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Dispersion of heavy particles in stably stratified turbulence

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The dispersion of heavy inertial particles in statistically stationary stably stratified turbulence is studied by means of direct numerical simulations. The following issues have been addressed: What distinguishes dispersion in such stratified flows from dispersion processes in statistically stationary homogeneous isotropic turbulence? How is the dispersion process affected by the Stokes number of the inertial particles ($0.1 \leq St = \tau_p/\tau_K \leq 10$, with $\tau_p$ the particle response time and $\tau_K$ the Kolmogorov time)? What is the interplay between buoyancy and the Stokes number? And what is the effect, if any, of particle settling, nonlinear drag, and lift forces (particularly relevant for stratified turbulence with its vertical shear layers) on particle dispersion? The long-time dispersion in isotropic turbulence is found to be maximum around $St=1$, in agreement with the observation of preferential concentration for $St=1$. In stably stratified turbulence such a maximum in the dispersion is only found for the horizontal direction. The horizontal and vertical dispersions in stably stratified turbulence show different behaviors due to the anisotropy of the flow, and in particular, vertical dispersion is strongly affected by the inertia of the particles. With increasing St the classical plateau found for vertical fluid particle dispersion becomes less pronounced and it even vanishes for Stokes numbers of $O(10)$ and higher. Furthermore, the long-time vertical dispersion increases with increasing St. The effects of gravity, nonlinear drag, and lift forces have been considered in more detail. It turned out that the settling enhancement of inertial particles, as observed in isotropic turbulence, is suppressed by stratification and by nonlinear drag effects. Moreover, nonlinear drag only affects the dispersion in the vertical direction in stably stratified turbulence. Finally, it is found that lift forces can safely be neglected for dispersion studies under the current parameter settings.

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I. INTRODUCTION

Particle dispersion plays an important role in industrial processes and in natural environments. A natural way to study the dispersion is from the Lagrangian point of view, in which the observer is moving along a particle trajectory. Inertial particles do not exactly follow the flow. When their density is larger than that of the surrounding fluid, they are transported out of the cores of vortical structures due to centrifugal forces. This leads to the so-called effect of preferential concentration; particles collect in regions of high strain rate and low vorticity and the resulting particle distribution is highly nonuniform. Inertial particles thus follow biased trajectories. For example, heavy particles that settle down in an isotropic turbulent flow are found to collect on the downward side of eddies. The dispersion of heavy particles, whose density $\rho_p$ is much larger than the fluid density $\rho_0$, in homogeneous isotropic turbulence has been a topic of interest for decades. Analytically, it has been studied by, among others, Tchen, Yudine, Csanady, and Reeks. Using grid-generated turbulence, Snyder and Lumley investigated the behavior of heavy particles experimentally in decaying isotropic turbulence. More recently, the topic has been studied by means of direct numerical simulations (DNSs). Elghobashi and Truesdell numerically reproduced the experimental results by Snyder and Lumley. Squires and Eaton examined heavy particle dispersion in both decaying and forced isotropic turbulence. These works show that the effect of particle inertia is to increase the eddy diffusivity over that of a fluid particle.

According to Taylor, the mean-squared displacement or single-particle dispersion of both fluid particles and heavy particles is a function of the rms velocity of the particle and of its velocity autocorrelation function. These two quantities have a competing effect on the dispersion of inertial particles. With increasing inertia the rms velocity of a particle decreases, but its memory (and thus the autocorrelation) increases. The overall effect of both terms on the dispersion behavior of inertial particles is not clear beforehand. In this work we will clarify the role of the two quantities in the different time regimes of heavy particle dispersion.

When gravitational forces act on the inertial particles a resulting mean drift velocity in the direction of gravity exists. This gives rise to the “crossing-trajectories” effect; heavy particles sink in more or less straight lines instead of following the flow, thereby spending less time within an eddy. The fluid neighborhood of a particle continuously changes and the correlation between the particle velocity and the velocity of the surrounding fluid is reduced. The excursions from the mean downward trajectory become smaller with increasing settling velocity and thus the particle disper-
sition is reduced in both the direction of gravity and in the lateral directions. Furthermore, it is found in most studies that the settling velocity of a heavy particle in an isotropic turbulent flow is enhanced compared to the Stokes settling velocity \(w_{st}\) in a quiescent fluid. This is due to the so-called preferential sweeping effect. Due to the interaction between the particles and the smallest turbulent flow structures clustering occurs (preferential concentration). This clustering of particles is found to be stronger on the downward side of vortical structures in the flow than on the upward side. The extent of this increased settling velocity depends strongly on parameters such as the particle time scale \(\tau_p\), the turbulence intensity (expressed using the Reynolds number \(Re_p\)), the particle Reynolds number \(Re_p\), the volume fraction of particles, the inclusion of particle-turbulence interaction (two-way coupling), and the relevance of the drift velocity \(w_p\) compared to velocity scales of the flow (vertical rms velocity \(w_{rms}\) or the Kolmogorov velocity \(v_k\)).

When the particle Reynolds number \(Re_p = |u - u_p| \rho_p / \nu\) is larger than 1, nonlinear drag effects may come into play. This occurs especially when a mean drift velocity is present. The nonlinear drag that acts on a particle is larger than the corresponding linear drag force. Hence, nonlinear drag effects have the opposite effect on the particle settling velocity as the preferential sweeping effect mentioned before; nonlinear drag diminishes the settling velocity in isotropic turbulence. The combined effect of the preferential sweeping, which enhances the settling velocity, and the nonlinearity of the drag force, which reduces the settling velocity, depends on properties of both the flow and the particles, such as \(Re_\lambda\) and \(Re_p\). Analytically and using Monte Carlo simulations Mei noticed that nonlinear drag forces reduce the settling velocity compared to \(w_{st}\). Small \(Re_p\), Stout et al. using a Markov-chain model to generate an isotropic turbulent flow, observed a decreased settling velocity already for \(Re_p = 1\) when using a nonlinear drag law. Most experimental and DNS studies measure increased settling velocities in turbulent flows. A small decrease compared to \(w_{st}\) was only found by Yang and Shy (experiments) for a small range of parameters (large \(Re_p\) and \(\tau_p\)) and by Wang and Maxey (DNS, using nonlinear drag) for their highest value of \(w_p\) and \(\tau_p\). Moreover, it might play a role that in numerical studies often one-way coupling is assumed, although Bosse et al. demonstrated that two-way coupling (which is more in line with experiments) enhances the settling velocity compared to one-way coupling.

In this work we study the dispersion of heavy particles in homogeneous stably stratified turbulence. Turbulent flows displaying stable density stratification are often encountered in nature, for example, the nocturnal atmospheric boundary layer, coastal areas, and lakes. In stably stratified flows a negative vertical density gradient is present; the average density of the fluid is decreasing with height. Strongly stratified flows typically display thin layers of large quasihorizontal vortical structures with strong shearing between these layers. Moreover, internal gravity waves are present in stably stratified flows. See, for example, the works by Riley and LeLong and Brethouwer et al. for an elaborate description of stably stratified turbulence.

The dispersion of fluid particles in decaying stably stratified flows is studied using DNS by Kimura and Herring and Liechtenstein et al. and in nondecaying stably stratified turbulence using kinematic simulations by Nicolleau and Vassilicos. In a previous paper we examined the dispersion of fluid particles in forced and therefore statistically stationary stably stratified turbulent flows. Fluid particles that are displaced from their original equilibrium height show a strong tendency to return to that equilibrium height due to a restoring buoyancy force. In the vertical direction, fluid particle dispersion in stably stratified turbulence is therefore reduced compared to that in isotropic turbulence. In isotropic turbulence the single-particle dispersion is proportional to \(t^2\) for short times and it has a slope proportional to \(t\) in the long-time limit. In stably stratified turbulence three successive regimes can be identified for the vertical mean-squared displacement of fluid particles: the classical \(t^2\) regime, a plateau which scales with the buoyancy frequency as \(N^2\), and a diffusion limit where the dispersion is proportional to \(O(t)\). In the horizontal direction the dispersion of fluid particles in stably stratified turbulence is similar to that in isotropic turbulence for short times, but for long times it is enhanced. In the long-time limit it is found to scale proportional to \(t^{2.1 \pm 0.1}\), larger than the classical linear diffusion limit.

In this work it will be discussed whether the dispersion of heavy particles in stably stratified turbulence displays the same characteristic scaling behavior as observed for fluid particles. Furthermore, the similarities and differences between the effects of inertia, particle settling, and nonlinear drag on particle dispersion in isotropic turbulence and in stratified turbulence will be examined.

The numerical method used in this work is introduced in Sec. II. Next, in Sec. III the heavy particle dispersion will be described. In Sec. III A the results for isotropic turbulence are discussed, which serve as a reference for the results obtained for stably stratified turbulence that are presented in Sec. III B. In Sec. IV the additional effects of gravity, nonlinear drag, and lift force will be considered. The influence of a mean sinking velocity resulting from gravity on the particle dispersion will be discussed in Sec. IV A. After that, in Sec. IV B it will be considered whether nonlinear drag effects need to be taken into account. The settling velocity of a heavy particle in a gravitational field is affected by turbulence and by nonlinear effects. This topic will come up for discussion in Sec. IV C. Finally, in Sec. IV D we will examine whether the strong vertical shear that is present in stratified flows causes lift forces to have a significant influence on heavy particle dispersion.

II. NUMERICAL METHOD

A. DNS of the Boussinesq equations

This study is performed by means of direct numerical simulations. A pseudospectral code is used that solves the full Navier–Stokes equations with Boussinesq approximation on a triple-periodic domain. An elaborate description of this code is given in Refs. 24 and 25.

Three different flows are studied: one homogeneous isotropic turbulent flow and two stably stratified flows, one
TABLE I. Properties of the three different flows. Case N0 is isotropic turbulence and cases N10 and N100 are moderately and strongly stratified turbulences, respectively. \( T_{E,3D} \) is the eddy turnover time of case N0. The values for \( k_{max} \eta \), with \( k_{max} \) the highest wavenumber resolved by the grid and \( \eta \) the Kolmogorov length scale, are a measure for the resolution.

<table>
<thead>
<tr>
<th>Case</th>
<th>( N ) (s(^{-1}))</th>
<th>Fr</th>
<th>( L_x/L_h )</th>
<th>( u_h/u_{rms} )</th>
<th>( w_{rms}/u_{rms} )</th>
<th>( T_E/T_{E,3D} )</th>
<th>Re(_h)</th>
<th>( k_{max} \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N0</td>
<td>0</td>
<td>—</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>85</td>
<td>1.13</td>
</tr>
<tr>
<td>N10</td>
<td>0.31</td>
<td>0.11</td>
<td>0.16</td>
<td>1.15</td>
<td>0.40</td>
<td>2.39</td>
<td>100</td>
<td>1.53</td>
</tr>
<tr>
<td>N100</td>
<td>0.98</td>
<td>0.04</td>
<td>0.08</td>
<td>1.21</td>
<td>0.16</td>
<td>2.02</td>
<td>170</td>
<td>1.27</td>
</tr>
</tbody>
</table>

moderately (case N10) and one strongly stratified (case N100). In the simulations of stably stratified turbulence a linear stable background density stratification is present, which is kept constant throughout a simulation. Density fluctuations are present on top of the linear profile. The total density of the fluid is given by \( \rho_f=\rho_0+\bar{\rho}(z)+\rho'_f(x,y,z,t) \) with \( \rho_0 \) a reference value, \( \bar{\rho} \) the time-independent background profile, and \( \rho'_f \) the fluctuations. The choice of a linear background profile implies a homogeneous stratification. The relative importance of the stratification can be expressed by the Froude number \( Fr=u_{rms}/(L_h N) \), with \( u_{rms} \) the root-mean-squared velocity, \( L_h \) the horizontal integral length scale, and \( N^2=-(g/\rho_0)\partial\bar{\rho}/\partial z \), the buoyancy frequency.

In order to reach statistically stationary turbulent flows, large-scale forcing is applied. A general description of the forcing method is given in Ref. 24. The forcing is applied in the three principal directions for isotropic turbulence and purely in the horizontal direction (velocity components \( u \) and \( v \), with vertical wavemode \( k_z=0 \)) for stably stratified turbulence. In this way inducing vertical fluid motion in the stratified flow by the artificial forcing is avoided; velocity fluctuations in the vertical direction are only created via nonlinear interaction with the horizontal velocity components. Purely horizontal forcing is chosen in order not to excite internal gravity waves.26 It has been tested for fluid particles that the degree of anisotropy of the flow can be derived from the ratios \( u_h/u_{rms}, w_{rms}/u_{rms}, \) and \( L_x/L_h \). The eddy turnover time \( T_E=L_h/u_{rms} \) is based on the horizontal integral length scale because in stratified turbulence the scales of the horizontal vortical structures are much larger than those of the vertical wavelike motions. A measure for the turbulence intensity is \( Re_h=u_{rms} \lambda/\nu \) with \( \lambda \) the Taylor length scale. Although stable stratification suppresses the turbulence, \( Re_h \) increases with increasing \( N \) due to an increase in the horizontal length scales.

To get an impression of the stratified flow field, the absolute value of the fluid velocity for case N100 is shown in Fig. 1 in a horizontal and a vertical cross section of the domain. Large horizontal structures and the layered vertical pattern can be identified.

The results presented in this work are derived from simulations with a resolution of 128\(^3\) to be able to track particles for very long times. However, as a check most cases are studied also at a higher resolution (256\(^3\)) and these simulations gave similar results for the time range that could be resolved at that resolution.

B. Particle tracking

When a statistically stationary flow field is obtained, particles are released at random positions in the domain. Next, their trajectories are calculated according to

\[
\frac{dx_p(t)}{dt} = u_p(t)
\]

together with the equation of motion

![FIG. 1. (Color online) Absolute value of the total fluid velocity for case N100 in (a) a horizontal and (b) a vertical cross section of the domain. The velocity is made nondimensional with \( u_{rms} \).](Image)
\[
\frac{du_p(t)}{dt} = \frac{1}{\tau_p} [u(x_p, t) - u_p(t)] - g + F_l.
\] (2)

Herein \(x_p(t)\) is the particle position, \(u_p(t)\) the velocity of the inertial particle, and \(u(x_p, t)\) the fluid velocity at the position of the particle (derived from the velocity field with use of cubic spline interpolation). The particle response time is \(\tau_p = d_p^3 (\rho_p / \rho_0) / 18 \nu\), with \(d_p\) and \(\rho_p\) the particle diameter and density, respectively. In the following the particle response time will be expressed using the Stokes number \(St = \tau_p / \tau_K\), with \(\tau_K\) the Kolmogorov time scale. The drag factor is given by \(\phi\) and \(F_l\) denotes the lift force, which will be discussed in detail in Sec. IV D. Equation (2), without the lift force, is a simplified version of the Maxey–Riley equation 28 in the limit of small \((d_p < \eta)\) and heavy \((\rho_p \gg \rho)\) rigid spherical particles with particle Reynolds number \(Re_p = |u - u_p|d_p / \nu \ll 1\). It takes into account drag and gravitational forces, successively. It has been tested that the other forces in the Maxey–Riley equation can be neglected for the particle parameters used in this work.29 When \(Re_p \ll 1\), Stokes drag can be assumed and \(\phi = 1\). For larger \(Re_p\) nonlinear drag effects need to be taken into account and \(\phi = 1 + 0.15Re_p^{0.87}\).2 For a particle in quiescent fluid—or Stokes settling velocity—resulting from gravitational forces is \(v_{st}\). It is defined as \(v_{st} = \tau_p g\) (Ref. 2) and it will be expressed by \(W = w_{st} / w_{rms}\), with \(w_{rms}\) the rms velocity of the fluid in the vertical direction. In order to be able to study particles with \(St = O(1)\) and \(W = O(1)\) simultaneously, a reduced gravitational acceleration \(g'\) acting on the particles is introduced instead of the normal \(g\) \((g' = g)\). Therefore, the Stokes settling velocity as used in this work is redefined as \(w_{st} = \tau_p g'\). The strength of the gravitational acceleration is denoted by \(g' = g' / g\). Both \(St\) and \(W\) are functions of \(\tau_p\) and they are related as \(W = f(g', St)\).

In each of the three flows \(N_p = 20,000\) particles are tracked for about 40 eddy turnover times for several particle response times. Particle diameters of order 0.1 \(\eta\) are chosen and their density ratio is kept fixed at \(\rho_p / \rho = 13,500\) throughout a simulation. This density ratio, which is higher than found in most practical applications, is chosen to obtain a desired particle response time while keeping \(d_p < \eta\). The exact value of \(\rho_p / \rho\), however, is not of importance for the results presented here, as long as \(\rho_p / \rho \gg 1\). The particle response time is adapted by changing the diameter and the range of investigated Stokes numbers is \(O(0.1) - O(10)\). The effect of a mean particle settling velocity is studied for cases N0 and N100. In these simulations three different values for \(g'\) are chosen such that the range of investigated values for \(W\) is \(O(0.1) - O(10)\).

Since we are using small particles \((d_p < \eta)\) and low particle volume fractions, the influence of the particles on the flow field and particle–particle interactions are assumed negligible and the system is called one-way coupled. The final dispersion results are insensitive to the choice of the initial particle conditions. For faster convergence we set the initial velocities of the particles equal to the local fluid velocity and after about two \(T_E\) the particles are completely adapted and reach a quasisteady distribution. From this time on the strength of the effect of preferential concentration remains constant. The calculation of statistical quantities starts only after this initial transient.

### III. PARTICLE DISPERSION

#### A. Heavy particle dispersion in isotropic turbulence

Single-particle dispersion, or mean-squared displacement, is given by Taylor’s relation

\[
\overline{X^2_{p,i}}(t) = 2u_{p,i}^2 \int_0^t (t-\tau) R_{L,i}(\tau) d\tau,
\]

(3)

with \(X_{p,i}(t) = x_{p,i}(t) - x_{p,i}(0)\).\(^{5,10}\) Here, \(u_{p,i}\) is the fluctuating component of the particle velocity in the \(i\) direction \((i \in \{x, y, z\})\). The Lagrangian velocity autocorrelation function is denoted by \(R_{L,i}(\tau)\), which is only a function of the time separation \(\tau = t - i'\) since the flow is stationary. Using the average value of the autocorrelation function over the three spatial components, \(R_L(\tau)\), the Lagrangian time scale \(T_L = \int_0^\infty R_L(\tau) d\tau\) can be computed.

The two quantities \(\overline{X^2_{p,i}}\) and \(R_L,\) in Eq. (3) have an opposite effect on the dispersion of heavy particles. With increasing inertia the rms velocity of a particle decreases, but its memory—expressed using the autocorrelation function—increases. Whereas Squires and Eaton\(^9\) reported that single-particle dispersion increases for increasing particle inertia (except for their largest \(\tau_p\)), He et al.\(^{30}\) found an optimum around \(\tau_p / \tau_K = 1\). In a microgravity environment, Groszmann and Rogers\(^{31}\) experimentally studied the effect of inertia on heavy particle dispersion. They reported a decreasing dispersion with increasing \(\tau_p\), but their smallest Stokes number is 0.9, around the optimum seen by He et al.\(^{30}\) Obviously, no complete agreement is reached in the literature as to the effect of particle inertia on heavy particle dispersion.

In Fig. 2(a) the mean-squared displacement as a function of time is presented for our simulations of statistically stationary isotropic turbulence for particles with different Stokes numbers. For the moment in Eq. (2) only the drag force is taken into account in order to study the pure effect of inertia on particle dispersion. The influence of a mean settling velocity or a lift force will be discussed in subsequent sections. The dispersion is computed from the position time series of the particles, according to

\[
\overline{X^2_{p,i}}(t) = \frac{1}{N_p} \sum_{p=1}^{N_p} (x_{p,i}(t) - x_{p,i}(0))^2,
\]

(4)

with \(N_p\) the number of particles, and then averaged over the three principal directions. The corresponding plot of the long-time limit of the particle diffusivity \(D_i(t) = (1/2) \times (d/dt) \overline{X^2_{p,i}}(t)\) as a function of the Stokes number is given in Fig. 2(b). Several conclusions can be drawn from the results. We retrieve the classical \(T^2\) and \(t\) regimes also for heavy particle dispersion in isotropic turbulence.\(^{8,10}\) By zooming in at very short time scales [inset in Fig. 2(a)], all the graphs are found to collapse for \(T^2 / T_E \leq 0.3\) when rescaled with the rms velocity of the particles instead of with the rms velocity of the fluid. In this short-time range \(R_L(\tau) \approx 1\) and the mean-
squared particle velocity is thus the term that determines the dispersion behavior, which is proportional to $t^2$.

For longer times the memory effect of the particles comes into play. The autocorrelation function, and thus the memory, is found to increase with increasing inertia. When the effect of the particle rms velocity is filtered out by rescaling Fig. 2(a) with $u_p^{rms}$ instead of with $u_p^{2rms}$, indeed for long times the dispersion increases with increasing St. The overall dispersion result in the long-time limit (when scaled with $u_p^{rms}$) is thus a combination of both the particle rms velocity and the autocorrelation function and it is found to be maximum around St=1. The dispersion in Fig. 2(a) is largest for St=0.67 and St=0.96 in the long-time limit. The dispersion optimum around St=1 becomes also clear from the graph of the long-time diffusivity $D_t(\infty)$, as plotted in Fig. 2(b).

The relative error in the results can be estimated from the number of particles, scaling according to $N_p^{1/2}$; or from the differences between the x, y, or z dispersion and the average dispersion. The latter values are larger and they give an estimated error of the order of 5% for the investigated Stokes numbers. However, the trend that the long-time dispersion increases with St for small St, that it reaches an optimum around St=1, and that it decreases for larger St is the same for the three spatial directions. Therefore, the value of 5% for the relative error most likely overestimates the uncertainty in the results. The difference between the dispersion at St=0.05 or St=4.83 and that at St=0.96 is of the order of 10%, after the initial period in which the particle rms velocities are dominant. The maximum value of the dispersion around St=1 is thus significant.

This conclusion of maximum dispersion around St=1 is in agreement with the result obtained by He et al., and presumably also with the results of Squires and Eaton. They observed the same effect, but because they only have one measurement for $\tau_p$ larger than the peak value they drew a different conclusion. Our results also agree with the large Stokes number results obtained by Groszmann and Rogers.

Not only the dispersion of heavy particles in isotropic turbulence displays a maximum for Stokes numbers around 1 but also for the effect of preferential concentration an optimum is found around St=1. A connection between the two maxima might be explained as follows. For Stokes numbers of order 1 the heavy particles do not remain trapped within vortices and, moreover, they collect in regions where high strain rates enhance their dispersion.

### B. Heavy particle dispersion in stably stratified turbulence

The dispersion of heavy inertial particles in statistically stationary stably stratified turbulence has not been reported in literature. Accordingly, our results will be compared with the results found for fluid particles in stratified turbulence and for heavy particles in isotropic turbulence.

For stratified turbulence the mean-squared displacement as a function of time is shown for cases N10 and N100 in Figs. 3–5. The horizontal dispersion is expressed as $X_p^{2h} = \frac{1}{2} \left( X_p^{2x} + X_p^{2y} \right)$, where $X_p^{2i}$ ($i \in \{x,y\}$) is calculated according to Eq. (4). Apart from the results for heavy inertial particles also the results for fluid particles in both stratified and isotropic turbulences are included for reference. The results for the cases N10 and N100 show qualitatively the same behavior. For horizontal dispersion the classical short-time ballistic $t^2$ regime is retrieved. This ballistic regime is plotted in the insets of Figs. 3(b) and 4(b), rescaled using the horizontal rms velocities of the particles $u_p^{rms}$ instead of the horizontal rms velocity of the fluid. Similar to what is found in isotropic turbulence, in this short-time regime the dispersion is fully determined by the particle rms velocity, of which the magnitude is decreasing with increasing St. The collapse of the different graphs seems to persist longer for the horizontal heavy particle dispersion in stratified turbulence (especially for case N100) than in isotropic turbulence, where the graphs start to separate at $t/T_L=0.3$. This can be related to the increased eddy turnover time in stratified turbulence compared to isotropic turbulence (note that the graphs in Figs. 3 and 4 are scaled using the Lagrangian time scale $T_L$ for fluid particles in isotropic turbulence.

For long times it can be seen that the dispersion of heavy particles in stably stratified turbulence is clearly enhanced compared to the dispersion in isotropic turbulence. Instead of the linear diffusive regime we obtain a long-term scaling proportional to $t^{2.0\pm0.1}$ [Figs. 3(a) and 4(a)]. This is consistent
with the results previously described for fluid particle dispersion in stratified turbulence (see Ref. 24), where a scaling proportional to \( t^{2.1 \pm 0.1} \) is reported. In strongly stratified turbulence the flow locally resembles shear flow. This strong local vertical shear causes the enhanced horizontal dispersion in stratified turbulence. When looking in more detail at the final part of our measurement range, shown in the insets in Figs. 3(a) and 4(a), different results are obtained for cases N10 and N100. For case N100 the same trend is found as for heavy particle dispersion in isotropic turbulence; it is maximum at \( \text{St} \approx 1 \). For case N10, however, the dispersion is found to decrease with increasing Stokes number for the whole range of investigated Stokes numbers (the origin of this behavior for case N10 is not completely understood).

Unfortunately, the alternative tool to quantify dispersion, the eddy diffusivity \( D_e \), cannot be used. Since the slope of \( X_{p,h}^2 / w_{r,h}^2 \) is larger than proportional to \( t \), the diffusivity \( D_e \) does not reach a constant value and cannot easily be used to draw a conclusion about the maximum dispersion for stratified turbulence, as was done for isotropic turbulence with use of Fig. 2(b).

The fact that a dispersion maximum around \( \text{St} = 1 \) is less clear in stratified turbulence than in isotropic turbulence can be related to the effect of preferential concentration. The correlation between maximum dispersion and preferential concentration was explained for isotropic turbulence in Sec. III A. It is shown in a previous study that the effect of preferential concentration is reduced in stratified turbulence compared to isotropic turbulence.\(^3\) Therefore, a weaker impact of the variation of \( \text{St} \) is expected for horizontal particle dispersion in stratified turbulence.

Also for vertical heavy particle dispersion the moderately (case N10) and strongly (case N100) stratified turbulent flows show the same trend. As can be seen in Fig. 5 the dispersion again starts with the classical \( t^2 \) regime. In this ballistic regime the dispersion decreases with increasing \( \text{St} \) due to a decreasing vertical particle rms velocity. When rescaled with the particle rms velocities (not shown) as in Figs. 3(a) and 4(a) again a collapse is found, although for shorter times than in isotropic turbulence or for horizontal dispersion in stratified turbulence. The strong influence of the rms velocities on the vertical dispersion only lasts for about \( t/T_L = 0.15 \).

After the initial \( t^2 \) regime the vertical dispersion is inhibited for all Stokes numbers. For fluid particle dispersion now a plateau is found around \( tN/2 \pi = 1 \) (corresponding to \( t/T_L = 0.5 \)), which scales as \( w_{rms}^2 / N^2 \). For heavy particle dispersion in stratified turbulence this typical plateau becomes less pronounced for higher \( \text{St} \) and eventually even vanishes. In
this intermediate time regime fluid particles feel a restoring buoyancy force and they oscillate around their initial equilibrium height. With increasing Stokes number the inertial forces become larger than this restoring force. As a consequence, particles overshoot and they keep on following their initial trajectory more and more as St increases. Furthermore, the onset of the plateau shifts toward larger times with increasing St.

The transition toward a final linear regime is found for all Stokes numbers. In this long-time regime, as opposed to the short-time behavior, vertical dispersion increases with increasing inertia. For fluid particles the origin of this linear scaling behavior is molecular diffusion of the active scalar (density), which is only observed for statistically stationary stably stratified turbulence.\(^\text{24,30}\) Heavy inertial particles with very small Stokes numbers have a fixed density, but they closely follow the flow. The density change in the fluid surrounding these particles due to the molecular diffusion therefore also affects their equilibrium height resulting in long-time diffusive behavior alike that observed for fluid particles.\(^\text{24}\) This is verified by repeating the simulation for case N100 with Sc=7 instead of Sc=1. For small Stokes numbers (St<1) indeed the final diffusive regime sets in at a later time for Sc=7 than for Sc=1 (cf. Ref.\(^\text{24}\)). Particles with higher inertia react slowly to fluctuations in the fluid velocity. Therefore, the reason for reaching a diffusive long-time regime is now different than for fluid particles. As in isotropic turbulence, heavy, high-inertial particles become uncorrelated from their initial vertical position which results in Brownian-like behavior with a vertical mean-squared displacement proportional to \(\mathcal{O}(t)\).

The velocity autocorrelation function shows the expected increasing memory effect for increasing St. Compared to the fluid particle results the horizontal autocorrelation increases with increasing St, similar to what is found for isotropic turbulence (see, for example, Ref.\(^\text{9}\)). The vertical autocorrelation function is plotted in Fig. 6 for cases N10 and N100. It can be seen that with increasing Stokes number less oscillating behavior is present for the inertial particles; the amplitude decreases and the period increases. For Stokes numbers larger than about St=5 the oscillations are almost absent; in this range the inertial forces are clearly stronger than the restoring buoyancy force. This Stokes number range corresponds to the range where the plateau in the vertical dispersion plot has vanished.

IV. GRAVITY, NONLINEAR DRAG, AND LIFT FORCE

A. Effect of a mean drift velocity on dispersion

Since in stably stratified turbulence gravity acts on the fluid, it is relevant to study the effect of gravitational forces on the particles too [one but last term on the right-hand side in Eq.\(^\text{(2)}\)]. It results in a mean drift velocity in the direction of gravity. This mean drift velocity gives rise to the crossing

![Graph](image-url)
trajectories effect mentioned in Sec. I. It reduces the heavy particle dispersion in isotropic turbulence in both the horizontal and the vertical direction. In isotropic turbulence this effect is especially evident for high Stokes number particles or high drift velocities.

1. Horizontal dispersion

First we will elucidate the dispersion behavior of heavy particles in the horizontal direction—the direction perpendicular to the drift velocity in the case of gravitational effects. As explained before, there are two parameters determining the overall dispersion behavior: the root-mean-squared velocity and the velocity autocorrelation function. In this work we find that a mean settling velocity decreases the horizontal particle rms velocity not only for isotropic turbulence (case N0) but also for strongly stratified turbulence (case N100). This decrease in the horizontal rms velocity with increasing drift velocity is mainly found for large Stokes numbers: St \( \geq 1 \) for case N0 and St \( \geq 5 \) for case N100 at \( W=1 \). For higher drift velocities the decrease is already found at smaller Stokes numbers. Increasing St or W reduces the ability of a particle to react to fluctuations of the fluid velocity.

The horizontal velocity autocorrelation decreases too with increasing W. This can be seen in Fig. 7, where the horizontal autocorrelation functions are plotted for cases N0 and N100. The settling particles continuously experience a new surrounding velocity field, which makes them lose the correlation with their previous horizontal velocity. Contrary to the previous results for W=0, where the particle rms velocity and the autocorrelation function displayed a counteracting effect when St was increased, here they both show the same—and thus amplifying—behavior with increasing W (while keeping St fixed).

The study of the effect of a mean settling velocity on particle dispersion in periodic computational domains requires special care under certain conditions. For large W (W \( \geq 1 \) for case N0 and W \( \geq 0.5 \) for case N100) the autocorrelation functions for both isotropic and stratified turbulences show strong oscillations. These oscillations are a consequence of tracking particles that repeatedly cross the domain in the vertical direction. For example, the period of the oscillations for W=6.93 in Fig. 7(a) can be associated with the time it takes for a particle to cross the domain vertically. Similar observations have been reported by Squires and Eaton, Elghobashi and Truesdell, and Snyder et al. (for rising bubbles in isotropic turbulence). Dispersion data can be extracted provided that the numerical data are carefully processed.

Horizontal single-particle dispersion in the presence of a mean vertical drift velocity is shown in Fig. 8 for different drift velocities for cases N0 and N100. The results are shown for St=0.96 and St=3.09, respectively, where the effect of preferential concentration is found to be maximum when
$W=0$ (see Ref. 33). As expected from the decreased rms velocity and autocorrelation function, for isotropic turbulence indeed a decreasing horizontal dispersion can be seen with increasing $W$, in correspondence with literature. Furthermore, it can be observed that the general dispersion behavior does not change; initially the slope in Fig. 8(a) is proportional to $t^2$ and the long-time regime scales as $O(t)$. Also for stably stratified turbulence a decrease in the horizontal dispersion is obtained with increasing drift velocity. However, here the long-time behavior differs considerably from that found for $W=0$. The slope becomes smaller than the $O(t^2)$ behavior found for $W=0$, and it even becomes smaller than proportional to $t$. For the larger values of $W$ it approaches a constant asymptotically. Since the particles do not stay within horizontal slabs as for $W=0$, the shear effect that caused the enhanced horizontal dispersion in stably stratified turbulence becomes ineffective. The particles continuously enter new layers at different heights. Roughly sketched, the horizontal fluid velocity in these layers is alternately positive and negative. For high fall velocities the particles spend a limited time in a certain layer and thus their direction of motion in the horizontal plane changes frequently. A simple test with particles falling through alternating layers of uniform flow results in a long-time horizontal dispersion proportional to $t^3$. When fluctuations are present on top of the mean, uniform flow this dispersion increases. Depending on the strength of the fluctuations the long-time dispersion grows as $t^3$ with $0 < \alpha < 1$. It is therefore anticipated that also the horizontal mean-squared displacement in strongly stratified turbulence for particles with large $W$ scales like $t^\alpha$ with $0 < \alpha < 1$, in agreement with our observations.

The results obtained for Stokes numbers other than those presented in Fig. 8 are very similar both for isotropic turbulence and for strongly stratified turbulence. For all $St$ in the studied range the graphs for case N0 deviate clearly from the case without a mean drift velocity when $W \approx 1$. For case N100 the influence of a mean drift velocity on the horizontal dispersion is observed for lower values of $W$. Already around $W = 0.1$ the graphs start to depart from the $W=0$ results. This difference between the values of $W$ at which the effect of the drift velocity becomes apparent can be related to the smaller vertical length scales that are present in stably stratified turbulence.

### 2. Vertical dispersion

In the vertical direction the rms velocity of the heavy particles in isotropic turbulence decreases with increasing $W$ too. For stratified turbulence, however, the vertical rms velocity of the particles is already small for $W=0$ and changes with increasing $W$ are almost negligible.

For isotropic turbulence the memory of the particle, expressed using the vertical velocity autocorrelation function, shows the same trend as for case N0 in the horizontal direction. It decreases with increasing $W$, but this decrease is less strong than for the horizontal velocity autocorrelation function. The reduced vertical autocorrelation for $W \neq 0$ is a result of the crossing-trajectories effect; particles continuously enter new regions in the flow. For stably stratified turbulence a different tendency is found for the autocorrelation function in the vertical direction. Now the autocorrelation hardly shows a dependence on $W$, except for the largest values of $W$ [$W=0.69$ in Fig. 9(a) and mainly $W=3.2$ in Fig. 9(b)]. Here, the autocorrelation is increased compared to the case where $W=0$. Due to the mean drift velocity a particle becomes less susceptible to the restoring buoyancy force exerted by the fluid on the particle. The fast drop in the autocorrelation function for $W=0$, that results from the wavelike motion of the particle around its equilibrium height, thus diminishes. A particle can follow its trajectory (downwards) for longer times whereby its velocity remains correlated with its previous velocity. This increased memory effect is similar to that encountered at $W=0$ when the Stokes number is increased (cf. Sec. III B).

The combined effect of the particle rms velocity and the autocorrelation function determines the dispersion behavior. For the vertical dispersion in isotropic turbulence a similar trend is expected as is found for the horizontal direction: decreasing vertical dispersion with increasing $W$. For stratified turbulence the prediction is mainly based on the behavior of the autocorrelation function, and thus the dispersion is likely to increase with increasing $W$ (provided that $W$ is larger than about 1).

For the calculation of the vertical dispersion of heavy particles in the presence of a mean drift velocity, an adapted version of Eq. (4) will be used.
and they do not remain trapped within a wavelike motion. Besides, for the particles with higher inertia the influence of the restoring buoyancy force, exerted by the fluid on the particles, is reduced just as for the zero-gravity case.

B. Effect of nonlinear drag

The linear Stokes drag law is only valid for particle Reynolds numbers $Re_p$ much smaller than 1. Especially for the simulations in which a large mean drift velocity is present, this requirement is not fulfilled and a validation of the use of a linear drag law is necessary. For larger $Re_p$ nonlinear drag effects may come into play and the $\phi$ in Eq. (2) is now replaced by the empirical drag law\(^7\)

$$\phi = 1 + 0.15 Re_p^{0.687}. \quad (6)$$

Not much work is reported about the difference between the use of a linear or a nonlinear drag law in dispersion studies. The available work mainly focuses on the mean settling velocity, a topic that will be discussed in Sec. IV B.

The introduction of the nonlinear drag law effectively leads to a decreased effective particle time scale $\tau_{p, eff} = \tau_p/\phi^2$. A particle with a certain $\tau_p$ can thus adapt more easily to fluctuations in the fluid velocity when the nonlinear drag law is used. This would result in a smaller relative velocity between the particle and the fluid than in the case of linear drag and thus in a smaller $Re_p$. For both isotropic and stratified turbulence the average particle Reynolds numbers $Re_p$ are computed for runs in which a linear and a nonlinear drag law are used. For the range of $St$ and $W$ presented in this work, $Re_p \leq O(1)$. Although $\phi$ differs for linear and nonlinear drag, $Re_p$ is only found to be changed by the use of a nonlinear drag law for settling particles in isotropic turbulence with $W>1$. It is indeed reduced, for example, in the run with $St=0.96$, $W=6.93$ [cf. Figs. 8(a) and 10(a)] by a factor of about 2 from $Re_p=1.2$ to $Re_p=0.65$. Since the total drag force is a function of both $|u-u_p|$ (which decreases for nonlinear drag) and $\phi$ (which increases for nonlinear drag), the overall effect of the use of the nonlinear drag law is not clear beforehand. For all parameters presented in this work, the drag force on the particles is found to be larger when the combinations of $\phi$ and $|u-u_p|$ (and thus $Re_p$) obtained for nonlinear drag are used.

FIG. 10. Vertical mean-squared displacement calculated according to both Eqs. (4) and (5) (denoted with an asterisk) for (a) case N0 and (b) case N100. The Stokes numbers are the same as in Fig. 8: $St=0.96$ and $St=3.09$, respectively.
The effect of the type of drag law on the dispersion of heavy particles is found to be small for the parameter range investigated here.

In isotropic turbulence it is found to be negligible for $W=0$. When a mean drift velocity is present the dispersion is altered by the use of the nonlinear drag law. For large settling velocities it is hard to draw a conclusion because of the wobbling behavior of the graphs [see Fig. 10(a), for $W=6.93$] but for $W=1$ the horizontal and vertical long-time dispersions are slightly increased (a factor of about 2%–3%) when the drag force is changed from linear to nonlinear. This increase might be related to a decrease in the mean settling velocity (see Sec. IV C) in the case of nonlinear drag. A lower settling velocity makes the particles more susceptible to local turbulent fluctuations.

In stratified turbulence only the dispersion in the vertical direction is found to be changed by the introduction of the nonlinear drag law. For Stokes numbers around 5 and $W=0$, where $Re_p<0.5$, the long-time vertical dispersion resulting from nonlinear drag is found to be smaller by about 7% than for the corresponding runs with linear drag. This decreased long-time vertical dispersion can be explained by means of the effective particle time scale $\tau_{p,eff}$. This $\tau_{p,eff}$ is lower than the corresponding $\tau_p$ for linear drag and, as shown in Sec. II B, a decrease in $\tau_p$ (or St) results in less vertical dispersion in stably stratified turbulence.

The decrease in the vertical heavy particle dispersion in stratified turbulence for nonlinear drag is also observed when gravitational forces act on the particles ($W \neq 0$). However, the effect is less clear, and it occurs only for large settling velocities and large St. The influence of gravitational forces thus dominates the influence of nonlinear drag effects for stably stratified turbulence.

The type of drag law that is chosen in dispersion studies with St and $W$ between $O(0.1)$ and $O(10)$ has thus only a small effect on the particle dispersion. Only for vertical dispersion in strongly stratified turbulence, with $W=0$, is a significant effect observed. However, neglecting the nonlinear drag in dispersion studies would lead to the same conclusions regarding the trend with St and/or $W$. The influence of the nonlinear drag on preferential concentration in stratified turbulence was also found to be negligible in a previous study.33

C. Mean settling velocity

As mentioned in Sec. I, several studies are performed on the influence of turbulence on the settling velocity of heavy particles in isotropic turbulence. Several values are reported in the literature for the increase in the settling velocity in isotropic turbulence, which depend among others on $Re_p$ of the flow.2,11 Moreover, the results obtained from numerical studies and from experiments differ. This is, at least partly, due to the commonly used assumption of one-way coupling in DNS studies. As shown by Bosse et al.,10 the settling velocity in the case of two-way coupling is increased compared to one-way coupling, and this stems from a collective effect of the particles. The particles, accumulated by the effect of preferential sweeping, accelerate the carrier fluid due to particle drag and this enhanced downward fluid motion, in turn, leads to a larger particle settling velocity in these regions. Therefore, also the volume fraction of the particles is a parameter of importance for experimental studies and for studies including two-way coupling. Since we assume that the influence of the particles on the flow can be neglected (one-way coupling), our DNS results might underpredict the settling velocity compared to corresponding experiments.

For all the runs performed with $W \neq 0$ the increase or decrease in the mean settling velocity, expressed as $(\bar{w}_p - w_{\text{st}})/w_{\text{st}}$, is plotted in Fig. 11 as a function of both St and $W$. It can be seen that the relative difference between the mean settling velocity and the settling velocity in a quiescent fluid decreases with increasing St or with increasing $W$, except for case N100 with linear drag. For that case $(\bar{w}_p - w_{\text{st}})/w_{\text{st}}$ remains more or less constant with St or $W$.

For isotropic turbulence and using linear Stokes drag, the mean settling velocity $\bar{w}_p$ is larger than the Stokes settling velocity and the relative increase can reach values of 30%–40%. For large St or large $W$ the difference approaches...
zero. This trend is in agreement with previous studies and also the values found for the relative increase are consistent with the literature values.\textsuperscript{2,11}

For the whole range of St and W studied here and at similar values of St and W, we obtain much smaller differences between \( w_p \) and \( w_{st} \) in stratified turbulence than in isotropic turbulence. The increase in the mean settling velocity compared to the Stokes settling velocity in stratified turbulence fluctuates between 0.7\% and 2.0\%. No clear trend is visible and the differences are small, so it is hard to draw firm conclusions about any dependency on St or W.

Considering nonlinear drag, it can be seen in Fig. 11 that for all cases the mean settling velocities derived using the nonlinear drag law are smaller than the corresponding values derived from linear drag. This was to be expected, since the stronger nonlinear drag force causes the particles to sink more slowly. For a considerable part of the results derived using the nonlinear drag law we even find a mean settling velocity that is smaller than the Stokes settling velocity. From Fig. 11 it can also be deduced that the effect of nonlinear drag on the mean settling velocity becomes important mainly for large St and/or large W [larger than \( O(1) \)], or correspondingly for large Re\(_p\).

Our results for isotropic turbulence are in agreement with the results obtained by Mei\textsuperscript{15} who reported a reduction in the settling velocity, which for large Re\(_p\) can even become smaller than \( w_{st} \) when a nonlinear drag law is used. Compared with experimental studies it only agrees with the work by Yang and Shy,\textsuperscript{17} who obtained a mean settling velocity smaller than \( w_{st} \) for their largest values of Re\(_p\) and \( \tau_p \).

For stably stratified turbulence, as opposed to the linear case, we find \( w_p < w_{st} \) when nonlinear drag effects are taken into account, even for small W. The reduction depends on St or W (or Re\(_p\), \( \tau_p \)), but the dependency is weaker than for isotropic turbulence. An explanation of the different influence of nonlinear drag on the settling velocity in isotropic turbulence and in stratified turbulence is the following. Even though for both flows the ranges of St and W are similar, the absolute value of the settling velocity in stratified turbulence is much smaller than in isotropic turbulence due to the smallness of the vertical rms velocity (about a factor of 10 smaller than in isotropic turbulence). The influence of the vertical component of the drag force is therefore also much smaller in stratified turbulence.

### D. Effect of the lift force

There are two sources for lift forces on a particle or a droplet: the Magnus force produced by a rotating particle (particle spin) and the Saffman force which occurs when the particle is placed in a flow with local shear.\textsuperscript{17} In this work it is assumed that the individual particles do not have spin. Maxey and Riley,\textsuperscript{28} neglected the Saffman lift force because of their small particle, low Reynolds number assumption. As described in Sec. I, in strongly stratified turbulence the local shear can be considerable. Therefore the Saffman lift force is expected to play a more prominent role there than in isotropic turbulence. The velocity gradients \( \partial u / \partial z \) and \( \partial v / \partial z \), which both give rise to a lift force in the vertical direction, are larger than the other components of the velocity gradient tensor by a factor of about 10. Therefore, only the vertical component of the lift force is incorporated. The lift force per unit mass \( F_l \) is given by

\[
F_l = \frac{3}{2} \frac{C_L}{\pi \varepsilon \rho_p} \left| \mathbf{v} \right|^{-1/2} \left[ (\mathbf{u} - \mathbf{u}_p) \times \mathbf{v} \right],
\]

with \( C_L \) a lift coefficient and \( \mathbf{v} = \nabla \times \mathbf{u} \) the vorticity. Saffman’s equation for the lift force is derived under the assumption that Re\(_p < Re^{1/2}_G\), where

\[
Re_G = \frac{G \mu_G}{\nu}
\]

is the shear Reynolds number with \( G \) the velocity gradient.\textsuperscript{38,39} Since the most important velocity gradient in stratified turbulence is the vertical shear of the horizontal velocity components, \( G \) is here defined as

\[
G = \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right]^{1/2}
\]

In some test runs it is found, however, that Re\(_p\) can have values up to one order of magnitude larger than Re\(_G^{1/2}\) during a considerable part of a simulation and thus the requirement Re\(_p < Re^{1/2}_G\) is not fulfilled in our simulations. Therefore, in the lift coefficient \( C_L = 6.46K \) the correction factor \( K \) is included. This \( K \) is given by\textsuperscript{40,41}

\[
K = \left\{ \begin{array}{ll}
[ -32 \pi^2 \varepsilon \ln(\varepsilon^{-3})]/2.255, & \varepsilon = 0.025, \\
[1.418 \arctan(2.8 e^{2.44})]/2.255, & 0.025 < \varepsilon \leq 20, \\
[2.255 - 0.6463 \varepsilon^{-2}]/2.255, & \varepsilon > 20,
\end{array} \right.
\]

Only the vertical component of the lift force is considered here because the strongest shear takes place between the different horizontal layers. Therefore only the difference between the fluid velocity and the particle velocity in the horizontal direction is included in Eq. (11). The derivation of this correction factor \( K \) is given by McLaughlin.\textsuperscript{41}

To study the effect of the lift force on heavy particle dispersion in stably stratified turbulence, three identical simulations are performed. For this purpose 4000 particles with St=3.09 are released in a case N100 flow. The first simulation is a reference run without the lift force; the particle equation of motion that is solved is the heavy particle limit given in Eq. (2), with \( g \) and \( F_l \) equal to zero. In the second run, the Saffman lift force (\( K=1 \)) is included additionally. The Saffman lift force is valid only for Re\(_p < 1 \) and Re\(_p < Re^{1/2}_G\).\textsuperscript{38,39} The first requirement is fulfilled for the St = 3.09 particles in this stratified flow, but the second requirement is not met for all the particles or at all time steps. The third run therefore solves the same equation, but now the correction factor \( K \) as given in Eq. (10) is taken into account.

Probability density functions of the ratio of the lift force and the vertical component of the drag force are depicted in
Fig. 12 for both the Saffman lift force and for the corrected lift force as proposed by McLaughlin.\textsuperscript{41} By comparing the horizontal axes of both graphs it can be seen that the relative importance of the lift force in the representation of Saffman is larger than that of the corrected lift force. This is consistent with the results of McLaughlin,\textsuperscript{41} who studied a particle settling in a linear shear flow. He reported that Saffman’s formula overestimates the magnitude of the migration velocity, and the error can be up to 25%.

For the Saffman lift force 17% of the data points presented in Fig. 12(a) has an absolute value larger than 0.1, and thus the lift force might be of importance. The corrected lift force, however, is small compared to the drag force. Only 6% of the data points have an absolute value larger than 0.1 and for almost 90% of the data points the relative importance of the lift force is less than 1/20.

Next to the forces themselves, also the influence of the two versions of the lift force on heavy particle dispersion is studied. The results obtained with inclusion of the lift force are compared to the results from the reference run. No clear differences can be observed between the results from the three simulations. Based on these results, it can be concluded that the lift force can safely be neglected for heavy particle dispersion in stably stratified turbulence, at least for Stokes numbers of order 1.

**V. CONCLUDING REMARKS**

The numerical study of heavy particle dispersion in forced isotropic and stably stratified turbulence is reported. A simplified version of the Maxey–Riley equation, including drag forces and gravity, is used as the equation of motion for the heavy particles.

The dispersion results in isotropic turbulence corroborate the findings of He et al.,\textsuperscript{30} that the long-time dispersion is maximum around $St=1$. For short times the dispersion behavior is purely determined by the particle rms velocity, whereas for longer times it results from a combination of the particle rms velocity and its velocity autocorrelation function.

In stably stratified turbulence the horizontal dispersion of heavy particles is comparable to that of fluid particles. In the long-time limit it is increased compared to isotropic turbulence and the mean-squared displacement is found to scale proportional to $t^{2.0\pm0.1}$. For large Stokes numbers the effect of inertia on the long-time horizontal heavy particle dispersion is the same as in isotropic turbulence; it decreases with increasing $St$. For Stokes numbers smaller than $O(1)$ no final conclusion regarding the relation between the horizontal dispersion and $St$ can be drawn from the present results. Either the horizontal dispersion shows a maximum around $St=1$ (case N100) or it decreases with increasing $St$ for all Stokes numbers (case N10).

The influence of the particle’s inertia on vertical dispersion in stratified turbulence, however, is clearly discernible. Increasing the particle’s inertia results in a less pronounced plateau which even vanishes for Stokes numbers of $O(10)$ and higher. In the long-time limit, the vertical dispersion increases with increasing $St$ and the scaling behavior becomes proportional to $t$. The increased vertical dispersion with increasing Stokes number is a result of the inertial forces that become larger than the restoring buoyancy force of the fluid.

The reaction of heavy particles on a mean settling velocity or on nonlinear drag is different in isotropic turbulence and in stratified turbulence. The difference between the mean settling velocity $w_p$ and the Stokes settling velocity $w_{st}$ is much larger in isotropic turbulence than in stably stratified turbulence. The nonlinear drag law reduces the mean settling velocity, especially for large values of $St$ or $W$. However, its effect on the mean-squared displacement can be neglected except for vertical dispersion in stably stratified turbulence. A mean settling velocity decreases the dispersion of heavy particles in isotropic turbulence (both horizontal and vertical direction) and in the horizontal direction in stably stratified turbulence. Contrary to what happens in isotropic turbulence, in stratified turbulence the long-time vertical dispersion of heavy particles increases with increasing $W$.

Finally, we studied the importance of the Saffman lift force (with and without correction factor) on heavy particles in strongly stratified turbulence, in which regions with strong local shear are observed. This force is found to be small compared to the Stokes drag, and its influence on the particle dispersion is negligible.
This work considered the dispersion of heavy particles, which would correspond to, for example, aerosol dispersion in air. The dispersion of so-called light particles, with densities of the same order of magnitude as the surrounding fluid (such as plankton, algae, and sand in marine environments), is left for future work. In order to study these light particles all forces in the Maxey–Riley equation need to be taken into account.

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