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Homogeneous water nucleation and droplet growth in methane and carbon dioxide mixtures at 235 K and 10 bar

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Homogeneous nucleation rates and droplet growth rates of water in pure methane and mixtures of methane and carbon dioxide were measured in an expansion wave tube at 235 K and 10 bar. The nucleation rate in pure methane is three orders of magnitude higher than literature nucleation rates of water in low-pressure helium or argon. Addition of carbon dioxide to the carrier gas mixture increases the rates even more. Specifically, rates in a mixture of methane and 3\% carbon dioxide are a factor of 10 higher than the rates in pure methane. With 25\% carbon dioxide, the rates are four orders of magnitude higher than the rates in pure methane. An application of the nucleation theorem shows that the critical cluster consists of 22 water molecules and 5 methane molecules, for nucleation in pure methane. Growth rates of water droplets were measured in methane and in methane-carbon dioxide mixtures at 243 K and 11.5 bar. At equal temperature, pressure and water vapor fraction, the growth rate of the squared droplet radius is about 20\% lower in the mixture with 25\% carbon dioxide than in pure methane. The lower growth rate is caused by a smaller diffusion coefficient of water in the mixture with carbon dioxide; the difference of the diffusion coefficients is qualitatively reproduced by the empirical Fuller correlation combined with Blanc’s law.


I. INTRODUCTION

Knowledge of the nucleation and droplet growth behavior of water in mixtures of methane and carbon dioxide is relevant for the natural gas industry. For example, it is useful for the design of devices that separate water from natural gas containing carbon dioxide. Previously, nucleation rates and droplet growth rates of water in methane at 10, 25, and 40 bar have been measured by Peeters et al.\textsuperscript{1,2} Nucleation in a sample of real natural gas was studied by Luijten et al.,\textsuperscript{3} but their sample did not contain water vapor, as it had been removed before the experiments. The effect of carbon dioxide on water nucleation has not been studied until now. Carbon dioxide is expected to influence nucleation rates because of its solubility in water at low temperatures (e.g., at 235 K and 1 bar, the equilibrium molar fraction of CO\(_2\) in liquid water is about 0.01). This article presents the first report of the influence of carbon dioxide on the nucleation and droplet growth rates of water in methane. A comparison is made of water nucleation and droplet growth in (1) pure methane, (2) a mixture of 97\% methane and 3\% carbon dioxide, and (3) a mixture of 75\% methane and 25\% carbon dioxide. All nucleation experiments take place at 235 K and 10 bar, and the droplet growth is measured at 243 K and 11.4 bar.

II. EXPERIMENTAL METHOD

The experimental setup of this study has been described before,\textsuperscript{4} and a summary will be given. Experiments were performed in a so-called expansion wave tube,\textsuperscript{5} which is a modified shock tube (see Fig. 1) in which the nucleation pulse principle is implemented. The high-pressure section (HPS) serves as a test section in which nucleation occurs. It contains the mixture of water vapor and the carrier gases methane and carbon dioxide, which is produced in a dedicated mixture preparation device.\textsuperscript{1} The purchased gases (Linde Gas) had a purity of 99.995\% or better, with no more than 5 ppm (by volume) water content. The water was purchased from Merck (GR grade) and had a resistivity of at least 1 M\(\Omega\) cm at 25 °C. The molar fraction of water vapor was varied between 240 and 340 ppm. The initial total pressure in the HPS was 25.0 bar. A diaphragm separates the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. The experiment is started by isolating the HPS from the low-pressure section (LPS) that contains the mixture of water vapor and the carrier gases methane and carbon dioxide, which is produced in a dedicated mixture preparation device.\textsuperscript{1} The purchased gases (Linde Gas) had a purity of 99.995\% or better, with no more than 5 ppm (by volume) water content. The water was purchased from Merck (GR grade) and had a resistivity of at least 1 M\(\Omega\) cm at 25 °C. The molar fraction of water vapor was varied between 240 and 340 ppm. The initial total pressure in the HPS was 25.0 bar. A diaphragm separates the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. The experiment is started by isolating the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. The experiment is started by isolating the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. The experiment is started by isolating the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. The experiment is started by isolating the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. The experiment is started by isolating the HPS from the low-pressure section (LPS) that contains a carrier gas without water, at a pressure of about 11 bar. Prior to the experiment, the HPS is flushed at 25 bar for 1 h with the test mixture, to ensure adsorption equilibrium with the walls. 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The pressure at a position of 5 mm from the HPS end wall is measured by a piezoelectric pressure transducer (Kistler 603B) and a piezoresistive transducer (Kistler 4073A50). The piezoelectric transducer has a fast response, but measures relative pressure changes only. The piezoresistive transducer measures absolute pressures and is used to calibrate the piezoelectric one in situ.

The initial temperature of the gas mixture is taken equal to the temperature of the HPS walls, which is measured before the start of the experiment with resistance thermometers accurate to 0.04 K. During the experiment, the temperature of the gas mixture changes so rapidly that it cannot be measured. However, the duration of the expansion is so short (2 ms) that the heat flow from the walls to the gas only influences the thermal boundary layers, and leaves the core to be adiabatic. The temperature can then be computed from the initial temperature and the pressure history, using a suitable equation of state (EOS). We selected the GREG-2004 EOS for this purpose. As an accuracy test, pulse temperatures of the experiments in pure methane were also computed with the EOS by Sychev et al., the differences with GREG-2004 were at most 0.01 K.

Close to the end wall of the HPS, the size and number density of droplets is determined by a combination of constant angle Mie scattering at 90° and extinction measurements of a laser beam with a wavelength of 532 nm. The growth of the droplets is monitored during a period of 40 ms after the pulse. The minimum observable droplet radius is about 0.17 μm, and the radius at the end of the observation period is at most 0.5 μm. Finally, the nucleation rate is found as the ratio of the droplet density and the pulse duration.

Taking into account real-gas behavior, the supersaturation of water in a high-pressure carrier gas mixture is given by

\[ S = \frac{y}{y_{eq}} = \frac{yp}{f_e p_e} \]  

(1)

Here \( y \) is the experimental molar vapor fraction of water and \( y_{eq} \) is its value at vapor-liquid equilibrium. Furthermore, \( p \) is the total pressure, \( f_e \) is the so-called enhancement factor, and \( p_e \) is the equilibrium vapor pressure. For the binary water-methane system, \( y_{eq} \) and \( f_e \) are functions of \( p \) and \( T \) only. For the ternary system with carbon dioxide, \( y_{eq} \) and \( f_e \) can be expressed as functions of \( p, T \), and the carbon dioxide molar fraction \( y_C \). The value of \( f_e \) was computed with a suitable EOS, as explained in the Appendix.

### III. NUCLEATION RATES

Figure 2 shows nucleation rates as a function of the supersaturation in the nucleation pulse. All data was obtained at approximately 235 K and 10 bar, with standard deviations of 0.7 K and 0.1 bar. The results have been corrected for the deviations of the nucleation temperature from the average. The average temperature and pressure of each series is listed as \( T \) and \( p \) in Table I. Tables with experimental data are available as supplementary material.

Four series of experiments were performed to study the effect of carbon dioxide, labeled A to D in Fig. 2. The first series (A) consists of nucleation experiments in pure methane. In the next series (B), nucleation took place in a carrier gas mixture of 3% carbon dioxide and 97% methane (the percentages are molar). Compared to the pure-methane data, the 3% carbon dioxide was found to increase the nucleation rate by about a factor of 10. To verify the reproducibility of the experiments, several experiments in pure methane and with 3% carbon dioxide were alternated.

After a 3-month break, the mixture preparation device was calibrated for experiments with a carrier gas mixture of 25% carbon dioxide and 75% methane. The reproducibility was then checked by performing an experiment with pure methane. The result deviated significantly from the earlier series A. After ruling out possible errors in the water vapor fraction and in measurements of pressure, temperature and nucleation rate, a new series with pure methane was performed (C). Finally, the effect of 25% carbon dioxide was studied by alternating several experiments with 25% carbon dioxide (D) and pure methane (C). The influence of 25% carbon dioxide on the nucleation rate is quite large. As a result, the only supersaturation common to series C and D is \( S = 9 \). At that supersaturation, the 25% carbon dioxide increases the nucleation rate by about four orders of magnitude.

In Fig. 3, the nucleation rate is plotted versus the reduced partial pressure, which is the ratio of the partial pres-
Our nucleation conditions are listed in Table I.

<table>
<thead>
<tr>
<th>A</th>
<th>0</th>
<th>235.0</th>
<th>10.09</th>
<th>11.77</th>
<th>22 ± 2</th>
<th>32 ± 3</th>
<th>80.00</th>
<th>1.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0</td>
<td>235.5</td>
<td>10.11</td>
<td>10.91</td>
<td>20 ± 1</td>
<td>34 ± 4</td>
<td>79.92</td>
<td>1.11</td>
</tr>
<tr>
<td>B</td>
<td>0.03</td>
<td>235.4</td>
<td>10.07</td>
<td>10.70</td>
<td>22 ± 1</td>
<td>34 ± 4</td>
<td>78.92</td>
<td>1.16</td>
</tr>
<tr>
<td>D</td>
<td>0.25</td>
<td>235.5</td>
<td>10.12</td>
<td>7.80</td>
<td>22 ± 1</td>
<td>40 ± 9</td>
<td>72.35</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Letters A to D refer to the labels of the series in Fig. 2.

The average of \( S \) is calculated from the mean of \( \ln S \), that is, \( \bar{S} = e^{\langle \ln S \rangle} \).

The uncertainty of the experimental \( n_w^* \) is the 90% confidence interval.

The uncertainty of the theoretical \( n_w^* \) is a standard deviation, taking into account the uncertainties of the surface tension \( \sigma \) and the enhancement factor \( f_e \).

Values of the surface tension \( \sigma \) and enhancement factor \( f_e \) are predictions (see the Appendix). At 235 K, the surface tension of pure water is estimated at 82.90 mN/m.

\[
n_w^* = \left( \frac{\partial \ln J}{\partial \ln S} \right)_e - 1,
\]

where \( J \) is the nucleation rate. As can be seen in the double-logarithmic plot of Fig. 2, all four experimental series have nearly the same slope, so the critical cluster size is also similar. Indeed, application of Eq. (2) yields sizes of about 22 molecules; see Table I. Only series B has a slightly smaller slope. A theoretical prediction of the critical cluster size is given by the Gibbs–Thomson equation

\[
n_w^* = \frac{32\pi \sigma^3 v_l^2}{3kT}n_w^* \sinh \Delta n_m^* + 2 \ln f_e,
\]

where \( \sigma \) is the surface tension, \( v_l \) is the molecular volume of liquid water and \( k \) is Boltzmann’s constant. A comparison of the theoretical and experimental values (Table I) shows that Eq. (3) overpredicts the critical size by 10 to 18 molecules. However, the Gibbs–Thomson equation holds for clusters containing water molecules only. For the experiments in pure methane, we show below that there are about five methane molecules in the critical cluster. For such cluster compositions, Eq. (3) does not hold.

The amount of methane molecules in the critical cluster \( n_m^* \) can be obtained with an extended version of the nucleation theorem, derived by Oxtoby and Laaksonen\(^{16}\) and modified by Luijten et al.\(^{13}\) Applied to the water-methane system, it reads

\[
\left( \frac{\partial \ln J}{\partial \ln p} \right)_{S,T} = \left( \frac{p v_w^f}{kT} - x_m^{eq} \right)n_w^* + Z_m \Delta n_m^* + 2 \ln f_e,
\]

where \( v_w^f \) is the molecular volume of pure liquid water, \( x_m^{eq} \) is the equilibrium molar fraction of dissolved methane in the liquid, \( Z_m \) is the compressibility factor of methane, and \( f_e \) is the enhancement factor in equilibrium. Furthermore, \( \Delta n_m^* \) is the excess number of methane molecules; that is, the number of methane molecules in the cluster minus the number of methane molecules in one critical cluster volume in the gas phase. A different form of the nucleation theorem was derived by Kalikmanov and Labetski;\(^{17}\) it is given by

![FIG. 3. Nucleation rate of water in methane and carbon dioxide at 10 bar and 235 K, as a function of the reduced partial pressure of water.](image-url)
FIG. 4. Effect of carrier gas pressure on the nucleation rate of water at 235 K. For water in methane, the two measurement series at 10.1 bar from this work are shown, as well as the data of Peeters et al. (Ref. 1) at 10 bar and 25 bar. For low carrier gas pressures, fits to the data from Holten et al. (Ref. 4) and Wölk and Strey (Ref. 18) are given.

\[ \frac{\partial \ln y}{\partial \ln p_{\text{JT}}} = - \frac{1}{n_m + 1} \left( \frac{p V_{\text{gas}}}{RT} \Delta n^*_m + Z_m \Delta n^*_n + \ln f_c + 1 \right), \]

where \( V_{\text{gas}} \) is the partial molar volume of water vapor in the gas phase and \( R \) is the gas constant.

To apply Eq. (4) or Eq. (5), measurements of the nucleation rate at different pressures are required. In Fig. 4, a collection of water nucleation rates at 235 K and several pressures is presented. Peeters et al. \(^1\) obtained nucleation rates in methane at 10 bar—as in this work—and at 25 bar. Note that the 10 bar results of Peeters et al. are approximately between our two series, which supports the validity of the data. At lower pressures, nucleation rates in helium by Holten et al. \(^4\) and rates in argon by Wölk and Strey \(^18\) are shown. Of course, the pressure nucleation theorem is intended to be used for measurements with the same carrier gas, but at low total pressures (1 bar or less) the effect of the carrier gas is expected to be small. For the following analysis, the data with helium or argon will therefore be treated as measurements in methane at 1 bar.

To determine the methane content of the critical cluster with Eq. (4) at 10 bar, the nucleation rate at a supersaturation of \( S=11.25 \) was taken, for all three available pressures. That value of \( S \) corresponds to the middle of the 10 bar data; to evaluate the nucleation rate at 25 bar, the 25 bar fit (Fig. 4) was extrapolated to \( S=11.25 \). In Fig. 5(a), the resulting nucleation rates are plotted as a function of the methane pressure. A quadratic interpolation through the points was used to obtain the derivative \( \left( \frac{\partial \ln J}{\partial \ln p} \right)_{S, T} \) at \( S=11.25 \), which was substituted in Eq. (4). The evaluation of the other quantities is described in the Appendix. Finally, the excess number of methane molecules in the critical cluster was found to be \( \Delta n^*_{m} = 5 \pm 2 \). Peeters et al. \(^1\) found the value \( \Delta n^*_{m} = 5 \pm 2 \) using a slightly different evaluation.

To apply Eq. (5), supersaturations were taken at a nucleation rate of \( 3 \times 10^{15} \text{ m}^{-3} \text{s}^{-1} \), which lies in the middle of our measurement range. The supersaturation values were then converted to vapor fractions with the help of Eq. (1), that is, \( y=5$f_{p,c}$/p$. For the low-pressure argon and helium data, an enhancement factor of $f_{p,c}=1$ was taken. The vapor fractions were plotted versus the pressure [Fig. 5(b)], and a quadratic interpolation was employed again to find the derivative \( \left( \frac{\partial \ln y}{\partial \ln p} \right)_{S, T} \) at \( S=11.25 \). The evaluation of the partial molar volume \( V_{\text{gas}}^m \) is described in the Appendix. Finally, the excess number of methane molecules in the critical cluster was found to be \( \Delta n^*_{m} = 5 \pm 2 \), which agrees with the result obtained by Eq. (4).

The actual number of methane molecules in the cluster is the sum of the excess number of methane molecules and the number of methane molecules in the gas phase that occupy a cluster volume. For our conditions (10 bar and 235 K) the actual number is about 0.3 molecules higher than the excess number. Taking into account the uncertainties of \( \pm 1 \) and \( \pm 2 \) in the results for the excess number, the absolute number of methane molecules can be taken equal to the excess number.

This means that at the mentioned conditions and a supersaturation of about 11, the critical cluster consists of approximately 22 water molecules and five methane molecules. Accordingly, the liquid fraction of methane is about 0.2, whereas the equilibrium methane fraction is \( 2 \times 10^{-3} \), 100 times smaller (see the Appendix).

For our experiments in methane and carbon dioxide, we expect that the critical cluster contains not only methane, but also carbon dioxide molecules. However, the number of carbon dioxide molecules cannot be evaluated from our current experiments, because the nucleation theorems [Eqs. (4) and (5)] are applicable to binary systems only. Therefore, to obtain the carbon dioxide content of critical water clusters, measurements of water nucleation rates in pure carbon dioxide at different pressures are required.

V. COMPARISON WITH THEORY

For carrier gases that do not enter the nucleating clusters significantly, the classical nucleation theory can be adjusted for the two influences of the carrier gas: vapor fraction enhancement and surface tension change. The influence of enhancement is also taken into account in the definition of the supersaturation, Eq. (1). With this definition, Luijten and van Dongen \(^19\) obtained the following expression for the nucleation rate:
Here $Z_g$ is the compressibility factor of the carrier gas ($Z_g = \rho / (\rho kT)$, with $\rho$, the total pressure and $\rho$, the number density of the carrier gas), $\rho_l$ is the density of liquid water, $M$ is its molar mass, and $N_A$ is the Avogadro constant. It is worth stressing that Eq. (6) does not treat the carrier gases as inert; it is only required that the carrier gas fraction in the clusters is small. Detailed validity conditions are given in Luijten and van Dongen\cite{19} and Peeters et al.\cite{1}

To find the enhancement factor $f_e$ and the surface tension $\sigma$ at our nucleation conditions, an EOS was selected and fitted to experimental data (see the Appendix). The resulting values of $f_e$ and $\sigma$ are listed in Table I. In Fig. 6, the theoretical nucleation rates are compared with the experimental results. There is a large discrepancy, of about four orders of magnitude, between the nucleation experiments and theory. On the other hand, the shift in supersaturation resulting from the carbon dioxide addition is predicted fairly well.

However, for the nucleation in pure methane, we found in Sec. IV that the methane fraction in the cluster is about 0.2, which is 100 times as large as in equilibrium. For such a carrier gas mixture, Eq. (6) cannot be expected to yield accurate results. A theory that aims to reproduce the experimental results should predict such a high fraction of methane (and also carbon dioxide, if it is present) in clusters. Probably, a modified form of binary nucleation theory is required, in which the carrier gas absorption and adsorption and their dependence on cluster size and composition are taken into account.

VI. DROPLET GROWTH RATES

The nature of droplet growth depends on the ratio of the mean free path of vapor molecules and the diameter of the droplet. For our nucleation experiments in methane at 10 bar, the molecular mean free path is about 5 nm, according to gas kinetic theory\cite{20}. In our setup, droplets can be observed when their diameter becomes larger than about 340 nm. Therefore, all observable droplets are much larger than the mean free path; for such droplets, the growth is controlled by diffusion of the vapor molecules through the carrier gas. In the case of diffusion-controlled growth, the square of the droplet radius is proportional to time, as long as no significant depletion of vapor molecules occurs. If depletion does occur, the growth can be described by the model of Muitjens et al.\cite{21}, which assumes constant temperature and pressure. In this model, the growth law (which gives the time $t$ at which a droplet reaches a radius $r$) can be written as

$$t - t_0 = \frac{2r_m^2}{c} G \left( \frac{r}{r_m} \right), \quad (7)$$

where $t_0$ is the time at which the droplet growth starts, $r_m$ is the maximum droplet radius (at which droplet growth stops), and $c = dr^2/dt|_{t = t_0}$ is the initial growth rate of the squared radius. The function $G$ is given by\cite{21}

$$G(x) = \int_0^x \frac{1}{1 - x^3} \, dx = \frac{\pi}{6} - \arctan \left( \frac{1 + 2x}{\sqrt{3}} \right)$$

$$+ \frac{1}{6} \ln \left( \frac{1 + x + x^2}{(1 - x)^2} \right). \quad (8)$$

For each experiment, the quantities $t_0$, $c$, and $r_m$ were obtained by least-squares fitting of the growth law to the experimental optical signals\cite{11}.

Before discussing the experimental growth rates, let us review the expected dependence on the water vapor fraction. At the beginning of the droplet growth, depletion is negligible, so the growth rate of the squared radius $dr^2/dt$ is approximately constant in time. Theoretically, it is given by\cite{2,22}

$$\frac{dr^2}{dt} = \frac{2\rho_g D}{x p_l} (y - y_{eq}), \quad (9)$$

where $\rho_g$ and $\rho_l$ are molar densities of the gas mixture and the liquid, respectively, $D$ is the diffusion coefficient of water in the gas mixture, $x$ is the water liquid fraction, $y$ is the water vapor fraction, and $y_{eq}$ is the equilibrium vapor fraction of water at the droplet growth conditions. According to Eq. (9), the initial growth rates $c$ of different experiments are a linear function of the water vapor fraction in the experiments, as long as the other quantities are held constant.

As expected, in Fig. 7 a linear trend is observed for experiments with the same carbon dioxide fraction. The presence of 25% carbon dioxide clearly decreases the growth rate. The growth rates of the experiments with 3% carbon dioxide are also slightly lower than the pure-methane ones, but the effect is small compared to the scatter. A remarkable result is the agreement of the growth rates of the two series with pure methane, series A and C. This agreement implies that the vapor fractions of both series are consistent, so the discrepancy in nucleation rates between the series (Fig. 2) cannot be due to an incorrect vapor fraction.

It is seen in Fig. 7 that the uncertainty of the growth rate is larger for experiments with a higher water fraction. In those experiments, depletion is important, and the strong decrease of the growth rate in time results in a less accurate value for the initial growth rate. Conversely, most experiments with water fractions below 280 ppm have negligible depletion and accurate growth rates.
Growth rates in the methane-water system (without carbon dioxide) have been measured earlier by Peeters et al.\textsuperscript{2} Their measurements agree with the values from this work to within the experimental uncertainties.

From the experimental growth rates, the diffusion coefficients were determined in the manner described by Peeters et al.\textsuperscript{2} which proceeds as follows. A straight line is fitted to measurements in Fig. 7 with a constant carbon dioxide fraction. According to Eq. (9), the slope of the line is equal to $2p_g D/(xp)$, so $D$ can be obtained when $p_g$, $p_t$, and $x$ are known or estimated. If the equilibrium water vapor fraction $y_{eq}$ is also known, the accuracy of the fit can be improved by forcing the line through the $y_{eq}$ on the horizontal axis, at which the growth rate is zero [Eq. (9)]. In our case, $y_{eq}$ was computed with a suitable EOS. Quantities from Eq. (9) are listed in Table II; sources are given in the Appendix.

The resulting fits are given as solid lines in Fig. 8, and the diffusion constants are listed in Table II. The fits through $y_{eq}$ agree with the experiments fairly well, although for the experiments without carbon dioxide the slope deviates somewhat; the slope of the constrained fit is 17% higher than the slope of the unconstrained fit. In fact, the slope of the unconstrained fit is so small that extrapolation down to $dr^2/dt=0$ yields $y_{eq}=10 \pm 7$. Such a value corresponds to an enhancement factor of about 0.2, which is impossible, given the known behavior of the methane-water system. In view of this inaccuracy, it is clear that a fit through $y_{eq}$ results in a more reliable value for the diffusion constant. The constrained fits were therefore used to determine the diffusion constants.

### TABLE II. Droplet growth conditions.

<table>
<thead>
<tr>
<th>$y_{eq}$</th>
<th>$T$</th>
<th>$\rho$</th>
<th>$D$ (mm$^2$/s)</th>
<th>$\rho_g$ (mol/m$^3$)</th>
<th>$x_m$ (%)</th>
<th>$x_c$ (%)</th>
<th>$\Delta \rho_l$ (%)</th>
<th>$y_{eq}$ (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>243.3</td>
<td>11.53</td>
<td>1.37 ± 0.02</td>
<td>1.5 ± 0.2</td>
<td>594.0</td>
<td>0.15</td>
<td>-0.2</td>
<td>49.7</td>
</tr>
<tr>
<td>0.03</td>
<td>242.9</td>
<td>11.41</td>
<td>1.37 ± 0.03</td>
<td>1.5 ± 0.2</td>
<td>589.4</td>
<td>0.15</td>
<td>-0.3</td>
<td>50.2</td>
</tr>
<tr>
<td>0.25</td>
<td>241.7</td>
<td>11.23</td>
<td>1.14 ± 0.04</td>
<td>1.4 ± 0.2</td>
<td>589.4</td>
<td>0.13</td>
<td>-1.4</td>
<td>58.6</td>
</tr>
</tbody>
</table>

\(\Delta \rho_l = (\rho_l/\rho_w) - 1\), with $\rho_l$ the molar density of the liquid and $\rho_w$ the molar density of pure water at the same temperature and pressure (see the Appendix for details).
water in a homogeneous mixture of methane and carbon dioxide, the effective diffusion coefficient can be approximated with Blanc’s law: 

\[ D_{w} = \left( \frac{y_{c}}{D_{cw}} + \frac{1 - y_{c}}{D_{mw}} \right)^{-1}, \]  

(11)

where \( y_{c} \) is the molar fraction of carbon dioxide and \( D_{cw} \) and \( D_{mw} \) are the diffusion coefficients of water in pure carbon dioxide and methane, respectively, evaluated at the same pressure and temperature as the mixture.

The resulting diffusion constants are listed in Table II. In addition, purely theoretical growth rate lines are shown dashed in Fig. 8. These lines intersect the computed equilibrium point and have a slope given by Eq. (9) with the Fuller/Blanc diffusion constant. Clearly, the Fuller/Blanc approximation overpredicts the experimental values. In the case of 25% carbon dioxide, the deviation is 26%, which is larger than the expected uncertainty of about 10% of the Fuller method.\(^23\) On the other hand, the qualitative behavior of lower growth rates with increasing carbon dioxide fraction is reproduced.

**VII. CONCLUSION**

We have measured nucleation rates of water at 235 K and 10 bar in methane and in mixtures of methane and carbon dioxide. Compared to the rates in pure methane, 3% of carbon dioxide increases the nucleation rate by a factor of 10; 25% carbon dioxide increases the rate by four orders of magnitude. The slope of the nucleation isotherms in the \( J-S \) diagram is unaffected by carbon dioxide; therefore, the number of water molecules in the critical cluster is also independent of the carbon dioxide concentration. In the case of nucleation in pure methane, the critical cluster contains 22 water molecules and five methane molecules, as an application of the pressure nucleation theorem shows. Remarkably, the methane fraction in the critical cluster is a hundred times as large as it is in the water-methane system in equilibrium. The nucleation rates in pure methane are three orders of magnitude higher than literature nucleation rates of water in low-pressure helium or argon.

Nucleation theory reproduces the influence of carbon dioxide qualitatively. The increase in nucleation rate is explained by the reduction in the water surface tension by methane and carbon dioxide. It is estimated that in the mixture with 25% carbon dioxide, the surface tension is 13% lower than the pure-water value; conversely, in pure methane at our conditions, its 3.5% lower than the pure-water value.

The influence of carbon dioxide is not only visible in the nucleation rates, but also in the growth rates. At equal temperature, pressure and water vapor fraction, the growth rate of the squared droplet radius is about 20% lower in the mixture with 25% carbon dioxide than in pure methane. Since the molar densities of liquid and gas are nearly independent of the carbon dioxide concentration, the lower growth rate is mainly caused by a smaller diffusion coefficient in the mixture with carbon dioxide. The decrease of the diffusion coefficient with increasing carbon dioxide concentration is qualitatively reproduced by the Fuller correlation in combination with Blanc’s law. Quantitatively, the experimental diffusion coefficient in the mixture with 25% carbon dioxide is 20% lower than the Fuller/Blanc prediction.

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**APPENDIX: PHYSICAL PROPERTIES**

To predict the vapor-liquid phase equilibrium in the water-methane and water-methane-carbon dioxide systems, an EOS is required. We selected the “cubic plus association” (CPA) EOS,\(^23\) implemented in the PE2000 program by Pfohl et al.\(^26\) We fitted the EOS to experimental literature data on phase equilibrium of the three pure substances,\(^27–29\) and the three binary systems water-methane,\(^30\) water-carbon dioxide,\(^31\) and methane-carbon dioxide.\(^32\) The reference equations for methane\(^28\) and carbon dioxide\(^29\) were evaluated with the NIST webbook.\(^33\) The literature data on water vapor fractions in the two binary systems with water is inaccurate and inconsistent, and the CPA prediction of the enhancement factor (Table I) is accurate to 5%, at best. On the other hand, the solubility of methane or carbon dioxide in water is reproduced well.

Another quantity that is required for the prediction of the nucleation rate is the surface tension of water, and the influence of methane and carbon dioxide on that quantity. Literature data\(^34–37\) were collected and summarized by empirical fits that are accurate to 0.5% in the range of experimental data. The equilibrium vapor pressure of water was taken from Murphy and Koop.\(^38\) For the evaluation of the nucleation rate of Eq. (6), the liquid density of Malila and Laaksonen\(^30\) was used.

For evaluating Eq. (4), \( \chi \) and \( f_{c} \) were computed with the CPA EOS. The compressibility factor of methane was taken from the NIST webbook,\(^33\) and \( v_{l} \) was computed from the liquid density of Malila and Laaksonen.\(^39\) For evaluating Eq. (5), the partial molar volume of water vapor in methane was calculated using the GREG-2004 EOS,\(^6,7\) and the relation \( \bar{V} = V + (\partial V/\partial y)_{T,P} \), with \( V \) the partial molar volume, \( \bar{V} \) the molar volume, and \( y \) the water vapor fraction, valid for \( y < 1 \).

Quantities from Eq. (9), listed in Table II, were obtained as follows. The molar density of pure methane was taken from Ref. 28 and the density of the methane-carbon dioxide mixture from the GREG-2004 EOS.\(^6,7\) The liquid fractions of methane, carbon dioxide and water were computed with the CPA EOS. The density of the liquid could not be taken directly from the CPA model because it is unable to reproduce the temperature dependence and the maximum of the density of water, as noted by Lundström et al.\(^40\) The CPA can, however, be used to approximate the relative density change caused by the solution of methane and carbon dioxide into water. This value was combined with the IAPWS (Ref. 27) density of pure water to obtain the density of the mixture.
Any of the following deviations is sufficient to explain the discrepancy

The correction is based on Eq. (15) of Ref. 13; in the correction of the current work, not only the temperature dependence of the surface tension but also the temperature dependence of the liquid density was also into account.

See supplementary material at http://dx.doi.org/10.1063/1.3432623 for tables with experimental nucleation and droplet growth data.

Any of the following deviations is sufficient to explain the discrepancy between series A and C: a 7% error in the water vapor fraction, 0.4 bar in initial pressure, 0.08% in pressure transducer sensitivity, and 0.05 K in initial temperature. Our investigation resulted in the following maximum possible deviations: 2% in vapor fraction, 0.01 bar in initial pressure, 0.08% in pressure transducer sensitivity, and 0.05 K in initial temperature.


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Any of the following deviations is sufficient to explain the discrepancy between series A and C: a 7% error in the water vapor fraction, 0.4 bar in initial HPS pressure, 1.3% in the pressure transducer sensitivity, or 1 K in initial temperature. Our investigation resulted in the following maximum possible deviations: 2% in vapor fraction, 0.01 bar in initial pressure, 0.08% in pressure transducer sensitivity, and 0.05 K in initial temperature.