Approximation of Transmission Line Parameters of Single-Core XLPE Cables

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Abstract - An accurate model of the high frequency behavior of a power cable is required for the precise simulation of the propagation of partial discharges in such a system. A cable model consists of the transmission line parameters: characteristic impedance, attenuation coefficient and propagation velocity. The semi-conducting screens of an XLPE power cable have a significant influence on the pulse propagation along the cable. Unfortunately, the dielectric properties of these layers are usually unknown and can vary hugely between cable types. This paper shows how the characteristic impedance and propagation velocity can be estimated using the cable geometry and the dielectric properties of XLPE. Typical uncertainties in the input parameters result in an uncertainty of a few percent in the approximation of the characteristic impedance and propagation velocity.

I. INTRODUCTION

A model that describes the propagation of partial discharge (PD) pulses through power cable systems is required for certain sensitive PD detection techniques [1]. Further, the understanding and interpretation of PD measurement on cable systems in general can be improved using a proper model. An essential part of a cable system model is a transmission line model of the cable itself.

An accurate transmission line model of a single-core XLPE cable, such as [2]-[4], requires detailed knowledge of the cable’s construction and material properties. Unfortunately, not all parameters are readily available, especially the high-frequency dielectric properties of the semiconducting layers are hard to obtain. This paper describes how the characteristic impedance $Z$, and the propagation velocity $v_p$ can be approximated with input parameters that are easy obtainable. The attenuation coefficient $\alpha$ can not be approximated with sufficient accuracy since the material properties of the semiconducting layers have a significant influence [3], [5].

II. SINGLE-CORE XLPE CABLE

In this paper the following typical construction of a single-core XLPE cable is assumed (see Fig. 1):

1. Aluminum or copper conductor with radius $r_c$.
2. Semiconducting layer extruded around conductor (conductor screen) with thickness $t_s$ and complex relative permittivity $\varepsilon_{r6} = \varepsilon'_{r6} - j\varepsilon''_{r6}$.
3. Insulation, most modern MV and HV cables use XLPE, with complex relative permittivity $\varepsilon_{rins} = \varepsilon'_{rins} - j\varepsilon''_{rins}$.
4. Semiconducting layer (insulation screen) with thickness $t_s$ and complex relative permittivity $\varepsilon_{r6} = \varepsilon'_{r6} - j\varepsilon''_{r6}$.
5. In many modern cables semiconducting swelling tapes are wrapped around the insulation screen. Because the electrical properties of this layer are similar to the insulation screen [2], we consider this layer to be part of the insulation screen.
6. Earth screen with mean radius $r_e$. Construction of this metallic screen is not the same for all cables. An often-used construction consists of copper wires wrapped helically around the cable. These wires are held in place by a counter-wound copper tape. An aluminum foil may be wrapped over the wires and tapes.
7. Oversheath, usually polyethylene (PE).

III. TRANSMISSION LINE MODEL

For high frequencies ($f \gg v_p / \text{cable length}$) a coaxial structure, such as a power cable, can be modeled as a two-conductor transmission line [6]. A two-conductor transmission line can be described in terms of the distributed series impedance $Z$ and the distributed shunt admittance $Y$. These parameters are expressed in terms of the resistance $R$, inductance $L$, conductance $G$, and (complex) capacitance $C$:

$$Z(\omega) = R(\omega) + j\omega L(\omega) \quad \text{and} \quad Y(\omega) = G(\omega) + j\omega C(\omega) \quad (1)$$

When an EM wave, e.g. a PD pulse, propagates through the cable the ratio between voltage and current is given by the characteristic impedance $Z_c$:
The propagation and deformation of a pulse traveling through a transmission line is described by the propagation coefficient \( \gamma \):

\[
\gamma(\omega) = \sqrt{Z(\omega)Y(\omega)} = \alpha(\omega) + j\beta(\omega)
\]  

(3)

The real part of \( \gamma \) is the attenuation coefficient \( \alpha \). This frequency dependent parameter describes how waves attenuate due to losses as they propagate through the transmission line. The propagation velocity \( V_p \) can be derived from the imaginary part of \( \gamma \):

\[
V_p(\omega) = \frac{\omega}{\beta(\omega)}
\]  

(4)

IV. PARAMETER APPROXIMATIONS

Accurate modeling of the transmission line parameters is discussed in several publications, such as [2]-[4]. Unfortunately, these models require detailed knowledge of the cable. Generally, the cable manufacturer can supply not all. Especially, the complex relative permittivity \( \varepsilon_r \) of the semiconducting layers at high frequencies is usually not available. Accurate measurement of \( \varepsilon_r \) is possible, but complicated [2], [7]. Approximations of the transmission line parameters, using only information that is readily available from the cable manufacturer, are described in this section. Experimental validation of these approximations can be found in [5].

A. Characteristic impedance

The series impedance \( Z \) is dominantly determined by the inductance \( L \) and the shunt admittance \( Y \) by \( C \). Assuming \( Z = j\omega L \) and \( Y = j\omega C \) reduces (2) to:

\[
Z_s(\omega) = \sqrt{L(\omega)/C(\omega)}
\]  

(5)

Substituting \( L \) and \( C \) with their equations for a coaxial structure [6] yields:

\[
Z_s(\omega) = \frac{1}{2\pi} \frac{\mu_o d}{\varepsilon_r \varepsilon_o} \ln \left( \frac{r_o}{r_i} \right)
\]  

(6)

The relative permeability \( \mu_r \) of the insulation and semiconducting layers is equal to one. The (complex) relative permittivity of the insulation \( \varepsilon_{r,insu} \) is not equal to the relative permittivity of the conductor screen \( \varepsilon_{r,con} \) and the insulation screen \( \varepsilon_{r,insu} \). Therefore, \( \varepsilon_r \) is replaced by an effective relative permittivity \( \varepsilon_{r,eff} \). This is the relative permittivity of the homogeneous insulation material of a fictive coaxial capacitor with the same total capacitance and inner and outer radius.

The capacitance of a single-core XLPE cable is a series of three (complex) capacitances: \( C_{eff} \) for the conductor screen, \( C_{insu} \) for the insulation and \( C_0 \) for the insulation screen. Fig. 2 depicts the capacitances of the insulation and semiconducting layers and their relation to the effective capacitance.

For the frequency range of interest (up to 30 MHz for most PD diagnostics on cable systems having at length of several hundred meter or more) \( C_{eff} \) is much smaller than \( C_{insu} \) and \( C_0 \) because (i) the relative permittivity (both \( \varepsilon_{r,con} \) and \( \varepsilon_{r,insu} \)) of the semiconducting layers is much larger than that of XLPE [2], [7]-[8], and (ii) the insulation is thicker. Therefore, \( C_{eff} \ll C_{insu} \) and \( C_0 \ll C_{insu} \) and thus \( C_{eff} \approx C_{insu} \). Because XLPE has extremely low losses \( \varepsilon_{r,insu} \approx \varepsilon_{r,con} \). As a result the effective relative permittivity can be expressed in terms of \( \varepsilon_{r,insu} \) and the dimensions:

\[
\varepsilon_{r,eff}(\omega) = \varepsilon_{r,insu}(\omega) \left( \ln \frac{r_o}{r_i} \right) \ln \left( \frac{r_o - t_{ins}}{r_i + t_{ins}} \right)
\]  

(7)

This equation shows that \( \varepsilon_{r,eff} \) is always larger than \( \varepsilon_{r,insu} \). For a typical 11/10 kV cable where \( r_i = 9.0 \text{ mm}, t_{ins} = 0.7 \text{ mm}, t_{con} = 0.7 \text{ mm}, r_o = 13.8 \text{ mm} \) \( \varepsilon_{r,eff} \) is 1.42 times larger than \( \varepsilon_{r,insu} \).

Note that for XLPE insulation \( \varepsilon_{r,insu} \) is frequency-independent for the frequency range of interest [9].

Combining (6) and (7) yields:

\[
Z_s(\omega) = \frac{1}{2\pi} \frac{\mu_o d}{\varepsilon_{r,eff}(\omega) \varepsilon_o} \ln \left( \frac{r_o}{r_i} \right) \ln \left( \frac{r_o - t_{ins}}{r_i + t_{ins}} \right)
\]  

(8)

B. Propagation velocity

Again we assume \( Z = j\omega L \) and \( Y = j\omega C \). This reduces (3) to:

\[
\gamma(\omega) = \sqrt{j\omega L(\omega)\cdot j\omega C(\omega)} = j\omega \sqrt{L(\omega)C(\omega)}
\]  

(9)

Thus the propagation velocity is approximated by:
\[ v_p(\omega) = 1/\sqrt{L(\omega)C(\omega)} \]  

(10)

For homogeneous media \( LC = \varepsilon_r\mu_r h \) [10]. Unfortunately, the material between conductor and (wire) screen is not homogeneous. Therefore, \( \varepsilon_r \) is replaced by \( \varepsilon_{r,\text{eff}} \) (7).

Another parameter that affects \( v_p \) is the helical lay of the wire screen. Because the coupling between the wires is not very strong the charges of a pulse in the wire screen will mostly follow the helical lay of the wires. Therefore, the pulse must travel some extra distance, resulting in a lower velocity in the direction of the cable axis. Assuming a helical wire screen with a “large” number of wires (> 10) and a straight conductor the multiplication factor for the velocity is given by [11]:

\[ F_M = \frac{1}{\sqrt{1 + \left( \frac{2\pi r_s}{l_1} \right)^2 \frac{1 - (r_s/r_c)^2}{2\ln(r_s/r_c)}}} \]  

(11)

where \( F_M \) is the velocity multiplication factor due to the helical lay of the wire screen and \( l_1 \) the lay length, this is the longitudinal distance along the cable required for one complete helical wrap. Note that \( F_M \) is always larger than the extra length of the helical lay relative to the axial length. This is in agreement with the simulation in [12]. Apparently, the pulses do not exactly follow the helical lay of the wire screen.

Note that (11) does not take into account the following situations:

- Semiconducting layers. The presence of semiconducting layers might have an influence on the factor \( F_M \) because charges can transfer from one wire to another more easily.
- Stranded conductors. These strands also have a helical lay. The capacitive coupling between these wires is much stronger than between the earth screen wires. Therefore, the helical lay of conductor strands is expected to have negligible influence on the propagation velocity.
- Some wire screens with a helical lay do not have a constant angle between wire and cable axis. Instead, the lay angle goes back and forth. In such a situation the propagation velocity is expected to be smaller than without helical lay, but larger than the value calculated by (11).

Combining (7), (10) and (11) gives the approximation of the velocity:

\[ v_p(\omega) = F_M \frac{c}{\sqrt{\varepsilon_{r,\text{ins}}(\omega)}} \ln \left( \frac{r_s - t_{\text{ins}}}{r_s + t_{\text{ins}}} \right) \ln \left( \frac{r_c}{r_s} \right) \]  

(12)

where \( c \) is the speed of light in vacuum \((c = 1/\sqrt{\varepsilon_0 \mu_0})\). Note that \( v_p \) is independent of the frequency if \( \varepsilon_{r,\text{ins}} \) is frequency-independent, which is the case for XLPE.

V. Parameter Sensitivity

The dimensions of the semiconducting layers have a major influence on the transmission line parameters of the cable. For a 12/20 kV single-core XLPE cable \((r_s = 9.6 \text{ mm}, r_c = 17.3 \text{ mm}, t_{\text{ins}} = 0.8 \text{ mm}, t_{\text{ct}} = 0.9 \text{ mm} \) and \( \varepsilon_{r,\text{ins}} = 2.26 \) ignoring the semiconducting layers introduces an error in \( Z_c \) and \( v_p \) of approximately 15%.

The values for the input parameters each have an uncertainty margin. This uncertainty results in error margins in the transmission line parameters. The sensitivity of the transmission line parameters to these uncertainties is studied below. The value of \( Z \) is often used to estimate the fraction of the signal which transmits at the cable end to the load impedance, e.g. for PD signal detection. An accuracy of 10% is usually enough to have a sufficient accurate notion of the PD shape and amplitude. If the estimated propagation velocity \( v_p \) is used to determine the PD origin, the relative accuracy should preferably be better than 1%.

A. Characteristic Impedance \( Z_c \)

Using (8) the influence of changes in the conductor radius \( r_c \), earth screen radius \( r_s \), conductor screen thickness \( t_{\text{ct}} \), insulation screen thickness \( t_{\text{ins}} \) and the insulation permittivity \( \varepsilon_{r,\text{ins}} \) on the impedance \( Z_c \) is plotted in Fig. 3. As a nominal situation a 12/20 kV single-core XLPE cable (similar as in previous the section) is used. The figure shows that \( Z_c \) is not very sensitive to changes in the thickness of the semiconducting layers. It is much more sensitive to changes in the conductor and earth screen radius. The relative uncertainty in these radii (in the order of 1%) is smaller than the uncertainty in the layer thicknesses (in the order of 10%).

B. Propagation Velocity \( v_p \)

Using (12) the sensitivity of \( v_p \) to changes in all five input parameters in plotted in Fig. 4. The sensitivity of the propagation velocity \( v_p \) to variations in \( t_{\text{ins}}, t_{\text{ct}} \) and \( \varepsilon_{r,\text{ins}} \) is the same as for \( Z_c \). Only the sensitivity to changes in \( r_c \) and \( r_s \) is different. This shows that \( v_p \) is not as sensitive as \( Z_c \) to changes

![Fig. 3. Relative change in characteristic impedance \( Z_c \) as a function of relative changes in \( t_{\text{ins}}, t_{\text{ct}}, r_c, r_s \) and \( \varepsilon_{r,\text{ins}} \).](image-url)
in the conductor and earth screen radius.

If the cable has a helical earth screen the propagation velocity is also influenced by the lay length \( l \). In Fig. 5 the multiplication factor \( F_{\text{lay}} \) is plotted as a function of the lay length using (11). If the lay length is short \( F_{\text{lay}} \) and consequently \( v_p \) is very sensitive to small changes in the lay length. For a larger lay length, on the other hand, the multiplication factor \( F_{\text{lay}} \) is virtually insensitive to changes in the lay length. For distribution cables a typical lay length is ten times the earth screen diameter, and a 10% error results in only about 0.5% error for the propagation velocity.

VI. CONCLUSIONS

The semi-conducting layers in a cable with polymeric insulation have a significant influence on high-frequency properties of the cable. Unfortunately, the dielectric properties of these layers at high frequencies are usually unknown and can vary significantly between cables. Accurate modeling of the transmission line properties is impossible without accurate measurements on samples of the semi-conducting layers. For an installed cable these samples are usually not available.

It is shown how the characteristic cable impedance \( Z_c \) and propagation velocity \( v_p \) can be estimated without sensitive measurements of the properties of the semi-conducting layers. The uncertainties in \( Z_c \) and \( v_p \) are limited to a few percent for typical uncertainties in the input parameters. This is sufficient for \( Z_c \). For \( v_p \) the desired accuracy of 1% is not quite reached, making it unsuitable for accurate location. For better location accuracy a measurement of the propagation time of the cable system under test is required. For other applications it can be sufficient, e.g. to identify reflections from joints since joints are usually much further apart.

ACKNOWLEDGMENT

This work was supported by KEMA Nederland B.V. and the Dutch utilities N.V. Continuon Netbeheer, ENECO Netbeheer B.V. and Essent Netwerk B.V.

REFERENCES