A novel approach to test the functional behaviour of cellular neural networks (CNNs) is proposed. The method attains 100% stuck-at fault coverage regardless of the array size without any extra hardware for its implementation. The Letter discusses the new fault model, presents the algorithmic procedures and shows simulated testing results.

The testing of CNNs has been scarcely addressed [1]. The approach described in this Letter overcomes the testing constraints of [1] and achieves 100% fault detection without any extra hardware. Our testing method is based on the concept of C-testability [2]. Using this approach it is possible to determine the functionality of a processing array by applying a constant number of predetermined vectors independent of the array size and then comparing the actual output values to the predicted output values.

**References**


**Functional testing for cellular neural networks**

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_**Indexing terms:** Cellular neural networks, Testing_
imaginary cells outside the processing array to ensure that the cells on the perimeter of the processing array achieve proper convergence. These imaginary cells are called border cells and form a ring around the processing array. The border cells are treated as members of the array for initialisation purposes and template implementation, but are not considered in the final state analysis.

Fault models: A CNN processor has only two output states. Commonly, in image processing applications these states appear as white or black pixels. A white pixel is associated to a cell whose output voltage is normalised to +1 V. A black pixel is associated to a cell whose normalised output voltage is +1 V. If a cell is unable to change from one state to the other, it is defined to be 'stuck-at-white' or 'stuck-at-black', depending on its current value. With the proposed test method it is possible to detect 100% of the stuck-at-faults in the processor.

Test methods: The test procedure has two separate methods to detect faulty cells, a local method using the A template and a propagation method using the A template. The entire array can be tested using either of these methods regardless of its size. The advantage of the A template method is that it verifies that each cell is responding correctly to its neighbour's output. The local method uses the input image and the B template to predict the final output state \( y_{ij} \) of each cell in the array. The algorithm for the local test procedure is shown below.

(i) let

\[
A = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
5 & 2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(ii) set the input image and border cells to white

\[ V_{ij} u_{ij} = -1 \quad i = 0,1,\ldots,n+1 \quad j = 0,1,\ldots,m+1 \]

(iii) set the initial conditions to white

\[ x_{ij}(0) = -1 \quad i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \]

(iv) allow the CNN to converge

(v) all cells \( C_{ij} \) remaining at white are considered faulty, e.g. 'stuck-at-white' cells

\[ V_{ij} y_{ij} = -1 \rightarrow C_{ij} \text{ is faulty} \quad i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \]

In step 2 the normalised value corresponding to a white input is -1. Notice that the B template values are also -1. The positive product of these two values results in current being injected into cell \( C_{ij} \) from each of its eight neighbours. As the current is injected into the cell, the integrator voltage rises and the cell output reaches the normalised value of 1, which corresponds to black. If cell \( C_{ij} \) fails to change under such overwhelming circumstances the conclusion that must be drawn is that the cell is 'stuck-at-black'. The same concept although the templates are different because we only want propagation and not 'full dragging' as described in [5]. In this case the output image does not matter and the border cells and initial conditions of the network are black. The template causes each cell, \( C_{ij} \), to look at the cell behind it, \( C_{i-1,j} \), and change to the colour of that cell. The algorithm for the propagation test is as follows:

(i) let

\[
A = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
5 & 2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(ii) set the border cells to black

\[
\forall \, i \quad u_{ij} = 1 \quad i = 0,1,\ldots,n+1 \quad j = 0,1,\ldots,m+1
\]

\[
\forall \, j \quad u_{ij} = 1 \quad i = 0,1,\ldots,n+1 \quad j = 0,1,\ldots,m+1
\]

(iii) set the initial conditions to black

\[ x_{ij}(0) = 1 \quad i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \]

(iv) allow the CNN to converge and save the results of rotation \( k \)

\[ V_{ij} x_{ij}(t) \rightarrow C_{ij}^{(k)} = 1,2,\ldots,n \quad j = 1,2,\ldots,m \]

(v) add 1 to \( k \) and rotate \( A \) template clockwise 45°

(vi) if \( k > 8 \) continue to step 7, otherwise go to step 2

(vii) perform the logical OR of the results

\[ \bigoplus_{k=1}^{8} V_{ij} C_{ij}^{(k)} \rightarrow C_{ij} \quad i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \]

(viii) all cells \( C_{ij} \) remaining at white are considered faulty, e.g. 'stuck-at-white' cells

\[ V_{ij} y_{ij} = -1 \rightarrow C_{ij} \text{ is faulty} \quad i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \]

The process starts at the left edge of the array and propagates across the network to the right side. Because the border cells, \( C_{ij}(0) \), are black, the predicted result should be an all black image. If a 'stuck-at-white' fault is detected, then all properly functioning cells to the right of the stuck cell should also remain white. The faulty cell in effect casts its 'shadow' across the array. The situation where two or more faulty cells lie on the same row can be detected by rotating the \( A \) template clockwise 45° and repeating the test. The template should be rotated in this manner 360° to ensure complete coverage of the array. The 'stuck-at-white' cells can be determined by performing the logical OR of the resulting eight output images. The same procedure can be used to detect 'stuck-at-black' cells by changing all occurrences of white to black and -1 to 1 in steps 2 to 8 in the propagation test algorithm and performing a logical AND in step 7.

Simulated results: Using a CNN simulation program the above tests were applied to a 5 × 5 CNN network. In both the local and propagation tests, cells \( C(3,2) \) and \( C(2,3) \) were intentionally forced into the 'stuck-at-white' state. Fig. 2a shows the input image used for both tests. Fig. 2b shows the resulting final image after the simulation of the local test. Fig. 3 shows the results of the simulation after the propagation test. Fig. 3a shows the 'shadow' effect discussed earlier. It is safe to assume by viewing Fig. 3a that cells \( C(3,2) \) and \( C(2,3) \) are faulty. Fig. 3b-h show the result after each rotation of the template by 45° and the new direction of propagation. The final results of the test are shown in Fig. 3i.

Conclusion: A testing method for NNs has been presented which provides 100% fault detection with no additional hardware required. Any size array can be tested using a constant size set of input vectors. The local testing method provides 100% fault isolation. There are some fault location configurations that could impede fault isolation using the propagation test, i.e. if the faults form a complete rectangle, the status of the cells inside the rectangle would be unknown due to the shadow effect. This fact lowers the fault isolation capabilities of the propagation method but does...
Relation between template spectrum and networks

Introduction:

Discrete-time cellular neural networks (CNNs) have been recently introduced as promising architectures for nonlinear and real-time image processing [1]. The stability analysis of CNNs is a main topic that has been investigated in recent contributions (see [2] for a survey). In [3] the convergence is proven for reciprocal discrete-time CNNs with continuous, monotonically increasing nonlinearities, and feedback operator which comply with

\[ A(i,j;i,j,i) \geq \sum_{k,l} |A(i,j;k,l)| \]

for every i and j.

This dominance constraint on the feedback operator is overly restrictive. Indeed, the convergence proof outlined in [3], that applies to every discrete-time feedback neural network, requires only that the interconnection matrix be symmetrical and positive semidefinite (PSD). It is therefore possible to exploit the peculiar interconnecting structure of CNNs to derive a design constraint which is less restrictive than that in eqn. 1. To simplify the notation, this improved constraint will be derived in the case of one-dimensional CNNs. However, the generalisation to two-dimensional networks is straightforward.

Assumptions and definitions: Assume a one-dimensional reciprocal discrete-time CNN described by the following state equation:

\[ x(n+1) = A x(n) + B u + d \]

In eqn. 2 x = \{x_i\}, where x_i is the state of the ith cell. y = \{y_j\}, where y_j = f(x_j) denotes the output of the jth cell. N is the number of cells. A(N x N) represents the symmetrical connection matrix. B is the matrix of control parameters. u is a threshold vector. The nonlinear function f(x) is a continuous, monotonically increasing function satisfying the following conditions: f(x) = +1, if x > +1; f(x) = -1, if x < -1; df/dx > 0, if -1 < x < +1; df/dx = 0 if x = \pm 1.

Let \( a(0), a(1), \ldots, a(r) \) denote the symmetrical cloning template, where r is the dimension of the cell neighbourhood; then, the connection matrix is given by

\[ A(i,j) = a(|i-j|) \]

\[ A(i,j) = 0 \quad \text{otherwise} \]

We define the template spectrum \( S(\lambda) \) as follows:

\[ S(\lambda) = \sum_{k=-r}^{r} a(k) \exp(j \lambda k) \lambda \]

so that

\[ a(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S(\lambda) \exp(-j \lambda k) d\lambda \]

Taking into account the symmetry of the template we can rewrite:

\[ S(\lambda) = a(0) + 2 \sum_{k=1}^{r} a(k) \cos(\lambda k) \]

Improved convergence criterion: As shown in [3], convergence of the model eqn. 2 can be proven under the assumption that matrix A is PSD. This condition can be formulated in terms of the (real) eigenvalues of A; namely it is required that all the eigenvalues of A be non-negative. Matrix A is a finite-order Toeplitz matrix, having the form

\[ A = \begin{bmatrix} a(0) & a(1) & \ldots & a(r) \\ a(1) & a(0) & \ldots & a(r-1) \\ \vdots & \vdots & \ddots & \vdots \\ a(r) & a(r-1) & \ldots & a(0) \end{bmatrix} \]

No analytic expression is known for the eigenvalues of matrix eqn. 5. However, in the case of CNNs we are interested in networks where the number of cells is far larger than the template dimension, i.e. N >> r. It is therefore possible to exploit an asymptotic property of finite-order Toeplitz matrices, to derive a design constraint which is satisfied by a broader class of cloning templates, with respect to that complying with eqn. 1. To this end, we introduce matrix \( C(N x N) \) defined by

\[ C(i,j) = a(|i-j|) \]

\[ C(i,j) = 0 \quad \text{otherwise} \]

where it is assumed N > 2r + 1. Matrix C is a circulant matrix, whose circulant elements are

\[ a(0), a(1), \ldots, a(r), 0, \ldots, 0, a(r) \]

It is well known that the eigenvalues of a circulant matrix can be