Direct numerical simulations of statistically steady, homogeneous, isotropic fluid turbulence with polymer additives
Perlekar, P.; Mitra, D.; Pandit, R.

Published in:
Physical Review E

DOI:
10.1103/PhysRevE.82.066313

Published: 01/01/2010

Document Version
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):
Direct numerical simulations of statistically steady, homogeneous, isotropic fluid turbulence with polymer additives

Prasad Perlekar,1,* Dhrubaditya Mitra,2,† and Rahul Pandit3,‡

1Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
2NORDITA, Roslagstullsbacken 23, 106 91 Stockholm, Sweden
3Centre for Condensed Matter Theory, Department of Physics, Indian Institute of Science, Bangalore 560012, India

(Received 31 July 2010; published 27 December 2010)

We carry out a direct numerical simulation (DNS) study that reveals the effects of polymers on statistically steady, forced, homogeneous, and isotropic fluid turbulence. We find clear manifestations of dissipation-reduction phenomena: on the addition of polymers to the turbulent fluid, we obtain a reduction in the energy dissipation rate; a significant modification of the fluid-energy spectrum, especially in the deep-dissipation range; and signatures of the suppression of small-scale structures, including a decrease in small-scale vorticity filaments. We also compare our results with recent experiments and earlier DNS studies of decaying fluid turbulence with polymer additives.

DOI: 10.1103/PhysRevE.82.066313 PACS number(s): 47.27.Gs, 47.27.Ak

I. INTRODUCTION

The addition of small amounts of polymers to a turbulent fluid leads to dramatic changes that include modifications of the small-scale properties of the flow [1–4] and, in wall-bounded flows, the phenomenon of drag reduction [5–7], in which polymer additives allow the maintenance of a given flow rate at a lower pressure gradient than is required without these additives. Several experimental, numerical, and analytical studies have investigated drag reduction [6–14] in wall-bounded flows. These studies have shown that the addition of polymers modifies the turbulence significantly in the region near the wall, and this leads to an increase in the mean velocity in the bulk. By contrast, there have been only some investigations of the effects of polymer additives on homogeneous isotropic turbulence. Examples include recent experiments [15–17], which have been designed to obtain a high degree of isotropy in the turbulent flow, and shell-model studies and direct numerical simulations (DNSs) [2–4,13,18,19]. These studies have shown that the addition of polymers to a turbulent flow leads to a considerable reduction in small-scale structures; and they have also discovered the phenomenon of dissipation reduction, namely, a reduction in the energy dissipation rate $\epsilon$, in decaying turbulence [2–4]. In this paper we first elucidate the phenomenon of dissipation reduction for the case of statistically steady, homogeneous, and isotropic turbulence with polymer additives; we then study the small-scale properties of such flows.

We do this by conducting a series of high-resolution DNS studies of the three-dimensional Navier-Stokes (NS) equation coupled to an equation for the polymer conformation tensor, which describes the polymer additives at the level of the finitely extensible nonlinear elastic-Peterlin (FENE-P) [2,3] model. Before we give the details of our study, it is useful to summarize our principal results; these are in two parts.

The first part contains results from our DNS of forced statistically steady fluid turbulence, with polymer additives, at moderate Reynolds numbers ($Re_\theta \approx 80$). The forcing is chosen such that the energy injected into the fluid remains fixed [20], both with and without polymers; this mimics the forcing scheme used in the experiments of Refs. [16,17]. We find that, on the addition of polymers, the energy in the statistically steady state and the energy-dissipation rate are reduced. This dissipation reduction increases with an increase in the polymer concentration $\epsilon$ at fixed Weissenberg number $We$. We, the ratio (see Table I) of the polymer time scale $\tau_\eta$ to a shearing time scale in the turbulent fluid. The dissipation reduction also increases with $We$ if we hold $\epsilon$ fixed. The dissipation reduction seen in our simulations should not be confused with the phenomenon of drag reduction seen in wall-bounded flows. In the fluid-energy spectrum we find that the energy content increases marginally at small wave vectors on the addition of polymers, but it decreases for intermediate wave vectors. In this part of our study we use 256$^3$ collocation points and attain a moderate Reynolds number $Re_\theta = 80$, but we do not resolve the deep-dissipation range. (We consider the deep-dissipation range in the following paragraph.) We also obtain the structural properties of the fluid with and without polymers and show that polymers suppress large-vorticity and large-strain events; our results here are in qualitative agreement with the experiments of Refs. [16,17]. Furthermore, we find, as in our study of decaying turbulence [3], that the polymer extension increases with an increase in the polymer-relaxation time $\tau_\eta$. We compare our results, e.g., those for the energy spectrum, with their counterparts in our earlier study of decaying fluid turbulence with polymer additives [3]. In such comparisons, we use averages over the statistically steady state of our system here; and, for the case of decaying turbulence, we use data obtained at the cascade-completion time, at which a plot of

* p.perlekar@tue.nl
† dhruva.mitra@gmail.com
‡ Also at Jawaharlal Nehru Centre for Advanced Scientific Research, Jakkur, Bangalore, India; rahul@physics.iisc.ernet.in
the energy dissipation rate versus time displays a maximum.

In the second part of our study we carry out the highest-resolution DNS, attempted so far, of forced statistically steady fluid turbulence with polymer additives; we drive the fluid by an external stochastic force as in Ref. [21]. This part of our study has been designed to uncover the effects of polymers on the deep-dissipation range, so the Reynolds numbers is small \( \text{Re}_k \approx 16 \). By comparing fluid-energy spectra, with and without polymers, we find that the polymers suppress the energy in the dissipation range but increase it in the deep-dissipation range. Finally, we calculate the second-order velocity structure function \( S_2(r) \) directly from the energy spectrum via a Fourier transformation; this shows that \( S_2(r) \) with polymers is smaller than \( S_2(r) \) without polymers in this range.

The remaining part of this paper is organized as follows. In Sec. II we present the equations we use for the polymer solution and describe the method we use for the numerical integration of these equations. Section III is devoted to a solution and describe the method we use for the numerical integration of these equations as in our DNS runs. The Taylor-microscale Reynolds number \( \text{Re}_t = \sqrt{2} \nu^2 / \delta \text{Re}_k \) and the Weissenberg number \( \text{We} = \tau_p / \delta \) are as follows: NSP-256A and NSP-256B: \( \text{Re}_t = 80 \) and NSP-512: \( \text{Re}_t = 16 \); the Kolmogorov dissipation length scale is \( \eta = (\nu^3 / \epsilon)^{1/4} \). For our runs NSP-256A-B, \( \eta = 1.07 \delta t \); and for run NSP-512, \( \eta = 19 \delta t \), where \( \delta t = L / \text{N} \) is the grid resolution of our simulations. The integral length scale \( \text{L}_\text{int} = (3 \pi / 4) \text{K} \text{E}(k) / [\text{K} \text{E}(k)] \) and \( \text{L}_\text{edd} = u_{\text{rms}} / \text{L}_\text{int} \) are as follows: NSP-256A and NSP-256B: \( \text{L}_\text{int} = 1.3 \) and \( \text{L}_\text{edd} = 1.2 \); and, for NSP-512, \( \text{L}_\text{int} = 2.05 \) and \( \text{L}_\text{edd} = 4.0 \).

### Table I

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \delta t )</th>
<th>( L )</th>
<th>( \nu )</th>
<th>( \tau_p )</th>
<th>( c )</th>
<th>( \text{We} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSP-256A</td>
<td>256</td>
<td>5.0 ( \times 10^{-4} )</td>
<td>100</td>
<td>5 ( \times 10^{-3} )</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>NSP-256B</td>
<td>256</td>
<td>5.0 ( \times 10^{-4} )</td>
<td>100</td>
<td>5 ( \times 10^{-3} )</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>NSP-512</td>
<td>512</td>
<td>10^{-3}</td>
<td>100</td>
<td>5 ( \times 10^{-2} )</td>
<td>1.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

We consider homogeneous isotropic turbulence, so we use periodic boundary conditions and solve Eq. (1) by using a pseudospectral method [22,23]. We use \( N^3 \) collocation points in a cubic domain (side \( L = 2 \pi \)). We eliminate aliasing errors by the 2/3 rule [22,23] to obtain reliable data at small length scales, and we use a second-order slaved Adams-Bashforth scheme for time marching. In earlier numerical studies of homogeneous isotropic turbulence with polymer additives, it has been shown that sharp gradients are formed during the time evolution of the polymer conformation tensor; this can lead to dispersion errors [19,24]. To avoid these dispersion errors, shock-capturing schemes have been used to evaluate the polymer-advection term \( [(u \cdot \nabla)C] \) in Ref. [24]. In our simulations we have modified the Cholesky-decomposition scheme of Ref. [19], which preserves the symmetric- positive-definite (SPD) nature of the tensor \( C \). We incorporate the large gradients of the polymer conformation tensor by evaluating the polymer-advection term \( [(u \cdot \nabla)C] \) via the Kurganov-Tadmor shock-capturing scheme [25]. For the derivatives on the right-hand side of Eq. (2) we use an explicit fourth-order central-finite-difference scheme in space, and the temporal evolution is carried out by using an Adams-Bashforth scheme. The numerical error in \( r_p \) must be controlled by choosing a small time step \( \delta t \); otherwise, \( r_p \) can become larger than \( L \), which leads to a numerical instability. This time step is much smaller than what is necessary for a
We resolve the sharp gradients in the polymer conformation tensor by discretizing the polymer-advection term using the Kurganov-Tadmor scheme [25]. Below we show the discretization of the advection term \( u \partial_x \ell \), where \( u = (u, v, w) \) and \( \ell \) is one of the components of \( \ell_{ij} \); the discretization of the other advection terms in Eq. (6) is similar:

\[
H_{i+1/2,j,k} = \frac{u_{i+1/2,j,k} [\ell_{i+1/2,j,k} + \ell_{i+1/2,j,k}^-]}{2} - a_{i+1/2,j,k} [\ell_{i+1/2,j,k}^- - \ell_{i+1/2,j,k}^+],
\]

\[
\ell_{i+1/2,j,k}^+ = \ell_{i+1,j,k} + \frac{\Delta t}{2} (\partial_x \ell_{1/2,i,j,k})_{i+1/2,j,k},
\]

where \( i, j, k = 0, \ldots, (N-1) \) denote the grid points and \( \Delta x = \Delta y = \Delta z \) is the grid spacing along the three directions.

We use the following initial conditions (superscript 0): \( C_{nm}^0(x) = \delta_{nm} \) for all \( x \) and \( u_{n}^0(k) = P_{nm}(k)v_{m}^0(k) \exp(i\theta_{nm}(k)) \), with \( m, n = x, y, z \), \( P_{nm} = (\delta_{mn} - k_m k_n / k^2) \) as the transverse projection operator, \( k \) as the wave vector with components \( k_m = (-N/2, -N/2 + 1, \ldots, N/2) \) and magnitude \( k = |k| \), \( \theta_{nm} \) as random numbers distributed uniformly between zero and \( 2\pi \), and \( v_m^0(k) \) chosen such that the initial kinetic-energy spectrum is \( E_0(k) = k^4 \exp(-2.0k^2) \). This initial condition corresponds to a state in which the fluid energy is concentrated, to begin with, at small \( k \) (large length scales), and the polymers are in a coiled state. Our simulations are run for 45T_{eddy} and a statistically steady state is reached in roughly 10T_{eddy}, where the integral-scale eddy-turnover time \( T_{eddy} = u_{rms}/l_{int} \), with \( u_{rms} \) as the root-mean-square velocity and \( l_{int} = \Sigma_k k^{-1}E(k)/\Sigma_k E(k) \) as the integral length scale. Along with our runs NSF-256A and NSF-256B we also carry out pure-fluid NS simulations until a statistically steady state is reached; this takes about 10T_{eddy}−15T_{eddy}. Once this pure-fluid simulation reaches a statistically steady state, we add polymers to the fluid at 25T_{eddy}, i.e., beyond this time we solve the coupled NSP equations (1) and (2) by using the methods given above. We then allow 5T_{eddy}−6T_{eddy} to elapse, so that transients die down, and then we collect data for fluid and polymer statistics for another 25T_{eddy} for our runs NSF-256A and NSF-256B.

III. RESULTS

We now present the results that we have obtained from our DNS. In addition to \( u(x,t) \), its Fourier transform \( \hat{u}(k) \), and \( \zeta(x,t) \), we monitor the vorticity \( \omega = \nabla \times u \), the kinetic-energy spectrum \( E(k,t) = \Sigma_i k^{-1/2} \langle e_{i}^{2} \rangle_{k} \hat{u}_{k}^{i} \langle 0 \rangle_{k} \), the total kinetic energy \( \dot{E}(t) = \Sigma_j E(k,j) \), the energy-dissipation rate \( E_{d}(t) = \nu \Sigma_j \dot{E}(k,j) \), the probability distribution of scaled polymer extensions \( P(U_{n}/L_{p}) \), the probability distribution...
provide evidence for dissipation reduction by polymer additives: obtain the following energy-balance equation for the fluid energy of the fluid without polymers. By using Eq. with polymers is reduced in comparison to the average energy injection fixed. Therefore, we can determine how the dissipation gets distributed between the fluid and polymer subsystems in forced statistically steady turbulence.

Before we present our results for the kinetic-energy dissipation rate, we first calculate the second-order structure function $S_2(r)$ via the following exact relation [26]:

$$S_2(r) = \int_0^\infty \left[ 1 - \frac{\sin(kr)}{kr} \right] E(k)dk.$$  (10)

In Fig. 2 we give a log-log plot of $S_2(r)$, compensated by $(r/L)^{-2}$, as a function of $r/L$. We find that, for small $r$, $S_2(r) \sim r^2$, which implies that our DNS resolves the analytical range, which follows from a Taylor expansion of $S_2(r)$ [27]; this guarantees that energy-dissipation rate has been calculated accurately. In Fig. 3 we present plots of $\epsilon_r(t)$ versus $t/T_{edd}$ for $We=3.5$ and $We=7.1$ with the polymer concentration $c=0.1$. We find that the average value of $\epsilon_r$ decreases as we increase We. This suggests the following natural definition of the percentage dissipation reduction for forced, homogeneous, and isotropic turbulence:

$\epsilon_p = \int_0^\infty \left[ 1 - \frac{\sin(kr)}{kr} \right] E(k)dk.$

In Fig. 1 we show a plot of the fluid energy $E$ versus the dimensionless time $t/T_{edd}$ (runs NSP-256A and NSP-256B) for Weissenberg numbers $We=3.5$ (blue circles) and $We=7.1$ (black dashed line). The corresponding plot for the pure-fluid case is also shown for comparison (red solid line). The polymers are added to the fluid at $t=27T_{edd}$.

FIG. 1. (Color online) A plot of the fluid energy $E$ versus the dimensionless time $t/T_{edd}$ (runs NSP-256A and NSP-256B) for Weissenberg numbers $We=3.5$ (blue circles) and $We=7.1$ (black dashed line). The corresponding plot for the pure-fluid case is also shown for comparison (red solid line). The polymers are added to the fluid at $t=27T_{edd}$.

FIG. 2. (Color online) Log-log (base 10) plots of the second-order structure function $S_2(r)$, compensated by $(r/L)^{-2}$, versus $r/L$, for our run NSP-256B (blue square) and for the pure-fluid case (red circle). The regions in which the horizontal black lines overlap with the points indicate the $r^2$ scaling ranges.

FIG. 3. (Color online) Plots of the energy dissipation rate $\epsilon_r$ versus $t/T_{edd}$ (runs NSP-256A and NSP-256B) for Weissenberg numbers $We=3.5$ (blue circles) and $We=7.1$ (black dashed line). The corresponding plot for the pure-fluid case is also shown for comparison (red solid line). The polymers are added to the fluid at $t=27T_{edd}$.

$\epsilon_r = -\frac{1}{V} \int \mathbf{u} \cdot \nabla^2 \mathbf{u},$

$$\epsilon_{poly} = \left( \frac{\mu}{\tau_p} \right) \frac{1}{V} \int \mathbf{u} \cdot \nabla [f(r_p)C],$$

$$\epsilon_{inj} = \frac{1}{V} \int \mathbf{f} \cdot \mathbf{u}.$$  (9)

In the statistically steady state $\frac{dE}{dt}=0$, and the energy injected is balanced by the fluid dissipation rate $\epsilon_r$ and the polymer dissipation $\epsilon_{poly}$. Our simulations are designed to keep the fluid energy injection fixed. Therefore, we can determine how the dissipation gets distributed between the fluid and polymer subsystems in forced statistically steady turbulence.
Here (and henceforth), the superscripts \( f \) and \( p \) stand, respectively, for the fluid without and with polymers and the angular brackets denote an average over the statistically steady state. The percentage dissipation reduction \( \text{DR} \) rises with \( \text{We} \); this indicates that \( \epsilon_p^f \) decreases with \( \text{We} \) [28]. Thus, in contrast to the trend we observed in our decaying-turbulence DNS [3], \( \text{DR} \) increases with \( \text{We} \): for \( \text{We}=3.5 \), \( \text{DR}=30\% \) and, for \( \text{We}=7.1 \), \( \text{DR}=50\% \). Our interpretation is that this increase in \( \text{DR} \) with \( \text{We} \) arises because the polymer extension and, therefore, the polymer stresses are much stronger in our forced-turbulence DNS than in our decaying-turbulence DNS (at least for the Reynolds numbers that we achieved in Ref. [3]). In Fig. 4 we show the cumulative PDF of the scaled polymer extension; this shows clearly that the extension of the polymers increases with \( \text{We} \). In general, the calculation of PDFs from numerical data is plagued by errors originating from the binning of the data to make histograms. Here, instead, we have used the rank-order method to calculate the corresponding cumulative PDF, which is free of binning errors [29].

C. Small-scale structures

We now investigate how polymers affect small-scale structures in homogeneous isotropic fluid turbulence, and we make specific comparisons with experiments [16,17]. We begin by plotting the PDFs of the modulus of the vorticity \( |\omega| \) and the local energy dissipation rate \( \epsilon_{\text{loc}}=\sum_i(\partial_i u_j+\partial_j u_i)^2/2 \) in Fig. 6. We find that the addition of polymers reduces regions of high vorticity and high dissipation (Fig. 6). Furthermore, we find that, on normalizing \( |\omega| \) or \( \epsilon_{\text{loc}} \) by their respective standard deviations, the PDFs of these normalized quantities for the fluid with and without polymers collapse.

B. Energy spectra

In this section we study fluid-energy spectra \( E_p(k) \), in the presence of polymer additives, for two different values of the Weisszenberg number \( \text{We} \) and fixed polymer concentration \( c=0.1 \) (Fig. 5). We find that the energy content at intermediate wave vectors decreases with an increase in \( \text{We} \). At small wave-vector magnitudes \( k \), we observe a small increase in the spectrum on the addition of the polymers, but this increase is within our numerical two-standard-deviation error bars. Because of the moderate resolution of our simulations we are not able to resolve the dissipation range fully in these simulations. We address this issue by conducting high-resolution low-Reynolds-number simulations in Sec. III D.

FIG. 4. (Color online) Log-log (base 10) plots of the cumulative PDF \( P(\epsilon_p^f) \) versus the scaled polymer extension \( r_p^f/L^2 \) for \( \text{We}=3.5 \) (blue dashed line for run NSP-256A) and \( \text{We}=7.1 \) (full black line for run NSP-256B). Note that as \( \text{We} \) increases, so does the extension of the polymers. These plots are obtained from polymer configurations at \( t=60\tau_{\text{eddy}} \).

FIG. 5. (Color online) Log-log plots (base 10) of the energy spectra \( E_p^f(k) \) versus \( k \) (runs NSP-256A and NSP-256B) for \( c=0.1 \) and \( \text{We}=3.5 \) (blue squares) or \( \text{We}=7.1 \) (black stars); we give two-standard-deviation error bars. The corresponding pure-fluid spectrum \( E_p(k) \) (red circles) is shown for comparison.

FIG. 6. (Color online) Semilogarithmic plots (base 10) of the PDFs \( P(|\omega|) \) versus \( |\omega| \) (top panel) and \( P(\epsilon_{\text{loc}}) \) versus \( \epsilon_{\text{loc}} \) (bottom panel), for our run NSP-256B, with \( [c=0.1, \text{We}=7.1] \) (blue dashed line) and without \( [c=0] \) (full red line) polymer additives.
and Fig. 3 of Ref. 

ability implies that 

ment with the results of Refs. 

middle panels of Fig. 8 

comes stronger as We increases 

strain matrix 

also affects the statistics of the eigenvalues of the rate-of-

deviations for 

for large values of 

where there is large strain, a result that is in qualitative 

agreement with the experiments of Ref. [16] (see Fig. 3(b) of Ref. [16]). Evidence for the suppression of small-scale structures on the addition of polymers can also be obtained by examining the attendant change in the topological properties of a three-dimensional turbulent flow. For incompressible ideal fluids in three dimensions there are two topological invariants: $Q = -\text{Tr}(A^2)/2$ and $R = -\text{Tr}(A^3)/3$, where $A$ is the velocity-gradient tensor $\nabla \mathbf{u}$ [32]. Topological properties of such a flow can be classified [33,34] by a $Q$-$R$ plot, which is a contour plot of the joint PDF of $Q$ and $R$. In Fig. 11 we
give $Q$-$R$ plots from our DNS studies with and without polymers; although the qualitative shape of these joint PDFs remains the same, the regions of large $R$ and $Q$ are dramatically reduced on the additions of polymers; this is yet another indicator of the suppression of small-scale structures.

D. Effects of polymer additives on deep-dissipation-range spectra

In the previous sections we have studied the effects of polymer additives on the structural properties of a turbulent fluid at moderate Reynolds numbers. We now investigate the effects of polymer additives on the structural properties of a turbulent fluid at moderate Reynolds numbers. We now investigate the effects of polymer additives on the deep-dissipation range. To uncover such deep-dissipation-range effects, we conduct effects of polymer additives on the deep-dissipation range. For homogeneous isotropic turbulence, the relationship between the second-order structure function and the energy spectrum is given in Eq. (10) [26]. Using this relationship and the data for the energy spectrum shown in Fig. 12, we have obtained the second-order structure function $S_2(r)$ for our run NSP-512. We find that the addition of polymers leads to a decrease in the magnitude of $S_2(r)$. Our plots for $S_2(r)$ are similar to those found in the experiments of Ref. [15]. In our simulations we are able to reach much smaller values of $r/\eta$ than has been possible in experimental studies on these systems [15]; however, we have not resolved the inertial range very well in these runs. Note that the spectra $E^h(k)$, with polymers, and $E^f(k)$, without polymers, cross each other as shown in Fig. 12. But such a crossing is not observed in the corresponding plots of second-order structure functions.
The corresponding plot for the pure fluid and combines large-
experimental results of Refs. PERLEKER, MITRA, AND PANDIT PHYSICAL REVIEW E run NSP=512 spectrum, introduced in Ref. This allows us to extend the definition of dissipation reduc-
ergy dissipation rate decreases on the addition of polymers.
thermore, our results compare favorably with several recent
crease in the Weissenberg number We at fixed polymer con-
We find that this dissipation reduction increases with an in-
homogeneous, and isotropic fluid turbulence with polymers.
Fig. 14. This can be understood by noting that $S_2(r)$ combines large- and small-k parts [35] of the energy spectrum.

IV. CONCLUSIONS

We have presented an extensive numerical study of the effects of polymer additives on statistically steady, homogeneous, and isotropic fluid turbulence. Our study complements, and extends considerably, our earlier work [3]. Furthermore, our results compare favorably with several recent experiments.

Our first set of results show that the average viscous energy dissipation rate decreases on the addition of polymers. This allows us to extend the definition of dissipation reduction, introduced in Ref. [3], to the case of statistically steady, homogeneous, and isotropic fluid turbulence with polymers. We find that this dissipation reduction increases with an increase in the Weissenberg number We at fixed polymer concentration c. We obtain PDFs of the modulus of the vorticity, of the eigenvalues $\lambda_n$ of the rate-of-strain tensor $\mathcal{S}$, and $Q-R$ plots; we find that these are in qualitative agreement with the experiments of Refs. [16,17].

Our second set of results deal with a high-resolution DNS that we have carried out to elucidate the deep-dissipation-range forms of (a) energy spectra and (b) the related second-order velocity structure functions. We find that this deep-dissipation-range behavior is akin to that in our earlier DNS of decaying, homogeneous, and isotropic fluid turbulence with polymers [3]. Furthermore, the results we obtain for the scaled second-order velocity structure $S_2(r)$ yield trends that are in qualitative agreement with the experiments of Ref. [15]. We hope that the comprehensive study that we have presented here will stimulate further detailed experimental studies of the statistical properties of homogeneous isotropic fluid turbulence with polymer additives.

ACKNOWLEDGMENTS

We thank J. Bec, E. Bodenschatz, F. Toschi, and H. Xu for discussions, Leverhume Trust, European Research Council under the AstroDyn Research Project No. 227952, CSIR, DST, and UGC (India) for support, and SERC (IISc) for computational resources. Two of us (R.P. and P.P.) acknowledge the International Collaboration for Turbulence Research and support from the COST Action Grant No. MP0806. P.P. acknowledges IISc.

DIRECT NUMERICAL SIMULATIONS OF STATISTICALLY...

PHYSICAL REVIEW E 82, 066313 (2010)

ence, edited by I. P. Castro, P. E. Hancock, and T. G. Thomas (CIMNE, Barcelona, 2002).

[32] Strictly speaking $Q$ and $R$ are not topological invariants of the (unforced and inviscid) NSP equations but only of the (unforced and inviscid) NS equation.