which is collected at time series \( t_1, t_2, \ldots, t_{M_t} \) (\( M_t \) is the total number of snapshots).

\[
X = \begin{bmatrix}
\begin{array}{cccc}
\chi_1(t_1) & \cdots & \chi_n(t_1) & \cdots & \chi_{M_x}(t_1) \\
\chi_1(t_2) & \cdots & \chi_n(t_2) & \cdots & \chi_{M_x}(t_2) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\chi_1(t_{M_t}) & \cdots & \chi_n(t_{M_t}) & \cdots & \chi_{M_x}(t_{M_t}) 
\end{array}
\end{bmatrix}. \tag{15}
\]

The term \( \Theta^T \) in Equation (14) is a library of the candidate dynamics that plays a crucial role in the performance of the SINDY algorithms. As explained in [30], the library includes mixed polynomials (e.g. \( X^2 \)), trigonometric (e.g. \( \sin(X) \)) terms, as well as their products. The feature library of SINDY plays an important role in determining its performance. The criteria to choose the components of the library depends on system characteristics and several other factors. For this research, a basic choice, such as polynomials based on the snapshots of estimated state variables and the input signals is used. Then, the complexity of the library is increased by adding other terms such as trigonometric functions. The whole feature library can be constructed and presented in the following:

\[
\Theta^T(X) = \begin{bmatrix}
- & 1 & - \\
- & X & - \\
\vdots & \vdots & \vdots \\
- & \sin(X) & - \\
- & X \sin(X) & -
\end{bmatrix}. \tag{16}
\]

As the library is constructed with \( M_t \) time horizon, this can be updated continuously to fit the purpose of an online application. For example, two candidate dynamics \( X^2 \) and \( \sin(X) \) in Equation (16) are as follows:

\[
X^2 = \begin{bmatrix}
\chi_1^2(t_1) & \cdots & \chi_n^2(t_1) & \cdots & \chi_{M_x}^2(t_1) \\
\chi_1^2(t_2) & \cdots & \chi_n^2(t_2) & \cdots & \chi_{M_x}^2(t_2) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\chi_1^2(t_{M_t}) & \cdots & \chi_n^2(t_{M_t}) & \cdots & \chi_{M_x}^2(t_{M_t})
\end{bmatrix}, \tag{17}
\]

\[
\sin(X) = \begin{bmatrix}
\sin(\chi_1(t_1)) & \cdots & \sin(\chi_n(t_1)) & \cdots & \sin(\chi_{M_x}(t_1)) \\
\sin(\chi_1(t_2)) & \cdots & \sin(\chi_n(t_2)) & \cdots & \sin(\chi_{M_x}(t_2)) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\sin(\chi_1(t_{M_t})) & \cdots & \sin(\chi_n(t_{M_t})) & \cdots & \sin(\chi_{M_x}(t_{M_t}))
\end{bmatrix}. \tag{18}
\]

In power system, the library of candidate dynamics is selected that contains a mix of nonlinear (trigonometric) multiplication of variables (e.g. \( V \sin(\delta - \theta) \) and \( V \cos(\delta - \theta) \)) [30, 31]. Each row in Equation (14) represents the row in Equation (1c), and the sparse coefficients \( \hat{\zeta}_k \) corresponding to the \( k\)-th row of the \( \Xi \) is identified from the initial values \( \hat{\zeta}_k \) using a sparse regression algorithm, such as least absolute shrinkage and selection operator (LASSO) [19]:

\[
\hat{\zeta}_k = \argmin_{\hat{\zeta}_k} \left\| \hat{X}_k - \hat{\zeta}_k \Theta^T(X) \right\|_2 + \alpha \left\| \hat{\zeta}_k \right\|_1, \tag{19}
\]

where \( \hat{X}_k \) represents the \( k\)-th row of \( \hat{X} \). The value of parameter \( \alpha \) is selected to balance the complexity and accuracy of the prediction model. The detail of the selection process can be found in [19].

#### 3.2.2 SINDY for model predictive control

In this subsection, the aforementioned SINDY algorithm is improved to use as a predictor model for long-term voltages in the proposed MPC. Generally, the MPC is closed-loop feedback control, which predicts the next state based on the current state and control input. Therefore, the feedback control signals
corresponding to the external forces is included in the original SINDY method and called SINDYc [19]. In this work, the feedback control signals are the LTC control action (\( T \)) and DER power outputs (\( P^\text{L}, Q^\text{L} \)). To be able to learn the dynamic voltage response, seven state variables are collected (i.e. \( V^\text{HV}, V^\text{MV}, p^\text{Gen}, Q^\text{Gen}, P^\text{Lac}, Q^\text{Lac} \)). Where \( p^\text{Gen} \), \( Q^\text{Gen} \), and \( I^\text{Gen} \) are the power outputs and field current of the generator, which is connected at Bus 0. The \( P^\text{Lac} \) and \( Q^\text{Lac} \) are the active and reactive power responses of the largest voltage dependent load in the feeder 1 (see Figure 3). Consequently, Equation (14) becomes:

\[
\dot{X} = \Xi \Theta^R(X, U),
\]

where \( U = (M_t \times N_u) \) dimensional of \( N_u \) control inputs. In this case, it requires measurements of the state \( X \) as well as the input signal \( U \). The collected data of \( U \) at time series \( t_1, t_2, \ldots, t_{M_t} \) is as follows:

\[
U = \begin{bmatrix}
\mu_1(t_1) & \cdots & \mu_{N_u}(t_1) \\
\mu_1(t_2) & \cdots & \mu_{N_u}(t_2) \\
\vdots & \ddots & \vdots \\
\mu_1(t_{M_t}) & \cdots & \mu_{N_u}(t_{M_t})
\end{bmatrix}.
\] (21)

The library of candidate nonlinear is extended to include the data \( X \) and \( U \):

\[
\Theta^R(X) = \begin{bmatrix}
-1 & \cdots & -1 \\
\vdots & \ddots & \vdots \\
\sin(X) & \cdots & \sin(U) \\
\sin(U) & \cdots & \sin(U) \\
\sin(U) & \cdots & \sin(U)
\end{bmatrix}.
\] (22)

The optimization problem Equation (19) coefficients

\[
\zeta_k = \arg \min_{\zeta_k} \| \hat{X}_k - \zeta_k \Theta^R(X, U) \|_2 + \alpha \| \hat{X}_k \|_1.
\] (23)

In order to find the coefficient vector \( \Xi \), Equation (23) is solved by using the sequential threshold least squares procedure, as can be seen from Algorithm 1. Once the coefficient matrix is learned, the dynamic voltage response model is obtained and replaced with the voltage prediction in Equation (12). It is worth mentioning that, the SINDY and SINDYc methods are robust to noise or disturbance, thanks to the included sparse regression framework. The robustness of these methods to different noise levels were tested in [19, 25].

### Algorithm 1

Sequentially threshold least squares to learn the active library terms

**Input:** Time data of \( \hat{X} \), library of \( \Theta^R(X, U) \), threshold \( \alpha \);

**Initialization:** \( \hat{X}^{0} \leftarrow (\Theta^R)^{T}X \);

**while not converged do**

1. \( k \leftarrow k + 1 \);
2. \( I_{\text{small}} \leftarrow (\text{abs}(\hat{X}) < \alpha) \);
3. \( \hat{X}^{k}(I_{\text{small}}) \leftarrow 0 \);
4. for all variable do
   - \( I_{\text{big}} \leftarrow I_{\text{small}}(:, ii) \);
   - \( \hat{X}^{k}(I_{\text{big}}, ii) \);
5. end for;

**end for**;

**Result:** Matrix of sparse coefficient vectors \( \Xi \)

## 4 Simulation and Discussion

In this section, to evaluate the effectiveness of the proposed method, three case studies have been investigated using MATLAB/Simulink. The numerical results were obtained on a ThinkPad Laptop with an Intel Core (TM) i7-8750 central processing unit (CPU), 2.20 GHz processing speed, and 16 GB random access memory (RAM). In the first case study, we validate the performance of the SINDY and SINDYc algorithms in predicting the voltage. Then, in the second case study, the performance of the proposed fixed prediction horizon MPC with SINDYc for long-term voltage coordination is evaluated. In the third case study, the adaptive prediction horizon MPC with the SINDYc method is tested and compared with the results in the second case study.

In all three case studies, the modified CIGRE medium voltage benchmark distribution system is used, as described [32, 33]. The single-line schematic diagram of the tested system is shown in Figure 3. The MV distribution system is supplied by an external grid via two parallel transmission lines, represented by a 110 kV/50 Hz three-phase voltage source, with a short-circuit power of 500 MVA and \( R/X \) ratio of 0.1. The nominal phase-to-phase voltage and frequency of the tested system are 20 kV and 50 Hz, respectively.

There are three DER units which are added at Buses 3, 14, and 9 with their rating power of \( S_1 = 4.3 \text{ MVA}, S_2 = 4.75 \text{ MVA}, \) and \( S_3 = 4.2 \text{ MVA}, \) with the power factor of 0.89, respectively. It should be noted that this power factor is chosen so that the output of the proposed coordination mechanism results in both the change in DER power outputs and LTC actions. In the case of a lower power factor, the number of LTC actions will be lower, and vice versa. In this case, the DERs can support 16% total power consumption of loads.

The transformer with an LTC controller is installed between Bus 0 and Bus 1, which is designed to keep the voltage at Bus 1 within a range from 0.985 p.u. to 1.015 p.u. A synchronous generator with OXL contributing to the long-term voltage issue is installed at the HV-MV substation. There is a mix of residential (\( L_{\text{res}} \)) and industrial load (\( L_{\text{ind}} \)) in the network (with \( i \) is the bus number, where a load is connected). The residential loads are modelled as constant power loads with \( \alpha = \beta = 0 \) (see Equations 7 and 8). The voltage dependent model is used to
present the industrial load. The details of line parameters of this benchmark network can be found in [28, 33].

4.1 Case study 1: SINDY- and SINDYc-based system identification

To show the performance of the data-based system identification methods, SINDY and SINDYc are used to predict the voltage responses (i.e. $V_{HV}$ and $V_{MV}$) in the modified CIGRE MV distribution system in two different scenarios.

Scenario 1.1: in this scenario, the SINDY model is used to predict the voltage responses without the voltage coordination algorithm from the upper control layer. We consider a three-phase fault at one of the two parallel transmission lines between the external bus and Bus 0. The response of $V_{HV}$, $V_{MV}$, and LTC are recorded. Figure 8 shows the $V_{HV}$, $V_{MV}$ training and prediction using the SINDY method. It is worth mentioning that, this work focuses on long-term voltage stability. Thus, the short-term voltage response is not considered (i.e. the area (1), in Figure 8). As the state of the transformer’s LTC ($T$) is not controlled in this scenario, it is considered as the eighth state variable in the SINDY model. These state variables are collected from $t_1 = 15$ s to $t_M = 55$ s with the time-step of $5 \times 10^{-5}$ (i.e. 800,000 snapshots for each signal). Thus, in total, 800,000 snapshots were collected for each state variable. The matrix $X_{(800000\times8)}$ is constructed as:

$$X = [V_{HV}, V_{MV}, p_{Gen}, q_{Gen}, I_{fGen}, p_{AC}, q_{AC}, T].$$

As can be seen from Figure 8, the time series data in area (2) is used to train the SINDY model. The predicted voltage is obtained and compared with the original signal in area (3). In Figures 8 b1 and 8 b2, the zoom-in prediction signals of $V_{HV}$ and $V_{MV}$ are plotted, respectively. The simulation results show that the SINDY model is able to predict the dynamic voltage response after the fault.

Scenario 1.2: in this scenario, the LTC and DERs are coordinated by the upper control layer. Thus, the SINDYc model is used to identify the dynamic voltage response when having the input control signals (i.e. the LTC, the active and reactive power output of three DERs). Similar to previous scenario, input control signals are collected from from $t_1 = 15$ s to $t_M = 55$ s with the time-step of $5 \times 10^{-5}$ (i.e. 800,000 snapshots for each signal). Then, the control inputs $U_{(800000\times7)}$ are constructed as follows:

$$U = [T, p_1, p_2, p_3, q_1, q_2, q_3],$$

where $p_i$, and $q_i$ are the active and reactive power outputs of DER$_i$ with ($i = 1, 2, 3$). The voltage responses of $V_{HV}$, $V_{MV}$ are presented in Figure 9a, and Figure 9c. To analyze the performance of SINDYc, the zoom-in prediction signals of $V_{HV}$ and $V_{MV}$ are plotted in Figure 9b1 and Figure 9b2. Meanwhile, Figure 9d, and Figure 9e show the input control signals of LTC and reactive power of DERs. As can be seen from Figure 9, the SINDYc model is trained in the area (2), and it is able to predict the voltage response in the next period following the input control signals with a minor error of 0.001 p.u., similar to the SINDY model.

4.2 Case study 2: Fixed prediction horizon MPC with SINDYc

To assess the performance of the proposed method, different scenarios were carried out in MATLAB/Simulink.
Scenario 2.1: in this scenario, the proposed method is used as a coordination mechanism for DERs and LTC. The performance of the proposed method is validated by comparing the long-term voltage stability in the case with and without an MPC controller. Figure 10 presents the time sequence of control actions. After a stable short-term period, the system voltage is entered the long-term voltage stability. The recorded pre-fault primary voltage is $V_{HV}^{0} = 0.996$ p.u. The proposed MPC method is activated based on the alarm signal with the objective is to bringing the primary voltage back to the pre-fault value. The simulation results show that the proposed method is able to smoothly bring the primary voltage back to the pre-fault value while keeping the secondary voltage within a predefined limit (i.e. [0.9 1.1] p.u.). In this case, the prediction horizon of MPC is 2 steps ahead. As can be seen from Figure 10, in the range of time $t = 22$ s to $t = 80$ s the voltage is slightly increased while the LTC is kept unchanged. This is the advantage of multi-objective-based control. In this period, the power from DERs is still available. Thus, the MPC kept the LTC unchanged and use only power from DERs to support the voltage. In the next period of time $t = 80$ s to $t = 140$ s, the powers from DERs reaches their limits. Therefore, the coordination mechanism must use the support from the LTC operation with a higher price. The LTC increased its position to increase the primary voltage of the transformer. Due to the increasing LTC position, the secondary voltage is decreased. However, the secondary voltage of the transformer is kept at the limit of [0.9 1.1] p.u. The results clearly show a superior performance of the proposed MPC method in solving the long-term voltage instability.

Scenario 2.2: in this scenario, the effect of the prediction horizon on the performance of the proposed method is observed. Figure 11 shows simulation results with different prediction horizons. With the increasing prediction horizon, the proposed MPC is able to predict the response of the voltage state corresponding to the control actions, together with their control limits. As shown, with $N = 2$, the MPC started to change the LTC set point at around $t = 80$ s, early than $N = 5$, and $N = 8$, at $t = 90$ s, and $t = 160$ s, respectively. As the result, the primary voltage is recovering faster when using least number of the prediction horizon. However, it required a higher number of LTC actions, which can be seen from Table 1 (the simulation time is set up to 200 s in Simulink). In the case of prediction horizon $N = 8$, the MPC needs only 4 LTC control actions, and less than 7 LTC control actions in the case of $N = 2$ to support the voltage, which can effectively extend the device lifespan. The reason is that the DER power is able to faster support the problem in case of a longer prediction horizon, which can be seen in Figure 12. Thus, it reduced the stress on the LTC operation. However, the computational burden is increased with the increasing prediction horizon. In this work, 7 control actions need to be obtained for each simulation step. Then, the optimizer needs to solve $7*N$ variables for each optimization step, the constraint matrices also increased together with the increasing number of the prediction horizons. Table 1

<table>
<thead>
<tr>
<th>Prediction step</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTC actions</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Simulation time (min)</td>
<td>16.27</td>
<td>25.51</td>
<td>38.22</td>
</tr>
</tbody>
</table>

FIGURE 11  MPC and different prediction horizon

TABLE 1  A comparison of different prediction horizons

FIGURE 12  Reactive power response of $DER_1$ with different prediction horizon
of fixed prediction horizon of $N_p = 8$. It results in a faster voltage recovery.

Meanwhile, the secondary voltage is kept closer to the nominal value with the adaptive MPC, in comparison with the cases using the fixed prediction horizon of $N_p = 2$. Furthermore, as shown in Table 2, the adaptive MPC used only 4 LTC actions which are less than 7, and 5 LTC actions corresponding to the case of fixed prediction horizon of 2 and 8, respectively. In overall, the performance of the proposed adaptive MPC was proven in solving the issue of LTC actions, while reducing the computational burden.

5 CONCLUSION

A sparse identification of nonlinear dynamics (SINDY) based model predictive control (MPC) has been proposed in this work for the purpose of long-term voltage support in transmission systems. The SINDY technique is used to predict the dynamic voltage response after the disturbance. It shows good performance when predicting the dynamic voltage evolution.

Then, the SINDY for MPC was designed to coordinate the LTC of the transformer and DERs in an active distribution network to support voltage at the primary side of the transformer. The simulation results verify the performance of the proposed method in regulating the voltage back to the pre-fault value. Furthermore, a simple adaptive rule enhanced the effectiveness of the proposed MPC in terms of LTC actions with reduced computational burden, which is the general problem of the optimization-based controller.

For further research direction, the proposed method can be tested with real-time simulations to show its suitability for online application.

### NOMENCLATURE

**Operators**

- $\mathbf{M}^T$ Transpose of matrix
- $\mathbf{M}^{-1}$ Inverse of matrix
- $\mathbf{M}^+$ Pseudo-inverse of matrix

**Parameter**

- $V_{HV}$ Primary voltage of transformer
- $V_{HV}^0$ Primary voltage of transformer before the disturbance
- $V_{MV}$ Secondary voltage of transformer
- $V_{MV}^0$ Secondary voltage reference for LTC operation
- $P_{DER_j}^k$ Active power of DER$_j$ at time step $k$
- $Q_{DER_j}^k$ Reactive power of DER$_j$ at time step $k$
- $T_k$ Tap position at time step $k$

### TABLE 2 A comparison between MPC and adaptive MPC

<table>
<thead>
<tr>
<th>Prediction step</th>
<th>8–2</th>
<th>2</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTC actions</td>
<td>4</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Simulation time (min)</td>
<td>31.02</td>
<td>16.27</td>
<td>38.22</td>
</tr>
</tbody>
</table>
\( P^a, Q^a \)  Active, reactive power reference inputs for DER controller
\( V_d', V_q', I_d', I_q' \)  Inverter voltage and current in dq frame, respectively
\( N_p, N_q \)  Prediction, control horizon of MPC
\( N_p, new \)  New prediction horizon of adaptive MPC
\( \alpha \)  Predefined value for LASSO algorithm
\( R_x, R_T \)  Weight matrices for voltage regulation and LTC actions
\( M_x, M_i \)  Number of state variables and number of snapshots
\( I_{db}, I_{dc} \)  Residential, industrial loads connected with Bus \( i \)
\( \Delta \delta, \Delta V_i \)  Deviation of voltage angle and magnitude at Bus \( i \)
\( \Delta P_i, \Delta Q_i \)  Deviation of active and reactive power at Bus \( i \)

**Abbreviations**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DERs</td>
<td>Distributed energy resources</td>
</tr>
<tr>
<td>LTC</td>
<td>Load tap changer</td>
</tr>
<tr>
<td>HV</td>
<td>High-voltage</td>
</tr>
<tr>
<td>MV</td>
<td>Medium-voltage</td>
</tr>
<tr>
<td>ENTSO</td>
<td>European network of transmission system operators for electricity</td>
</tr>
<tr>
<td>ISGAN</td>
<td>International smart grids action network</td>
</tr>
<tr>
<td>NERC</td>
<td>North American electric reliability corporation</td>
</tr>
<tr>
<td>T-D</td>
<td>Transmission-distribution</td>
</tr>
<tr>
<td>MPC</td>
<td>Model predictive control</td>
</tr>
<tr>
<td>PMUs</td>
<td>Phasor measurement units</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial neural network</td>
</tr>
<tr>
<td>SINDY</td>
<td>Sparse identification of nonlinear dynamics</td>
</tr>
<tr>
<td>SINDYc</td>
<td>Sparse identification of nonlinear dynamics with control</td>
</tr>
<tr>
<td>OXL</td>
<td>Over-excitation limiter</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>DAEs</td>
<td>Differential algebraic equations</td>
</tr>
<tr>
<td>CIGRE</td>
<td>International council on large electric systems</td>
</tr>
<tr>
<td>CC</td>
<td>Current controller</td>
</tr>
<tr>
<td>NSF</td>
<td>Negative sequence filter</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage sourced converter</td>
</tr>
<tr>
<td>LASSO</td>
<td>Least absolute shrinkage and selection operator</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of common coupling</td>
</tr>
<tr>
<td>DMD</td>
<td>Dynamic model decomposition</td>
</tr>
<tr>
<td>EDMD</td>
<td>Extended dynamic model decomposition</td>
</tr>
</tbody>
</table>

**AUTHOR CONTRIBUTIONS**

Minh-Quan Tran: Conceptualization, formal analysis, investigation, methodology, software, validation, visualization, writing - original draft, writing - review and editing. Trung Thai Tran: Conceptualization, investigation, methodology, software, supervision, writing - original draft, writing - review and editing. Phuong H. Nguyen: Conceptualization, funding acquisition, methodology, project administration, supervision, writing - original draft, writing - review and editing. Guus Pemen: Project administration, supervision, writing - review and editing.

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**CONFLICT OF INTEREST**

The authors have declared no conflict of interest.

**DATA AVAILABILITY STATEMENT**

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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