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Citation for published version (APA):

Kunnen, R. P. J., Clercx, H. J. H., & Geurts, B. J. (2008). Enhanced vertical inhomogeneity in turbulent rotating convection. *Physical Review Letters*, 101(17), 174501-1/4. [174501].
<https://doi.org/10.1103/PhysRevLett.101.174501>

DOI:

[10.1103/PhysRevLett.101.174501](https://doi.org/10.1103/PhysRevLett.101.174501)

Document status and date:

Published: 01/01/2008

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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- The final published version features the final layout of the paper including the volume, issue and page numbers.

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Enhanced Vertical Inhomogeneity in Turbulent Rotating Convection

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(Received 28 April 2008; published 20 October 2008)

In this Letter we report experimental evidence that rotation enhances vertical inhomogeneity in turbulent convection, in spite of the increased columnar flow ordering under rotation. Measurements using stereoscopic particle image velocimetry have been carried out on turbulent rotating convection in water. At constant Rayleigh number $Ra = 1.11 \times 10^9$ several rotation rates have been used, so that the Rossby number takes values from $Ro = \infty$ (no rotation) to 0.09 (strong rotation). The three-component velocity data, obtained at two vertical positions, are used to investigate the anisotropy of the flow through the invariants of the Reynolds-stress anisotropy tensor and the Lumley triangle, as well as to correlate the vertical velocity and vorticity. In the center plane rotation causes the turbulence to be “rodlike,” while closer to the top plate a trend toward isotropy is observed.

DOI: [10.1103/PhysRevLett.101.174501](https://doi.org/10.1103/PhysRevLett.101.174501)

PACS numbers: 47.55.pb, 47.32.Ef, 47.80.Cb

The convective motions in a fluid that is heated from below and cooled from above are modulated considerably when rotation is introduced. Rotation and convection are important ingredients in many geophysical and astrophysical flows, as well as industrial applications. Key examples include open-ocean convection [1] and the convective outer layer of the Sun [2].

Only very few experiments involving *in situ* velocity measurements in rotating convection are reported in the literature [3–5]. In these studies a fluid seeded with tracer particles was illuminated with a sheet of light. A camera recorded the movement of tracer particles in the light sheet: the in-plane velocity components were measured.

In the current study we expand this configuration: using a stereoscopic view (two cameras) it is possible to resolve particle motion in the direction normal to the light sheet in addition to the in-plane displacement. Thus three-component two-dimensional velocity fields are measured. This technique is known as stereoscopic particle image velocimetry (SPIV) [6]. This is, to our knowledge, the first utilization of SPIV in rotating convection.

The inclusion of the third velocity component allows for consideration of many valuable statistics that were previously unavailable. Here we characterize anisotropy of the turbulence with the invariants of the Reynolds-stress anisotropy tensor [7–10]. We also consider the correlation of the axial velocity component with the vertical component of vorticity. This quantity is useful in characterization of the vortex-column state observed in visualizations, e.g., Refs. [11–13], and numerical simulations [14,15] of rotating convection.

The ordering of the flow into coherent structures is strongly dependent on the Rossby number Ro , indicating the ratio of buoyancy and Coriolis forces [5,14]. A rough

division can be made into two regimes. For $Ro \gtrsim 1$ the rotational influence is small. The ordering into a large-scale circulation cell (LSC) is observed, typical for confined convection (see, e.g., Refs. [16,17] and references therein). The orientation of the LSC can precess against the sense of rotation [18]. When $Ro \lesssim 1$ the rotation forces the vertical transport of fluid and heat into vortical columns [5,12,14,19]. Convergent fluid near the plates spins up cyclonically (gaining positive vorticity) before penetrating the bulk. Approaching the vertically opposite plate the fluid radially diverges, thereby spinning down anticyclonically with negative vorticity.

Our convection cell is the same as in [20], except for its placement on a rotating table for this investigation. We repeat the most important characteristics here. The cell is a Plexiglas cylinder of diameter and height $D = H = 23$ cm. Its axis is vertically aligned with gravity and coincides with the rotation axis. The bottom plate is made of copper and is heated from below with an electric resistance heater. At the top cooling water is circulated through a transparent cooling chamber. This arrangement sustains a temperature difference of $\Delta T = 5^\circ\text{C}$ over the working fluid, water, that is at an average temperature of 24°C .

The measurements are carried out with two corotating cameras (1 megapixel resolution; 10 bits dynamic range) placed above the convection cell. The effective stereoscopic angle [6] in the fluid is 51° . The water is seeded with $50\text{-}\mu\text{m}$ -diameter neutrally buoyant tracer particles. A laser light sheet approximately 2 mm thick transects the fluid horizontally. Two different vertical positions have been used: $z = 0.5H$ at half-height, and $z = 0.8H$ near the top plate. The effective measurement area after processing covers roughly $9 \times 12\text{ cm}^2$ with 49×57 velocity vectors at $z = 0.5H$, and about $12 \times 15\text{ cm}^2$ with 53×55

velocity vectors at $z = 0.8H$. At zero rotation 4×10^3 velocity snapshots are recorded, one per second. In experiments with rotation 1×10^4 velocity snapshots are recorded at 15 frames per second. The typical uncertainty in the velocity measurement is about 5%. However, the inherent temporal fluctuations found in the quantities of interest for this Letter dominate in their statistical uncertainty rather than the measurement errors.

The flow can be described with three dimensionless parameters. The Rayleigh number $Ra \equiv g\alpha\Delta TH^3/(\nu\kappa)$ is the relative strength of buoyancy to dissipation, with g the gravitational acceleration and ν , κ , and α are kinematic viscosity, thermal diffusivity, and thermal expansion coefficient of the fluid, respectively. In this case $Ra = 1.11 \times 10^9$. The Prandtl number $\sigma \equiv \nu/\kappa$ describes the diffusive properties of the fluid; here $\sigma = 6.37$. The Taylor number $Ta \equiv (2\Omega H^2/\nu)^2$ is a dimensionless measure of the rotation; it takes values between $Ta = 0$ and 2.15×10^{10} . The Rossby number is defined as $Ro \equiv \sqrt{Ra/(\sigma Ta)}$. Ro takes values from $Ro = \infty$ (no rotation) to $Ro = 0.09$ (Coriolis-dominated flow).

With the three-component velocity data available, it is possible to calculate the Reynolds stress tensor $R_{ij} \equiv \overline{u_i u_j}$, with u_i the i th velocity component (i and j take integer values 1–3; $i = 3$ is the vertical direction); here the overbar indicates spatial averaging.

Anisotropy can be characterized with the deviatoric part of R_{ij} , designated here with

$$b_{ij} \equiv \frac{R_{ij}}{R_{kk}} - \frac{1}{3} \delta_{ij}, \quad (1)$$

where δ_{ij} is the second-order Kronecker tensor and summation is implied over repeated indices. b_{ij} is a symmetric tensor with zero trace. Plots of the second and third invariants of b_{ij} , denoted here as II and III , allow for a graphical evaluation of the anisotropy [7–10]:

$$II \equiv -b_{ij}b_{ji}/2 \quad \text{and} \quad III \equiv b_{ij}b_{jk}b_{ki}/3 = \det(b_{ij}). \quad (2)$$

All realizable states are found within a triangular region in the $(III, -II)$ space: the so-called Lumley triangle (Fig. 1, after Fig. 1 of Ref. [10]). Three-component (3C) isotropic turbulence is found for $II = III = 0$. The left corner is the limit of two-component (2C) axisymmetric turbulence, the right corner of one-component (1C) turbulence. The line connecting the left and right extreme points is 2C turbulence. The curve connecting the left extreme point to the origin is designated pancake-shaped turbulence in [9] and disklike turbulence in [10]: of the three eigenvalues (EV) of b_{ij} one is smaller than the other two, so that one-component of the turbulent kinetic energy is smaller than the other two. On the right-hand side b_{ij} has one EV larger than the other two, designated cigar-shaped turbulence in [9] and rodlike turbulence in [10]. This analysis and its relation with the actual turbulent eddy shape is delicate, as is mentioned by Refs. [9,10].

FIG. 1. Map of realizable turbulence states (Lumley triangle) in terms of the invariants II and III (after Fig. 1 of [10]).

We have calculated the invariants as a function of time for experiments at several rotation rates and at both vertical measurement positions. Time traces of trajectories in the Lumley triangle are shown in Fig. 2 at $Ro = \infty$, 1.44, and 0.09, for both vertical positions.

Without rotation there are strong variations in time, especially at $z = 0.5H$. There is some anisotropy observed, as fluctuations of vertical velocity are somewhat stronger compared with the horizontal components. When rotation is added a concentration toward the limit of cigar-shaped turbulence is found at $z = 0.5H$, indeed pointing to a columnar flow structuring [21]. At $z = 0.8H$ another effect may be recognized: at this height a trend toward 3C isotropy is found, as the trajectory is confined to a small region around the origin. Because of proximity of the wall vertical fluctuations are damped and the horizontal fluctuations increase [21].

To quantify these observations we have calculated temporal averages $\langle II \rangle$ and $\langle III \rangle$ of the (spatially averaged) invariants II and III at several Ro and for both heights. In Fig. 3 the trajectories are plotted in the Lumley map as a function of Ro . The trajectory at $z = 0.5H$ is rapidly pressed against the limiting curve of axisymmetric turbulence where b_{ij} possesses one large EV (cigar-shaped or rodlike turbulence [9,10]). Indeed, a highly anisotropic state is reached at the lowest $Ro = 0.09$.

At position $z = 0.8H$, after an upward excursion the trajectory rapidly approaches the point (0, 0) as Ro decreases: the horizontal fluctuations are approximately equal to the vertical fluctuations, near the 3C isotropic limit. At the lowest Rossby number under consideration ($Ro = 0.09$) $\langle III \rangle$ crosses zero to the negative side while $\langle II \rangle$ has a larger value than at $Ro = 0.18$. These observations may be an indication of a transition toward the range of axisymmetric turbulence where b_{ij} possesses one small EV (pancake-shaped or disklike turbulence [9,10]), a trend opposite to that found at $z = 0.5H$. Measurements at even

FIG. 2. Temporal evolution of the invariants II and III at (a, b) $Ro = \infty$; (c, d) $Ro = 1.44$; (e, f) $Ro = 0.09$. The figures (a, c, e) concern vertical position $z = 0.5H$ while (b, d, f) are for $z = 0.8H$. The gray lines are the limiting curves of the Lumley map, cf. Fig. 1.

lower Rossby numbers are required to verify this statement.

As mentioned before, we know from studies like [5] that in rotating convection the vertical components of velocity and vorticity are related; i.e., vertical transport takes place in vorticity-dominated regions. Using the results from the current experiments we can identify this correlation. In Fig. 4 we present the joint probability density function (PDF) of u_3 (the subscript 3 refers to the vertical direction) and $\omega_3 \equiv \partial_1 u_2 - \partial_2 u_1$ (vertical vorticity component) at $Ro = 0.09$ for both vertical positions. For reference, for $Ro \ll 1$ Vorobieff and Ecke [5] reported symmetric PDFs of ω_3 at the midplane, while near the top plate a positively skewed distribution was found. At $Ro = \infty$ these were both symmetric. These findings of [5] have been verified for this Letter.

At height $z = 0.5H$ the joint PDF has a clear circular shape and hence there is no correlation to be detected between u_3 and ω_3 , but obviously we still expect a correlation between u_3 and temperature. However, this cannot be confirmed with the present measurement. At $z = 0.8H$ the circular shape is replaced by an elliptic patch, elongated along the diagonal. This negative correlation points out that downward (negative) velocity is preferentially coupled with cyclonic (positive) vorticity, and vice versa.

We use PDFs of the quantity $\xi \equiv u_3 \omega_3$ to further quantify the relation between u_3 and ω_3 . If there is zero correlation between u_3 and ω_3 the distribution function for ξ

FIG. 3. Trajectories in the Lumley map of the space-and-time-averaged invariants $\langle II \rangle$ and $\langle III \rangle$ as a function of Ro for $z = 0.5H$ (filled symbols) and $z = 0.8H$ (open symbols). Typical errors in the symbol locations are from the size of the symbols in the bottom left, to 15% in the top right. The thin solid lines connect the symbols in Rossby-number order from $Ro = 0.09$ (left triangles) to 0.18 (stars), 0.36 (down triangles), 0.72 (diamonds), 1.44 (squares), 2.89 (circles), and ∞ (up triangles). The gray lines are the limiting curves of the Lumley map, cf. Fig. 1.

will be symmetric; for the situation as depicted in Fig. 4(b) we expect a strongly asymmetric distribution with a large negative tail. In Fig. 5 PDFs of this quantity are shown for the two vertical positions and at two Rossby numbers $Ro = \infty$ and 0.09. The PDFs have strong tails. This signals an intermittent distribution with strongly localized events. For reference, a fit to the PDF of crosses in Fig. 5(b) of the shape $P(\xi) \sim \exp[-(|\xi|/\xi_0)^\beta]$ returned an exponent $\beta = 0.44$. Such a stretched-exponential PDF is expected for distributions of products of variables [22].

Asymmetry of the PDF is quantified by its skewness $S \equiv \langle \xi^3 \rangle / \langle \xi^2 \rangle^{3/2}$. We have calculated S for all Ro values considered and for both vertical positions. These are depicted in Fig. 6. Starting on the right-hand side of the figure, it is found that $S \approx 0$ at $Ro = \infty$ for both heights considered.

FIG. 4. Joint PDFs of u_3 and ω_3 at $Ro = 0.09$ at positions: (a) $z = 0.5H$ and (b) $z = 0.8H$. The axes have been normalized by the respective standard deviations.

FIG. 5 (color online). Normalized PDFs of ξ at (a) $z = 0.5H$ and (b) $z = 0.8H$. The PDF at $Ro = \infty$ is shown with crosses; triangles are for $Ro = 0.09$. The black lines are reference exponential distributions.

We can conclude that the distributions of ξ are symmetric at $Ro = \infty$, and thus that without rotation there is no correlation between vertical velocity and vertical vorticity, just like the circular shape of Fig. 4(a).

With rotation added, for $z = 0.5H$ the skewness remains close to zero (squares in Fig. 6), and symmetry is approximately maintained. At $Ro = 2.89$ (and less so at $Ro = 1.44$) the skewness is slightly positive. We expect that this observation has a relation to the LSC and its mean azimuthal motion for small rotation rates [18]. The LSC precesses anticyclonically, thereby providing a background of negative vorticity (the average vorticity is indeed negative at this Ro). We do not know the exact mechanism yet. At $Ro \lesssim 1$ the LSC is no longer present and instead cyclonic vorticity is dominant [5], leading in a similar way to negative skewness. The trend of S toward zero at the smallest Rossby values stems from the trend toward symmetry in the vorticity distribution [5].

At $z = 0.8H$, however, with decreasing Ro a growing negative skewness is found, which attains a value $S \approx -3.5$ at $Ro = 0.09$. The strong skewness emphasizes the strong coupling of positive vertical velocity to negative

FIG. 6. Skewness S of the PDF of ξ as a function of Ro for $z = 0.5H$ (squares) and $z = 0.8H$ (triangles). The filled symbols on the right-hand side represent $Ro = \infty$. In the bottom right a representative error bar is shown.

vertical vorticity and vice versa near the top plate, by the vortex formation mechanism mentioned before.

In conclusion, we have for the first time experimentally evaluated anisotropy in turbulent rotating convection, and quantified the correlation between vertical velocity and vertical vorticity. It is found that rotation increases inhomogeneity. The considerable variation in turbulence phenomenology within the sample volume exemplifies some of the challenges in the description and modeling of turbulent rotating convection, which we hope to have given valuable input.

R. P. J. K. wishes to thank the Foundation for Fundamental Research on Matter (Stichting voor Fundamenteel Onderzoek der Materie, FOM) for financial support.

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