ARQ by subcarrier assignment for OFDM-based systems

Citation for published version (APA):

Ho, C. K., Yang, H., Pandharipande, A., & Bergmans, J. W. M. (2008). ARQ by subcarrier assignment for OFDM-based systems. *IEEE Transactions on Signal Processing*, 56(12), 6003-6016.
<https://doi.org/10.1109/TSP.2008.2005093>

DOI:

[10.1109/TSP.2008.2005093](https://doi.org/10.1109/TSP.2008.2005093)

Document status and date:

Published: 01/01/2008

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

ARQ by Subcarrier Assignment for OFDM-Based Systems

Chin Keong Ho, Hongming Yang, Ashish Pandharipande, and Jan W. M. Bergmans, *Senior Member, IEEE*

Abstract—We consider two automatic repeat request (ARQ) schemes based on subcarrier assignment in orthogonal frequency-division multiplexing (OFDM)-based systems: single ARQ subcarrier assignment (single ARQ-SA) and multiple ARQ-SA. In single ARQ-SA, data transmitted on a subcarrier in a failed transmission are repeated on a *single assigned* subcarrier in the ARQ transmission. In multiple ARQ-SA, the data are repeated on *multiple assigned* subcarriers in the ARQ transmission. At the receiver, maximum ratio combining is performed on subcarriers that carry the same data. Our goal is to optimize certain system utility functions (such as to minimize bit error rates or to maximize sum capacity) through the choice of the subcarrier assignment. We show that a large class of reasonable system utility functions that we wish to maximize are characterized as Schur-concave. For this class of utility functions, we obtain the optimum subcarrier assignment for single ARQ-SA and propose a suboptimum (heuristic) subcarrier assignment scheme for multiple ARQ-SA. Further, to lower the overhead of signaling the subcarrier assignment information, we consider subcarrier grouping methods. Numerical results indicate that substantial throughput improvement can be achieved by appropriate assignments, especially with the use of incremental redundancy at high signal-to-noise ratios.

Index Terms—Automatic repeat request (ARQ), majorization, orthogonal frequency-division multiplexing (OFDM), subcarrier assignment.

I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) is an effective solution for delivering high data rates over wireless channels with frequency-selective fading. A number of wireless standards such as IEEE 802.11a [1], WiMax [2], and long-term 3G evolution [3] have adopted OFDM-based solutions for physical-layer transmission. An OFDM-based system combats multipath fading with the use of

a cyclic prefix that, in conjunction with the Fourier transform, converts the frequency-selective fading channel into a set of parallel subcarriers experiencing flat fading. It is common to use an automatic repeat request (ARQ) mechanism [4], [5] in OFDM systems when a packet transmission fails. In this mechanism, the transmitter retransmits the data when it fails to receive an acknowledgment (ACK) or receives an explicit negative ACK. We shall study ARQ schemes involving subcarrier assignment in OFDM-based systems.

The system under consideration is a general OFDM system where a linear unitary pretransform may be applied before the application of the inverse discrete Fourier transform at the transmitter. Such pretransformed OFDM (PT-OFDM) systems have been known to offer various advantages such as improved block error rates [6] and reduced transmitter complexity [7]. A PT-OFDM system can also be shown to be equivalent to a system with parallel subcarriers in the frequency domain. In such systems, under the ARQ mechanism, the data in the failed transmission have to be retransmitted over the parallel subcarriers in the event of a packet failure. For clarity, we shall call the failed transmission the original transmission and the associated subcarriers the original subcarriers. The retransmission will be called ARQ transmission and the associated subcarriers will be termed ARQ subcarriers. In this paper, we consider two ARQ schemes: single ARQ subcarrier assignment (ARQ-SA) scheme and multiple ARQ-SA. In single ARQ-SA, data on an original subcarrier are repeated on a single ARQ subcarrier, which may be different from the original subcarrier. We say that the ARQ subcarrier is *assigned* to the original subcarrier. In multiple ARQ-SA, however, zero, one, or more ARQ subcarriers may be assigned to an original subcarrier. At the receiver, maximum ratio combining (MRC) is performed on the original subcarrier, and all ARQ subcarriers that carry the same data. Subsequently, a single stage of equalization and decoding is carried out.

Our goal is to optimize a certain system metric by choosing the assignment, under the assumption that full channel state information (CSI) is available at both the transmitter and the receiver. We first phrase this optimization problem as one of maximizing a utility function and show that many utility functions of practical interest that we wish to maximize are Schur-concave. Examples of such utility functions are the sum capacity and the probability of correct reception. Under single ARQ-SA, we show that for Schur-concave utility functions, the optimum assignment is to assign the ARQ subcarrier with the strongest signal-to-noise ratio (SNR) to the original subcarrier with the weakest SNR, and so on. That is, if the original subcarriers are ordered with their respective SNRs in decreasing order, then

Manuscript received April 03, 2007; revised July 01, 2008. First published August 29, 2008; current version published November 19, 2008. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Gerald Matz.

C. K. Ho was with the Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands, and also with Philips Research Laboratories Eindhoven, The Netherlands. He is now with the Institute for Infocomm Research, A*STAR, Singapore (e-mail: hock@i2r.a-star.edu.sg).

H. Yang and J. W. M. Bergmans are with the Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands. They are also with Philips Research Laboratories Eindhoven, 5656 AE Eindhoven, The Netherlands (e-mail: h.m.yang@tue.nl; j.w.m.bergmans@tue.nl).

A. Pandharipande is with Philips Research Laboratories Eindhoven, 5656 AE Eindhoven, The Netherlands (e-mail: ashish.p@philips.com).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TSP.2008.2005093

Fig. 1. An OFDM system with M_0 subcarriers.

the assigned ARQ subcarriers should be ordered with their respective SNRs in increasing order. Similar results have been obtained recently in the context of relay-assisted communications [8], [9]. To obtain our results, we employ the theory of majorization [10]; see [11] and [12] for a recent review and also [13] and [14] for applications of majorization theory. Under multiple ARQ-SA, however, we find that determining the optimum assignment is an NP-hard problem. Hence, we propose a heuristic scheme.

The ARQ-SA schemes we propose here are aimed at exploiting CSI effectively, so as to realize the maximum gain offered through an appropriate assignment. Fixed assignments that do not exploit CSI have been considered in the literature [15]–[17]. In these works, a diversity effect is realized by retransmitting data on an ARQ subcarrier with a channel that is independent from that of the original subcarrier. To this end, *cyclic assignments* are used, where ARQ subcarriers are cyclic-shifted versions of the original subcarriers. If the cyclic shift is larger than the coherence bandwidth, then data are transmitted on subcarriers that experience independent channel fades, even in quasi-static channels. This clearly offers an improvement over a simple scheme where no assignment is done. However, in cyclic assignments, for some channel realizations data on a weak original subcarrier may again be retransmitted on a weak ARQ subcarrier, resulting in poor performance. We address this shortcoming by using the available CSI to develop better assignment strategies. In [18], CSI was also employed to develop an ARQ scheme based on Chase combining. Although [18] considered the problem of choosing the *group* of ARQ subcarriers for retransmission (specifically those with SNRs above a certain threshold), the problem of how to exactly perform the assignment was not addressed.

This paper is organized as follows. In Section II, we describe the general OFDM system with ARQ-SA and consider a number of utility functions that characterize system performance. The assignment problems for single ARQ-SA and multiple ARQ-SA are formulated in Section III. Algorithms for subcarrier assignment that solve these problems and their complexity are presented in Section IV. The optimality of these algorithms is investigated in Section V. The overhead of signaling the subcarrier assignment is analyzed in Section VI, where subcarrier grouping techniques for reducing this overhead are presented.

Section VII presents throughput analysis for the subcarrier assignment schemes. In Section VIII, simulation results are presented to test the efficacy of our algorithms. Conclusions are drawn in Section IX.

Notation: We use bold lower case letters to denote column vectors and bold upper case letters to denote matrices. The superscripts $*$, T , and H denote complex conjugate, transpose, and Hermitian, respectively. The (m, n) th element of matrix $\mathbf{W}$ is denoted by w_{mn} and the m th element of vector $\mathbf{w}$ is denoted w_m . The identity matrix is denoted as $\mathbf{I}$, while the all-ones vector is denoted as $\mathbf{1}$.

II. SYSTEM DESCRIPTION

A. OFDM Systems

In Fig. 1, we show a general OFDM system including a possible pretransform. We consider a system with M_0 symbols, each of unit power, represented as $\mathbf{x} = [x_1, x_2, \dots, x_{M_0}]^T$. We shall see that each symbol x_m may either carry data or redundancy that is used to improve the probability of detecting previously failed transmissions. The vector $\mathbf{x}$ is linearly transformed into M_0 subcarriers in the frequency domain as $\mathbf{s} = [s_1, s_2, \dots, s_{M_0}]^T = \mathbf{W}\mathbf{x}$, where $\mathbf{W}$ is an $M_0 \times M_0$ transformation matrix. For simplicity, we consider either an OFDM system where $\mathbf{W}$ is an identity matrix or the *pretransformed OFDM* system, where $\mathbf{W}$ is unitary with constant-amplitude entries [19]. These choices are prevalent in current wireless systems [1]–[3]. The block of modulation symbols $\mathbf{s}$ is then passed through an inverse discrete Fourier transform, usually implemented using the inverse fast Fourier transform (IFFT). After performing a parallel-to-serial (P/S) conversion (its inverse operation is denoted as S/P), we insert a cyclic prefix with duration not shorter than the maximum channel delay spread so as to avoid inter-OFDM symbol interference. Finally, the PT-OFDM symbol is transmitted. At the receiver, the samples of the received signal corresponding to the cyclic prefix are removed. After FFT, the received subcarrier vector in the frequency domain $\mathbf{r} = [r_1, r_2, \dots, r_{M_0}]^T$ can be expressed as

$$\mathbf{r} = \mathbf{\Gamma}\mathbf{s} + \mathbf{v} = \mathbf{W}\mathbf{x} + \mathbf{v} \quad (1)$$

where $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is independent identically distributed (i.i.d.) circularly symmetric complex additive white Gaussian noise (AWGN) with zero mean and unit variance, while $\mathbf{\Gamma} = \text{diag}(h_1, h_2, \dots, h_{M_0})$ is a diagonal matrix. Here, $h_m = \sum_l \tilde{h}_l \exp(-j2\pi lm/M_0)$, $m = 1, \dots, M_0$, is the m th channel response coefficient in the frequency domain, assuming a sample-spaced L_h th order finite-impulse response channel model with coefficients $\{\tilde{h}_l, l = 0, \dots, L_h\}$.

B. Transmission Scheme

We define an *ARQ round* to consist of an original transmission and the subsequent ARQ transmissions, before the next original transmission begins. The ARQ round ends if all the data sent so far have been recovered, or if a maximum number of ARQ transmissions has been reached. After the ARQ round has ended, all

Fig. 2. Transmission structure for the original and the first ARQ transmission. Redundancy for the original data symbols is sent generally by using (a) incremental redundancy symbols or, as a special case, (b) full redundancy symbols.

past transmissions are discarded from memory. To initiate another ARQ round, an independent original transmission is sent. Hence, the transmissions within any ARQ round are *independent* of those in other ARQ rounds. Fig. 2 shows the transmission structure of an ARQ round consisting of the original transmission and the first ARQ transmission. We assume that each transmission uses one OFDM symbol with subcarriers in the set $\mathcal{S} = \{1, \dots, M_0\}$; extensions to multiple OFDM symbols are straightforward and are not treated in this paper.

For exposition, let us first focus on Fig. 2(a). Here, subcarriers used for a common purpose are grouped for clarity, but they need not be neighboring subcarriers. In general, each symbol x_i in (1) is used for transmission either as a data symbol (DS) or as a redundancy symbol (RS). Specifically, in the original transmission, *original DSs* composed of $\{x_i, i \in \mathcal{S}_0\}$ are used to send data, where $\mathcal{S}_0$ is the set of subcarriers used. Clearly, all subcarriers should be used for transmission; hence $\mathcal{S}_0 = \mathcal{S}$ and the size of the set is $|\mathcal{S}_0| = M_0$. When at least one bit error occurs in the original transmission, the ARQ transmission is triggered, for example, by feeding back a negative ACK (NACK) to the transmitter. In the ARQ transmission, RSs composed of $\{x_i, i \in \mathcal{S}_1\}$ with size $|\mathcal{S}_1| = N_0$ are sent as redundancy for the original DSs. We refer to these RSs generally as *incremental RSs*. If $\mathcal{S}_0 = \mathcal{S}_1$ and thus $N_0 = M_0$, we refer to these RSs specifically as *full RSs* and Fig. 2(a) becomes Fig. 2(b) as a special case. In general, the remaining $M_0 - N_0$ subcarriers in the ARQ transmission are then split into two disjoint sets $\mathcal{S}'_0$ of size M_1 and $\mathcal{S}'_1$ of size M_1 . The set $\mathcal{S}'_0$ carries *ARQ DSs* that are used to send more data. The set $\mathcal{S}'_1$ carries *ARQ RSs* that are used as redundancy for the ARQ DSs. Clearly, $M_0 = N_0 + M_1 + N_1$.

If the original DSs or ARQ DSs are still not recovered after the first ARQ transmission is sent, a *second* ARQ transmission consisting of more RSs and DSs can be sent, by a straightforward generalization of Fig. 2(a). For clarity of presentation, henceforth we allow at most one ARQ transmission to be sent, as depicted in Fig. 2(a). With this restriction, simulation results in Section VIII show that substantial performance can already be achieved; better performance would be achieved with more ARQ transmissions.

In this paper, our use of incremental redundancy is more general (with arbitrary N_0, M_1, N_1), with full redundancy as a special case (with $N_0 = M_0, M_1 = N_1 = 0$). The ARQ schemes in the literature typically employ only full redundancy, e.g., [15]–[17] and [20], although the redundancy is referred to as incremental redundancy.

C. Incremental RSs for Original Data Symbols

We consider how to assign incremental RSs (and full RSs as a special case) to original DSs. Recall that $\mathcal{S}_0$ and $\mathcal{S}_1$ are the sets of subcarrier indexes used by the original DSs and incremental RSs, respectively, where $\mathcal{S}_0 = \mathcal{S}$ and $\mathcal{S}_1 \subseteq \mathcal{S}$. The channel coefficients in the frequency domain in the original and ARQ transmissions are denoted as $h_m, m \in \mathcal{S}_0$ and $g_n, n \in \mathcal{S}_1$, respectively. We call h_m the m th original subcarrier and g_n the n th ARQ subcarrier. For a time-invariant channel, assuming full redundancy is used, we have $h_m = g_n$ for $m = n$. This scenario is commonly considered in the literature and is covered in our formulation. We denote the power of the original subcarrier and ARQ subcarrier by $\alpha_m = |h_m|^2$ and $\beta_n = |g_n|^2$, respectively. Since the noise variance is set to one, α_m and β_n are also the SNRs of the original and ARQ subcarrier, respectively.

The received signal (1) can be expressed on a per-subcarrier basis as

$$r_m = h_m \mathbf{w}_m^H \mathbf{x} + v_m, \quad m \in \mathcal{S}_0 \quad (2)$$

where $\mathbf{w}_m^H$ is the m th row of the transform $\mathbf{W}$. Suppose that at least one bit in $\mathbf{x}$ is not received correctly and ARQ is triggered. The signal $\mathbf{w}_m^H \mathbf{x}$ carried by original subcarrier m is then repeated in the *assigned* ARQ subcarriers in the ARQ transmission. The set of indexes of the ARQ subcarriers assigned to original subcarrier m is denoted as $\mathcal{A}(m) \subseteq \mathcal{S}_1$. These ARQ subcarriers are received as

$$r'_n = g_n \mathbf{w}_m^H \mathbf{x} + v'_n, \quad n \in \mathcal{A}(m) \quad (3)$$

where $v'_n \sim \mathcal{CN}(0, 1)$. To detect $\mathbf{x}$, we employ MRC on all the received signals that carry $\mathbf{w}_m^H \mathbf{x}$ to give

$$\tilde{r}_m = h_m^* r_m + \sum_{n \in \mathcal{A}(m)} g_n^* r'_n, \quad m \in \mathcal{S}_0. \quad (4)$$

Since the noise in all received signals is independent, it follows that the *effective SNR* of $\tilde{r}_m$ in the frequency domain is given by summing the SNRs of the original subcarrier and the assigned ARQ subcarriers, which gives

$$\gamma_m = \alpha_m + \sum_{n \in \mathcal{A}(m)} \beta_n, \quad m \in \mathcal{S}_0. \quad (5)$$

Besides the assignment carried out via $\mathcal{A}(m)$, we note that the effective SNR depends also on the subcarrier sets $\mathcal{S}_0$ and $\mathcal{S}_1$ used for (re)transmissions.

We detect $\mathbf{x}$ based on $\tilde{r}_m, m \in \mathcal{S}_0$, at the receiver. Various detectors can be used, such as the maximum likelihood detector (MLD), iterative detectors, or linear-equalizer based detectors. For linear-equalizer based detectors, the signal before the slicers is denoted as $\tilde{\mathbf{x}}$; see Fig. 1. It can be expressed as $\tilde{\mathbf{x}} = \mathbf{G}\mathbf{r}$, where $\mathbf{G}$ is a linear equalizer. It is common to use the zero-forcing (ZF) or minimum mean-square-error (MMSE) equalizer, given, respectively, by

$$\mathbf{G}_{\text{ZF}} = \mathbf{\Omega}^{-1} \quad (6a)$$

$$\mathbf{G}_{\text{MMSE}} = (\mathbf{\Omega}^H \mathbf{\Omega} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{\Omega}^H \quad (6b)$$

where $\mathbf{\Omega} \triangleq \text{diag}(\sqrt{\gamma_1}, \dots, \sqrt{\gamma_{M_0}}) \mathbf{W}$.

D. Redundancy for ARQ Data Symbols

In the previous section, we saw that incremental RSs are assigned to original DSs via the assignment $\mathcal{A}(m)$. The assignment of ARQ RSs to ARQ DSs is carried out in essentially the same way, but with the “retransmission” always triggered. In particular, (2)–(5) apply for the case of assigning ARQ RSs to ARQ DSs by replacing $\mathcal{S}_0$ with $\mathcal{S}'_0$ and $\mathcal{S}_1$ with $\mathcal{S}'_1$, and a new assignment in place of $\mathcal{A}(m)$ is used. Henceforth, it suffices to consider the problem of assigning incremental RSs to original DSs as considered in Section II-C, since the problem of assigning ARQ RSs to ARQ DSs is similar.

Although we consider at most one ARQ transmission, (2)–(5) can be easily generalized to any arbitrary number of ARQ transmissions, so that the subcarrier assignment problem remains essentially unchanged. Details are provided in Appendix A.

E. Utility Functions

We describe several utility functions $\phi(\boldsymbol{\gamma})$ commonly used to reflect system performance, as a function of the effective-SNR vector $\boldsymbol{\gamma}$ with elements γ_m from (5). We seek to maximize these utility functions by appropriately choosing the subcarrier assignment. To illuminate this problem, we define the utility functions with respect to the original DSs. The problem remains essentially the same if we instead maximize the utility with respect to the ARQ DSs. This is because, to reflect the new utility functions, we only need to replace $\mathcal{S}_0$ with $\mathcal{S}'_0$ and $\mathcal{S}_1$ with $\mathcal{S}'_1$.

1) *OFDM and PT-OFDM*: For OFDM and PT-OFDM systems, we consider these utility functions

$$\phi_{\min}(\boldsymbol{\gamma}) = \min\{\gamma_1, \dots, \gamma_M\} \quad (7a)$$

$$\phi_{\text{MI}}(\boldsymbol{\gamma}) = \sum_{m=1}^M \log(1 + \gamma_m). \quad (7b)$$

In (7a), $\phi_{\min}$ is the minimum of the effective SNR over all subcarriers. The uncoded symbol error performance is often dominated by weak subcarriers experiencing deep fades. To reduce the effects of fading, the effective SNR should be made as flat as possible across the subcarriers; one way to do this is to maximize $\phi_{\min}$. To give an intuitive explanation of our subsequent results, we will make frequent use of $\phi_{\min}$. In (7b), ϕ_{MI} is the sum of the mutual information between x_m and $\tilde{r}_m$ in (4) after MRC is performed, assuming that $\mathbf{x}$ is i.i.d. Gaussian distributed. We note that ϕ_{MI} indicates the number of bits that can be reliably transmitted with a Gaussian codebook if ideal channel coding is carried out.

2) *PT-OFDM*: For PT-OFDM systems, to simplify implementations, we may use either the ZF or MMSE equalizer (6). Let $\tilde{x}_m$ be the equalized signal before slicing. The SNR of $\tilde{x}_m$ after ZF equalization and the signal-to-interference noise ratio (SINR) after MMSE equalization is denoted as $\phi_{\text{PT-ZF}}$ and $\phi_{\text{PT-MMSE}}$, respectively. Both are appropriate measures to maximize since error probabilities typically decrease as SNR or SINR increases. When $\mathbf{W}$ is unitary with constant-amplitude entries, we obtain (see, for example, [19]) the SNR and SINR for subcarrier m as

$$\phi_{\text{PT-ZF}}(\boldsymbol{\gamma}) = \frac{M}{\sum_{m=1}^M \gamma_m^{-1}} \quad (7c)$$

$$\phi_{\text{PT-MMSE}}(\boldsymbol{\gamma}) = \frac{1}{\delta^{-1} - 1}, \quad \delta \triangleq \frac{1}{M} \sum_{m=1}^M \frac{1}{1 + \gamma_m^{-1}} \quad (7d)$$

respectively. We note that the SNR or SINR is independent of m .

We refer to all the original DSs and their redundancy symbols, or all the ARQ DSs and their redundancy symbols, as a *block*. The block error rate (BLER) is defined as the probability that at least one bit error occurs in a block. This is appropriate if each block uses a separate error detection code, and a block is discarded when any bit error occurs. Two important measures of performance are the BER and BLER. We use quadrature phase-shift keying (QPSK) modulation throughout this paper. For ZF equalization, the noise after equalization is Gaussian distributed, and the BER (on any subcarrier) is thus given by $P_e = Q(\sqrt{\phi_{\text{PT-ZF}}/2})$, where $Q(x) = \int_x^\infty \exp(-y^2/2)/\sqrt{2\pi} dy$. The BLER is given by $1 - (1 - P_e)^{2M}$, since there are $2M$ bits in an OFDM symbol with QPSK modulation. We see that both the BER and BLER indeed decrease monotonically as $\phi_{\text{PT-ZF}}$ increases.

3) *OFDM*: For OFDM systems, the effective SNR γ_m for subcarrier m remains the same after ZF or MMSE equalization, since these equalizations involves only a scalar multiplication. An appropriate measure to *minimize* is the expected BER,

$P_e(\gamma_m)$, summed over all m . The utility function to *maximize* is then the negative of this measure. Hence, the utility function is

$$\phi_{\text{OFDM-BER}}(\boldsymbol{\gamma}) = - \sum_{m=1}^M Q(\sqrt{\gamma_m/2}). \quad (7e)$$

We can alternatively minimize the BLER. This is equivalent to *maximizing* the probability that all the bits in the block are successfully detected, given by

$$\phi_{\text{OFDM-BLER}}(\boldsymbol{\gamma}) = \prod_{m=1}^M \left(1 - Q(\sqrt{\gamma_m/2})\right). \quad (7f)$$

III. PROBLEM FORMULATION

We consider the problem of finding the optimal subcarrier assignment for a *given* choice of subcarrier sets $\mathcal{S}_1, \mathcal{S}'_0, \mathcal{S}'_1$ (this choice also fixes the parameters N_0, M_1, N_1). In practice, the choice of these subcarrier sets can be predetermined to optimize system performance for a given average SNR, while the subcarrier assignment is optimized during run time whenever small-scale fading leads to changes in the channel. Since our problem formulation holds for arbitrary $\mathcal{S}_1, \mathcal{S}'_0, \mathcal{S}'_1$, we defer the choice of the subcarrier sets to Section VIII. Briefly, we have chosen the subcarrier sets such that, as much as possible, each retransmitted symbol experiences an uncorrelated channel, while N_0, M_1, N_1 are optimized via simulations.

We now describe two ARQ-SA schemes, namely, single ARQ-SA and multiple ARQ-SA.

We assume that the CSI in the form of the SNRs of the original and ARQ subcarriers is known to the transmitter. The CSI can be estimated at the transmitter when the channel is reciprocal, such as in IEEE 802.11a system [1], or by an explicit CSI feedback, which is used in long-term 3G evolution systems [3]. In practice, if the time duration between an original transmission and its ARQ transmissions is less than the channel coherence time, it is reasonable to assume that the CSI is known.

A. ARQ Subcarrier Assignment (ARQ-SA)

To simplify the description, we define $a_{mn} = 1$ if ARQ subcarrier n is assigned to original subcarrier m , i.e., if $n \in \mathcal{A}(m)$; we define $a_{mn} = 0$ otherwise. The ARQ-SA is completely described by an $M_0 \times N_0$ matrix $\mathbf{A}$ with binary entries a_{mn} . Hence, the effective SNR (5) can be written in vector form as

$$\boldsymbol{\gamma}(\mathbf{A}) = \boldsymbol{\alpha} + \mathbf{A}\boldsymbol{\beta} \quad (8)$$

where $\boldsymbol{\beta}, \boldsymbol{\alpha}$ are the vectors of β_m, α_m , respectively. In (8), we emphasize the dependence of $\boldsymbol{\gamma}$ on $\mathbf{A}$ explicitly.

B. ARQ-SA Schemes

For maximum performance gain, clearly each ARQ subcarrier should be assigned to at least one original subcarrier. To employ MRC directly, an ARQ subcarrier cannot be assigned to two or more original subcarriers; otherwise, more advanced techniques, such as MLD or interference cancellation, would

be required for detection. Hence, to keep the receiver processing simple, in all our schemes each ARQ subcarrier is assigned to exactly one original subcarrier. Since $\mathbf{A}$ is a binary matrix, this constraint is equivalent to imposing the condition $\sum_{m \in \mathcal{S}_0} a_{mn} = 1, n \in \mathcal{S}_1$, i.e.,

$$\mathbf{A}^T \mathbf{1} = \mathbf{1}. \quad (9)$$

Two ARQ-SA schemes are possible depending on whether multiple ARQ subcarriers can be assigned to an original subcarrier.

1) *Single ARQ-SA*: In single ARQ-SA, we impose the condition that each original subcarrier is either assigned to one ARQ subcarrier or not assigned at all. That is, we do not allow multiple ARQ subcarriers to be assigned to any original subcarriers. We shall see in Section V that this condition simplifies an otherwise NP-hard problem to a problem that has an optimal solution with polynomial-order complexity.

Together with constraint (9), the imposed condition under single ARQ-SA implies that we must have $N_0 \leq M_0$. For convenience of description, henceforth we add $M_0 - N_0$ *virtual subcarriers* with zero SNRs to the set of ARQ subcarriers $\mathcal{S}_1$. Note that an original subcarrier is *not* physically assigned to any ARQ subcarrier if the ARQ subcarrier turns out to be a virtual subcarrier. Equivalently, we pad $\boldsymbol{\beta}$ with zeros so that its length becomes M_0 , and so $\mathbf{A}$ becomes an $M_0 \times M_0$ square matrix. By including virtual subcarriers, the imposed condition is equivalent to setting the row sums of $\mathbf{A}$ to one, i.e.,

$$\mathbf{A}\mathbf{1} = \mathbf{1}. \quad (10)$$

From (9) and (10), $\mathbf{A}$ is therefore the permutation matrix, and single ARQ-SA reduces to finding an optimal permutation (not necessarily unique) that maximizes the utility function.

As an example, consider Fig. 3(a). We see that each ARQ subcarrier, say, with SNR β_n , is assigned to one original subcarrier, say, with SNR α_m , to give an effective SNR of $\gamma_m = \alpha_m + \beta_n$.

Under single ARQ-SA, the optimization problem becomes the following.

Problem Single ARQ-SA:

Find $M_0 \times M_0$ $\mathbf{A}$ that solves

$$\begin{aligned} & \text{maximize} \quad \phi(\boldsymbol{\gamma}), \text{ where } \boldsymbol{\gamma} = \boldsymbol{\alpha} + \mathbf{A}\boldsymbol{\beta} \\ & \text{subject to} \quad \mathbf{A}\mathbf{1} = \mathbf{1} \\ & \quad \mathbf{A}^T \mathbf{1} = \mathbf{1} \\ & \quad a_{mn} \in \{0, 1\}, \forall m, n. \end{aligned} \quad (11)$$

2) *Multiple ARQ-SA*: If an original subcarrier is already very strong, with high probability the data would be recovered. Hence, we should not assign any ARQ subcarrier to it. Instead, we could assign multiple ARQ subcarriers to boost the performance of an original subcarrier that is very weak. To improve system performance and to provide a more general framework, in the multiple ARQ-SA scheme we allow zero, one, or more ARQ subcarriers to be assigned to an original subcarrier. To this

Fig. 3. An ARQ subcarrier n is assigned to an original subcarrier m if $a_{mn} = 1$. The effective SNR γ_m is the sum of the SNR of the original subcarrier α_m and all SNRs of the assigned ARQ subcarriers β_n , where $a_{mn} = 1$. (a) Only one ARQ subcarrier is assigned to each original subcarrier. (b) Multiple ARQ subcarriers can be assigned to each original subcarrier.

end, we remove the constraint (10) under multiple ARQ-SA. Thus, multiple ARQ-SA applies for any N_0 and M_0 , unlike for single ARQ-SA, which can be used only if $N_0 \leq M_0$.

An example is given in Fig. 3(b), where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \quad (12)$$

In this case, two, one, and no ARQ subcarriers are assigned to the first, second, and third original subcarriers, respectively.

Without the constraint of (10), under multiple ARQ-SA the optimization problem becomes the following.

Problem Multiple ARQ-SA:

Find $M_0 \times N_0$ $\mathbf{A}$ that solves

$$\begin{aligned} & \text{maximize} \quad \phi(\boldsymbol{\gamma}), \text{ where } \boldsymbol{\gamma} = \boldsymbol{\alpha} + \mathbf{A}\boldsymbol{\beta} \\ & \text{subject to} \quad \mathbf{A}^T \mathbf{1} = \mathbf{1} \\ & \quad \quad \quad a_{mn} \in \{0, 1\}, \forall m, n. \end{aligned} \quad (13)$$

IV. ALGORITHMS FOR ARQ-SA SCHEMES

We begin by providing Algorithm 1 and Algorithm 2 to solve problem single ARQ-SA (11) and problem multiple ARQ-SA (13), respectively, and then discuss their complexity. Algorithm 1 is optimal, while Algorithm 2 is suboptimal; a detailed discussion on their optimality will be given in the Section V.

A. Algorithm 1 for Problem Single ARQ-SA

In Algorithm 1, we order the original subcarriers increasingly and the ARQ subcarriers decreasingly according to their SNRs. Then, we assign the m th strongest ARQ subcarrier to the m th weakest original subcarrier for all m . The ordering among subcarriers with the same value is arbitrary (this does not change the effective SNR nor the utility function). By pairing strong ARQ subcarriers with weak original subcarriers, Algorithm 1 produces effective SNRs that do not fluctuate significantly across

subcarriers. Consequently, we expect that the minimum effective SNR $\phi_{\min}$ is increased as compared to random pairings of ARQ and original subcarriers. Algorithm 1 is given as follows.

Algorithm 1 For solving problem single ARQ-SA.

Initialization with inputs $\boldsymbol{\alpha}, \boldsymbol{\beta}$:

- set $\mathbf{A} = \mathbf{0}$;
- order $\boldsymbol{\beta}$ decreasingly to obtain $\boldsymbol{\beta}_{\downarrow}$, so that $\beta_{n(1)} \geq \dots \geq \beta_{n(N_0)}$, where $n(l)$ is the ordered index;
- order $\boldsymbol{\alpha}$ increasingly to obtain $\boldsymbol{\alpha}_{\uparrow}$, so that $\alpha_{m(1)} \leq \dots \leq \alpha_{m(M_0)}$, where $m(l)$ is the ordered index.

Iteration $l = 1, 2, \dots, N_0$:

- assign ARQ subcarrier $n(l)$ to original subcarrier $m(l)$, i.e., set $a_{m(l),n(l)} = 1$.
-

B. Algorithm 2 for Problem Multiple ARQ-SA

We iteratively assign ARQ subcarriers to the original subcarriers, subcarrier by subcarrier. For initialization, we set the effective SNR as that of the original subcarriers. In each iteration, the strongest ARQ subcarrier that has not been assigned so far is assigned to the original subcarrier with the smallest effective SNR. After assignment, the effective SNR is updated in each iteration to include the contribution from the additional ARQ subcarrier. Notice that Algorithm 2 imitates Algorithm 1 so as to maximize the effective SNR in a greedy manner. In Algorithm 2, we explicitly allow multiple ARQ subcarriers to be assigned to an original subcarrier; otherwise Algorithms 1 and 2 are clearly equivalent.

Algorithm 2 For solving problem multiple ARQ-SA.

Initialization with inputs $\boldsymbol{\alpha}, \boldsymbol{\beta}$:

- set $\mathbf{A} = \mathbf{0}$ and $\boldsymbol{\gamma} = \boldsymbol{\alpha}$;
- order $\boldsymbol{\beta}$ decreasingly to obtain $\boldsymbol{\beta}_{\downarrow}$, so that $\beta_{n(1)} \geq \dots \geq \beta_{n(N_0)}$, where $n(l)$ is the ordered index.

Iteration $l = 1, 2, \dots, N_0$:

- find smallest effective SNR in $\boldsymbol{\gamma}$ and denote its index as $m(l)$;

- assign ARQ subcarrier $n(l)$ to original subcarrier $m(l)$, i.e., set $a_{m(l),n(l)} = 1$;
- update the effective channel power as

$$\gamma_{m(l)} = \gamma_{m(l)} + \beta_{n(l)}.$$

C. Complexity

For the cyclic assignment considered in the literature [15]–[17], practically no complexity is required in determining the assignment during run time, since it is a fixed assignment independent of the CSI. In Algorithms 1 and 2, the assignment has to be recomputed during run time, whenever the channel changes. It is thus important to consider this assignment complexity. To this end, we let $M_0 = N_0 = M$ and consider how the complexity scales with M . We note that the complexity of ordering or sorting M items is $O(M \log M)$ by using, for example, merge sort [21].

1) *Algorithm 1*: The initialization requires two sorting operations for α and β . The actual assignments are linear in complexity, involving a simple recording of the assignment solution in memory. Hence, Algorithm 1 has a complexity of $O(M \log M)$.

2) *Algorithm 2*: The initialization requires one sorting operation for β . Suppose that in the initialization we also sort γ (equals α). We now consider the complexity of the first step of iteration l in Algorithm 2, namely, to find the smallest effective SNR in γ ; the remaining steps of iteration l are less complex and incur only a linear complexity. For $l = 1$, γ has already been sorted, so the weakest subcarrier can be found immediately. For $l = 2, \dots, M$, γ has already been sorted *except* for the effective SNR $\gamma_{m(l)}$ that has just been updated in the previous iteration (corresponding to the third step of iteration $l - 1$ in Algorithm 2). To resort γ , we only need to remove $\gamma_{m(l)}$ and appropriately insert¹ it back to γ . Since the vector γ is already sorted if we exclude $\gamma_{m(l)}$, at most $M - 1$ comparisons are required to appropriately insert $\gamma_{m(l)}$. This insertion is similar to the insertion operation in the sorting algorithm insertion sort [21], where the worst case complexity for each insertion is given by $O(M)$, even after taking into account the number of comparisons and shifts required to adjust the storage of the output. Since there are $M - 1$ iterations that require re-sorting, a total complexity of $O(M^2)$ is required. As the sortings in the initialization require a smaller complexity of $O(M \log M)$, we conclude that Algorithm 2 has an overall complexity of $O(M^2)$.

V. OPTIMALITY OF PROPOSED ALGORITHMS

In this section, we show that for utility functions (7), Algorithm 1 solves problem single ARQ-SA (11) optimally. We also make comments on the suboptimality of Algorithm 2 for solving problem multiple ARQ-SA (13).

A. Algorithm 1 for Problem Single ARQ-SA

In order to prove optimality, we need a few results from the theory of majorization [10]. We first introduce the notions of majorization and Schur-concavity.

¹In implementation, this insertion operation is carried out by updating a list of pointers that tracks the orderings.

Definition of Majorization: For any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^M$, we say that $\mathbf{x}$ is *majorized* by $\mathbf{y}$ (or $\mathbf{y}$ majorizes $\mathbf{x}$), denoted as $\mathbf{x} \prec \mathbf{y}$, if

$$\sum_{m=1}^k x_{[m]} \leq \sum_{m=1}^k y_{[m]}, \quad 1 \leq k \leq M - 1 \quad (14a)$$

$$\sum_{m=1}^M x_m = \sum_{m=1}^M y_m. \quad (14b)$$

Here, the subscript $[m]$ denotes a decreasing ordering such that $x_{[1]} \geq x_{[2]} \geq \dots \geq x_{[M]}$.

Definition of Schur-Concave Functions: A real-valued function ϕ defined on a set $\mathcal{A} \subseteq \mathbb{R}^M$ is said to be *Schur-concave* on $\mathcal{A}$ if

$$\mathbf{x} \prec \mathbf{y} \text{ on } \mathcal{A} \Rightarrow \phi(\mathbf{x}) \geq \phi(\mathbf{y}). \quad (15)$$

Thus, a Schur-concave function ϕ defined on vectors from a set $\mathcal{A}$ achieves its maximum at a vector that is majorized by all other vectors in the set. To test for Schur-concavity, the following two lemmas in [10] can be used. By definition, a function is symmetric if it is invariant under permutation of its variables.

Lemma 1 [10, Ch. 3.A.4]: Let $\mathcal{A} \subset \mathbb{R}^M$ be a symmetric, nonempty convex set and let $\phi : \mathcal{A} \rightarrow \mathbb{R}$ be continuously differentiable. The function ϕ is Schur-concave on $\mathcal{A}$ if and only if

$$\phi \text{ is symmetric on } \mathcal{A}, \quad (16a)$$

$$(x_i - x_j)(\phi'_i(\mathbf{x}) - \phi'_j(\mathbf{x})) \leq 0 \text{ for all } \mathbf{x} \in \mathcal{A} \quad (16b)$$

where $\phi'_i(\mathbf{x}) = \partial\phi(\mathbf{x})/\partial x_i$ denotes the partial derivative of ϕ with respect to x_i .

Lemma 2 [10, Ch. 3.C.2]: If ϕ is symmetric and concave on $\mathcal{A}$, then ϕ is Schur-concave on $\mathcal{A}$.

Without loss of generality, we can assume that the original subcarriers are ordered increasingly based on their SNRs. Our main result can then be summarized as follows.

Theorem 1: Let $\alpha_{\uparrow}$ be increasingly ordered such that $\alpha_1 \leq \dots \leq \alpha_{M_0}$ and $\beta_{\downarrow}$ be decreasingly ordered such that $\beta_1 \geq \dots \geq \beta_{M_0}$. For a Schur-concave function ϕ

$$\phi(\alpha_{\uparrow} + \beta_{\downarrow}) \geq \phi(\alpha_{\uparrow} + \beta_{\pi}) \quad (17)$$

for any β_{π} which are permutations of β . Alternatively, we have $\arg \max_{\beta_{\pi}} \phi(\alpha_{\uparrow} + \beta_{\pi}) = \beta_{\downarrow}$.

Proof: From [10, Ch. 6.A.2], we have $\alpha_{\uparrow} + \beta_{\downarrow} \prec \alpha_{\uparrow} + \beta_{\pi}$ for any β_{π} which is a permutation of β . It follows from the definition of Schur-concavity that (17) holds. ■

Theorem 1 shows that, for a Schur-concave function ϕ , the optimal single ARQ-SA is to assign decreasingly ordered ARQ subcarriers to the original subcarriers, which is equivalent to Algorithm 1. Theorem 2 particularizes this result to the utility functions (7), which we have considered in this paper.

Theorem 2: Algorithm 1 optimally solves ARQ-SA for the utility functions (7).

Proof: By using Theorem 1, it is sufficient to show that the functions (7) are Schur-concave. We note that (7) are all symmetric functions. It is well known that ϕ_{MI} and ϕ_{min} are concave functions. Using Lemma 2, it then follows that these functions

are Schur-concave. Using standard calculus, the partial derivatives of the remaining utility functions (7) with respect to γ_m are

$$\begin{aligned}\phi'_{\text{PT-ZF},m}(\gamma) &= \frac{1}{\gamma_m^2} f_1(\gamma) \\ \phi'_{\text{PT-MMSE},m}(\gamma) &= \frac{1}{(1 + \gamma_m)^2} f_2(\gamma) \\ \phi'_{\text{OFDM-BER},m}(\gamma) &= \frac{\exp(-\frac{\gamma_m}{4})}{\sqrt{\gamma_m}} f_3(\gamma) \\ \phi'_{\text{OFDM-BLER},m}(\gamma) &= \frac{\exp(-\gamma_m/4)}{\sqrt{\gamma_m} (1 - Q(\sqrt{\gamma_m/2}))} f_4(\gamma).\end{aligned}$$

Here, we define $f_1 = M(\sum_i \gamma_i^{-1})^{-2}$, $f_2 = M^{-1}(1 - \delta)^{-2}$, $f_3 = 1/(2\sqrt{\pi})$, and $f_4 = \phi_{\text{OFDM-BLER}}/(2\sqrt{\pi})$. Clearly, f_1, f_2, f_3, f_4 are all symmetric functions of γ , so it can be easily verified that (16b) is valid. Using Lemma 1, $\phi_{\text{PT-ZF}}$, $\phi_{\text{PT-MMSE}}$, $\phi_{\text{OFDM-BER}}$, and $\phi_{\text{OFDM-BLER}}$ are thus Schur-concave functions. ■

B. Algorithm 2 for Problem Multiple ARQ-SA

We now establish that problem multiple ARQ-SA is at least NP-hard for the utility function $\phi_{\min}$. To do so, we relate this problem with a simpler problem that is known to be NP-hard. In [22], the following task allocation problem is considered: assign N_0 tasks with processing time β to M_0 processors, so that the minimum processor time required to complete all the tasks is maximized. This is analogous to problem multiple ARQ-SA: assign N_0 ARQ subcarriers with SNRs β to M_0 original subcarriers, so that the minimum SNR across all effective SNRs is maximized. In the task allocation problem, however, the processor are identical, while in problem multiple ARQ-SA, the original subcarrier cannot be treated identically since their SNRs α may not be the same. If $\alpha = \alpha \mathbf{1}$ for some α , our problem is exactly equivalent to the task allocation problem for any β . This implies that knowing the optimal solution for our problem allows the task allocation problem to be solved, but not vice versa. Our problem is therefore harder. Since the task allocation problem is NP-hard [22], multiple ARQ-SA is at least NP-hard. Algorithm 2, which can be implemented with a complexity of $O(M^2)$, is therefore not likely to be optimal for solving problem multiple ARQ-SA. We remark that the longest processing time first algorithm considered in [22] is a special case of Algorithm 2; both are equivalent if $\alpha = \alpha \mathbf{1}$.

A numerical counterexample confirms that Algorithm 2 is suboptimal for *all* utility functions (7). Let $\alpha = [2, 1, 6]^T$, $\beta = [2, 3, 4]^T$. Using Algorithm 2 leads to an ARQ-SA given by (12), and so $\gamma^* = [7, 5, 6]^T$. However, if we interchange the first two rows of (12), we obtain $\gamma^o = [6, 6, 6]$. Clearly, $\gamma^o \prec \gamma^*$ according to the definition of majorization, and so $\phi(\gamma^o) \geq \phi(\gamma^*)$ for all Schur-concave functions. It can be easily verified that $\phi(\gamma^o) \neq \phi(\gamma^*)$ for all utility functions (7). Hence, we have $\phi(\gamma^o) > \phi(\gamma^*)$, and thus we conclude that Algorithm 2 is suboptimal.

TABLE I
AMOUNT OF SIGNALLING REQUIRED IN BIT PER SUBCARRIER FOR ALGORITHMS 1–3 AND VARIOUS NUMBER OF SUBCARRIERS M . WE FIX $M = M_0 = N_0$

	M	32	64	1024
Algorithm 2:	$\mathcal{N}_m/M$	5.00	6.00	10.00
Algorithm 1:	$\mathcal{N}_s/M$	3.68	4.62	8.56
Algorithm 3, $G = 2$:	$\mathcal{N}_g/M$	1.38	1.84	3.78
Algorithm 3, $G = 4$:	$\mathcal{N}_g/M$	0.48	0.69	1.64

VI. GROUPING OF SUBCARRIERS

The assignment obtained by Algorithm 1 or Algorithm 2 can be determined by the transmitter and made known to the receiver, or determined by the receiver and made known to the transmitter. Independent of the mechanism actually used, we propose a method to reduce the amount of signaling information required to convey the assignment.

A. Amount of Signalling Required

For Algorithm 1, each ARQ subcarrier is assigned to a different original subcarrier. Since there are N_0 ARQ subcarriers (excluding virtual subcarriers) and M_0 original subcarriers, there are in total $P_{N_0}^{M_0} = M_0!/(M_0 - N_0)!$ possible permutations. Hence, $\mathcal{N}_s = \log(P_{N_0}^{M_0})$ bits of signalling are required to communicate a chosen assignment. We use base 2 for all logarithms. For Algorithm 2, each ARQ subcarrier can be assigned to any original subcarrier. There are in total $M_0^{N_0}$ possible assignments. Hence, $\mathcal{N}_m = N_0 \log(M_0)$ bits of signalling are required to communicate a chosen assignment.

For many applications, the channel is quasi-static over a period of time. The assignment needs only to be updated every L OFDM symbols (say), when the channel has sufficiently changed. Typically, L is on the order of hundreds in wireless local-area network applications. A useful measure of the overhead used is therefore the *fractional signalling overhead* (FSO) given by $\mathcal{N}/(M_0 L)$ in bit per subcarrier per update, where $\mathcal{N} = \mathcal{N}_s$ for single ARQ-SA and $\mathcal{N} = \mathcal{N}_m$ for multiple ARQ-SA. For simplicity, let $M_0 = N_0 = M$. Table I shows the amount of signalling required for Algorithms 1 and 2 for $L = 1$.

Obviously, $\mathcal{N}_m \geq \mathcal{N}_s$. We consider the saving in FSO for using Algorithm 1 compared to Algorithm 2 for large M . By using Stirling's formula [23], we have $M! = \sqrt{2\pi M}(M/e)^M(1 + O(1/M))$ and so $\mathcal{N}_s = \log(M!) = M \log M - M \log(e) + O(\log(M))$. Since $\mathcal{N}_m = M \log M$, we obtain the saving as

$$\frac{\mathcal{N}_m - \mathcal{N}_s}{ML} = \frac{\log(e)}{L} + O\left(\frac{\log(M)}{ML}\right). \quad (18)$$

which approaches $\log(e)/L \approx 1.44/L$ for large M . This shows that using Algorithm 1 can lead to a significant saving. Table I numerically confirms that as M increases, the difference in FSO approaches 1.44 with $L = 1$. Nevertheless, we see that the absolute amount of signalling is still quite large for Algorithm 1 and Algorithm 2; hence more efficient algorithms are desirable.

B. Method of Grouping

To reduce the number of possible subcarrier assignments, we group G contiguous subcarriers and apply Algorithm 1 or Algorithm 2 on these groups. For each group, we use a group-equivalent SNR to represent the SNR of the group, e.g., the minimum SNR, arithmetic mean, or geometric mean of the group. If G is much smaller than the coherence bandwidth of the channel, the subcarriers within each group have approximately the same SNR. Performing assignment based on these grouped subcarriers would likely result in only negligible performance loss. For each group of G ARQ subcarriers that has been assigned to a group of G original subcarriers, a second (deeper) level of assignment is carried out for every subcarrier. This level of assignment may be performed dynamically during run time depending on the CSI, but at the expense of incurring additional signalling bandwidth.

Although the technique of grouping applies to both multiple ARQ-SA and single ARQ-SA, we focus on the latter since our main motivation is to reduce the amount of signalling. For simplicity, we assume that M_0, N_0 is divisible by G . Simulations reveal that using the *minimum SNR* as the group-equivalent SNR, and a *fixed* (arbitrary) assignment for the second level of assignment typically gives good BER performance. Hence, we propose Algorithm 3, as follows.

Algorithm 3 For solving problem single ARQ-SA.

First-level of assignment:

- Group the original and ARQ subcarrier into groups of G contiguous subcarriers. Each group uses the minimum SNR in the group as the group-equivalent SNR.
- Apply Algorithm 1, but with γ, α, β replaced by their group-equivalent counterparts.
That is, $\mathbf{A}$ becomes $\tilde{\mathbf{M}}_0 = M_0/G$ by $\tilde{N}_0 = N_0/G$ matrix.

Second level of assignment (independent of channel):

- For each group of G ARQ subcarriers assigned to a group of G original subcarriers, assign the ARQ subcarrier with the g th smallest subcarrier index to the original subcarrier with the g th smallest subcarrier index for $g = 1, \dots, G$.
-

Algorithm 3 reduces to Algorithm 1 when $G = 1$.

We denote the amount of signalling required for Algorithm 3 as $\mathcal{N}_g$. Since the second level of assignment is fixed and does not require signalling, we get $\mathcal{N}_g = \log(P_{\tilde{N}_0}^{M_0})$, by replacing M_0, N_0 in $\mathcal{N}_s$ with $\tilde{M}_0, \tilde{N}_0$, respectively. Let $M_0 = N_0 = M$. We consider the saving in FSO with grouping compared to no grouping for large M . By using Stirling's formula again, we obtain the saving as

$$\frac{\mathcal{N}_s - \mathcal{N}_g}{ML} = \frac{\log(M)(1 - 1/G)}{L} + O\left(\frac{\log(M)}{M}\right)$$

which approaches $\log(M)(1 - 1/G)/L$ for large M . As expected, the saving is zero when no grouping is carried out, i.e., when $G = 1$. For large M , we observe that the saving increases slowly (due to the logarithm) with M but increases relatively quickly with G even for small G . Table I shows the values

Fig. 4. The original transmission and ARQ transmission over time. The random variables t_i and T_i denote the end and the period of ARQ round i , respectively.

of $\mathcal{N}_g$ for $G = 2, 4$, which numerically supports the above conclusions.

For illustration, consider $L = 100$, $M = 64$, and using Algorithm 3 with $G = 2$. From Table I, the FSO is $1.84/L = 1.84\%$. To offer a strong error protection for the signalling bits, one may use additional redundancy bits, but overall the amount of overhead remains small.

VII. THROUGHPUT

This section obtains the throughput of the ARQ system based on the transmission structure shown in Fig. 2(a). We recall from Section II-B that an ARQ round consists of an original transmission and an additional ARQ transmission if the original transmission is erroneous, i.e., if a NACK is received. An example of the transmissions over time is shown in Fig. 4, where thicker vertical lines mark the boundaries of the ARQ rounds. Let t_i be the discrete time at the end of ARQ round i , where $i = 1, 2, \dots$. By definition, $t_0 = 0$. Without loss in generality, we assume that each transmission takes one unit time. For example, $t_1 = 1$, $t_2 = 3$, $t_3 = 5$, $t_4 = 6$ in Fig. 4. The duration of ARQ round i is given by $T_i = t_i - t_{i-1}$. Since the data symbols transmitted in different ARQ rounds are decoded independently, T_i is i.i.d. and hence a renewal process.

Let s_i be the number of bits recovered in ARQ round i , which takes the role of a *reward* for utilizing T_i units of time. The throughput is given by the total number of recovered bits normalized by the total time spent, over an infinite time horizon. We note from Section II-B that the transmissions in one ARQ round are independent of other ARQ rounds, and hence the renewal-reward theorem [20], [24] applies. The throughput can hence be determined as

$$S \triangleq \lim_{K \rightarrow \infty} \frac{1}{M_0} \frac{\sum_{i=1}^K s_i}{\sum_{i=1}^K T_i} \quad (19)$$

$$= \frac{1}{M_0} \frac{\mathbb{E}[s]}{\mathbb{E}[T]} \quad \text{with probability one.} \quad (20)$$

The throughput can now be computed if the expectations of T and s are known (the indexes are dropped for brevity). To this end, it is convenient to define the following error events. Let:

- $E_1(M_0)$ be the event that at least one original DS is erroneous, after decoding from M_0 original DSs (equivalently the event that the original transmission fails);
- $E_2(M_0, N_0)$ be the event that at least one original DS is erroneous, after *jointly* decoding from M_0 original DSs and N_0 incremental RSs;
- $E_3(M_1, N_1)$ be the event that at least one *new* data symbol is erroneous, after *jointly* decoding from M_1 ARQ DSs and N_1 ARQ RSs.

Fig. 5. Probability tree over the original and ARQ transmissions in an ARQ round. Below all possible events that result in the termination of the ARQ round, we provide $(T, s/n_b)$ indicating the duration and normalized throughput of that ARQ round.

For brevity, we drop these arguments. The complementary event of E_i is denoted as E_i^c . For example, E_1^c is the event that *all* the original DSs are successfully decoded in the original transmission.

Let n_b be the number of bits transmitted in each data symbol. Since QPSK modulation is used, we have $n_b = 2$. Since we employ error detection on a per block basis, none of the bits in a block (of original DSs or ARQ DSs) is considered to be recovered if at least one of the data symbols is erroneous; otherwise, all the bits in the block are recovered. The relationship of T and s/n_b can then be represented as shown in Fig. 5. The ARQ round begins with an original transmission. This original transmission is successful (i.e., event E_1^c occurs) with probability $1 - \Pr(E_1)$, for which the ARQ round then terminates with $T = 1$, $s/n_b = M_0$. On the other hand, the original transmission is erroneous with probability $\Pr(E_1)$, and an ARQ transmission will be sent; hence $T = 2$. Depending on whether E_2 occurs and whether E_3 occurs, s/n_b can then take four possible values: $M_0 + M_1$, M_0 , M_1 , or 0, as shown in Fig. 5. From Fig. 5, we obtain

$$\mathbb{E}[T] = \Pr(E_1^c) + 2\Pr(E_1) = 1 + \Pr(E_1) \quad (21)$$

$$\begin{aligned} \mathbb{E}[s]/n_b &= M_0 (\Pr(E_1^c) + \Pr(E_1, E_2^c)) \\ &\quad + M_1 (\Pr(E_1, E_2^c, E_3^c) + \Pr(E_1, E_2, E_3^c)) \\ &= M_0 (1 - \Pr(E_1, E_2)) \\ &\quad + M_1 \Pr(E_1) (1 - \Pr(E_3|E_1)). \end{aligned} \quad (22)$$

So, the throughput (20) becomes

$$\begin{aligned} S(\bar{\gamma}, M_0, N_0, M_1, N_1) \\ = n_b \frac{(1 - \Pr(E_1, E_2)) + \frac{M_1}{M_0} \Pr(E_1) (1 - \Pr(E_3|E_1))}{1 + \Pr(E_1)}. \end{aligned} \quad (23)$$

The throughput depends, via the error events, on the average SNR $\bar{\gamma} \triangleq \mathbb{E}[\alpha] = \mathbb{E}[\beta]$ and on the size of the subcarrier sets M_0, N_0, M_1, N_1 . This dependence is written explicitly in (23).

We now consider the scenario that full redundancy is used, i.e., $M_0 = N_0$ and $M_1 = N_1 = 0$. This is a typical case considered in the literature, e.g., [15]–[17] and [20]. The throughput is then upper bounded as

$$S(\bar{\gamma}, M_0, M_0, 0, 0) \leq S^*(\bar{\gamma}; M_0) \triangleq \frac{n_b}{1 + \Pr(E_1)} \quad (24)$$

since $\Pr(E_1, E_2) \geq 0$. If $\Pr(E_2|E_1)$ approaches zero, the throughput S approaches the upper bound S^* . This scenario occurs if the effective SNR is high after applying Algorithm 1, 2, or 3, which is typical when the average SNR is high. On the other hand, if we employ incremental redundancy so that some ARQ DSs are sent, i.e., $M_1 > 0$, it is possible for the throughput to exceed $S^*(\bar{\gamma}; M_0)$. Specifically, if

$$M_1 (1 - \Pr(E_3|E_1)) > M_0 \Pr(E_2|E_1) \quad (25)$$

is satisfied, it can be easily shown that the throughput (23) must exceed $S^*(\bar{\gamma}; M_0)$, which is already the largest possible throughput with full redundancy. This implies that not sending any data in the ARQ transmission can limit throughput. In the simulations in Section VIII, we demonstrate that at high SNR, full redundancy indeed poses as a severe limitation to the significant throughput gain that can otherwise be achieved with incremental redundancy.

Intuitively, the condition (25) can be explained as follows. Given that the original transmission is erroneous, i.e., E_1 occurs, the left-hand side of (25) reflects the throughput gained by the ARQ DSs after the ARQ transmission, while the right-hand side reflects the throughput lost by the original DSs. If the gain is larger than the loss, then one should transmit ARQ DSs to achieve an overall gain in throughput.

VIII. NUMERICAL RESULTS

We first use a case study to illustrate the difference of our proposed algorithms with optimal solutions. Then, we show the improvement in block error rate and throughput achieved.

A. Case Study

To better illustrate the case study, we evaluate the solutions numerically for a small number of subcarriers with $M_0 = N_0 = 10$, and we use the minimum-SNR utility function $\phi_{\min}$ in (7a). The (arbitrary) SNRs of the original and ARQ subcarriers α, β that we consider are generated independently from an i.i.d. exponential distribution and are shown in Fig. 6. For clarity, the original subcarriers are sorted increasingly according to their SNRs, while the ARQ subcarriers are sorted decreasingly.

To solve problem single ARQ-SA and problem multiple ARQ-SA optimally, we formulate them as mixed-integer linear programs (MILPs) and solve these MILPs using GNU Linear Programming Kit,² a standard linear programming software; we leave out the details.

Algorithms 1 and 2 are also implemented. Fig. 6 shows the resulting minimum effective SNRs. All the above four approaches clearly give a significantly higher minimum effective SNR compared to the minimum SNR of the original subcarriers (without

²<http://www.gnu.org/software/glpk>.

Fig. 6. The effective SNRs (not in decibels, shown as dotted lines) after applying Algorithm 1 and Algorithm 2, based on typical SNR realizations of original subcarriers and ARQ subcarriers (shown as full lines). For clarity, the SNRs of the original subcarriers are ordered increasingly, while the ARQ subcarriers are ordered decreasingly.

ARQ). Moreover, Algorithm 2 performs (slightly) better than Algorithm 1, which is expected as fewer constraints have been imposed. For single ARQ-SA, the MILP solution gives the same minimum effective SNR as Algorithm 1,³ as expected due to Theorem 1. For multiple ARQ-SA, the MILP solution is marginally better than Algorithm 2. Specifically, the smallest effective SNR based on MILP (at subcarrier 6) is slightly higher than that based on Algorithm 2 (at subcarrier 3).

We note that using Algorithm 1 already improves the worst case SNR gain significantly, while the additional improvement offered by other solutions is relatively small. Although we study a particular case here, this conclusion holds typically. This is because for wireless channels that are frequency selective, the weakest subcarriers are already significantly boosted by the strongest subcarriers using Algorithm 1. Hence, any further gain by using a more sophisticated algorithm is likely to be small.

B. Performance Evaluation

1) *Scenario*: For our performance evaluation, we consider a QPSK modulated system and a PT-OFDM system with $M_0 = 64$ subcarriers at different average SNR $\bar{\gamma}$. We use the transform [25]

$$\mathbf{W} = \mathbf{F} \times \text{diag}\{1, \lambda, \dots, \lambda^{M_0-1}\} \quad (26)$$

before applying IFFT, where $\lambda = \exp(-j\pi/(2M_0))$ and $\mathbf{F}$ is the FFT matrix. Notice that the FFT and the IFFT cancel out, so the transformation is equivalent to rotating the data symbol in the time domain. The transform $\mathbf{W}$ is unitary and has constant-amplitude elements. Although it has been designed for maximum likelihood decoding in [25], simulation studies (not shown here) indicate that it also results in good error performance generally when used with other detection schemes. For

³The other effective SNRs can be different because the utility function $\phi_{\min}$ only considers the minimum SNR.

illustration, we use the ZF equalizer given by (6a); more advanced detectors such as MLD [25] or iterative subcarrier reconstruction [19] can also be used. We model the wireless channel with $L_h + 1 = 16$ i.i.d. time domain channel taps. The channel used for transmission is assumed to be time invariant in each ARQ round. We assume an error-free channel is available to signal the ACK bit and the assignment used.

The ARQ subcarriers are classified as incremental RSs, ARQ DSs, or ARQ RSs via subcarrier sets $\mathcal{S}_1, \mathcal{S}'_0, \mathcal{S}'_1$, respectively; see Fig. 2(a). In our simulations, we select $\mathcal{S}_1$ as N_0 subcarriers that are roughly spaced uniformly apart, so that these subcarriers experience independent channels as much as possible. From the remaining subcarriers, similarly we select $\mathcal{S}'_0$ as M_1 subcarriers that are spaced uniformly apart. Finally, $\mathcal{S}'_1$ is made up of the remaining subcarriers. Unlike [18], we do not reserve stronger ARQ subcarriers only for $\mathcal{S}_1$, since this will result in weaker subcarriers for $\mathcal{S}'_0$ and $\mathcal{S}'_1$.

2) *Block Error Rate (BLER)*: We consider these BLERs that are used to calculate the throughput S in (23):

- $\Pr(E_1)$, the BLER for the original DSs without using ARQ;
- $\Pr(E_1, E_2)$, the BLER for the original DSs by using ARQ;
- $\Pr(E_3|E_1)$, the BLER for the ARQ DSs, given that the original transmission fails.

Monte Carlo simulations are used to obtain the BLERs for Algorithms 1–3, as analytical expressions for the BLERs involve order statistics and often do not yield closed-form results.

For benchmarking, we use a (fixed) cyclic assignment considered in the literature [15]–[17], in which CSI is not exploited. In this cyclic assignment, we cyclically shift the indexes of the ARQ subcarrier with respect to the original subcarriers by 16 subcarriers, which allows each DS and its corresponding RS to experience channels that are close to independent.

Lower bounds based on idealistic conditions are used to check how good our schemes perform. Since much of the throughput gain comes from the original DSs, lower bounds are considered only for $\Pr(E_1, E_2)$. First, we use the *averaged matched filter bound* (MFB) to provide a lower bound, based on the same frequency-selective channel model. We assume that there is no interference from other data symbols and hence a matched filter is used for detection; moreover, ARQ is always activated. Secondly, we employ the *AWGN bound*, where an AWGN channel is used, i.e., the original and ARQ subcarriers do not experience fading. Moreover, ARQ is always activated. Details of both bounds are provided in Appendix B.

$N_0 = 64$: We first consider that full redundancy is employed, i.e., $M_0 = N_0 = 64$ and $M_1 = N_1 = 0$; see Fig. 2(b). Since ARQ DS is not sent, $\Pr(E_3|E_1)$ is not relevant here. From Fig. 7, the BLER $\Pr(E_1, E_2)$ obtained from using Algorithm 1 is almost the same as Algorithm 2. Both algorithms perform substantially better than the BLER $\Pr(E_1)$ when ARQ is not used, and provide more than 2 dB of SNR gain compared to using a cyclic assignment. Surprisingly, Algorithm 1 and Algorithm 2 are only about 0.5 dB away from the averaged MFB bound, even though only a simple ZF equalizer has been employed.

Finally, we observe that using Algorithm 3 with grouping of $G = 2$ achieves a performance that is only a few tenths of decibels away from that of Algorithm 1 (without grouping). When

Fig. 7. BLER performance using full redundancy, $M_0 = N_0 = 64$.

$G = 4$, the BLER is about 1 dB away from Algorithm 1 but is still about 1 dB better than using cyclic permutation. This suggests that using grouping of $G = 2$ is adequate for this scenario; if further reduction of signalling is desired, using grouping of $G = 4$ can be a good compromise.

$N_0 = 32$: Next, we consider $N_0 = 32$ in Fig. 8. Since $N_0 < M_0$, incremental redundancy is strictly used (instead of full redundancy); see Fig. 2(a). We first consider the BLER for the original DSs $\Pr(E_1, E_2)$. The performance of Algorithm 2 is very close to that of Algorithm 1 and is not shown. As in Fig. 7, Fig. 8 shows that using more signalling for the assignment leads to better performance. However, these performance gaps between different algorithms widen for $N_0 = 32$ in Fig. 8, compared to $N_0 = 64$ in Fig. 7. In particular, the gaps between cyclic permutation and proposed algorithms are more significant. Hence, when redundancy is limited, the proposed algorithms become more important in maintaining a reasonable system performance. We now consider the BLER for the ARQ DSs $\Pr(E_3|E_1)$ by using Algorithm 3 with $G = 2$. We transmit $M_1 = 24$ ARQ DSs and $N_1 = 8$ ARQ RSs; these parameters have been optimized to maximize the throughput, as explained in Section VIII-B-3). At a sufficiently high SNR of 12 dB, for example, the original DSs are received with low error probability, yet we can additionally send ARQ DSs with an error probability of only around 0.06.

3) *Throughput*: Fig. 9 illustrates the throughput obtained with and without ARQ. We focus on using Algorithm 3 with $G = 2$, which gives good performance with small overhead. With full redundancy, we observe that a substantial improvement is obtained compared to when ARQ is not used. However, we note that this improvement is limited by the upper bound S^* given in (24) for $\text{SNR} \leq 10$ dB. To obtain further improvement at high SNR, incremental redundancy must therefore be used. This implies that the number of incremental RSs N_0 should be reduced, but this has little effect at high SNR since the original DSs can usually still be recovered.

So far we have fixed N_0, M_1, N_1 . We now optimize these parameters with the use of incremental redundancy to maxi-

Fig. 8. BLER performance of the original DSs using incremental redundancy, with $M_0 = 64, N_0 = 32$. An illustrative case of the BLER $\Pr(E_3|E_1)$ when ARQ DSs are sent, where $M_1 = 24, N_1 = 8$, is also shown.

Fig. 9. Throughput using Algorithm 3 with subcarrier grouping of $G = 2$.

mize the throughput for $\text{SNR} \geq 10$ dB. In our simulations, we vary the parameters in steps of four and obtain the maximized throughput as shown in Fig. 9, where the optimized parameters are indicated as $[N_0, M_1, N_1]$. We observe that a significant gain, up to about 4 dB, can be realized at high SNR. On the other hand, in the low-SNR regime, full redundancy should preferably be used to ensure reliable packet recovery. To improve throughput in this regime, an option is to increase the number of ARQ transmissions to more than one, but at the expense of incurring higher delay.

IX. CONCLUSION

We propose two ARQ schemes based on subcarrier assignment for general OFDM systems with a possible pretransform. For the single ARQ-SA scheme, we propose an optimum subcarrier assignment algorithm that optimizes the class of Schur-concave utility functions. For the multiple ARQ-SA scheme, we propose a suboptimum algorithm. In order to

keep the amount of feedback required to communicate the assignment low, we consider subcarrier grouping techniques by grouping contiguous subcarriers and performing subcarrier assignment on these groups. Numerical results have shown an improvement of the error performance even when limited redundancy is available for ARQ, which has led to significant throughput gains over a wide range of SNR. Even though in this paper we restrict to the case of at most one retransmission, our ARQ schemes can be generalized easily to any number of retransmissions.

APPENDIX A

ARQ-SA FOR TWO OR MORE ARQ TRANSMISSIONS

We show that the subcarrier assignment problem for the original DSs for two ARQ transmissions is fundamentally the same as for one ARQ transmission, and so both problems can be solved similarly.

Consider that a second ARQ transmission is activated. In this second ARQ transmission, suppose that the subcarrier set $\mathcal{S}_2$ is available to provide redundancy for the original DSs. Let $\{\beta'_n, n \in \mathcal{S}_2\}$ be the SNRs corresponding to $\mathcal{S}_2$, and let $\mathcal{A}'(m) \in \mathcal{S}_2$ be the assignment of the ARQ subcarriers for the m th original subcarrier. After MRC, the effective SNR at subcarrier m becomes

$$\gamma_m = \alpha_m + \sum_{n \in \mathcal{A}(m)} \beta_n + \sum_{n \in \mathcal{A}'(m)} \beta'_n, \quad m \in \mathcal{S}_0. \quad (27)$$

Compared to (5), we have included the additional contribution of the second ARQ transmission. Due to causality, past assignments $\{\mathcal{A}(m)\}$ are fixed before the current assignments $\{\mathcal{A}'(m)\}$ are carried out. Consequently, $(\alpha_m + \sum \beta_n)$ can be treated as a single *fixed* term, similar to α_m , which is fixed in (5). Hence, the subcarrier assignment problem, either of selecting $\{\mathcal{A}'(m)\}$ given (27) or of selecting $\{\mathcal{A}(m)\}$ given (5), is fundamentally the same problem and can be solved similarly. This conclusion also holds for more than two ARQ transmissions by treating all previous assignments as fixed (and similarly if we consider the assignment for the ARQ DSs instead of the original DSs).

APPENDIX B

BOUNDS FOR ARQ IN FADING CHANNELS

Typically, the MFB gives a lower bound for an error probability *without ARQ for a AWGN channel*. Here, we modified it for ARQ systems in a fading channel. To this end, we assume that ARQ is always activated and we take the expectation of the error probability over the fading channel.

Consider a unitary transform with constant-amplitude elements, such as (26), where $|w_{mn}|^2 = 1/M_0$. To obtain a lower bound, without loss of generality we assume that only the first symbol is transmitted, i.e., $\mathbf{x} = [x_1, \mathbf{0}_{M_0-1}]$. This symbol is transmitted over the original subcarriers and the ARQ subcarriers with SNRs α, β , respectively. A matched filter is used at the receiver, which equivalently collects the SNR over all original and ARQ subcarriers. Thus, the equivalent SNR for x_1 is $\gamma_{\text{MFB}}(\alpha, \beta) = (\sum_{m \in \mathcal{S}_0} \alpha_m + \sum_{n \in \mathcal{S}_1} \beta_n)/M_0$. For QPSK modulation, the BER is $P_e(\alpha, \beta) = Q(\sqrt{\gamma_{\text{MFB}}(\alpha, \beta)/2})$.

Since any bit error constitutes a block error and there are $2M$ bits in a block, the BLER is

$$\text{BLER}_{\text{MFB}} = \mathbb{E}_{\alpha, \beta} \left[1 - (1 - P_e(\alpha, \beta))^{2M} \right]. \quad (28)$$

Here, the expectation is performed over α, β . A semianalytical method can be used to obtain numerical results by averaging the term within the expectation operator over realizations of α, β generated by Monte Carlo simulations.

The AWGN bound provides a looser bound, but in closed form. It is obtained similarly as the MFB, except that the channel is always fixed as the average SNR, i.e., $\alpha_m = \beta_n = \bar{\gamma}$ for all m, n . Thus, the equivalent SNR for x_1 becomes $\gamma_{\text{MFB}} = \bar{\gamma}(M_0 + N_0)/M_0$, a constant. Hence, the BLER provided by the AWGN bound is given by $\text{BLER}_{\text{AWGN}} = 1 - (1 - Q(\sqrt{\gamma_{\text{MFB}}/2}))^{2M}$.

ACKNOWLEDGMENT

The authors gratefully acknowledge discussions with L. Tolhuizen and S. Serbetli.

REFERENCES

- [1] *Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: High Speed Physical Layer in the 5 GHz Band*, IEEE 802.11a Standard, 1999.
- [2] A. Ghosh, D. R. Wolter, J. G. Andrews, and R. Chen, "Broadband wireless access with WiMax/802.16: Current performance benchmarks and future potential," *IEEE Commun. Mag.*, vol. 43, pp. 129–136, Feb. 2005.
- [3] *Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation*, 3GPP Std. 3GPP TS 36.211, Rev. 8.0.0, Sep. 2007 [Online]. Available: <http://www.3gpp.org/ftp/Specs/htmlinfo/36211.htm>
- [4] P. Sindhu, "Retransmission error control with memory," *IEEE Trans. Commun.*, vol. COM-25, pp. 473–479, May 1977.
- [5] J. D. J. Costello, J. Hagenauer, H. Imai, and S. B. Wicker, "Applications of error-control coding," *IEEE Trans. Inf. Theory*, vol. 44, pp. 2531–2560, Oct. 1998.
- [6] Y. Wu, C. K. Ho, and S. Sun, "On some properties of Walsh-Hadamard transformed OFDM," in *Proc. 56th IEEE Veh. Technol. Conf.*, Vancouver, BC, Canada, Sep. 2002, vol. 4, pp. 2096–2100.
- [7] D. Falconer, S. L. Ariyavistakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless system," *IEEE Commun. Mag.*, vol. 40, pp. 58–66, Apr. 2002.
- [8] A. Hottinen and T. Heikkinen, "Optimal subchannel assignment in a two-hop OFDM relay," in *Proc. IEEE 8th Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Helsinki, Finland, Jun. 2007, pp. 1–5.
- [9] A. Pandharipande and C. K. Ho, "Spectrum pool reassignment for a cognitive OFDM-based relay system," in *Proc. 2nd Int. Conf. Cognitive Radio Oriented Wireless Networks Commun. (CrownCom)*, Orlando, FL, Jul. 2007.
- [10] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*. New York: Academic, 1979.
- [11] D. P. Palomar and Y. Jiang, "MIMO transceiver design via majorization theory," *Found. Trends Commun. Inf. Theory*, vol. 3, no. 4–5, pp. 331–551, 2006.
- [12] E. Jorswieck and H. Boche, "Majorization and matrix-monotone functions in wireless communications," *Found. Trends Commun. Inf. Theory*, vol. 3, no. 6, pp. 553–701, 2007.
- [13] D. Palomar, J. Cioffi, and M. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2381–2401, Sep. 2003.
- [14] A. Pandharipande and S. Dasgupta, "Optimum DMT-based transceivers for multiuser communications," *IEEE Trans. Commun.*, vol. 51, pp. 2038–2046, Dec. 2003.

- [15] T. Kumagai, M. Mizoguchi, T. Onizawa, H. Takanashi, and M. Morikura, "A maximal ratio combining frequency diversity ARQ scheme for OFDM signals," in *Proc. IEEE Int. Conf. Personal, Indoor Mobile Radio Commun.*, Sep. 1998, vol. 2, pp. 528–532.
- [16] M. Gidlund and P. Ahag, "Enhanced HARQ scheme based on rearrangement of signal constellations and frequency diversity for OFDM systems," in *Proc. 59th IEEE Veh. Technol. Conf.*, May 2004, vol. 1, pp. 500–504.
- [17] X. Peng, F. Chin, and A. S. Madhukumar, "Performance studies of a VSF-OFCDM system using a symbol relocated scheme during retransmission," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Mar. 2005, vol. 2, pp. 1138–1143.
- [18] L. Cai, Y. Wan, P. Song, and L. Gui, "Improved HARQ scheme using channel quality feedback for OFDM systems," in *Proc. 59th IEEE Veh. Technol. Conf.*, May 2004, vol. 4, pp. 17–19.
- [19] C. K. Ho, Z. Lei, S. Sun, and W. Yan, "Iterative detection for pre-transformed OFDM by subcarrier reconstruction," *IEEE Trans. Signal Process.*, vol. 53, pp. 2842–2854, Aug. 2005.
- [20] G. Caire and D. Tuninetti, "The throughput of hybrid-ARQ protocols for the Gaussian collision channel," *IEEE Trans. Inf. Theory*, vol. 47, pp. 1971–1988, Jul. 2001.
- [21] D. Knuth, *The Art of Computer Programming*, 2nd ed. Reading, MA: Addison-Wesley, 1998.
- [22] B. L. Deuermeier, D. K. Friesen, and M. A. Langston, "Scheduling to maximize a multiprocessor system," *SIAM J. Alg. Discrete Meth.*, vol. 3, no. 2, pp. 190–196, Jun. 1982.
- [23] *Handbook of Mathematical Functions*, M. Abramowitz and I. A. Stegun, Eds. New York: Dover, 1972.
- [24] R. G. Gallager, *Discrete Stochastic Processes*. Boston, MA: Kluwer Academic, 1996.
- [25] Z. Liu, Y. Xin, and G. B. Giannakis, "Space-time-frequency coded OFDM over frequency-selective fading channels," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2465–2476, Oct. 2002.

Chin Keong Ho received the B.Eng. (first-class honors) and M.Eng. degrees from the Department of Electrical Engineering, National University of Singapore, in 1999 and 2001, respectively, and the Ph.D. degree from Eindhoven University of Technology, Eindhoven, The Netherlands, in 2007.

Since 2001, he has been with the Institute for Infocomm Research, Singapore. During his doctoral work, he conducted joint research with Philips Research Laboratories, Eindhoven. His research interest lies in adaptive wireless communications

and signal processing for multicarrier and space-time communications.

Hongming Yang received the B.S. and M.S. degrees from the Department of Electronic Engineering, Tsinghua University, Beijing, China, in 2000 and 2003, respectively, and the M.E. degree from the Department of Electrical and Computer Engineering, National University of Singapore, Singapore, in 2005. He is now pursuing the Ph.D. degree at Eindhoven University of Technology, Eindhoven, The Netherlands.

He currently is conducting joint research with Philips Research Laboratories, Eindhoven. His research interest lies in signal processing for digital communications, recording systems, and illumination systems.

Ashish Pandharipande received the B.E. degree in electronics and communications engineering from Osmania University, Hyderabad, India, in 1998. He received the M.S. degree in electrical and computer engineering and in mathematics and the Ph.D. degree in electrical and computer engineering from the University of Iowa, Iowa City, in 2000, 2001, and 2002, respectively.

Since then, he has been a Postdoctoral Researcher with the University of Florida, Tallahassee, and a Senior Researcher with Samsung Advanced Institute of Technology, Suwon, South Korea. He has held visiting positions with AT&T Laboratories and the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore. He is currently a Senior Scientist with Philips Research, Eindhoven, The Netherlands. His research interests are in the areas of cognitive wireless networks, sensor signal processing, multicarrier and MIMO wireless communications, and signal-processing applications.

Jan W. M. Bergmans (SM'91) received the Elektrotechnisch Ingenieur degree (*cum laude*) and the Ph.D. degree from Eindhoven University of Technology (TU/e), Eindhoven, The Netherlands, in 1982 and 1987, respectively.

From 1982 to 1999, he was with Philips Research Laboratories, Eindhoven, working on signal-processing techniques and IC architectures for digital transmission and recording systems. In 1988 and 1989, he was an Exchange Researcher with Hitachi Central Research Labs, Tokyo, Japan. Since 1999,

he has been a full Professor and Chairman of the Signal Processing Systems Group, TU/e. He has published extensively in refereed journals, is author of *Digital Baseband Transmisison and Recording* (Norwell, MA: Kluwer Academic, 1996), and has received around 40 U.S. patents.