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Effect of Carrier Frequency Offset on Offset QAM Multicarrier Filter Bank Systems Over Frequency-Selective Channels

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Abstract—This paper presents an analysis of the effect of carrier frequency offset (CFO) on offset QAM (OQAM) multicarrier filter bank (MCFB) systems, also known as staggered modulated multitone (SMT), over frequency-selective channels. This effect may be quantified by signal-to-interference ratio (SIR). We drive an accurate expression for the interference power and the desired signal power. Then SIR of SMT systems is calculated. Next, we drive an approximated SIR when the maximum delay of the channel is small in comparison with symbol spacing. The approximated SIR show that the SIR over frequency-selective channels converges to that over additive white Gaussian noise (AWGN) channels. Numerical results show that with increasing number of subcarriers, both of accurate and approximated forms of SIR as a function of CFO over frequency-selective channels converge to that over AWGN channels. Also, we compare SMT and orthogonal frequency division multiplexing (OFDM). It is shown that OFDM has better SIR in small CFO over frequency-selective channels. But with large number of subcarriers or high CFO, SMT outperforms OFDM.

I. INTRODUCTION

Multicarrier filter bank (MCFB) is an attractive technique for multicarrier communication over broadband channels. The available bandwidth is divided into N subchannels, also known as subcarriers. Since each subchannel only occupies a relatively narrow band, the subchannel frequency response can be considered approximately flat. On each subchannel, the input data symbols with symbol spacing T are passed through a pulse shaping filter and then modulated to the subchannel frequency. Orthogonal frequency division multiplexing (OFDM), with rectangular pulse shaping, is a very popular MCFB system widely used in wireless communications. It is well known to prevent intersymbol interference (ISI) by using a cyclic prefix (CP) longer than the maximum delay of the channels. By using CP, equalization of the frequency-selective channels can be performed easily using a one-tap equalizer on each subchannel. However, CP doesn't carry useful information and thus reduces spectral efficiency. Other multicarrier filter bank systems have been suggested as an alternative to OFDM with better spectral efficiency without CP [1]-[4]. Offset QAM (OQAM) MCFB system is an MCFB system that does not require CP. It utilizes well designed pulse shaping filters, with normally longer length than symbol

spacing, and achieve more spectral efficiency than OFDM [5]-[7]. There is a time-staggering between the real and the imaginary parts in OQAM MCFB system and therefore it is also referred to as staggered modulated multitone (SMT) system[3].

The SMT system, like OFDM, is sensitive to carrier frequency offset (CFO), which results from the Doppler shift in the channel or from the difference between local oscillators in the receiver and the transmitter [8]. The effect of CFO on SMT systems was investigated for additive white Gaussian noise (AWGN) channel extensively [8]-[11]. It was shown that CFO produces interference consisting of both ISI and intercarrier interference (ICI) and SMT is more robust to CFO than OFDM over AWGN channel. In [9], the SNR degradation of SMT in the presence of CFO over flat fading channels was investigated by simulations and it was shown that SMT is more robust to CFO than OFDM. The performance of SMT was studied in the absence of CFO over multipath fading channel [12]-[15]. It was shown that, different from OFDM, the frequency selective channel introduces interference in SMT systems. The effect of CFO on SMT systems over frequency-selective channels has received less attention in the literature. While the MCFB system, such as SMT, is mostly used in frequency-selective channels, it is important to investigate the effect of CFO over frequency-selective channels.

In this paper, we present a study on the effect of CFO, in terms of signal-to-interference ratio (SIR), on SMT systems over frequency-selective channels. We first model the estimated data symbols of the SMT system before decision device and then derive accurate expressions for the powers of the desired signal and the interference. We show that the interference in SMT is caused both by CFO and the frequency-selective channel. The SIR over frequency-selective channel is then derived as the ratio of the desired signal power and the interference power. By assuming that the maximum delay of the channel is small in comparison with symbol spacing, we further derive an approximated expression for SIR and show analytically that the SIR of SMT systems over frequency-selective channels and flat fading/AWGN channels are the same when the assumption is valid. Finally, numerical results are obtained for the same transmission bandwidth, frequency-

Fig. 1. Baseband equivalent of an SMT transceiver with N subcarriers.

selective channel, and CFO. Our results show that with increasing number of subcarriers, SIR of SMT over frequency-selective channels converges to that over AWGN channels. When the number of subcarriers in given transmission bandwidth is increased, the subcarrier spacing is reduced. Then the channel frequency response over each subcarrier is approximately flat and the one-tap equalizer works better. In this case, for SMT systems, the interference due to frequency-selective channel becomes very small and the total interference is dominated by the interference caused by the CFO. Moreover, numerical results show that the approximated SIR converges to its accurate form with increasing number of subcarriers. In the end, a comparison between SMT and OFDM is provided in the same channels. We show that OFDM has larger SIR than that of SMT for small CFO over frequency-selective channels. But by increasing CFO, SMT shows better SIR than that of OFDM. The SMT system with large number of subcarriers outperforms OFDM due CFO over frequency-selective channels. Different from OFDM, SMT suffers from interference over frequency-selective channels in the absence of CFO. Then in small CFOs, SMT suffer from more interference due to channel. But the interference power in OFDM is very small. With increasing CFO, the dominant interference in SMT comes from CFO and in such cases SMT is found more robust than OFDM.

This paper is organized as follows. Section II describes the system model of SMT. In Section III, the effect of CFO on SMT systems over frequency-selective channels is analyzed in detail and we derive accurate and approximated forms for the SIR as a function of CFO. Section IV presents some numerical results and finally the conclusions are drawn in Section V.

II. SYSTEM MODEL

The baseband equivalent of an SMT transceiver with N subcarriers is depicted in Figure 1 [1] [3]. The complex input data signal $s^k(t)$ with in-phase and quadrature components $s_I^k(t)$, $s_Q^k(t)$ on the k th subcarrier is an impulse train corresponding

to the complex input data symbols with symbol spacing T

$$s^k(t) = \sum_{l=-\infty}^{\infty} (a_l^k + jb_l^k) \delta(t - lT), \quad (1)$$

where T and l are the symbol spacing and the symbol index respectively. Also, a_l^k and b_l^k denote the real and imaginary parts of the complex input data symbol transmitted on the k th subcarrier. We assume that the real and imaginary parts a_l^k and b_l^k are independent and identically distributed with $E\{a_l^k\} = E\{b_l^k\} = 0$ and $E\{|a_l^k|^2\} = E\{|b_l^k|^2\} = \sigma_s^2/2$ for all values of k and l . In the transmitter, the real and imaginary parts of each data symbol are staggered by half a symbol spacing passed through pulse shaping filters with impulse responses $h(t)$ and $h(t - T/2)$, respectively. The pulse shaping filter $h(t)$ has length KT . In SMT systems, it needs to be guaranteed that there is spectrum overlapping only between adjacent subcarriers [3]. This is achieved by proper design of the pulse shaping filter $h(t)$ [3]. The filter outputs are summed together and are modulated by $e^{jk(\frac{2\pi t}{T} + \frac{\pi}{2})}$ with k being subcarrier index. The transmitted baseband signal is thus obtained as [1] [3]

$$x(t) = \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} \left(a_l^k h(t - lT) + jb_l^k h(t - lT - \frac{T}{2}) \right) e^{jk(\frac{2\pi t}{T} + \frac{\pi}{2})}. \quad (2)$$

Equation (2) shows that the l th SMT symbol is the sum of N complex input data symbols shaped by $h(t)$ and modulated by N carrier frequencies with $1/T$ Hz subcarrier spacing.

The signal $x(t)$ transmitted through a equivalent baseband frequency-selective channel $c(t)$ is affected by a CFO and AWGN $n(t)$, which independent of $x(t)$ and $c(t)$ and has zero mean and variance σ_n^2 . The received signal thus may be written as

$$r(t) = (x(t) \star c(t)) e^{j(2\pi \epsilon t + \phi)} + n(t), \quad (3)$$

where $\star$ denote convolution. Also, $c(t) = \sum_{d=0}^{D-1} g_d \delta(t - \tau_d)$ is a frequency-selective channel with D paths, in which

$\tau_0 < \tau_1 < \dots < \tau_{D-1}$, and $\sum_{d=0}^{D-1} |g_d|^2 = 1$. The CFO and a constant phase offset between the transmitter carrier and the receiver carrier are denoted by ε and ϕ respectively.

At the receiver, the received signal is demodulated and passed through filters $h(t)$ and $h(t + T/2)$. The filtered received signal is sampled by a sampler working at a rate $1/T$ and then is equalized in the real and imaginary paths using one-tap equalizer coefficients ($C_{n,r}^m$) and ($C_{n,i}^m$) as shown in Figure 1. Finally, the real and the imaginary parts of the equalized signals are passed through $\Re\{\cdot\}$ and $\Im\{\cdot\}$ blocks that take real part and imaginary part of input samples respectively. The estimated data symbol on subcarrier m in n th symbol interval is

$$\begin{cases} \hat{a}_n^m = \Re\left\{C_{n,r}^m \left(r(t)e^{-jm(\frac{2\pi t}{T} + \frac{\pi}{2})} \star h(t)|_{t=nT}\right)\right\} \\ \hat{b}_n^m = \Im\left\{C_{n,i}^m \left(r(t)e^{-jm(\frac{2\pi t}{T} + \frac{\pi}{2})} \star h(t + T/2)|_{t=nT}\right)\right\}. \end{cases} \quad (4)$$

For simplicity, let us define

$$c_k(t) \triangleq c(t)e^{-j2\pi kt/T}, \quad (5)$$

$$g_k(t) \triangleq h(t) \star c_k(t) = \sum_{d=0}^{D-1} g_d e^{-j2\pi k\tau_d/T} h(t - \tau_d). \quad (6)$$

Using the above definitions, the real part and the imaginary part of the estimated data symbol in the presence of CFO over frequency-selective channel can be obtained using (4) as

$$\begin{aligned} \hat{a}_n^m = \Re\left\{C_{n,r}^m e^{j(2\pi\varepsilon nT + \phi)} \sum_{l=-\infty}^{+\infty} \sum_{k=0}^{N-1} \int_{-\infty}^{\infty} (a_{l+n}^k h(t) g_k(t - lT) + \right. \\ \left. j b_{l+n}^k h(t) g_k(t - lT - \frac{T}{2})) e^{j(\Phi^{k-m}(t) + 2\pi\varepsilon t)} dt\right\} + \hat{n}_{n,R}^m, \end{aligned} \quad (7)$$

$$\begin{aligned} \hat{b}_n^m = \Im\left\{C_{n,i}^m e^{j(\pi\varepsilon T(2n+1) + \phi)} \sum_{l=-\infty}^{+\infty} \sum_{k=0}^{N-1} \int_{-\infty}^{\infty} (j b_{l+n}^k h(t) g_k(t - lT) \right. \\ \left. + a_{l+n}^k h(t) g_k(t - lT + \frac{T}{2})) e^{j(\Phi^{k-m}(t) + 2\pi\varepsilon t)} dt\right\} + \hat{n}_{n,I}^m, \end{aligned} \quad (8)$$

where $\Phi^k(t) = k(\frac{2\pi t}{T} + \frac{\pi}{2})$. Also, $\hat{n}_{n,R}^m$ and $\hat{n}_{n,I}^m$ are the real part and the imaginary part of the complex Gaussian noise $\hat{n}_n^m$ that

$$\begin{aligned} \hat{n}_n^m = \Re\left\{C_{n,r}^m \left(n(t)e^{-jm(\frac{2\pi t}{T} + \frac{\pi}{2})} \star h(t)|_{t=nT}\right)\right\} + \\ j \Im\left\{C_{n,i}^m \left(n(t)e^{-jm(\frac{2\pi t}{T} + \frac{\pi}{2})} \star h(t + \frac{T}{2})|_{t=nT}\right)\right\}. \end{aligned} \quad (9)$$

Moreover, it can be shown that $\hat{n}_n^m$ is complex Gaussian noise. Equations (7) and (8) show that the estimated data symbols suffer from phase rotation due to CFO, ε , and constant phase offset, ϕ . Hence, the real part of each estimated data symbol has a phase rotation $2\pi\varepsilon nT + \phi$, while its imaginary part has a phase rotation $2\pi\varepsilon nT + \pi\varepsilon T + \phi$. Also, a phase rotation is induced due to complex coefficient of the channel. The SMT system is very sensitive to phase offset [11]. For coherent demodulation, the receiver should be able to estimate the phase

of the desired section of the estimated data symbol in order to decode it correctly. We assume that the estimation of this phase is perfect. Any phase compensation using one-tap equalizer coefficients $C_{n,r}^m$ and $C_{n,i}^m$ has to be done before signal separation in real and imaginary paths in the m th subcarrier respectively [11]. The phase compensation due to CFO and constant phase offset are different in the real part and the imaginary part and are equal to $e^{-j(2\pi\varepsilon nT + \phi)}$ and $e^{-j(\pi\varepsilon T(2n+1) + \phi)}$ according to (7) and (8), respectively. Also, we denote $e^{j\phi_m}$ as the phase rotation due to the channel. The detailed calculation of phase rotation ϕ_m is presented in Section III. Here the phase compensator coefficients include $e^{-j\phi_m}$. As a result, one-tap equalizer coefficients for real and imaginary paths are $C_{n,r}^m = e^{-j(\phi_m + 2\pi\varepsilon nT + \phi)}$ and $C_{n,i}^m = e^{-j(\phi_m + \pi\varepsilon T(2n+1) + \phi)}$, respectively.

Estimated data symbols suffer from ISI and ICI due to CFO [8]-[11]. Furthermore, the frequency-selective channels destroy orthogonality between subcarriers and the Nyquist property of the pulse shaping filter $h(t)$. The estimated data symbol on the m th subcarrier in the n th symbol interval, $\hat{s}_n^m$, consisting of both ISI and ICI can be written as

$$\begin{aligned} \hat{s}_n^m = a_n^m \Re\{e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_m(t) e^{j2\pi\varepsilon t} dt\} + \\ j b_n^m \Re\{e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_m(t) e^{j2\pi\varepsilon t} dt\} + \text{ISI} + \text{ICI} + \hat{n}_n^m, \end{aligned} \quad (10)$$

where ISI and ICI are given by (11) and (12) at the top of the next page. The estimated data symbol on the m th subcarrier suffer from ICI due to other subcarriers and ISI due to the m th subcarrier in adjacent symbols. If $h(t)$ is chosen to be real and even with root-Nyquist property $\int_{-\infty}^{+\infty} h(t) h(t - lT) dt = \delta_l$ [16], estimated data symbols is received free of ISI as well as ICI in an ideal channel $c(t) = \delta(t)$ in the absence of CFO [1] [3]. It results that the output and input data symbols are equal in the absence of noise. Finally, estimated data symbols are passed to a decision device to estimate the transmitted symbols.

III. SIR ANALYSIS

In this section, an analysis of SIR for SMT due to CFO over frequency-selective channels is provided. Before the analysis, we first need to calculate the phase rotation $e^{j\phi_m}$ due to complex channel coefficients. With regards to (10), the desired section of the estimated data symbol before signal separation is attenuated by $\int_{-\infty}^{\infty} h(t) g_m(t) e^{j2\pi\varepsilon t} dt$ that is equal to

$$\int_{-\infty}^{\infty} h(t) g_m(t) e^{j2\pi\varepsilon t} dt = G_m \int_{-\infty}^{\infty} h(t) h(t - \tau_d) e^{j2\pi\varepsilon t} dt, \quad (13)$$

where $G_m = \sum_{d=0}^{D-1} g_d e^{-j2\pi m\tau_d/T}$ can be interpreted as the channel frequency response on the m th subcarrier. Therefore the phase offset due to complex channel on the m th subcarrier, ϕ_m , is equal to the phase of G_m .

For a given channel, SIR of the m th subcarrier is given by

$$\text{SIR}_m(\varepsilon) = \frac{P_{s_m}(\varepsilon)}{P_{i_m}(\varepsilon)}, \quad (14)$$

$$\begin{aligned} \text{ISI} = & \sum_{\substack{l=-\infty \\ l \neq 0}}^{+\infty} \left(a_{l+n}^m \Re \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_m(t-lT) e^{j2\pi\epsilon t} dt \right\} + j b_{l+n}^m \Im \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_m(t-lT) e^{j2\pi\epsilon t} dt \right\} \right) \\ & - \sum_{l=-\infty}^{+\infty} \left(b_{l+n}^m \Im \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_m(t-lT - \frac{T}{2}) e^{j2\pi\epsilon t} dt \right\} - j a_{l+n}^m \Re \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_m(t-lT + \frac{T}{2}) e^{j2\pi\epsilon t} dt \right\} \right), \end{aligned} \quad (11)$$

$$\begin{aligned} \text{ICI} = & \sum_{\substack{k=0 \\ k \neq m}}^{N-1} \sum_{l=-\infty}^{+\infty} \left(a_{l+n}^k \Re \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_k(t-lT) e^{j(\Phi^{k-m}(t)+2\pi\epsilon t)} dt \right\} + j b_{l+n}^k \Im \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_k(t-lT) e^{j(\Phi^{k-m}(t)+2\pi\epsilon t)} dt \right\} \right) \\ & - b_{l+n}^k \Im \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_k(t-lT - \frac{T}{2}) e^{j(\Phi^{k-m}(t)+2\pi\epsilon t)} dt \right\} + j a_{l+n}^k \Re \left\{ e^{-j\phi_m} \int_{-\infty}^{\infty} h(t) g_k(t-lT + \frac{T}{2}) e^{j(\Phi^{k-m}(t)+2\pi\epsilon t)} dt \right\} \right). \end{aligned} \quad (12)$$

where $P_{s_m}(\epsilon)$ and $P_{i_m}(\epsilon)$ are the desired signal power and the interference power of the m th subcarrier due to CFO ϵ conditioned on the given frequency-selective channel $c(t)$ respectively.

Since there is spectrum overlapping only between two adjacent subcarriers in SMT, we can assume that ICI comes only from two adjacent subcarriers ($k = m \pm 1$). By this assumption and some algebraic manipulations, the SIR can be written as

$$\begin{aligned} \text{SIR}_m(\epsilon) = & \frac{\sum_{l=-\infty}^{+\infty} \left\{ \sum_{\substack{k=m-1 \\ k \neq m \vee l \neq 0}}^{m+1} (\Re \{ G_m^* \Lambda_l^{k-m}(\epsilon) \})^2 + \sum_{k=m-1}^{m+1} (\Im \{ G_m^* \Gamma_l^{k-m}(\epsilon) \})^2 \right\}}{(\Re \{ G_m^* \Lambda_0^0(\epsilon) \})^2} \end{aligned} \quad (15)$$

where $\Lambda_l^{k-m}(\epsilon)$ and $\Gamma_l^{k-m}(\epsilon)$ (see (25) and (26) in the Appendix) are related to pulse shaping filter, channel coefficients, and CFO. The detailed calculation including the interference power and the desired signal power can be found in the Appendix. The average SIR across all subcarriers in the presence of CFO over given frequency-selective channel is obtained as

$$\overline{\text{SIR}}(\epsilon) = \frac{1}{N} \sum_{m=0}^{N-1} \text{SIR}_m(\epsilon). \quad (16)$$

Equation (15) is a complicated expression. For more clear insight, we simplify it using some reasonable assumptions. Given the same transmission bandwidth, when the number of subcarriers is large, the maximum delay of the channel τ_{D-1} is small in comparison with the symbol spacing T . Then, we can approximate $\lambda_l^{k-m}(\epsilon, \tau_d) \approx \lambda_l^{k-m}(\epsilon, 0) = \lambda_l^{k-m}(\epsilon)$ and $\gamma_l^{k-m}(\epsilon, \tau_d) \approx \gamma_l^{k-m}(\epsilon, 0) = \gamma_l^{k-m}(\epsilon)$ for $d = 0, \dots, D-1$ (see (23) and (24) in the Appendix). While $k = m \pm 1$, we can use the first-order Taylor series expansion $e^{-j\frac{2\pi(k-m)\tau_d}{T}} \approx 1 - j\frac{2\pi(k-m)\tau_d}{T}$ in $\Lambda_l^{k-m}(\epsilon)$ and $\Gamma_l^{k-m}(\epsilon)$ (see (25) and (26)

in the Appendix). Thus, with large subcarriers, we have

$$\Re \{ G_m^* \Lambda_l^{k-m}(\epsilon) \} \approx |G_m|^2 \Re \{ \lambda_l^{k-m}(\epsilon) \} + (k-m) \Im \left\{ \lambda_l^{k-m}(\epsilon) G_m^* \sum_{d=0}^{D-1} \frac{2\pi g_d \tau_d}{T} e^{-\frac{j2\pi m \tau_d}{T}} \right\}, \quad (17)$$

$$\Im \{ G_m^* \Gamma_l^{k-m}(\epsilon) \} \approx |G_m|^2 \Im \{ \gamma_l^{k-m}(\epsilon) \} - (k-m) \Re \left\{ \gamma_l^{k-m}(\epsilon) G_m^* \sum_{d=0}^{D-1} \frac{2\pi g_d \tau_d}{T} e^{-\frac{j2\pi m \tau_d}{T}} \right\}. \quad (18)$$

With more increasing number of subcarriers in a given transmission bandwidth (increasing T), we can approximate $\Re \{ G_m^* \Lambda_l^{k-m}(\epsilon) \} \approx |G_m|^2 \Re \{ \lambda_l^{k-m}(\epsilon) \}$, $\Im \{ G_m^* \Gamma_l^{k-m}(\epsilon) \} \approx |G_m|^2 \Im \{ \gamma_l^{k-m}(\epsilon) \}$. If we substitute these approximations into (15), the SIR on the m th subcarrier can be approximated by

$$\begin{aligned} \text{SIR}_m(\epsilon) \approx & \frac{\sum_{l=-\infty}^{+\infty} \left\{ \sum_{\substack{q=-1 \\ q \neq 0 \vee l \neq 0}}^{+1} (\Re \{ \lambda_l^q(\epsilon) \})^2 + \sum_{q=-1}^{+1} (\Im \{ \gamma_l^q(\epsilon) \})^2 \right\}}{(\Re \{ \lambda_0^0(\epsilon) \})^2}. \end{aligned} \quad (19)$$

From (19), it can be seen that $\text{SIR}_m(\epsilon)$ is independent of the subcarrier index. Therefore the $\overline{\text{SIR}}(\epsilon)$ is also equal to (19). When channel is AWGN, $c(t) = \delta(t)$ with $D = 1$ and $G_m = 1$. With regard to (25) and (26), $\Lambda_l^{k-m}(\epsilon) = \lambda_l^{k-m}(\epsilon)$, $\Gamma_l^{k-m}(\epsilon) = \gamma_l^{k-m}(\epsilon)$. Then $\overline{\text{SIR}}(\epsilon)$ is equal to (19). With large subcarriers, the average SIRs in the presence of CFO over AWGN and frequency-selective channels are the same.

IV. NUMERICAL RESULTS

In this section SIR graphs of SMT due to CFO over frequency-selective channels are depicted and compared with that of OFDM. The SMT system studied has a total bandwidth of $BW = 0.5$ MHz and a spectral efficiency $\eta = 1.0$. We use a frequency-selective channel with $L = 4$ taps $[0.7826e^{j\frac{\pi}{8}} \ 0.4973e^{j\frac{\pi}{5}} \ 0.3160e^{j\frac{\pi}{3}} \ 0.2008e^{j\frac{2\pi}{3}}]$. The truncated square-root raised-cosine (SRRC) filter with 8T length and roll-off factor $\alpha = 1$ is used as the pulse shaping filter.

Fig. 2. SIR versus CFO for different number of subcarriers.

Fig. 3. SIR versus CFO for SMT and OFDM systems.

We provide SIR graphs for SMT with $N = 64, 128$, and 256 subcarriers where symbol spacings are equal to $T = 128, 256$, and $512 \mu\text{sec}$ respectively. Figure 2 shows three sets of SIR graphs as a function of CFO, including accurate and approximated SIRs over frequency-selective channels and accurate SIR over AWGN channel with $64, 128$, and 256 subcarriers. As seen in this figure, there is big gap between SIR for $64, 128$, and 256 . The SMT system with 64 subcarriers has more SIR than two other cases because of large subcarrier spacing and then less sensitivity to CFO. With increasing CFO, the SIR over frequency-selective channels converges to that over AWGN channels. It comes from the fact that SMT suffers from interference due to both the frequency-selective channels and the CFO. With increasing CFO, the interference power due to CFO dominates the interference power. In each set, SMT has better SIR over AWGN channels in comparison with SIR for frequency-selective channels. This difference becomes smaller when the number of subcarriers increases.

When the number of subcarriers increases, the assumption of flat gain over each subcarrier is approximately true thus makes the equalization of the channel better. So the interference due to frequency-selective channel becomes negligible. Also, comparison between accurate and approximated SIR in each set shows that the approximated SIR converges to its accurate value with increasing the number of subcarriers, which verifies our simplification.

The OFDM system and the SMT system are different forms of multicarrier systems. It is useful to compare two systems in the same channels. In OFDM, we choose the length of CP longer than the maximum delay of the channel and thus OFDM is free of ISI. Figure 3 shows a comparison between SMT and OFDM in the same frequency-selective channel. For 64 subcarriers, OFDM with spectral efficiency $\eta = 0.94$ has better SIR in comparison with SMT for CFO values up to 300 Hz. For $\text{CFO} > 300$ Hz, SMT outperforms OFDM. Finally, by Increasing the number of subcarriers, SMT is more robust than OFDM. Different from OFDM, SMT suffers from interference over frequency-selective channels in the absence of CFO. Then, in small CFOs, SMT suffer from some interference due to channel, while OFDM does not. With increasing CFO, the dominant interference in SMT comes from CFO and, hence, it outperforms OFDM. With subcarriers $128, 256$, the SIR over frequency-selective and AWGN channels are approximately the same (Figure 2) and also SMT outperforms OFDM.

V. CONCLUSION

In this paper, the SIR of SMT in the presence of CFO over frequency-selective channels was investigated. Firstly, we modeled the received signal and expressed the interference power and the desired signal power as a function of CFO over frequency-selective channels accurately. We used SIR to evaluate the effect of CFO. By assuming that the maximum delay of the channel is small in comparison with symbol spacing, an approximated SIR was derived. We showed that with increasing number of subcarriers, SIR over frequency-selective channels converges to the SIR over AWGN channels. Numerical results showed that accurate and approximated SIRs with large number of subcarriers converge to that over AWGN channel. Also, this convergence happened with increasing CFO. Finally, we made a comparison between OFDM and SMT in the same frequency-selective channel and CFO. It was shown that OFDM has better SIR in small CFO over frequency-selective channels. But with large number of subcarriers or high CFO, SMT outperformed OFDM.

APPENDIX

It is easy to show that

$$\sum_{l=-\infty}^{+\infty} \int_{-\infty}^{\infty} h(t) g_k(t - lT - \frac{T}{2}) e^{j(\Phi^{k-m}(t) + 2\pi\epsilon t)} dt = \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{\infty} h(t) g_k(t - lT + \frac{T}{2}) e^{j(\Phi^{k-m}(t) + 2\pi\epsilon t)} dt. \quad (20)$$

According to (11) and (12), the interference power consists of both ISI and ICI due to CFO over given frequency-selective

channel. We use $e^{-j\phi_m} = \frac{G_m^*}{|G_m|}$ for phase compensation due to the channel. The interference power can be written as

$$P_{i_m} = E \left\{ |\text{ISI} + \text{ICI}|^2 \right\} = \sigma_s^2 \sum_{l=-\infty}^{+\infty} \frac{1}{|G_m|^2} \times \left(\sum_{\substack{k=m-1 \\ k \neq m \vee l \neq 0}}^{m+1} \left\{ \left| \Re \left\{ G_m^* \int_{-\infty}^{\infty} h(t) g_k(t-lT) e^{j(\Phi^{k-m}(t)+2\pi\epsilon t)} dt \right\} \right|^2 \right\} + \sum_{k=m-1}^{m+1} \left\{ \left| \Im \left\{ G_m^* \int_{-\infty}^{\infty} h(t) g_k(t-lT - \frac{T}{2}) e^{j(\Phi^{k-m}(t)+2\pi\epsilon t)} dt \right\} \right|^2 \right\} \right). \quad (21)$$

Moreover $g_k(t) = \sum_{d=0}^{D-1} g_d e^{-j2\pi k\tau_d/T} h(t - \tau_d)$, then

$$P_{i_m} = \sigma_s^2 \sum_{l=-\infty}^{+\infty} \frac{1}{|G_m|^2} \times \left(\sum_{\substack{k=m-1 \\ k \neq m \vee l \neq 0}}^{m+1} \left\{ \left| \Re \left\{ G_m^* \sum_{d=0}^{D-1} g_d \lambda_l^{k-m}(\epsilon, \tau_d) e^{-j2\pi k\tau_d/T} \right\} \right|^2 \right\} + \sum_{k=m-1}^{m+1} \left\{ \left| \Im \left\{ G_m^* \sum_{d=0}^{D-1} g_d \gamma_l^{k-m}(\epsilon, \tau_d) e^{-j2\pi k\tau_d/T} \right\} \right|^2 \right\} \right), \quad (22)$$

where

$$\lambda_l^k(\epsilon, \tau_d) = \int_{-\infty}^{\infty} h(t) h(t - \tau_d - lT) e^{j(\Phi^k(t)+2\pi\epsilon t)} dt, \quad (23)$$

$$\gamma_l^k(\epsilon, \tau_d) = \int_{-\infty}^{\infty} h(t) h(t - \tau_d - lT - \frac{T}{2}) e^{j(\Phi^k(t)+2\pi\epsilon t)} dt. \quad (24)$$

When τ_d is small in comparison with T , $\lambda_l^k(\epsilon, \tau_d) \approx \lambda_l^k(\epsilon, 0) = \lambda_l^k(\epsilon)$ and $\gamma_l^k(\epsilon, \tau_d) \approx \gamma_l^k(\epsilon, 0) = \gamma_l^k(\epsilon)$. This is because of length $h(t)$ that is naturally several times of T . By defining

$$\Lambda_l^k(\epsilon) \triangleq \sum_{d=0}^{D-1} g_d \lambda_l^k(\epsilon, \tau_d) e^{-j2\pi k\tau_d/T} e^{-j2\pi m\tau_d/T}, \quad (25)$$

$$\Gamma_l^k(\epsilon) \triangleq \sum_{d=0}^{D-1} g_d \gamma_l^k(\epsilon, \tau_d) e^{-j2\pi k\tau_d/T} e^{-j2\pi m\tau_d/T}, \quad (26)$$

the interference power is equal to

$$P_{i_m}(\epsilon) = \sigma_s^2 \times \frac{\sum_{l=-\infty}^{+\infty} \left\{ \sum_{\substack{k=m-1 \\ k \neq m \vee l \neq 0}}^{m+1} \left(\Re \left\{ G_m^* \Lambda_l^{k-m}(\epsilon) \right\} \right)^2 + \sum_{k=m-1}^{m+1} \left(\Im \left\{ G_m^* \Gamma_l^{k-m}(\epsilon) \right\} \right)^2 \right\}}{|G_m|^2}. \quad (27)$$

Similarly, it is easy to show that the desired signal power is

$$P_{s_m}(\epsilon) = \sigma_s^2 \frac{\left(\Re \left\{ G_m^* \Lambda_0^0(\epsilon) \right\} \right)^2}{|G_m|^2}. \quad (28)$$

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