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Analytical Equations for the Analysis of Folded Dipole Array Antennas

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Abstract— An accurate analytical model has been derived for a linear array of wire folded dipole antennas. The model combines closed form analytical equations for the folded dipole antenna, the re-entrant folded dipole antenna, the two-wire transmission line, the mutual coupling between two folded dipole antennas and the mutual coupling between two thin dipole antennas.

I. INTRODUCTION

Wire antennas may still be found in numerous applications, ranging from large broadcasting antennas, [1] to in-clothing-integrated antennas for use in the ISM frequency bands, [2]. Single wire radiators may be employed for small-band applications. Broadband wire antennas may be realised as log periodic arrays of monopole or dipole elements, [1], [3] and may be used, for example, for broadcast applications, [1]. To enhance the range in UWB communication systems, higher gain antennas are needed. Log periodic dipole array antennas may then be used if OFDM-like modulation schemes are being employed, [4]. When, for UWB communication, different length dipole elements are being used in an array, a better correlation will be achieved as compared to one single dipole, [5]. For low-power applications, not only in UWB, but also in the context of RFID and rectenna systems, an accurate estimation of the antenna input impedance must be made or a desired impedance value must be synthesised [6]. So, a regained interest in the analysis of wire array antennas has been created. Although these antennas may be analysed by (commercial) full-wave methods, e.g. the Method of Moments (MoM), it is still opportune to develop analytical analysis methods, especially if we want to employ these methods in multi-analysis-iteration optimisation schemes for automatically designing wire array antennas.

For the above mentioned log periodic array antennas, the feeding at successive junctions of the dipoles with the feeding line should be reversed, [7], to provide a 180° phaseshift between adjacent dipoles. By employing *folded* dipoles to create a log-periodic antenna, the cumbersome twist of the feeder line between two adjacent dipoles is avoided by providing the required 180° phase shift through the folded dipole configuration, [8].

In the following we will review analytical equations and improvements for single folded dipole radiators and derive

equations for these radiators combined into series arrays of folded dipoles.

II. ANALYSIS

In the equations to be derived, we will use analytical expressions for the input impedance of an ordinary dipole antenna. To include mutual coupling effects in the array antenna analysis, we will separate the radiating elements from the feeding structure and analyse them separately. For the wire folded dipole antenna coupling, use will be made of analytical equations for the coupling between thin ordinary dipole antennas.

A. Single Folded Dipole

A single folded dipole radiator is shown in Figure 1.

Fig. 1 Decomposition of an equal radius wire folded dipole antenna (left) into a transmission line mode (middle) and an antenna mode (right).

In [9] it is shown that the current on the folded dipole radiator may be considered as composed of a transmission line mode and a dipole mode, see Figure 1. The input impedance is then calculated as

$$Z_{in} = \frac{4Z_T Z_D}{Z_T + 2Z_D}, \quad (1)$$

where Z_D is the impedance of a cylindrical, ordinary, dipole antenna with effective radius a_e , where

$$\ln(a_e) = \ln(a) + \frac{1}{2} \ln\left(\frac{D}{a}\right). \quad (2)$$

Herein, a is the radius of the wire and D is the separation of the wires, see Figure 1. In our analyses we use the analytic equation for the dipole impedance as given in [7], that is based on the work of C.T. Tai

$$Z_D = \begin{bmatrix} 122.65 - 204.1k_0 \frac{L}{2} + 110 \left(k_0 \frac{L}{2}\right)^2 \\ -j \left[120 \left(\ln\left(\frac{L}{a}\right) - 1 \right) \cot\left(k_0 \frac{L}{2}\right) - 162.5 + \right. \\ \left. 140k_0 \frac{L}{2} - 40 \left(k_0 \frac{L}{2}\right)^2 \right] \end{bmatrix}, \quad (3)$$

where $k_0 = 2\pi/\lambda_0$ is the free space wave number and L is the length of the radiator.

Z_T is the impedance of a short-circuited two-wire transmission line of length $L/2$

$$Z_T = -jZ_0 \tan\left(\frac{k_0 L}{2}\right), \quad (4)$$

where Z_0 is the characteristic impedance of the two-wire transmission line, that is given by

$$Z_0 = 120 \ln \left[\frac{D + \sqrt{D^2 + 4a^2}}{2a} \right]. \quad (5)$$

To broaden the range of wire separations, the folded dipole length is replaced by an equivalent length according to [10], so that wire separations up to $\lambda_0/6$ instead of $\lambda_0/100$ are allowable. The equivalent length is given by, [11]¹

$$L_{eq} = L + 0.39D. \quad (6)$$

B. Series Array of Folded Dipoles

If the folded dipole antennas are arranged into a series array, see Figure 2, the folded dipole antenna needs to be modified into a so-called re-entrant folded dipole [12], see also Figure 2.

¹ This length extension allows for larger wire separations in the folded dipole. For two-wire transmission lines connecting folded dipoles in a series array, the wire separation should still be limited to $\lambda_0/100$. The reason that the wires may be separated further in the folded dipole antenna is due to the dominant character of the dipole mode, see equation (1).

The array antenna may be analysed by constructing ABCD matrices for the re-entrant folded dipoles [12] and for the interconnecting transmission lines [13], after which a chain matrix analysis may be applied to the array antenna. This analysis however, does not include mutual coupling effects, [2], [12].

Fig. 2 Series array of re-entrant folded dipole antennas (left) and modification of a folded dipole antenna into a re-entrant folded dipole antenna.

For the inclusion of mutual coupling effects, the radiating elements and the feeding structure are separated as described in [14], [15].

The admittance matrix of an N -elements folded dipole array may be written as a $2N \times 2N$ array $[Y]$

$$[Y] = [Y_F] + [Y_A], \quad (7)$$

where $[Y_F]$ is the admittance matrix of the feed network, that has the form

$$[Y_F] = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & [Y_{F1}] & 0 & \dots & 0 & 0 \\ 0 & 0 & [Y_{F2}] & \dots & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & & [Y_{F(N-1)}] & 0 \\ 0 & 0 & 0 & \dots & 0 & Y_L \end{bmatrix}. \quad (8)$$

In Equation (8), Y_L is the load admittance of the array. For the array shown in Figure 2, $Y_L = \infty$. The 2×2 submatrices $[Y_{Fi}]$, $i=1, 2, \dots, N-1$, are defined by, [13]

$$\begin{aligned} Y_{Fi_{11}} &= Y_{Fi_{22}} = -jY_{0i} \cot(k_0 l_i) \\ Y_{Fi_{12}} &= Y_{Fi_{21}} = jY_{0i} \csc(k_0 l_i), \end{aligned} \quad (9)$$

where Y_{0i} and l_i are, respectively, the characteristic admittance and length of two-wire transmission line i .

$[Y_A]$ is the admittance matrix of the network of re-entrant folded dipoles and has the form

$$[Y_A] = \begin{bmatrix} [Y_{A11}] & [C_{A12}] & \cdots & [C_{A1N}] \\ [C_{A21}] & [Y_{A22}] & \cdots & [C_{A2N}] \\ \vdots & \vdots & \ddots & \vdots \\ [C_{AN1}] & [C_{AN2}] & \cdots & [Y_{ANN}] \end{bmatrix}, \quad (10)$$

where the 2×2 submatrices $[Y_{Aii}]$, $i=1,2,\dots,N$, are defined by [11]

$$\begin{aligned} Y_{Aii11} &= Y_{Aii22} = \frac{1}{2} Y_{Ti} + \frac{1}{4} Y_{Di} \\ Y_{Aii12} &= Y_{Aii21} = -\frac{1}{2} Y_{Ti} + \frac{1}{4} Y_{Di} \end{aligned}, \quad (11)$$

with Y_{Ti} and Y_{Di} , respectively, the transmission line admittance and the dipole admittance of re-entrant folded dipole i . They are calculated using Equations (3) and (4).

The 2×2 submatrices $[C_{Aij}]$, $i,j=1,2,\dots,N$, $i \neq j$, contain the mutual admittances between the folded dipole elements. Following [16], the mutual admittance between two folded dipole antennas may be calculated from the mutual admittance between two equivalent dipole antennas

$$Y_{21 \text{ folded-dipole-to-folded-dipole}} = \frac{1}{4} Y_{21 \text{ dipole-to-dipole}}. \quad (12)$$

In our effort to solve the folded dipole array with analytical equations, we take for the equivalent dipole antennas, infinitely thin dipole antennas having the same lengths as the folded dipole antennas they represent. The mutual impedance between two non-staggered dipoles of half-lengths l_1 and l_2 , separated by a distance d , is then given by, [17]

$$Z_{12} = R_{12} + jX_{12}, \quad (13)$$

where

$$R_{12} = 30 \left\{ \begin{aligned} &\cos k_0(l_1 + l_2) \begin{bmatrix} Ci(u_0) + Ci(v_0) \\ -Ci(u_1) - Ci(v_1) \\ -Ci(w_1) - Ci(y_1) \\ +2Ci(k_0 d) \end{bmatrix} + \cos k_0(l_1 - l_2) \begin{bmatrix} Ci(u_0') + Ci(v_0') \\ -Ci(u_1) - Ci(v_1) \\ -Ci(w_1) - Ci(y_1) \\ +2Ci(k_0 d) \end{bmatrix} \\ &+ \sin k_0(l_1 + l_2) \begin{bmatrix} -Si(u_0) + Si(v_0) \\ +Si(u_1) - Si(v_1) \\ -Si(w_1) + Si(y_1) \end{bmatrix} + \sin k_0(l_1 - l_2) \begin{bmatrix} -Si(u_0') + Si(v_0') \\ +Si(u_1) - Si(v_1) \\ +Si(w_1) - Si(y_1) \end{bmatrix} \end{aligned} \right\}, \quad (14)$$

and

$$X_{12} = 30 \left\{ \begin{aligned} &\cos k_0(l_1 + l_2) \begin{bmatrix} -Si(u_0) - Si(v_0) \\ +Si(u_1) + Si(v_1) \\ +Si(w_1) + Si(y_1) \\ -2Si(k_0 d) \end{bmatrix} + \cos k_0(l_1 - l_2) \begin{bmatrix} -Si(u_0') - Si(v_0') \\ +Si(u_1) + Si(v_1) \\ +Si(w_1) + Si(y_1) \\ -2Si(k_0 d) \end{bmatrix} \\ &+ \sin k_0(l_1 + l_2) \begin{bmatrix} -Ci(u_0) + Ci(v_0) \\ +Ci(u_1) - Ci(v_1) \\ -Ci(w_1) + Ci(y_1) \end{bmatrix} + \sin k_0(l_1 - l_2) \begin{bmatrix} -Ci(u_0') + Ci(v_0') \\ +Ci(u_1) - Ci(v_1) \\ +Ci(w_1) - Ci(y_1) \end{bmatrix} \end{aligned} \right\}. \quad (15)$$

In Equations (14) and (15), $Si(x)$ and $Ci(x)$, are the sine and cosine integral of argument x , respectively and

$$\begin{aligned} u_0 &= k_0 \left(\sqrt{d^2 + (l_1 + l_2)^2} - (l_1 + l_2) \right) \\ v_0 &= k_0 \left(\sqrt{d^2 + (l_1 + l_2)^2} + (l_1 + l_2) \right) \\ u_0' &= k_0 \left(\sqrt{d^2 + (l_1 - l_2)^2} - (l_1 - l_2) \right) \\ v_0' &= k_0 \left(\sqrt{d^2 + (l_1 - l_2)^2} + (l_1 - l_2) \right) \\ u_1 &= k_0 \left(\sqrt{d^2 + l_1^2} - l_1 \right) \\ v_1 &= k_0 \left(\sqrt{d^2 + l_1^2} + l_1 \right) \\ w_1 &= k_0 \left(\sqrt{d^2 + l_2^2} - l_2 \right) \\ y_1 &= k_0 \left(\sqrt{d^2 + l_2^2} + l_2 \right) \end{aligned} \quad (16)$$

The mutual admittance follows from the mutual impedance from

$$Y_{12} = \frac{Z_{12}}{Z_{D1}Z_{D2} - Z_{12}}, \quad (17)$$

where Z_{D1} and Z_{D2} are the dipole impedances of the two coupled dipoles.

III. VERIFICATION

The above analysis has been implemented in software and several wire folded dipole array antennas have been analysed. Comparisons have been made with a Method of Moment analysis. Typical analysis results are shown for a four element wire folded dipole array antenna, see Figure 3.

Fig. 3 Four elements series array of re-entrant folded dipole antennas.

The real part of the input impedance as a function of frequency is shown Figure 4, together with Method of Moments (MoM) analysis results. The imaginary part of the input impedance is shown in Figure 5, again with MoM analysis results.

The array dimensions are: $L_1=11\text{mm}$, $L_2=15\text{mm}$, $L_3=19\text{mm}$, $L_4=23\text{mm}$, $D=0.15\text{mm}$, $TL=10\text{mm}$, $a=3\mu\text{m}$. The analysis has been performed for frequencies ranging from 9GHz to 12GHz.

The Figures clearly show a very good agreement between the analysis results of our model and those obtained with a Method of Moments for the first resonance. Although MoM is more accurate and more versatile, our Transmission Line (TL) method, by being a dedicated tool for this type of antenna, is much faster and therefore more suited for synthesis problems.

Fig. 4 Real part input impedance as a function of frequency for a four elements folded dipole array antenna.

Fig. 5 Imaginary part input impedance as a function of frequency for a four elements folded dipole array antenna.

IV. CONCLUSIONS

An accurate analytical model has been derived for a linear array of wire folded dipole antennas. The model combines closed form analytical equations for the folded dipole antenna, the re-entrant folded dipole antenna, the two-wire transmission line, the mutual coupling between two folded dipole antennas and the mutual coupling between two thin dipole antennas. A technique wherein the analysis of the

antennas and the feeding network is separated is successfully applied to combine all the aforementioned closed-form analysis equations. The analysis may be employed in an optimisation scheme to synthesise desired input impedance characteristics.

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