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Distributed, Price-based Control Approach to Market-based Operation of Future Power Systems

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Abstract—In this paper we present, discuss and illustrate on examples the price-based control paradigm as a suitable approach to solve some of the challenging problems facing future, market-based power systems. It is illustrated how global objectives and constraints are optimally translated into time-varying prices. The real-time varying price signals are guaranteed to adequately reflect the state of the physical system and present the signals that optimally shape, coordinate and synchronize local, profit driven behaviors of producers/consumers to mutually reinforce and guarantee global objectives and constraints. As an illustrative example, the real-time price-based power balance control with congestion management is presented.

Index Terms—Power System Markets; Distributed control; Network congestion management; Optimization

I. INTRODUCTION

Electrical power systems are going through some major restructuring processes and it has been widely recognized that the feasibility, reliability and efficiency of the future European power grid cannot be achieved by a simple extrapolation of the current state. Regarding market, operational and control structures, national power grids in Europe currently exhibit rather significant diversity of employed solutions, many of which are posed on ad-hoc basis, are characterized by insufficient coordination among involved parties, have inconsistencies, non-optimality and lack of stability and robustness proofs. In spite of vast research efforts, see e.g. [1], [2], [3], [4], [5], and commonly shared vision for the future, we can say that there is yet no unifying and fundamental scientific theory to shape the optimal management and control structure of the future EU power grid.

In this paper, our main goal is to present distributed control systems theory as a scientific framework which is suitable to adequately formulate and cope with some of the major challenges facing future power systems. The leitmotiv and the core idea of the presented approaches is in utilization of real or near real-time varying price signals to shape the behaviors of local subsystems so that the desired overall system objectives and constraints are met. Depending on the considered time-scale and global objectives of interest, the term local subsystem can mean either producer or consumer as an active player (market agent) in economical layers, or, in other cases, generator or some other physical device as local dynamical system in the physical layer. It is a nice observation that different problems and on different time-scales can be treated with the same mathematical tools, where prices, which in some cases have purely mathematical interpretations, act as crucial control signals.

Indeed, the idea of price-based control is long present in power system community, dating back to work of Fred Schweppe and his co-workers, see e.g. [1], [6], [7], [8]. In this paper we further explore this idea by taking a control systems view on the problems and suitable emerging solutions. Although analogy between market operation and dual formulation of optimization problems has been often noted in the past, a novel approach and one of the main contributions of this paper is to present how specific structure of power flow equations can be preserved in devising efficient and flexible distributed control solutions, as alternative to centralized solutions. In addition to addressing some general theoretical notions and results, we will present several concrete examples to illustrate their potential for real-life applications.

II. OPTIMIZATION DECOMPOSITION: A MATHEMATICAL THEORY OF FUTURE POWER SYSTEM CONTROL

Modern control control systems theory is grounded on the following remarkable fact: virtually all control problems can be casted as optimization problems. It is insightful to realize that the same, far reaching statement, holds as well for the power systems: virtually all global operational goals of a power system can be formulated as constrained, time-varying optimization problems. Similarly as modern control theory accounts for efficiently solving these optimization problems (which is in most cases a far from trivial task), the same mathematical framework provides a systematic and rigorous scientific approach to shape operational and control architectures of power systems\(^1\). For illustration, in mathematical terms, a shift from monopolistic, one utility controlled system, to the market-based system is seen as a shift from explicitly solving primal problem (e.g. economic dispatch at the control center) to solving its dual problem (e.g. operating real-time energy market). The former case can be called the cost-based operation, while the latter can be called the price-based operation. Before continuing with consideration of some specific problems in power systems, and their price-based solutions, we will first recall some basic notions from optimization theory. For closely related subjects and the state-of-the-art results on the distributed optimization

\(^1\)The interested reader is refereed to the excellent paper [9] where the role of alternative ways for solving optimization problems is reflected in devising alternative operational structures for communication networks.
and control, the interested reader is referred to [10], [11], [9], [12], [13] and the references therein.

Consider the following structured, time varying\(^2\), optimization problem

\[
\begin{align*}
\min_{x_1, \ldots, x_N} & \quad \sum_{i=1}^{N} J_i(x_i), \\
\text{subject to} & \quad x_i \in X_i, \quad i = 1, \ldots, N, \\
& \quad G(x_1, \ldots, x_N) \leq 0, \\
& \quad H(x_1, \ldots, x_N) = 0,
\end{align*}
\]

where \(x_i \in \mathbb{R}^{n_i}, \ i = 1, \ldots, N\) are the local decision variables, the functions \(J_i : \mathbb{R}^{n_i} \to \mathbb{R}, \ i = 1, \ldots, N\), denote the local objective functions, while each set \(X_i \subseteq \mathbb{R}^{n_i}\) defines local constraints on the corresponding local variable \(x_i\). The functions \(G\) and \(H\), which respectively take values in \(\mathbb{R}^k\) and \(\mathbb{R}^l\), define global inequality and equality constraints. The vector valued inequality (1c) should be interpreted elementwise.

Note that the optimization problem (1) is defined on the overall, global system level, where global objective function is sum of local objectives as indicated in (1a). Furthermore, note that if the global constraints (1c) and (1d) are omitted, the optimization problem (1) becomes separable in a sense that it is composed of \(N\) independent local problems which can be solved separately. For such a completely separable case, we say that the optimization problem can be solved in a decentralized way. For the future reference, we will call the problem (1) the primal problem.

Next, from (1) we formulate the dual problem as follows

\[
\begin{align*}
\max_{\lambda, \mu} & \quad l(\lambda, \mu) \quad \text{(2a)} \\
\text{subject to} & \quad \mu \geq 0, \quad \text{(2b)}
\end{align*}
\]

where

\[
l(\lambda, \mu) := \min_{x_1 \in X_1, \ldots, x_N \in X_N} \left( \sum_{i=1}^{N} J_i(x_i) - \lambda^T H(x_1, \ldots, x_N) + \mu^T G(x_1, \ldots, x_N) \right). \quad \text{(3)}
\]

In (2) and (3) \(\lambda \in \mathbb{R}^k\) and \(\mu \in \mathbb{R}^l\) are the dual variables (Lagrange multipliers) and have an interpretation of prices for satisfying the global constraints (1c) and (1d). If (1) is a convex optimization problem, it can be shown that the solutions of the primal and the dual problem coincide\(^3\).

**Remark II.1** Suppose that the functions \(H\) and \(G\) have an additive structure in local decision variables \(x_i\), meaning that \(H(x_1, \ldots, x_N) = \sum_{i=1}^{N} h_i(x_i)\) and \(G(x_1, \ldots, x_N) = \sum_{i=1}^{N} g_i(x_i)\) with some given functions \(h_i, g_i, \ i = 1, \ldots, N\).

\(^2\)For notational convenience, we have omitted the explicit reference to the time dependence.

\(^3\)In fact, an additional mild condition, the so-called Slater’s constraints qualification, is required for the solutions to coincide, see e.g. [14] for more details.

Then for a fixed \(\lambda\) and \(\mu\) the optimization problem in (3) is separable and can be solved in a decentralized way. \(\square\)

**Remark II.2** Updating the the dual variables (prices) \(\lambda, \mu\) to solve the maximization problem in (2) can be achieved in a centralized way on a global level, e.g. at the central market operator which calculates the market clearing price. In some important cases, as it will be presented in Section III, the optimal prices \((\lambda, \mu)\) can also be efficiently calculated in a distributed way. This means that they can be calculated even if there is no one central unit that gathers information and communicates with all the subsystems in the network, but the optimal price calculations are based only on the locally available information and require only limited communication among neighboring systems. \(\square\)

**A. Example 1.**

Loosely speaking, and as already mentioned in the introduction, the market-based power system can be seen as solving the dual optimization problem (2). When the power limits in transmission network are not considered, this can be more precisely described as follows.

Suppose that for each \(i\), local decision variables \(x_i\) is a scalar and represents the power production \((x_i > 0)\) of a power plant \(i\), or a power consumption \((x_i < 0)\) if the subsystem \(i\) is a consumer. Furthermore, let \(J_i(x_i)\) denote power production costs when the \(i\)-th subsystem is a producer, and its negated benefit function when the \(i\)-th subsystem is a consumer. Since we do not consider the transmission network limits, the only global constraint is the power balance constraint \(\sum_{i=1}^{N} x_i = 0\), i.e. in (1) and (2) we have that \(H(x_1, \ldots, x_N) := \sum_{i=1}^{N} x_i \).

Obviously, the primal problem (1) now corresponds to minimization of total production costs and maximization of total consumers benefit, while the power balance constraint is explicitly taken into account via (1d).

Let us now consider the price-based solution through the corresponding dual problem. First note that minimization problem in (3) is in this case given by

\[
l(\lambda) := \min_{x_1, \ldots, x_N} \sum_{i=1}^{n} \left( J_i(x_i) - \lambda x_i \right). \quad \text{(4)}
\]

In the above equation each term in the summation, i.e. \(J_i(x_i) - \lambda x_i\), denotes the benefit of a subsystem \(i\) where \(\lambda\) denotes the price for electricity. Obviously, the dual problem (2) then accounts to maximizing the total benefit of the system. Note that in solving the dual problem, the power balance constraint is not explicitly taken into account. However, mathematics tells us that the corresponding maximum in (2) is attained precisely when the price \(\lambda\) is such that for the solution to the corresponding minimization problem in (4) it holds that \(\sum_{i=1}^{N} x_i = 0\). In other words, the price \(\lambda\) which maximizes the total benefit of the system is precisely the price for which the system is in balance.

To summarize, while in primal solution the global constraints were explicitly taken into account, in the dual solution they are enforced implicitly through the price \(\lambda\).
Remark II.3 The observation from the above presented example can be generalized to the core idea of the price
based control paradigm:

*In the price-based control, a price (Lagrange multiplier) is
assigned to each crucial global constraint (i.e. each row in
(1c) and (1d), see (3)) and is used to implicitly enforce this
constraint.*

With this interpretation in mind, the main objective of the
market operator in a power system is to adequately translate
global constraints into price signals so that the reaction of
local subsystems to this price (via minimization problem in
(3)) will result in a situation when these global constraints
are necessarily satisfied.

It is also insightful to interpret the price-based solutions
as *incentives-based* solutions, as prices \( \lambda \) are used to give
incentives to the local subsystems so that their local objec-
tives will make them behave in a way which serves global
needs. \( \square \)

**Remark II.4** When the transmission network constraints are
completely neglected, as it was done in the above example,
the global equality constant

\[
H(x_1, \ldots, x_N) := \sum_{i=1}^{N} x_i = 0 \quad (5)
\]

has an additive structure as described in Remark II.1. There-
fore, for a fixed \( \lambda \) the solution to the corresponding optimization
problem in (3) is separable and it can be *decomposed*
into a set of \( N \) optimization independent problems. The
prices can then be interpreted as the variables that are used to
coordinate these independent problems with the goal that their solution coincide with the solution of the “original”
primal problem. In such a cases, we say that the optimization
problem is solved through the *dual decomposition.* \( \square \)

The simplicity of the problem in the above presented example was instrumental to illustrate the main “philosophy”
of using prices as the curtail signals in control of power
systems. The examples presented in the following section are
based on the same general idea as outlined in Remark II.3,
however, there the underlying problems and solutions are
much more complex and involved. Therefore, we will restrict
our presentation to only some of their basic and the most
illustrious features, while for detailed treatment and all the
proofs the interested reader is referred to [15], [16].

**III. PRESERVING THE STRUCTURE: DISTRIBUTED
CONTROL FOR TRANSMISSION SYSTEM MANAGEMENT**

When the optimal power balance problem from the pre-
vious section (Example 1) is complemented to include the
security limits on the power flows in tie-lines of the transmis-
sion network, the solution becomes much more complex, see
e.g. [17], [4], [15]. In fact, devising efficient operational and
control schemes to optimally cope with the transmission net-
work limits in competitive environment of restructured power
systems is considered to be one of the toughest problems
in market structure design [3]. In this section we illustrate,
by using appropriate examples, how this problem can be
solved in a *distributed* way. The crucial points on the path
towards the solution are, firstly, adequate formulation of the
problem as structured optimization problem, and secondly,
resetting the structure of the problem in the solution. The
latter is crucial as it allows for a far more flexible, scalable
and robust distributed solutions, as opposed to centralized
solutions which depend on extensive communications among
distant subsystems in the network, are less robust and become
increasingly complex as the size of the system increases.

Let us complement the optimal power balance problem
discussed in Example 1 by adding the global inequality
constraints (1c) which represents the security limits imposed
on the power flows in tie-lines of the transmission network.
In this case, the local variables \( x_i \) from Example 1 have
to be extended to include at least the voltage phase angles
each bus in the system, see e.g. [17]. The voltage phase
angles are necessary variables to describe the power flows in
the transmission network. For simplicity, suppose that \( i \) is an
index of a bus in the transmission network, and that at each
bus there is connected either one producer or consumer with
the local objective function \( J_i \). As the transmission system
is taken in the account, it is now not sufficient any more to
represent the global power balance as a single scalar equality
constraint, as it was done before in equation (5). The power
balance constraint is now a vector valued equality constraint

\[
H(x_1, \ldots, x_N) = 0, \quad (6)
\]

where \( H(x_1, \ldots, x_N) \in \mathbb{R}^N \) and the \( i \)-th row in (6)
represents the power balance of the \( i \)-th bus in the network.
Furthermore, suppose that the network has a set of \( M \) lines
for which, and due to security and reliability issues, there are
determined the upper limits on the allowable power flows.
In other words, we have to add \( M \) inequality constraints of the
form

\[
G(x_1, \ldots, x_N) \leq 0, \quad (7)
\]

where \( G(x_1, \ldots, x_N) \in \mathbb{R}^M \) and the \( j \)-th row in (6) states
that the power flow limit violation in the \( j \)-th line has to be
less than zero, i.e. that the line must not be congested.

With the above definitions and interpretations of the global
constraints, the optimal power balance problem with congestion
management can be formulated as a primal problem (1),
or the corresponding dual problem (2). Note that in the dual
problem \( \lambda \) is now a vector, i.e. \( \lambda \in \mathbb{R}^N \), where \( i \)-th entry in
\( \lambda \) corresponds to the price of power balance constraint at
the \( i \)-th bus. Similarly, the \( j \)-th entry in \( \mu \) (note that \( \mu \in \mathbb{R}^M \))
is interpreted as a price for not overloading the \( j \)-th line.

According to the price-based control paradigm, see Re-
mark II.3, the goal of the system operator is to calculate the
prices \( \lambda \) and \( \mu \), which will result in satisfaction of the
constraints (6) and (7). The challenge of solving this problem
originates from the complicating fact that the functions \( H \)
and \( G \) have no longer an additive structure, as it was the
case in Example 1. However, and as it is illustrated in
the following example, the functions \( H \) and \( G \) are highly
structured and this structure can be further exploited in the solution.

A. Example 2.

Consider a simple network depicted in Figure 1 and suppose that the line connecting bus 1 and bus 2 has an upper power flow limit set to \( \bar{p}_{12} \), i.e. with \( p_{12} \) denoting the power flow in the corresponding line we have that \( p_{12} - \bar{p}_{12} \leq 0 \). With \( \mu_{12} \) denoting the corresponding price (Lagrange multiplier) for the power balance at bus \( i \) \((i \in \{1, 2, 3, 4\})\), it can be shown that when the optimum of the corresponding dual problem (2), the prices necessarily satisfy the following equation:

\[
\begin{pmatrix}
    b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} \\
    -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} \\
    -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 \\
    0 & 0 & -b_{34} & b_{34} & 0
\end{pmatrix}
\begin{pmatrix}
    \lambda_1 \\
    \lambda_2 \\
    \lambda_3 \\
    \lambda_4 \\
    \mu_{12}
\end{pmatrix}
= 0,
\]

(8)

where \( b_{12,13} = b_{12} + b_{13} \) (and so on) and where \( b_{pq} \) is a susceptance of the line connecting buses \( p \) and \( q \). The above relations among the prices are derived from the so-called Karush-Kuhn-Tucker (KKT) optimality conditions for the corresponding optimization problems (1) and (2), see e.g. [14] for the KKT conditions in general, and [15] for detailed derivation for this particular problem. What is important is that the optimality condition (8) preserves the structure of the power flow equations present in \( H \) and \( G \). This is structure is more precisely described as follows.

Each row in (8) represents an equality related to the corresponding bus in the network, i.e. the first row is related to the first bus etc. Note that the \( i \)-th row directly relates the price \( \lambda_i \) only with the prices of its neighboring busses, i.e. in the \( i \)-th row of (8) there explicitly appears \( \lambda_j \) if and only if there is a line connecting bus \( i \) with the bus \( j \). Similarly, only the prices \( \lambda_i \) and \( \lambda_j \) in the busses corresponding to the congested line \( i \rightarrow j \) are directly related to the corresponding Lagrange multiplier \( \mu_{ij} \).

Note that in practice transmission network graph is usually sparse in a sense that the number of neighboring busses for most of the busses is small, e.g. two to four.

Remark III.1 The highly structured relations among nodal prices, as illustrated above in the equation (8), are a consequence of an appropriate problem formulation, i.e. special care was taken already at the initial stage of formulating the global power system objectives as an optimization problems (1) and (2). More precisely, and for all the details we refer the interested reader to [15], to obtain the highly structured relations (8) it was necessary to induce the voltage phase angles explicitly in (1) in (2) as decision variables. Another possibility, common in the literature, is to introduce a “slack bus” with zero voltage phase angle and to solve the equations for the line flows, completely eliminating voltage phase angles from the problem formulation [17], [1]. However, in that case a specific structure, i.e. sparsity, of the power flow equations is lost.

![An example of a simple congested network.](image1.png)

These highly structured relations from the optimality conditions can be used in devising distributed real-time price-based power balance control and congestion management solution as shortly described in the following subsection and further illustrated in the example in Subsection III-C.

B. Distributed price-based control

When the violations of global system contracts (6) and (7) can be measured in real-time, in [16], [15] it was shown that a suitable dynamic extension of the optimality condition (8) can be used as a control law for real-time price update of the nodal prices \( \lambda_i \). This dynamical extension of the optimality conditions was appropriately named the KKT controller [18], [15]. When the prices are updated according to the KKT control law, and when each producer or consumer is adjusting its power production/consumption according to its benefit maximization strategy, i.e.

\[
p_i = \arg \min_{\tilde{p}_i} (J_i(\tilde{p}_i) - \lambda_i \tilde{p}_i)
\]

with \( p_i \) denoting the power production/consumption of a subsystem at bus \( i \), it was proven that the system will settle in the operating point where the total benefit of is maximized, i.e. that the operating point will coincide with the corresponding solution of (2).

Furthermore, in [15] it was shown that the KKT controller further preserves the highly structured relations present in the optimality conditions. The implication of this fact is that the price-based KKT control laws can be implemented through a set of nodal controllers, where a nodal controller (NC) is assigned to each node (bus) in the network, and each NC communicates only with the NC’s of the neighboring nodes. The distributed implementation of the propose price-based control structure is graphically illustrated in Figure 2.

![Distributed control scheme for power balance and congestion control.](image2.png)
Note that the communication network graph among NC’s is the same as the graph of the underlying physical network. Any change in the network topology requires only simple adjustments in NC’s at the location of the change. A distributed control structure is specially advantageous taking into account the large-scale of electrical power systems.

Finally, it is worth to mention that the only system parameters that are explicitly included in the price update rules are the transmission network parameters, i.e. the network topology and line impedances. To provide the correct prices, the controller requires no knowledge of cost/benefit functions $J_i(\cdot)$ of producers/consumers in the system (neither is it based on their estimates).

C. Example 3.

To illustrate the potential of the price-based methodology for practical application we consider the widely used IEEE 39-bus New England test network. The network topology, generators and loads are depicted in Figure 3. The complete network data, including reactance of each line and load values can be found in [19]. All generators in the system are modeled using a third order model consisting of governor, turbine and rotor dynamics. This is a standard model used in “automatic generation control” studies [20]. The parameter values, in per units, are taken to be in the ±20% interval from the values given in [21], pp. 545. Each generator is taken to be equipped with a proportional feedback controller for frequency control with the gain in the interval [18, 24]. We have used quadratic functions to represent the variable production costs, i.e.

$$J_i(p_i) = \frac{1}{2} c_{g,i} p_i^2 + b_{g,i} p_i,$$

with the values of parameters $c_{g,i}$, $b_{g,i}$, for $i = 1, \ldots, 10$ as listed in Table 5 in [22]. The lower saturation limit and the upper saturation limit for each generator was set to 0 and 10, respectively. All loads are taken to be price-inelastic, with the values from [19].

The distributed price-based controller was implemented in the simulation and for simplicity of exposition, the line power flow limit was assigned only for the line connecting nodes 25 and 26. The simulation results are presented in Figure 4 and Figure 5. In the beginning of the simulation, the line flow limit $P_{25,26}$ was set to infinity, and the corresponding steady-state operating point is characterized by the unique price of 39.28 for all nodes. At time instant 5s, the line limit constraint $P_{25,26} = 1.5$ was imposed. The solid line in Figure 4 represents the simulated trajectory of the line power flow $p_{25,26}$. In the same figure, the dashed line indicates the limits on the power flow $P_{25,26}$. The solid lines in Figure 5 are simulated trajectories of nodal prices for the generator buses, i.e. for buses 30 to 39, which is where the generators are connected. In the same figure, dashed lines indicate the off-line calculated values of the corresponding steady-state optimal nodal prices. For clarity,
the trajectories of the remaining 29 nodal prices were not plotted. In the simulation, all these trajectories converge to the corresponding optimal values of nodal prices as well. The optimal nodal prices for all buses are presented in Figure 6. In this figure, the nodal prices corresponding to generator buses 30-39 are emphasized with the gray shaded bars. The obtained simulation results clearly illustrate the efficiency of the proposed distributed control scheme.

IV. CONCLUSION AND FUTURE WORK

In this paper we have presented and illustrated on examples the price-based control paradigm as a suitable approach to solve some of the challenging problems facing future, market-based power systems. It was illustrated how global objectives and constraints, updated from the on-line measurements of the physical power system state, can be optimally translated into time-varying prices. The real-time varying price signals are guaranteed to adequately reflect the state of the physical system, present the signals that optimally shape, coordinate and in real or near real-time synchronize local, profit driven behaviors of producers/consumers to mutually reinforce and guarantee global objectives and constraints. Current research is devoted to modification of the devised price-based control scheme so that the prices are updated on the time scale of 5-15 minutes, rather than on unrealistically fast scale of seconds. In that approach, instead of using rapidly changing network frequency deviations as indication of power imbalance in the system, we use are using deviation of power production reference values to the generators which originate from (slightly modified) secondary control loops (automatic generation control) over the sampling period (i.e. over 5 - 15 minutes). These deviations are used as a measure of imbalance in the system.

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