Peak restoration in OFDM receiver with clipping A/D converter

Peak Restoration in OFDM Receiver with Clipping A/D Converter

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***Abstract*—OFDM signals suffer from a large Peak to Average Power Ratio, which requires large power back-offs in the transmit and receive chains. This paper studies two digital post-processing methods of different complexity that mitigate clipping by the analog-to-digital converter in the receiver. Clipped peaks cause spurious signals on empty subcarriers, which can be used to eliminate clipping artifacts and to recover the original signal. Simulations show that a significant reduction of 3 dB in the headroom of the A/D converter (ADC) is possible, when an elaborate MMSE clip correction algorithm is used. A simple algorithm still allows for 1 dB reduction of the headroom. As the ADC is consuming an ever increasing fraction of the total receiver power, the results are believed to be relevant for low–power design of OFDM receivers, for instance to prolong battery life during digital television reception on mobile phones.**

***Index Terms*—OFDM, analog-to-digital conversion, clipping, IEEE 802.11, DVB-T/H.**

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) modulation has a number of distinct advantages for transmission over multipath channels. However, the nearly Gaussian distribution of the OFDM transmit signal is seen as one of its main disadvantages. Practical systems require a margin of about 8 to 9 dB headroom between the average signal power and the clipping level of the transmitter and the receiver. Many papers have been written about the effect of the Peak to Average Power Ratio (PAPR) of OFDM, and on methods to mitigate this problem. Mostly, the high PAPR is seen as a problem for the linearity and the power consumption of the power amplifier at the transmitter. Several methods have been proposed to reduce the PAPR of the transmitted and many papers have been written on receiver algorithms for mitigating clipping noise, see e.g., [1].

Here we take a receiver-oriented direction. We develop a receiver algorithm that compensates modest clipping *by the ADC*. This approach is relevant for instance in an OFDM broadcasting system, such as DVB-H where the transmitter is built according to professional standards with sufficient power available, while the receiver may need to be integrated in a mobile phone, which is highly limited in available power. Further, for wireless personal area network systems which transmit at gigabits per second over a very short distance, the power consumption of the ADC in the receiver may exceed the power consumed by the transmit power amplifier. If a moderate amount of clipping could be compensated digitally, the ADC's saturation level could be lowered, and hence its power consumption reduced. This has been a motivation for us to develop a signal processing algorithm that attempts to reconstruct clipped peaks of the OFDM signal. Obviously such an algorithm does not give any performance gain (in terms of packet error rate) when the ADC's saturation level is so high that no clipping occurs, but rather it allows us to reduce the power consumption by lowering the ADC's saturation level, without detrimental effects on the performance.

In order to effectively shape the emitter spectrum, all wireless OFDM standards use empty, i.e. unmodulated, subcarriers at the outskirts of their spectrum. We pursue the idea that clipping causes spurious components which occur as non-zero subcarrier signals. Hence these can assist the receiver to reconstruct the original (not clipped) signal components.

The idea to exploit spurious signals received at otherwise empty subcarriers has been considered for various other reasons, e.g, mitigation of man-made burst noise, [2]; refinement of frequency synchronization, [3], [4].

The contribution of this letter is two-fold. First, we apply these concepts to the existing standards for WLANs and digital video transmission, initially to simulate the performance for an idealized receiver and later to include practical constraints. The second ambition is to formalize the mathematical model, to cover a broader class of situations, for instance when not only empty subcarriers are considered but also when interference signals with known variance are present, for instance from adjacent channel users. The variance of the interference may be estimated for instance during the reception of a known wanted signal, a further discussion of this topic lies outside the scope of this letter.

More specifically, in Section II we give a baseband model of an OFDM communication system and a statistical model of the OFDM signal at the receiver, including the receiver's ADC. Then, in Section III, we describe our clipping correction algorithms and evaluate their performance for an idealized DVB-H system. In Section IV we present the results of an 802.11a WLAN system simulation in which we included our simple clip correction algorithm.

II. MODEL OF THE COMMUNICATION SYSTEM

We give a baseband model for the transmitter, the channel and the receiver of an OFDM system, including the receiver's ADC. The system diagram is given in fig. 1.

The number of subcarriers is denoted by N_s, which is assumed to be even. The transmitted OFDM symbol has duration $T_\text{symbol} = 1/f_\mathrm{s}$, where f_s is the subcarrier spacing. The subcarriers are indexed from $-N_\mathrm{s}/2$ to $N_\mathrm{s}/2 - 1$, and subcarrier m has subcarrier frequency mf_s. In an OFDM

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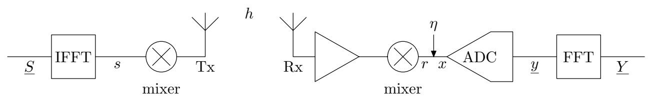

Fig. 1. System diagram at baseband level

symbol, N_{s} complex numbers $\underline{S} = (S_{-N_{\mathrm{s}}/2}, \ldots, S_{N_{\mathrm{s}}/2-1})$ are transmitted, by transmitting S_m on subcarrier m. Actually only a subset of the subcarriers is used: most standards require that the leftmost M_L and the rightmost M_R subcarriers be unmodulated, so that the corresponding $S_m = 0$. For IEEE 802.11a, $N_{\mathrm{s}} = 64$, $M_L = 6$ and $M_R = 5$, while for the 2K-mode of DVB-T, $N_{\mathrm{s}} = 2048$, $M_L = 172$ and $M_R = 171$. The set of empty subcarriers is denoted by $\mathcal{M}_0$, its size $|\mathcal{M}_0| = M_L + M_R$ is denoted by M.

The transmitter adds a cyclic prefix of duration T_{guard}, so the transmitted signal can be written in complex baseband representation as

$$s(t) = \sum_{m=-N_{\mathrm{s}}/2}^{N_{\mathrm{s}}/2-1} S_m e^{2\pi \mathrm{i} m f_{\mathrm{s}} t}, \quad t \in [-T_{\text{guard}}, T_{\text{symbol}}). \quad (1)$$

The signal passes through a channel with impulse response $h(\tau)$, where it is assumed that $h(\tau)$ is zero for τ outside of $[0, T_{\text{guard}})$. The analog transmit and receive filters are included in $h(\tau)$. Noise from the receiver's low-noise amplifier and interference from adjacent channel users is denoted by η. The receiver ignores the signal received in the interval $[-T_{\text{guard}}, 0)$, since it suffers from inter-symbol interference, and samples and discretizes the in-phase and quadrature components of the complex signal in the interval $[0, T_{\text{symbol}})$ at the ADC. The complex baseband signal entering the ADC is $x(t) = \eta(t) + r(t)$ with

$$r(t) = \sum_{m=-N_{\mathrm{s}}/2}^{N_{\mathrm{s}}/2-1} R_m e^{2\pi \mathrm{i} m f_{\mathrm{s}} t}, \quad t \in [0, T_{\text{symbol}}) \quad (2)$$

where

$$R_m = S_m \int_0^{T_{\text{guard}}} h(\tau) e^{-2\pi \mathrm{i} m f_{\mathrm{s}} \tau}\, d\tau =: S_m H_m. \quad (3)$$

Assuming sampling at the Nyquist frequency, $T_{\text{sample}} = (N_{\mathrm{s}} f_{\mathrm{s}})^{-1}$ and the n-th sample is $x_n = x(nT_{\text{sample}}) = r_n + \eta_n$, $n = 0, \ldots, N_{\mathrm{s}} - 1$. The set of all N_{s} samples is denoted by $\underline{x} = (x_0, \ldots, x_{N_{\mathrm{s}}-1})$. The in-phase and quadrature components of each sample x_n are discretized. Discretization of the signal value w gives the best approximation in a discrete set D with L different quantisation levels, $D = \{d_1, \ldots, d_L\} \subset \mathbb{R}$ with $d_1 < d_2 < \ldots < d_L$, according to

$$q(w) = \arg\min_{d \in D} |d - w|.$$

After the ADC, the complex baseband representation of the sample is

$$y_n = q(\mathrm{Re}(x_n)) + \mathrm{i} q(\mathrm{Im}(x_n)) =: Q(x_n), \quad (4)$$

or

$$\underline{y} = Q(\underline{x}), \quad (5)$$

where $\underline{y} = (y_0, \ldots, y_{N_{\mathrm{s}}-1})$.

The discretized samples are Fourier transformed, which yields the received signal on the subcarriers:

$$Y_m = \frac{1}{N_{\mathrm{s}}} \sum_{n=0}^{N_{\mathrm{s}}-1} y_n e^{-2\pi \mathrm{i} m n / N_{\mathrm{s}}}, \quad m = -N_{\mathrm{s}}/2, \ldots, N_{\mathrm{s}}/2 - 1. \quad (6)$$

We now return to the quantization of the signal. We take $L > 2$ and a uniform level spacing: $d_\ell = (2\ell - L + 1)C/(L-2)$, so that $(d_1 + d_2)/2 = -C$, $q(w) = d_1$ if $w < -C$, and $q(w) = d_L$ if $w > C$. If the signal value w is less than C in absolute value, the discretization error $|q(w) - w|$ is at most equal to $C/(L-2)$, but if the input sample $w > C$, or $w < -C$ the error is unbounded and the ADC is said to *clip* the sample. For an unclipped signal, the conditional expected squared error $W_{\mathrm{u}} = \mathbb{E}[|w - q(w)|^2 \,||w| < C]$ is well approximated by $C^2/3(L-2)^2$, for a clipped signal the conditional expected squared error depends on the tail of the distribution of w.

From the central limit theorem it follows that in the limit $N_{\mathrm{s}} \to \infty$ each of the $n = 2N_{\mathrm{s}}$ samples is distributed as a zero-mean Gaussian with variance $P/2$. Therefore we approximate the probability that a given sample is clipped by

$$p_{\mathrm{c}} = \frac{2}{\sqrt{\pi P}} \int_C^\infty \exp[-w^2/P]\, \mathrm{d}w = \mathrm{erfc}(C/\sqrt{P}). \quad (7)$$

The expected squared error, given that a sample is clipped, equals

$$W_{\mathrm{c}} = \frac{\int_C^\infty (w - d_L)^2 \exp[-w^2/P]\, \mathrm{d}w}{\int_C^\infty \exp[-w^2/P] \mathrm{d}w}. \quad (8)$$

If we ignore the small effect of the correlation between the samples due to the empty subcarriers, the expected total squared error is given by

$$\begin{aligned} W &= \sum_{k=0}^{n} \binom{n}{k} (1 - p_{\mathrm{c}})^{n-k} p_{\mathrm{c}}^k [k W_{\mathrm{c}} + (n-k) W_{\mathrm{u}}] \\ &= n[p_{\mathrm{c}} W_{\mathrm{c}} + (1 - p_{\mathrm{c}}) W_{\mathrm{u}}]. \end{aligned} \quad (9)$$

Note that the relative errors W_{u}/P and W_{c}/P depend on the power P and clip level C only through the ratio P/C^2.

Now suppose that we have an algorithm that is able to fully mitigate the effect of up to m clippings, e.g., as in (14) in the next section. Then that algorithm reduces the expected error W to

$$W = n W_{\mathrm{u}} + \sum_{k=m+1}^{n} \binom{n}{k} (1 - p_{\mathrm{c}})^{n-k} p_{\mathrm{c}}^k (k - m)(W_{\mathrm{c}} - W_{\mathrm{u}}). \quad (10)$$

As an example, we have plotted in, fig. 2, the relative squared error per complex sample $2W/nP$ (in dB) as function of P/C^2 (in dB) for $n = 128$, $L \in \{2^8, 2^{10}\}$ and $m \in \{0, 20\}$. We want to operate the ADC in a regime where this relative error is well below the relative error due to thermal noise, for instance $2W/nP \ll -30\,\mathrm{dB}$. The graphs show that the lower limit of this regime depends on the number of quantization levels, L, whereas the upper limit appears to be independent of L. It also appears that a clip mitigation algorithm can increase the upper limit of the regime by an amount that does not depend on L, whereas the lower limit is unaffected. Furthermore the results suggest that a good clip correction

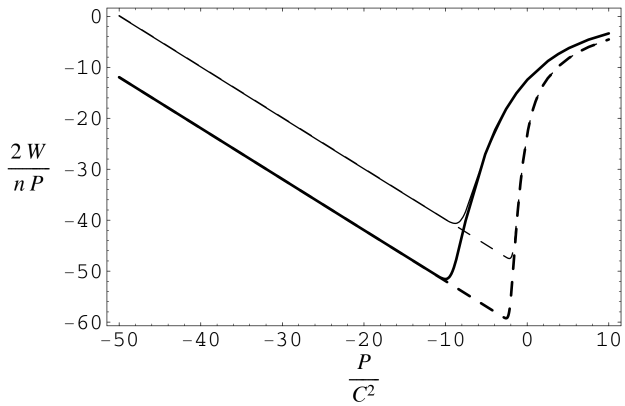

Fig. 2. The relative error due to clipping and quantization according to (10) (in dB) as a function of P/C^2 (in dB) for $n = 2N_\mathrm{s} = 128$ and $(L = 256, m = 0)$ (thin line), $(L = 1024, m = 0)$ (thick line), $(L = 256, m = 20)$ (thin dashed line) and $(L = 1024, m = 20)$ (thick dashed line).

algorithm may give a gain (at the upper limit) that is similar to the gain (at the lower limit) that would be obtained if the number of bits of the ADC were increased by two. In other words: a good clipping algorithm may allow for a reduction in the power consumption of the ADC by 6 dB (by reducing the number of bits) without significantly lowering its dynamic range.

We assume from now on that L is large enough to consider only the error introduced by clipping, i.e.:

$$q(w) = \begin{cases} -C & \text{if } w \leq -C \\ w & \text{if } -C < w < C \\ C & \text{if } w \geq C. \end{cases} \tag{11}$$

III. Clipping compensation algorithms

In this section we describe two methods to reconstruct a clipped signal. The first one estimates the degree of clipping independently for all subcarriers involved and weighs the measured out-of-band artifacts equally.

The second method exploits the statistical properties of the signal and noise to reconstruct the expected received signal before clipping. The model is worked out for signals and noise with Gaussian distributions.

A. First clip correction algorithm

Initially we assume that there is no noise, i.e., $\eta = 0$ so that any sample that is not clipped by the ADC is not distorted at all. Then we can write $r_n = y_n + c_n$ with

$$\begin{aligned} \mathrm{Re}(c_n) = 0 \text{ if } |\mathrm{Re}(y_n)| < C, \\ \mathrm{Im}(c_n) = 0 \text{ if } |\mathrm{Im}(y_n)| < C. \end{aligned} \tag{12}$$

Denoting the number of clippings by N_c, there are N_c non-zero parameters, one for each clipped real or imaginary part of a sample.

After the Fourier transform, we have that $R_m = Y_m + C_m$. Since the real and imaginary parts of R_m are zero if m corresponds to an empty subcarrier, we have that $C_m = -Y_m$ for $m \in \mathcal{M}_0$. These equations are equivalent to $2M$ equations for N_c unknown parameters, which can be written in matrix form as

$$\underline{v} = A\underline{u}, \tag{13}$$

where $\underline{v}$ is a known $2M$-vector with the real and imaginary parts of $-Y_m$ for the subcarriers where $R_m = 0$, $\underline{u}$ is an N_c-vector with the unknown values of $\mathrm{Re}(c_n)$ or $\mathrm{Im}(c_n)$ at a clipped sample and A is an $2M \times N_\mathrm{c}$-matrix of which the elements are the appropriate real and imaginary parts of the Fourier-transform matrix. If $N_\mathrm{c} \leq 2M$, $\underline{u}$ can be uniquely determined by solving any subset of N_c equations, or by

$$\underline{u} = (A^T A)^{-1} A^T \underline{v}. \tag{14}$$

If $N_\mathrm{c} > 2M$, $A^T A$ has $N_\mathrm{c} - 2M$ eigenvalues equal to zero, so $\underline{u}$ cannot be determined uniquely and this equation-solving clip correction algorithm doesn't work. The above results suggest that each empty subcarrier allows for the correction of two clippings, so that at most $2M$ clippings can be corrected for.

We return to the discussion of the clip correction algorithm. In [2], Wolf studied an equivalent algorithm to repair the effects of impulsive noise. Impulsive noise typically has a heavy-tailed probability distribution, possibly even to the extent that for each n either $\mathrm{Re}(\eta_n) = 0$ or $|\mathrm{Re}(\eta_n)| \gg C$, and similarly for the imaginary part of η_n. The algorithm in [2] also appeared robust against a small amount of random noise.

We now make an approximation that simplifies the clip correction significantly. Each of the diagonal elements of $A^T A$ is equal to M/N_s^2. The off-diagonal elements are *partial* correlations of an integer number of rotations of complex exponentials, so these are usually non-zero. However, off-diagonal components are smaller than the diagonal elements. Approximating $A^T A$ by its diagonal leads to the simpler approximation

$$\tilde{u} = \frac{(N_\mathrm{s})^2}{M} A^T v. \tag{15}$$

The corresponding clip-corrected complex-valued signal in the frequency domain is given by

$$\tilde{R} = Y - \frac{N_\mathrm{s}}{M} F P_{\mathrm{clip},\underline{y}} F^{-1} P_{\mathrm{empty}} Y, \tag{16}$$

where F is the Fourier transform matrix, with elements $F_{mn} = \exp(-2\pi i mn/N_\mathrm{s})/N_\mathrm{s}$, F^{-1} its inverse, P_{empty} projects onto the M empty subcarriers, i.e., $(P_{\mathrm{empty}}Y)_m = Y_m$ if $m \in \mathcal{M}$ and zero otherwise, and $P_{\mathrm{clip},\underline{y}}$ projects the real and imaginary parts of its argument onto the positions where the corresponding real or imaginary parts of $\underline{y}$ are clipped, i.e.,

$$\begin{aligned} \mathrm{Re}(P_{\mathrm{clip},\underline{y}}x)_n &= \begin{cases} \mathrm{Re}(x_n) & \text{if } |\mathrm{Re}(y_n)| = C \\ 0 & \text{otherwise} \end{cases} \\ \mathrm{Im}(P_{\mathrm{clip},\underline{y}}x)_n &= \begin{cases} \mathrm{Im}(x_n) & \text{if } |\mathrm{Im}(y_n)| = C \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{17}$$

Intuitively, this algorithm is equivalent to making the approximation that the out-of-band clip artifacts u for every clip location are orthogonal, so that clipping can be corrected by inverting the clip artifacts one by one. Note that this algorithm only uses simple digital processing and is of order $O(N_\mathrm{s}\log(N_\mathrm{s}))$,

B. MMSE algorithm for Gaussian signal and noise

In this section we give an elaborate clip correction algorithm for Gaussian signals and noise.

If the output of the ADC, $\underline{y}$, is observed, the minimum mean squared error (MMSE) estimate of the input, $\underline{x}$ is given by

$$\hat{\underline{x}}(\underline{y}) = \mathbb{E}_{\underline{x}}[\underline{x}|Q(\underline{x}) = \underline{y}]. \tag{18}$$

Here the expectation value is calculated with the *a priori* probability distribution of the input $\underline{x}$. Similarly, if $\underline{x}$ is a signal $\underline{r}$ distorted by additive noise $\underline{\eta}$, the MMSE estimate of the signal is

$$\hat{\underline{r}}(\underline{y}) = \mathbb{E}_{\underline{r},\underline{\eta}}[\underline{r}|Q(\underline{r}+\underline{\eta}) = \underline{y}] \tag{19}$$

where the expectation is calculated with the *a priori* distributions of $\underline{r}$ and $\underline{\eta}$.

If the quantizer (11) were of the Max–Lloyd type and thus would be optimal in terms of MMSE quantization error, see [5], [6], and if all samples were uncorrelated, then the ADC output $\underline{y}$ as given by (5) would be the best estimate of $\underline{x}$. This paper particularly discusses how we can improve the estimate of $\underline{x}$ when this assumption does not hold.

The distribution of the time domain signals at the sample moment becomes Gaussian if the number of OFDM subcarriers, N_s, goes to infinity, even if each subcarrier signal is non-Gaussian. For large but finite N_s, we therefore make the approximation that $\underline{r} = (r_0, \ldots, r_{N_s-1})^T$ has a circularly symmetric Gaussian distribution with mean zero, $\mathbb{E}[\underline{r}] = 0$, $\mathbb{E}[\underline{r}\underline{r}^T] = 0$, and correlation matrix

$$\mathbb{E}[\underline{r}\underline{r}^H] =: G_r.$$

Our particular case of an OFDM signal implies $\underline{r} = F^{-1}\underline{R}$, so that

$$G_r = F^{-1} G_R (F^{-1})^H,$$

where G_R is a diagonal $N_s \times N_s$ matrix with $\mathbb{E}[|R_m|^2]$ on the position corresponding to subcarrier m, so the diagonal elements of G_R corresponding to empty subcarriers are equal to zero.

We assume that the noise is Gaussian as well, with correlation matrix G_n, so that $\underline{x}$ is Gaussian with correlation matrix $G_x = G_r + G_n$. The MMSE estimate of the received signal $\underline{r}$, given the ADC output $\underline{y}$ is then given by

$$\hat{\underline{r}}(\underline{y}) = G_r G_x^{-1} \mathbb{E}_{\underline{x}}[\underline{x}|Q(\underline{x}) = \underline{y}]. \tag{20}$$

If we approximate the mean of $\underline{x}$ over $Q^{-1}(\underline{y})$, the pre-image of $\underline{y}$ under Q, by the point in $Q^{-1}(\underline{y})$ where the Gaussian distribution of $\underline{x}$ takes on its maximum value, this estimate reduces to

$$\hat{\underline{r}} = G_r G_x^{-1} \arg \min_{\underline{x} \in Q^{-1}(\underline{y})} \underline{x}^H G_x^{-1} \underline{x}. \tag{21}$$

The pre-image $Q^{-1}(\underline{y})$ is a semi-infinite space with dimension equal to the number of clippings N_c. The vectors $\underline{x} \in Q^{-1}(\underline{y})$ can be written as

$$\underline{x} = \underline{y} + E(\underline{y})\underline{c}. \tag{22}$$

Here $\underline{c}$ is an N_c-dimensional vector with non-negative real valued components and the $N_s \times N_c$ matrix $E(\underline{y})$ describes the embedding of the clipping into the signal, i.e., $E_{m_k,k} = 1, -1, \mathrm{i}, -\mathrm{i}$ if the k-th clipped sample is at position m_k and $\mathrm{Re}(y_{m_k}) = C, -C$ or $\mathrm{Im}(y_{m_k}) = C, -C$, respectively, and all other elements are equal to zero. The MMSE estimate of the clipping vector is then given by

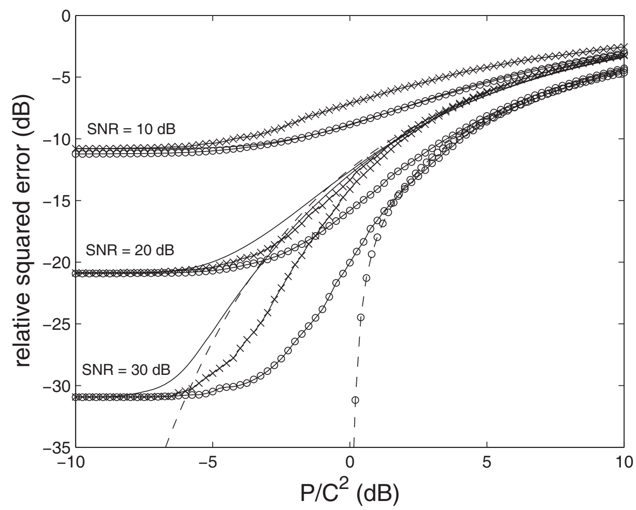

Fig. 3. Performance evaluation results of the simple and the MMSE algorithm.

$$\hat{\underline{c}} = \arg \min_{\underline{c} \in \mathbb{R}_+^{N_c}} [\underline{c}^T B \underline{c} - 2\underline{b}^T \underline{c}], \tag{23}$$

where the $N_c \times N_c$ matrix B and the N_c-vector $\underline{b}$ are given by

$$B = \mathrm{Re}(E^H G_x^{-1} E) \tag{24}$$

$$\underline{b} = -\mathrm{Re}(E^H G_x^{-1} \underline{y}). \tag{25}$$

The constraints imply that $\hat{\underline{c}}$ satisfies

$$\forall_i (\hat{\underline{c}})_i > 0 \wedge (B\hat{\underline{c}} - \underline{b})_i = 0) \vee (\hat{\underline{c}}_i = 0 \wedge (B\hat{\underline{c}} - \underline{b})_i \geq 0). \tag{26}$$

Finding the solution could require testing up to 2^{N_c} parts of the boundary of $\mathbb{R}_+^{N_c}$, the examination of each part of the boundary takes $O(N_c)$ steps in an iterative algorithm, each step having $O(N_c)$ operations. Hence the cost of this algorithm is above $O(N_c^2)$. Minimization algorithms, based on the conjugate gradient method and rules to determine which lower dimensional edge to try next, are described in [7].

C. Simulation Results

In the simulation we use DVB-H as a reference, so we consider a system with $N_s = 2048$ subcarriers, out of which $M = 343$ subcarriers are empty. We use a frequency-flat channel: $H_m = 1$, although the algorithm does not require this. Then each of the $N_s - M$ non-empty subcarriers carries a signal $\sqrt{P}\,R_m$ where the R_m are i.i.d. circularly symmetric complex Gaussian distribution with mean zero and variance $1/(N_s - M)$, and where P is the received signal power. The noise, with power P/SNR, is evenly distributed among all N_s subcarriers, so each subcarrier also carries an i.i.d. complex Gaussian noise signal with variance $P/(N_s\mathrm{SNR})$. For a given SNR, the effect of clipping depends on the clipping level C only through the ratio P/C^2.

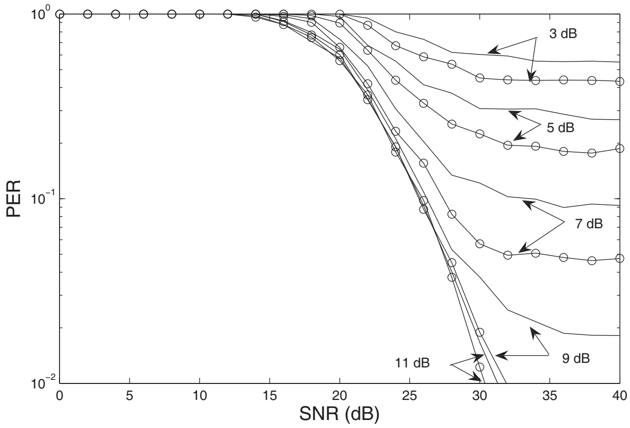

Fig. 4. Packet error rate (PER) versus SNR for an 802.11a WLAN system simulation for various values of the AGC back-off from 3 to 11 dB, without using any clip correction (lines) and using the simple clip correction algorithm (lines with circles).

In fig. 3 we have plotted the relative squared error $\sum_{m\in\mathcal{M}} |\hat{R}_m - R_m|^2/P$, where the sum runs over the non-empty subcarriers, (in dB) versus received signal power divided by the clipping threshold squared, P/C^2 (in dB), for SNR values of $10\,\mathrm{dB}$, $20\,\mathrm{dB}$ and $30\,\mathrm{dB}$. The lines give the uncorrected results, i.e. $\hat{R} = Y$, the lines with crosses give the results of the simple clip correction algorithm, i.e., $\hat{R} = \tilde{R}$ from (16), and the lines with circles give the results of the MMSE algorithm. Finally, the theoretical relative clipping error $W_{\text{total}}/N_{\text{s}}P$, see (10), without corrections (dashed line) and with up to $2M$ corrections (dashed line decorated with circles) are also shown.

First consider the curves for SNR $= 30\,\mathrm{dB}$. For low P, there is no clipping, so the error is determined by the noise η: in the non-empty subcarriers the signal-to-noise ratio is $30.8\,\mathrm{dB}$, where the additional $10\log_{10}(2048/1704) = 0.8\,\mathrm{dB}$ arises because the noise in the empty subcarriers is discarded. Without clip correction, the clipping causes the relative error $||\hat{R} - R||^2/P$ to rise above $-30\,\mathrm{dB}$ at $P/C^2 = -7\,\mathrm{dB}$, with simple clip correction this happens at $P/C^2 = -6\,\mathrm{dB}$, so the clipping level may be reduced by about $1\,\mathrm{dB}$. The MMSE algorithm performs better and allows a reduction of approximately $3\,\mathrm{dB}$. For higher values of P/C^2, the effectiveness of the simple clip correction algorithm decreases rapidly, whereas the MMSE algorithm remains effective. When SNR $= 20\,\mathrm{dB}$, the performance of both clip correction algorithms is reduced, for SNR $= 10\,\mathrm{dB}$, the use of any of our algorithms cannot be justified anymore Simulations, not presented here, for systems with fewer subcarriers show the same behavior if the fraction of empty subcarriers, M/N_{s} is approximately the same.

IV. Clip correction in an IEEE 802.11a simulation

In order to evaluate the effectiveness of the simple clip correction algorithm in a somewhat more realistic scenario, we introduced the correction (16) in the simulation code of an 802.11a WLAN. In the simulation we assume perfect synchronisation of transmitter and receiver, perfect channel estimation at the receiver and an ADC with enough bits so that only clipping needs to be considered. The transmitter sends 8000 bit packets at the highest data rate of $54\,\mathrm{Mb/s}$, the channel is modeled as having an exponential delay profile with RMS delay of $50\,\mathrm{ns}$. The automatic gain controller in the receiver scales the received signal power to $10^{-\mathrm{BackOffdB}/10}$, where BackOffdB is the back-off in decibel. Fig. 4 show the packet error rate (PER) versus the SNR for various values of the back-off, both with and without clip correction. The PER was obtained by simulating random channel and noise realizations until 100 packet errors were observed, hence the estimates of PER are accurate to about 10% at low PER, which explains the non-smoothness of the graphs al high SNR. Nonetheless, these graphs suggest that the simple clip correction algorithm allows for a reduction of back-off by about 1 dB at a SNR of 30 dB while delivering the same PER.

V. Concluding Remarks

The advantage of a clip compensation algorithm is that the design specifications of the ADC can be relaxed. We found that for systems operating at high SNR the clipping threshold can be reduced by up to 3 dB. So for a specified signal-to-quantization noise, one may use only 70% of the number of quantization steps needed hitherto. As the power consumption of an ADC is proportional to the number of quantization steps, this corresponds to a reduction of power consumption by the ADC by 30%. Nonetheless our solution demands more digital signal processing operations. We have developed a much simpler algorithm that gives less gain, about 1 dB, but does so for very little signal processing cost. Our proposed algorithms do not require any modification to existing OFDM standards.

Acknowledgments

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