Control of Dynamics and Hysteresis in Electromagnetic Lenses

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus, prof.dr.ir. C.J. van Duijn, voor een commissie aangewezen door het College voor Promoties in het openbaar te verdedigen op woensdag 25 mei 2011 om 16.00 uur

doors

Patrick Jacobus van Bree

geboren te Weert
This work is carried out as part of the Condor project, a project under the supervision of the Embedded Systems Institute (ESI) and with FEI company as the industrial partner. This project is partially supported by the Dutch Ministry of Economic Affairs under the BSIK program.

A catalogue record is available from the Eindhoven University of Technology Library
ISBN: 978-90-386-2475-4
Summary

Control of Dynamics and Hysteresis in Electromagnetic Lenses

A comparison of control strategies is presented for fast and accurate switching of the operating point of electromagnetic lenses as used in electron microscopy. Electron microscopes are valuable tools for inspection and manipulation of specimens at the micrometer down to the nanometer scale. They enable further development of the next generation semiconductors, solar panels, fuel cells and chemical production processes of for instance polymers and medicines. From its invention in 1931 the electron microscope has traditionally been an imaging instrument used by highly experienced operators. More and more new applications arise in which the microscope is transformed from an imaging instrument into an automated measurement and manipulation tool. Next to a high quality image at a high magnification, the throughput of automated applications has become important.

One of the throughput limiting factors is the time involved with switching the operation point of the electromagnetic lenses. Such transitions are required when images are recorded at different magnifications, with different electron energies or using different imaging and specimen manipulation principles. A transition consists of a two step procedure: In step 1, which is the main topic of this research, the magnetic flux density within the lens is brought as fast as possible to a steady level very close to the new operating point. In step 2, the focal settings of the lens are optimized using image based feedback techniques. The primary aim of research is to design and compare control strategies that are able to decrease both the maximum transition error and the maximum transition time involved with switching the operating point of electromagnetic lenses. To guarantee performance of image based focus optimization the error made in step 1 has to be smaller than 1% of the full range of possible set points. This bound was estimated by means of experiments carried out on a state-of-the-art scanning electron microscope. Feed forward controlled set point changes were evaluated with the help of the recorded image series. Besides experiments, the requirements for control were extracted from first principle electron optical models in combination with an analysis of the most dominant magnetization dynamics.

An electromagnetic lens taken from a scanning electron microscope is extended with power electronics, magnetic flux density sensors and a data acquisition and rapid prototyping system. With this setup the controller performance can be evaluated experimentally. Instead of image quality the performance is based on measured behavior of the electromagnetic field. To meet the specifications for electron microscopy applications, the most accurate, large range, high bandwidth magnetic-flux-density-sensors available were placed within the lens geometry. Feed forward control is presently used in many microscopes and serves as the benchmark situation. The open loop magnetization dynamics in combination with ferromagnetic hysteresis result in a maximum transition time around 0.5s and a maximum transition error of 5% of the full range. Since the maximum error allowed is 1% there is a need for more advanced control.

Analysis and design of control strategies is complicated due to spatially distributed dynamics and hysteresis in combination with both the demand for high accuracy and the restrictions on sensor positioning in the lens geometry. The implemented feedback controller reduces the maximum transition time down to 50ms, an improvement of a factor 10 when compared to feed forward. Next to that, feedback control is capable of dealing with the error introduced by hysteresis. However, restrictions on the sensor positioning imply that the sensor may not be placed in, or very close to, the electron optic volume during online operation of electron microscope. Because of this restriction the relation
between the magnetic flux density at the position of the sensor and the magnetic flux density in the electron optical volume has to be controlled in feed forward. The experimental results obtained with this controller scheme show that hysteresis is again the dominant cause of the transition error. Due to the restrictions on the sensor position in combination with the spatially distributed hysteresis effect, the performance of this controller layout in terms of maximum transition error is at the critical boundary of 1%. Despite these restrictions, very fast switching is still guaranteed since the maximum transition times estimated at the sensor position and in the electron optic volume are both equal to 50\,ms.

Feed forward initialization is introduced as a technique that specifically deals with reducing the error involved with hysteresis. By means of a forced reset of the state of the system, the error level is brought down to 0.05\% of the full range, an improvement of 100 times when compared with conventional feed forward. The price being paid is the extra time (0.1\,s to 0.5\,s) needed for the applied input profile to enable the reset. The requirements on initialization trajectories for hysteretic systems are investigated by means of a model based analysis in combination with experiments carried out at both the electromagnetic lens setup and a scanning electron microscope. The optimal initialization trajectory for a specific trade-off between the duration of initialization and the level of error reduction is obtained by an experimental procedure. The performance of all the different control techniques along with the performance limiting factors are indicated in a mapping of maximum transition time versus maximum transition error.
# Contents

1 **Introduction** ........................................... 7  
   1.1 Electron Microscopy .................................. 7  
   1.2 Trigger: From Imaging to Measurement Tool .......... 8  
   1.3 Switching the Operating Point ........................ 9  
   1.4 The Condor Project .................................. 9  
   1.5 Research Goal ........................................ 11  
   1.6 Thesis Outline .................................... 11  

2 **Scanning Electron Microscopy and Electron Optics** .......... 13  
   2.1 Introduction into Scanning Electron Microscopy ....... 13  
   2.2 Electromagnetic Lenses ................................ 14  
   2.3 Focal Distance ....................................... 15  
   2.4 Image Based Defocus Control .......................... 19  
   2.5 Applications: Changes of Operating Point .......... 23  
   2.6 Conclusions .......................................... 24  
   2.7 Beyond the Scope .................................... 25  

3 **Dynamics and Hysteresis in Electromagnetic Lenses** ....... 27  
   3.1 Transients and Image Acquisition ........................ 27  
   3.2 Transient Effects in Electromagnetic Actuation ....... 28  
   3.3 Hysteresis Effects in Electromagnetic Actuation ...... 37  
   3.4 Formulation of Control Objective ..................... 42  
   3.5 Beyond the Scope .................................... 46  

4 **SEM Experiments** ........................................ 47  
   4.1 Sharpness ........................................... 47  
   4.2 SEM setup .......................................... 48  
   4.3 SEM Experiment: Sensitivity in an Operating Point .... 49  
   4.4 SEM Experiment: Hysteresis and Switching Effects .... 51  
   4.5 SEM Experiment: Feed Forward Initialization ........ 55  
   4.6 SEM Experiment: Transients ........................... 58  
   4.7 SEM Experiment: Hysteresis and Circular Symmetry .... 59  
   4.8 Conclusions .......................................... 60  

5 **The Electromagnetic Lens Setup** .......................... 63  
   5.1 Magnetic Flux Sensing .................................. 64  
   5.2 Controller Configurations ............................... 67  
   5.3 Recommendations and Remarks ........................... 70
Chapter 1

Introduction

In this thesis feed forward and feedback strategies for the control of the magnetic flux density in electromagnetic lenses as used in electron microscopy are compared and experimentally validated. There is a recent demand for automated microscopy applications that measure specimen characteristics or manipulate the specimen. These new applications require a frequent change of the operating points which implies a need for fast and accurately controlled changes of the electromagnetic fields within the lenses. The goal of the project is to decrease both the transition error and the transition time by means of control techniques, and to indicate the performance limiting factors. The performance of the different strategies will be represented in a performance map of time versus error. This map provides a clear overview of the required balance between a small error and a small transition time. It will be shown that the open loop benchmark situation shows transition errors in the order of 5% and transition times of 0.5s. The maximum error allowed to guarantee overall machine performance is around 1%.

Electromagnetic lenses as used in charged particle optics are high precision electromagnetic actuators which are controlled by the current through the lens coil. Most important for imaging at micro to nanometer scale is a low level of optical aberrations. In automated measurement applications their behavior as a dynamical system becomes significant since transition times and transition errors involved with switching the operating point limit the performance of the machine in terms of throughput.

The characteristics of the system to be controlled show spatially distributed dynamical and hysteresis effects. Modeling and control of hysteresis coupled with dynamics is a major theme in this work. The available sensor types for feedback control have significant limitations due to a trade-off between resolution, range, bandwidth and geometry. Besides that, restrictions hold on the sensor positioning within the lens geometry since it may not disturb the imaging process. It is the combination of all these factors that makes improving the performance a challenging task. The control objective is to accurately control the magnetic flux density at a point in the lens geometry which is highly related to the optical performance. The presented approach is based on lumped models.

1.1 Electron Microscopy

Electron microscopes are used for automated micro-analysis of minerals, rocks and man-made materials, where they provide quantitative information on particle size and shape, [31]. In semiconductor, solar and MEMS industries, laboratories and data storage production facilities, high-quality characterization and metrology data are provided. Research and development of structure-property-function relationships of a wide range of materials and processes, such as next generation fuel cell and solar cell technologies; catalyst activity and chemical selectivity; energy-efficient solid state lighting; and lighter, stronger and safer materials, are made possible by these imaging and measurement tools for micro to nanometer sub-scales.

The electron microscope was invented in 1931, [72]. For decades most efforts in charged particle optics were made towards improving the optical quality of such systems from micrometer down to sub-
nanometer scale. The major limiting factor for this is aberration (e.g. spherical or chromatic aberration [67]). To prevent image deformation, the variation in the electromagnetic field over time should be very low during image recording. As a result the design of the microscope is highly optimized for static use.

A human eye can resolve two objects 0.2mm apart, if the objects are closer to each other only one object is observed, [30]. The resolving power of a microscope is the capability to provide an image from which a human or an image processing procedure can resolve objects that are much closer together (down to sub-nanometer scale). The maximum resolving power is mainly limited by the aberrations of the lens which are beyond the scope of this work. Note that in light microscopy the resolving power is limited by the wave-length of light. In electron microscopy the limitation on wave length is not reached, since the significant limitations come from lens aberrations, and e.g. disturbing vibrations, positioning accuracies of the sample and electron beam. Alternatives for electron microscopes concerning imaging at the smallest scale are scanning tunneling microscopes, atomic force microscopes, scanning probe microscopes. Which type is favorable depends on the application.

1.2 Trigger: From Imaging to Measurement Tool

For decades, electron microscopes were tools almost exclusively used by researchers studying material properties. The result was mainly a high quality strongly magnified image. However, next to research new markets have evolved in which the image itself is no longer the main result. A quantitative analysis of specimen features at micro- to nanometer scale is the new machine output. The electron microscope is becoming an automated measurement tool, [84]. An example is feature extraction, e.g. detection of the number and size (nm) of particles within 1mm² of the specimen. Such a procedure requires large series of images obtained on different settings. The recorded images are internally used for processing and feedback control to obtain an accurate measurement. Examples are found in e.g. [74], [18] and [79].

The system’s quality is now expressed in accuracy (e.g. standard deviation of the estimated number and size of particles) and throughput (the time it takes to analyze the complete specimen). Image quality is only important for the image-based feature extraction algorithms. There is no strict need to work with images that have the optimal quality. If other criteria, like throughput, can be increased while accuracy decreases only slightly, less image quality can be acceptable depending on the specific application. The machine settings result from a trade-off between accuracy and throughput which is different for each application. However, to make this trade-off, an objective framework to reveal the relation between the different performance criteria has to be available.

The performance of image processing procedures for e.g. autofocus or specimen characterization decreases if there are any deformations in the image. Lens aberrations are a known and extensively studied cause of deformations. However, these aberrations are studied when all systems involved with electron optical imaging are in steady state, e.g. the specimen stage is not moving anymore, the electron acceleration voltage is constant and the magnetic flux distribution within the lenses is constant. Many high accuracy measurement applications cannot obtain the required information out of one single image. Changes in operating points are required to obtain different views of the specimen. This can be a change of magnification, a change of the electron energy, a different area of the specimen, etc. Variation in the magnetic flux distribution also causes image deformation. The accuracy of applications required to count the number of particles decreases if transient effects are present. To keep the required accuracy the system has to wait for the transients of the magnetic flux distribution to decay, which limits the throughput of the machine. The behavior of the microscopy system as a dynamical system becomes important when the time involved with switching between settings becomes a significant factor in the overall process time.
1.3 Switching the Operating Point

In this research the behavior of the electromagnetic objective lens, as a subcomponent of a scanning electron microscope, is studied. The performance of the overall system is a balance between accuracy and throughput. Considering the nature of the application, the accuracy is preset and fixed. E.g. when estimating the number and the size of particles for quality control of a medicine production process, the number of particles has an allowed error of 1% and the size an allowed deviation of 1 nm. First of all the machine has to be capable of achieving these specifications, after which the throughput can be optimized.

The performance of the lens subsystem for set point changes is expressed in the transition time and the transition error. During a transition the application has to wait. Directly after a transition a possible error in optical settings has to be corrected, e.g. by image-based focusing procedures, which again take time. Therefore, both transition errors and transition times involved with a change of the operating point limit the throughput of the application.

The transition time of the lens is determined by the physical process of magnetization. An electromagnetic lens consists of a coil surrounded by a solid yoke made of electrically conducting ferromagnetic material. The amplitude of the current running through the lens coil determines the amplitude of the magnetic flux distribution within the electron optical volume of the lens geometry. The optical properties of the lens, such as focal distance, can be varied by changing the lens current. The variation of the magnetic flux density in the lens gap lags behind the variation in lens current due to magnetization dynamic effects such as eddy currents.

The transition error is mainly determined by ferromagnetic hysteresis present in the relation between the applied magnetic field strength determined by the lens current and the actual magnetic flux density in the electron optic volume. As a result of hysteresis the obtained flux density is a function of both the lens current and the previous settings instead of the lens current only. As a result one constant input current can result in a range of possible magnetic flux densities. The obtained magnetic flux value after a transition depends on the applied input current but also on the previous excitation. Fig. 1.1 illustrates that when the current is taken as a control input and the hysteresis effect is not taken into account, the resulting image quality is uncertain. If the transition error is too large, it cannot be corrected by image based focus optimization techniques.

There are two basic types of electron microscopes: Transmission Electron Microscopes (TEM) and Scanning Electron Microscopes (SEM). TEM is particularly suitable for extremely high magnification down to sub-Ångström level. However, the specimens viewed with TEM have to be flat and really thin, since the electrons have to travel through the specimen. In SEM the surface of large 3D structures can be viewed. The choice for SEM or TEM is application dependent.

This research is based on experiments carried out using an actual SEM or an SEM objective lens. However, this research applies to all applications based on charged particle optics where ferromagnetic hysteresis and magnetization dynamics form a significant performance limiting factor during changes in operating point of the lenses. The developed methodology can be translated one-to-one to TEM. However, the electromagnetic behavior in terms of dynamics and hysteresis differs with the material choice and geometry of the lens. Besides that, the requirements for control can be different for each specific microscope type, lens type or application.

1.4 The Condor Project

The development of the next generation automated electron microscopes is the leading theme of the Condor project. The carrying industrial partner is FEI Company, an expert company in the domain of electron microscopes and owner of the industrial problem. Academic partners are Eindhoven University of Technology, Delft University of Technology, Katholieke Universiteit Leuven and University of Antwerp. Second participating industrial partner is Technolution, a company on technical automation and embedded systems. The Embedded Systems Institute (ESI) has the responsibility for the project.
Chapter 1. Introduction

Figure 1.1: The influence of hysteresis, present in the relation between the lens current and the resulting magnetic flux density in the lens, on the quality of the obtained images: Corresponding to one single input value a range of image qualities can be obtained. When hysteresis is not explicitly taken into account in the control scheme, the resulting image quality after a transition is uncertain.

management and knowledge dissemination.

The focus of the project is on performance and evolvability. Performance is defined as high-end image quality and measurement accuracy, productivity (fast response times), ease-of-use, and instrument autonomy (autotuning and calibration). Evolvability is the adaptability to various applications and different (and changing) requirements during the planned life cycle. A sample of the publications that are directly linked to Condor is: image sharpness evaluation [70], [71] (TU/e Applied Mathematics), control of a motion stage of a TEM [69] (TU/e Mechanical Engineering), three-dimensional characterization on the atomic scale using STEM [96] (University of Antwerp), the system and software architecture, [54], [45] (KU Leuven), image based defocus control in TEM [97], [82], [84], [83] (TU Delft), and the control of electromagnetic lenses [90], [93], [91], [92] (TU/e Electrical Engineering).
1.5 Research Goal

The topic of research is fast and accurate switching of the operating point of electromagnetic lenses as used in electron microscopy applications. An electromagnetic lens consists of a coil (or set of coils) surrounded by a ferromagnetic yoke. The relevant quantity to control is the magnetic flux distribution within the electron optic volume. The flux distribution is a function of the input current of the coil, the previous excitation, the geometry and the lens material. In an operating point the variation of the flux density has to be extremely low not to disturb the imaging process. Microscopy applications require to change operating points in order to enable different views of the specimen under study. The requirement for one lens to work in different operating points results in a ratio, between the allowed variation and the total range required to reach all operating points, which is smaller than $10^{-5}$. It will be shown in this dissertation that the maximum transition error with conventional control is in the order of 5% of the total range of operating point values. After a transition, the image quality is further optimized using image based feedback techniques. However, the performance is only guaranteed if the transition error is smaller than about 1%, depending on the application.

In this research different feed forward and feedback control strategies for fast and accurate switching of the operating point of electromagnetic lenses are designed, implemented, analyzed and compared. The main objective is to minimize both the maximum transition time and the maximum transition error. The second objective is to show the relation between the performance of the designed controllers and the:

- magnetization dynamics and nonlinearities, e.g. eddy current and hysteresis,
- sensor limitations, e.g. accuracy, resolution, bandwidth, range,
- restrictions on sensor positioning within the lens geometry,
- actuator limitations, e.g. accuracy, bandwidth, range,
- constraints for control, e.g. limited input current,
- controller structure,
- application requirements,
- model accuracy and lack of full understanding of physical processes.

This is realized by means of implementation and analysis of the various controller structures on a developed setup consisting of an electromagnetic lens, magnetic flux density sensors, actuators and a rapid prototyping system.

1.6 Thesis Outline

1.6.1 Chapter 2

In chapter 2 the need for high precision is derived from first principle electron optical models. The electromagnetic lens has to be considered as a subsystem of a larger control scheme that regulates the level of defocus. It is illustrated that both a decreased error and a decreased transition time are beneficial for the throughput of automated applications.

1.6.2 Chapter 3

The control objective is formulated in chapter 3. On one hand there is the time involved with set point transitions, on the other hand the transition error. The transition time involved with stepwise changes in magnetization is illustrated with the analysis of the Maxwell equations for a linear problem. The effects and cause of hysteresis are discussed.
Chapter 1. Introduction

1.6.3 Chapter 4

Simulators for electron microscopy are focused on electron optics and neglect electromagnetic side effects. To be sure to work with realistic criteria and assumptions the effects introduced by hysteresis and magnetization dynamics are measured on a state-of-the-art scanning electron microscope. A microscope at FEI Company (Eindhoven, Netherlands) has been extended with a rapid prototyping system for the control of the objective lens. Transition time and errors are estimated from the obtained image series. All images are synchronized with the control actions.

1.6.4 Chapter 5

This chapter describes the second setup consisting of an objective lens extended with magnetic flux density sensors, a data acquisition and a rapid prototyping system. A feedback controller is designed and implemented.

1.6.5 Chapter 6

An experimental controller evaluation framework is presented in chapter 6. As a benchmark conventional feed forward control is implemented. Next, it is shown that feedback control of the magnetic flux density is capable of solving the problem if there are no restrictions on the sensor position within the lens. It is shown that the position of the sensor is extremely important for controlling the error. However, the sensor position is less relevant for the transition time.

1.6.6 Chapter 7

Feed forward initialization is introduced as an approach to reduce the error as a result of hysteresis. For various hysteresis models input trajectories are designed which reduce the difference between trajectories starting from different initial conditions. Requirements for initialization strategies valid for the models are projected on the behavior of the electromagnetic lens.

1.6.7 Chapter 8

In chapter 8 the feed forward initialization approach is evaluated using the electromagnetic lens setup. Throughout the thesis a controller performance map, representing the transition time versus the transition error, is build up. Bounds are derived which indicate the application benefits and the physical constraints coming from actuator and sensor limitations. The performance comparison of feed forward, feedback and feed forward initialization strategies is presented. Since the memory/dynamical/hysteretic structure of the electromagnetic actuator is not known in detail the optimal initialization trajectory remains unknown. By focusing on periodic inputs the performance is studied as a function of the number of periods and the frequency. At the cost of the duration of initialization the transition error is reduced.
Chapter 2

Scanning Electron Microscopy
and Electron Optics

Each specimen, out of the very large variety of specimens that can be analyzed using electron microscopes, requires a different combination of electron optical settings. As a result the lenses in a typical microscope are required to have an ratio of the resolution divided by the amplitude range $< 10^{-5}$. These requirements are derived in this chapter using first principle models.

2.1 Introduction into Scanning Electron Microscopy

Fig. 2.1 shows the basic components of a scanning electron microscope. In scanning electron microscopy an electron beam is scanned over the specimen under study. The energy of the incoming primary electrons generates secondary electrons of which the number varies with the morphology and composition of the specimen. The resulting image is composed from the scan pattern versus the obtained electron intensity. The basic principle is as follows (e.g. [67], [34], [99]):

- The specimen-stage positions $(xyz)$ the specimen in the vacuum chamber.
- An electron gun generates free electrons which are accelerated by means of a high voltage.
- The probe forming or objective lens focuses the approximately parallel electron beam such that the diameter of the beam that is projected on the specimen corresponds to the scanning pattern, Fig. 2.4. In general a demagnification of the beam diameter from $\mu m$ to $nm$ scale (factor 1000) is required.
- The projected spot position $(xy)$ is controlled by the deflection system which ensures that the beam travels through a virtual pivot point in the electron optic area of the lens, Fig. 2.1.
- The deflector system controls the $xy$-movement and with that the magnification; A higher magnification results from scanning a smaller surface. The magnification is therefore not a property of the objective lens. However, the highest possible magnification is limited by the smallest projected spot size, controlled by the lens.
- Due to the high energy of the incoming electron beam, secondary electrons are generated. The electron intensity is captured by the electron detector. The intensity varies with the material-type and the composition of the specimen. For example, at edges more secondary electrons can escape from the sample and, therefore, the electron-intensity corresponding to an edge will appear white in the image.
- The scan pattern versus the electron intensity provides the image. In the image, a cell is represented by a pixel with a certain uniform intensity. During the time that the beam is within a cell, the detected electron energy is accumulated. Therefore, the pixel intensity represents an average over a very small area.
2.2 Electromagnetic Lenses

An electromagnetic lens consists of a cylinder shaped coil surrounded by a ferromagnetic (e.g. NiFe) pole-piece (yoke), Fig 2.2. In first approximation such a lens can be considered circular symmetric. By varying the amplitude of the current running through the lens coil, the magnetic flux density in the electron optic volume can be varied. Therefore, the magnetic flux density $B[T]$ observed by the electrons is a function of the geometry and material of the pole-piece, and the input current applied to the coil $I[A]$.

With finite element analysis the magnetic flux density distribution at the optical axis is calculated for different coil currents in a 2D-circular symmetric lens geometry, Fig. 2.3. The electrical conductivity in the example is $\sigma = 10^7 \, A/(V \, m)$ and the relative permeability is $\mu_r = 5000$. This represents a solid yoke made of soft ferromagnetic material. The height $z$ is 130mm with a radius $r$ of 40mm. The magnetic flux density $B_z$ at the optical axis $r = 0$ is shown for different coil currents. The number of windings is chosen $n = 900$. The internal geometry of the coil, windings and isolation, is not taken into account. The coil is represented by a volume with equal current distribution.
2.3. Focal Distance

The magnetic flux density is thus a time varying 3D-distribution within the lens geometry. Fig. 2.3, shows a one-dimensional distribution at a certain time instance for \(0 \leq z \leq 100\text{mm}\) at \(r = 0\text{mm}\). The maximum is found around \(z = 43\text{mm}\). The amplitude of the distribution important for electron optics is then defined as \(B_z(t, z = 43\text{mm}, r = 0\text{mm})T\). The static gain between current and magnetic flux density \(B_z(t, z = 43\text{mm}, r = 0\text{mm})T\) resulting from this example is \(B_z = 5.5 \cdot 10^{-3} I\), a factor 180 [A/T]. Note that this factor scales linearly with the number of turns in the coil.

The electromagnetic objective lens controls the focal distance \(f\). The level of defocus \(\Delta f\) is then determined by the difference between the focal distance and the position of the specimen stage \(z_{\text{stage}}\). Fig. 2.4 shows the relation between the \(f, \Delta f, z_{\text{stage}}\) and the projected spot size which is the main quantity influencing image sharpness (or quality in general). In a simplified view, the diameter of the electron beam projected at the specimen (spot size) should be smaller than the cell dimensions to obtain a sharp image. As soon as the beam diameter is larger, the image will be blurred. In this approximation the beam intensity is considered uniform; in practice it will have a distribution with a non-constant gradient. The highest possible magnification is thus limited by the smallest possible spot size, mainly limited by the aberrations of the lens which are beyond the scope of this research.

A sharp image is obtained with a low level of defocus. Fig. 2.5 shows the geometric relation between focus, defocus and projected spot size for a so-called thin lens approach which implies that only the asymptotes of the electron trajectories are taken into account assuming the lens is infinitely thin. It is derived that the ratio between the projected spot radius \(r_{\text{spot}}\) and the incoming electron beam radius \(r_0\) is equal to the ratio between the absolute level of defocus \(|\Delta f|\) and the focal distance \(f\):

\[
\frac{r_{\text{spot}}}{r_0} = \frac{|\Delta f|}{f}.
\]

2.3.1 Example: Requirements on the Level of Defocus

Given a specimen of which a scan area of \(2\mu m^2\) is under study. The resulting image is divided into \(512^2\) pixels. The radius of the incoming parallel electron beam \(r_0 = 200\mu m\). The specimen is positioned at \(z_{\text{stage}} = 10\text{mm}\). The scan area of \(2\mu m^2\) is divided in \(512^2\) cells which results in a
Chapter 2. Scanning Electron Microscopy and Electron Optics

Figure 2.3: Lens geometry and the magnetic flux density at the symmetry axis for different current amplitudes.

cell dimension of 3.9 nm. The maximum radius of the projected spot size is equal to the dimension of the cell divided by two: $r_{\text{spot}} < 1.95 \text{nm}$. For a desired focal distance of 10 mm the allowed variation of focus $\Delta f < 98 \text{nm}$. The ratio $r_{\text{spot}}/r_0 = |\Delta f|/f \approx 10^{-5}$. The next step is to derive the equations of motion as a function of the magnetic flux density such that an allowed deviation $\Delta B$ can be calculated.

2.3.2 Electron Trajectory: Equations of Motion

To illustrate the link between the flux density $B_z(z)$ and the focal distance a first principle model is derived from the equations of motion for charged particles in electromagnetic fields. Table 2.1 provides an overview of the physical quantities used for the analysis of the electron motion. The differential equation describing the spiral-alike movement of an electron in a circular symmetric magnetic field (e.g. [34],[19]) is derived from the Lorentz force and the acceleration force:

$$ F_L = q(E + v \times B), \quad E = 0, q = -e \quad (2.2) $$

$$ F_a = m\dot{v} \quad (2.3) $$

$$ \Rightarrow m\dot{v} = -e(v \times B). \quad (2.4) $$

The charge of an electron is denoted by $q = -e$ and $\dot{v}$ represents the acceleration. In the electromagnetic lens no electric field $E$ is present. The equations for the motion of an electron in a cylindrical coordinate system describing the radial displacement $r$ and the angular displacement $\phi$ are derived:

$$ r'' + r \frac{e^2}{4m^2v_z^2} B_z^2(z) = 0, \quad r(z_0) = r_0 \quad (2.5) $$

$$ \phi(z) = \phi_0 + \frac{e}{2mv_z} \int_{z_0}^{z} B_z(z)dz. \quad (2.6) $$

Due to the quadratic term in (2.5) the electron is always contracted towards the optical axis. This implies that the same optical properties can be obtained with positive or negative magnetic flux density. The differential equation (2.5) is expressed only as a function of $B_z$, which is possible due
2.3. Focal Distance

Figure 2.4: Schematic representation of the electron trajectory with underfocus (lower magnetic field) and overfocus (higher magnetic field) in a lens. The projected spot size for the two cases can be equal.

Figure 2.5: Relation between focus and spot size.

to the assumptions that the magnetic flux density is perfectly circular symmetric and that the electron velocity \( v_z \) is not influenced by the electromagnetic field. Note that (2.5) is not a function of time \((t)\), but a function of space \((z)\). Time derivatives are indicated with \(dr/dt = \dot{r}\), space derivatives with \(dr/dz = r'\).

Acceleration of the electrons to a speed \( v_z \) is achieved using a potential drop \( U \), where the potential energy is converted into kinetic energy:

\[
e U = \frac{1}{2} m v_z^2, \quad \Rightarrow v_z^2 = 2 \frac{e}{m} U.
\]  

(2.7)

In table 2.2 the electron velocity \( v_z \) is calculated using (2.7). These values are in the order of 1 to 35% of the speed of light. The higher the velocity, the more relevant a relativistic correction becomes, [19, p. 90-92]. For the sake of simplicity this correction is neglected here.

The differential equation for the radial position can now be expressed as a function of the 1D magnetic flux distribution at the optical axis, the material parameters and the electron acceleration voltage: The initial condition of (2.8) is the velocity \( v_r, v_\phi, v_z \) with which the electron enters the optical volume and the initial position \( r, \phi, z \)

\[
\Rightarrow r'' = -r \frac{e}{8 m U} B_z^2(z).
\]  

(2.8)
Chapter 2. Scanning Electron Microscopy and Electron Optics

Table 2.1: Physical quantities involved with the electron trajectory for a typical setting of the FEI Helios SEM.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>C</td>
<td>charge of an electron</td>
<td>$1.6022 \cdot 10^{-19}$</td>
</tr>
<tr>
<td>$U$</td>
<td>kV</td>
<td>acceleration voltage</td>
<td>$[0.5, \ 30]$</td>
</tr>
<tr>
<td>$m_0$</td>
<td>kg</td>
<td>rest mass electron</td>
<td>$9.101 \cdot 10^{-31}$</td>
</tr>
<tr>
<td>$c$</td>
<td>m/s</td>
<td>velocity of light</td>
<td>$3 \cdot 10^8$</td>
</tr>
<tr>
<td>$v_z$</td>
<td>m/s</td>
<td>velocity of electrons</td>
<td>$[0.1, \ 1.5] \cdot 10^8$</td>
</tr>
<tr>
<td>$B$</td>
<td>T</td>
<td>magnetic flux density</td>
<td>$[-0.2, \ 0.2]$</td>
</tr>
<tr>
<td>$z$</td>
<td>mm</td>
<td>vertical position</td>
<td>$[1, \ 10]$</td>
</tr>
<tr>
<td>$I_{beam}$</td>
<td>A</td>
<td>beam current</td>
<td>$\approx 0.2$</td>
</tr>
</tbody>
</table>

For a certain distribution $B_z(z)$ as for instance presented in Fig. 2.3 the trajectories can be calculated using numeric solvers. With the help of a distribution which allows an analytical solution for the electron trajectory $r(z)$ further insight is gained in $f$ as a function of the amplitude of $B_z$.

### 2.3.3 Analytical Expression for a Constant Magnetic Flux Distribution

An analytical expression for the focal distance $f[m]$ is obtained as a function of $B_z[T]$ and $U[V]$. The differential equation (2.8), describing the radial displacement, represents a linear system for the case $B_z(z)$ is constant. This allows for an analytical expression. In [67] and references, analytical solutions are also obtained for other choices of $B_z(z)$, e.g. bell-shaped symmetric fields. In Fig. 2.3 the shape of $B_z$ for the electromagnetic lens under study is shown.

If $B_z$ is taken to be constant, the set of eigenvalues $\lambda$ of (2.5) is the complex conjugate pair:

$$\lambda = \pm i \sqrt{\frac{e}{8m}} \sqrt{\frac{1}{U} B_z}.$$  \hspace{1cm} (2.9)

This solution corresponds to an oscillator. For initial radial velocity $v_r(z_0) = 0$, and an initial distance from the optical axis $r_0$, the result is:

$$r(z) = r_0 \cos \left( \sqrt{\frac{e}{8m}} \sqrt{\frac{1}{U} B_z} \right) z.$$  \hspace{1cm} (2.10)

The focal distance $f$ is defined by the point at which the optical axis is intersected for the first time (at a quarter period length) in relation to the magnetic flux distribution in the lens geometry. From (2.10) it follows that $f$ is independent of the initial distance $r_0$ from the optical axis. The focal distance as a
function of the magnetic flux density amplitude and vice versa are now expressed as:

\[ f = 2\pi \sqrt{\frac{m}{2e}} \sqrt{\frac{U}{|B_z|}} \]  
(2.11)

\[ B_z = \pm 2\pi \sqrt{\frac{m}{2e}} \sqrt{\frac{U}{f}} \]  
(2.12)

Using (2.11) the allowed variation of the flux density in an operating point \( \Delta B_z \) can be calculated:

\[ \frac{df}{dB_z} = -2\pi \sqrt{\frac{m}{2e}} \sqrt{\frac{U}{B_z^2}} \{ B_z^{-2} \} = -\frac{f}{|B_z|} \]  
(2.13a)

\[ \Rightarrow \frac{\Delta f}{f} = -\frac{\Delta B_z}{|B_z|}. \]  
(2.13b)

2.4 Image Based Defocus Control

2.4.1 Lower Bound

The allowed error in steady state is determined by the scan settings: The magnification (the scan area) is controlled by the deflection system, number of pixels in the resulting image, and the diameter of the incoming electron beam. As long as the projected spot size is smaller than the dimensions of one cell of the scan raster the image is considered in focus. A relative allowed deviation \( \epsilon_{rel} \) in an operating point is defined by the relation between the maximum allowed spot size and the incoming beam diameter. This ratio also provides the bounds for variation of the quantities \( f, B \) and \( I \):

\[ \frac{r_{spot}}{r_0} \leq \epsilon_{rel}, \quad \frac{\Delta f}{f} \leq \epsilon_{rel}, \quad \frac{\Delta B}{B} \leq \epsilon_{rel}, \quad \frac{\Delta I}{I} \leq \epsilon_{rel}. \]  
(2.14)

For the typical settings, as discussed in 2.3.1, the requirements for a sharp image imply \( \epsilon_{rel} = 10^{-5} \), table 2.3.

2.4.2 Example: Requirements on Magnetic Flux Density

Example 2.3.1 is now extended for calculation of the allowed variation \( \Delta B \) in an operating point. A scan area of \( 2\mu m^2 \) is divided into \( 512^2 \) cells. The radius of the incoming parallel electron beam \( r_0 = 200\mu m \). The specimen is positioned at \( z_{stage} = 10mm \) and the acceleration voltage \( U = 1kV \).

With the derived formula (2.12) that provides \( B_z \) as a function of \( f \) and \( U \), the required \( B_z = 33.5mT \) and the allowed variation \( \Delta B \leq 0.335\mu T \). Fig. 2.6 shows the relation between \( B, f, \Delta f \) and \( \Delta B \) using the derived formulas. For comparison the relation is also shown for \( U = 10kV \) and \( U = 30kV \). In all situations the required focal distance is \( f = 10mm \). The plot of \( \Delta B \) vs. \( \Delta f \) for the three acceleration voltages illustrates that the lower the acceleration voltage, the higher the requirements on the absolute \( \Delta B \). Besides the dashed lines in Fig. 2.6 that indicate the bound on \( \Delta f \) imposed by the cell size, the upper bounds for defocus control are also shown. These are explained next.

<table>
<thead>
<tr>
<th>operating point</th>
<th>lower bound</th>
<th>variable</th>
<th>upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_0 = 200\mu m )</td>
<td>( 2nm )</td>
<td>( \leq r_{spot} \leq 2\mu m )</td>
<td></td>
</tr>
<tr>
<td>( f = 10mm )</td>
<td>( 100nm )</td>
<td>( \leq \Delta f \leq 0.1mm )</td>
<td></td>
</tr>
<tr>
<td>( B_z = 100mT )</td>
<td>( 1\mu T )</td>
<td>( \leq \Delta B \leq 1mT )</td>
<td></td>
</tr>
<tr>
<td>( I = 1A )</td>
<td>( 10\mu A )</td>
<td>( \leq \Delta I \leq 10mA )</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: Bounds on the various quantities involved with the focal settings.
Chapter 2. Scanning Electron Microscopy and Electron Optics

2.4.3 Upper Bound on Defocus Error

If the projected spot size is too large the resulting image is out of focus and appears blurred. For a low level of defocus the level of defocus can be estimated from recorded image data. An image based defocus feedback control corrects the error. Before discussing this controller structure, an upper bound is derived for the error in the focal distance for these image based methods to work.

If the projected spot in relation to the cell size is so large that a small variation $\Delta r_{\text{spot}}$ cannot be observed from the images, image based defocus will fail or at least have a major drop in performance. This bound is set to the point where the spot diameter is 1000 times the cell width. Fig. 2.7 illustrates this point for the settings of example 2.4.2.

The upper bound for defocus to work is set on 1000 times the bound $B = 1000 \cdot r_{\text{spot}}/r_0 = 10^{-2}$. This result implies that an error of 1% of the operating point is the bound for defocus to work. These bounds are also indicated in Fig. 2.6, Fig. 2.9 and table 2.3.

2.4.4 Overall Focus Procedure

Fig. 2.8 shows the overall control scheme for lens control. In a simplified view the overall controller structure is composed of 3 layers:

1. **Application control layer**: The application determines the required magnification, electron acceleration voltage, brightness, etc. In this layer it is decided how many images have to be recorded and at which operating point.
2.4. Image Based Defocus Control

![Defocus Control Diagram]

Figure 2.7: (left.) Typical scan settings in which the scan $2\mu m^2$ is divided into $512^2$ cells. The total scan time is $70ms$. (right.) Illustration of the scan pattern and a spot size that is 1000 too large. This spot size is taken as the maximum error where image based defocus will work.

2. Defocus control layer: With use of image based feedback control the level of defocus is minimized to the level defined by the lower bound. This layer requires defocus or quality estimation from images.

3. Lens control layer: The lowest layer is the lens controller which obtains set points from defocus control.

Switching of the operating point is required when images are recorded at different magnifications, with different electron energies or using different imaging and specimen manipulation principles. A transition consists of a two step procedure:

- In step 1, which is the main topic of this research, the magnetic flux density within the lens is brought as fast as possible to a steady level very close to the new operating point. This is the job of the lens controller.

- In step 2, the level of defocus is minimized using image based feedback techniques. The control objective of the defocus layer is to reach $\Delta f < \epsilon_{rel f}$ as fast as possible since e.g. feature extraction can then start at the application layer.

To guarantee performance of image based focus optimization the error made in step 1 has to be smaller than the upper bound.

In the expressions for the focal distance, the magnetic flux density distribution, the electron acceleration voltage and the stage position provide the operating point. The reference $R_B$, Fig. 2.8, represents the desired set point for one point in the lens geometry $B_z(t, r, \phi, z)$. However, the part providing the reference $R_B$ contains uncertainty since especially $z_{stage}$ is only known with limited accuracy. Even if a lens controller can be developed in which the transition error is guaranteed to be below the lower bound, this is no guarantee for sharp images since the reference value for this controller is also uncertain. The coupling between the design of defocus control layer and the lens controller is not further investigated. The objective is to design lens controllers for the case where the set point is known. If the performance for such a controller is known, the next step is to integrate it in the overall design.
Chapter 2. Scanning Electron Microscopy and Electron Optics

Figure 2.8: Overall control scheme consisting of 3 layers: Application control, defocus control and lens control.

Fig. 2.9 presents the performance map for controller evaluation in terms of the transition error. In this map, the upper and lower bounds are indicated for the transition error after switching the operating point. The lower bound is based on the presented analysis of the maximum spot size deviation in relation to the dimensions of a single cell in the scan pattern. A lower error does not result in an improved image quality. Minimizing the error below the lower bound is of no use for the overall machine performance. The upper bound is based on the assumption that if the spot size is 1000 times larger than the cell dimension, image based defocus control will fail. In chapter 4 this range will be validated using experiments on a scanning electron microscope. The full range of all possible operating point values is set to 100%. Despite the fact that the upper and lower bound depend on the absolute value of the operating point (as illustrated in Fig. 2.6), they will from now on be expressed as a fixed percentage of the full range. The lower bound is set to 0.001% and the upper bound for the maximum transition error is set to 1%.
2.5 Applications: Changes of Operating Point

This section presents an overview of different cases in which the electromagnetic lens has to change operating point.

2.5.1 SEM: Change in Acceleration Voltage

Depending on the application, images are recorded at different electron acceleration voltages. This example illustrates the requirements for images recorded at $U = 20kV$ and $U = 2kV$. The focal distance in the two cases is the same. The expression for the focal distance for a uniform $B_z$ was derived in (2.11) and is repeated here:

$$f = 2\pi \sqrt{\frac{m}{2e}} \sqrt{\frac{U}{|B_z|}}$$

It follows that $B_2\sqrt{U_1}/\sqrt{U_2} = B_1$. In this example $B_U=20kV = 3.16B_U=2kV$. The switch in acceleration voltage requires a switch of 300%. Note that the range in which image based focus correction has a guaranteed performance is only 1%.

2.5.2 SEM: Survey and Immersion Lenses

The SEM lens discussed so far is mainly used for lower magnifications, e.g. $< 2000$ and is called the survey lens. A modern SEM can image down to 1nm. For this range the immersion lens principle is more suitable, details on the design are found in e.g. [46] and [88]. Here the specimen is placed in the electromagnetic field. This is where the name immersion comes from. Such lenses show a lower amount of aberrations. As a result they have a better performance at high magnifications. However,
Figure 2.10: Illustration of a SEM configuration with a survey and an immersion lens mode. In the immersion lens modes the vacuum chamber is part of the magnetic circuit. The resulting magnetic flux density at the optical axis is shown on the left for both modes.

the immersion lens mode to obtain an overview of the sample at low magnifications. Therefore, two different lenses are required. The switch from one lens mode to another, because of a threshold in the magnification, should be fast.

Fig. 2.10 shows a SEM configuration with two lenses. Note that a part of the magnetic circuit is shared by both lenses. A switch from one lens mode to another is obtained by enabling current to the next mode and disabling the other. For the immersion mode, the specimen and the stage are within the electromagnetic field. Also the vacuum chamber itself is part of the magnetic circuit. While the lens-yoke is made of soft ferromagnetic material, the vacuum chamber is made of steel without any special magnetic properties.

2.5.3 SEM and Focused Ion Beam

In so-called dual beam systems an electron optic imaging system is combined with a focused ion beam (FIB) system that is capable of manipulating the specimen, Fig. 2.11. It can for instance perform milling operations at micro to nanometer scale [89]. In [35] the machine is used for tomography applications. The major application area is found in the semiconductor industry.

Switching of magnetic lenses is required when imaging is carried out using electron optics, while sample manipulation is carried out by the ion beam system. The magnetic immersion lenses have to be switched off during the milling process since their fields influence the trajectory of the ion beam. The reference trajectory for the magnetic lens system is then on-off-on with variable times between transitions.

2.6 Conclusions

In this section the requirements on the maximal deviation of the magnetic flux density relevant for electron optics have been derived. From first principal models for the trajectories of charged particles in magnetic fields the relations between focal distance $f$, electron acceleration voltage $U$ and magnetic
Figure 2.11: Illustration of a combination of a SEM and a FIB configuration within one vacuum chamber. During FIB operation the SEM immersion lens has to be switched off. For electron optic imaging it has to be switched on again.

flux density $B$ have been derived. With the choice of a uniform magnetic flux distribution analytical expressions are derived for the required amplitude $B$ for a desired focal distance.

With a specific choice of imaging settings, e.g. scan area, scan pattern, acceleration voltage, stage position, the maximum relative variation that has no significant effect on the image quality is $\Delta B/B < 10^{-5}$ (0.001%). The magnetic lens controller is part of a larger defocus control structure. An error in focal distance is corrected by image based feedback techniques. For this control loop to work, the upper bound on the transition error made by the lens controller is $\Delta B/B < 10^{-2}$ (1%).

2.7 Beyond the Scope

Defocus control

In an operating point the level of defocus can possibly be controlled by image based feedback control. This provides a different set of requirements than with change of operating points. It will be shown that performance highly depends on the defocus detection methods which vary with machine type e.g. SEM or TEM, application settings, e.g. magnification, and specimen type, e.g. amorphous versus sharp edges. Defocus control for TEM is considered in other parts of the Condor project, e.g. [97], [82],[84],[83]. Defocus detection and optimization methods for SEM within Condor are considered in e.g. [70], [71].

Combined Defocus and Magnetic Flux Density Control

The ability of image based feedback control to work over a large range of defocus, to be robust for all types of specimen and to converge to the optimum fast greatly determines the requirements on transitions of the electromagnetic field. The two control layers have to work together. In this research the combination of the two controllers is not considered.
Chapter 3

Dynamics and Hysteresis in Electromagnetic Lenses

In this chapter transient effects occurring during magnetization of electrically conducting materials are discussed. In the previous chapter the steady state relation between the magnetic flux density in a point within the lens geometry and the optical properties of the lens was derived. That $B_z$ is a function of the coil’s input current was shown with the help of finite element analysis. In this section it will be illustrated that magnetization of the lens material takes time. First the effect of transients on recorded images is presented. Next, the equations for magnetization in time are derived from the Maxwell equations for a simplified geometry. Using finite element analysis the response of the lens is established. Further in this chapter the effect of hysteresis is discussed after which the control objective for switching operating point of the electromagnetic lens is formulated.

3.1 Transients and Image Acquisition

3.1.1 Time Involved with Image Acquisition

Due to the scanning principle of operation, significant time is involved with recording a single image. Fig. 3.1 shows the scan pattern which consists of $512^2$ cells. To construct the image, the electron intensities corresponding to the individual cells is converted into a gray scale pattern. The time that the electron beam is actually positioned within a cell is called the dwell-time and is at least $25\, \text{ns}$. In combination with the defined number of cells this results in $6.6\, \text{ms}$ to record a single image, about $153\, \text{images/s}$. However, an electron beam current of $0.2\, \text{nA}$ (table 2.1) in combination with the scan pattern implies 31 electrons per cell which results in a poor signal to noise ratio. A longer dwell time contributes to a higher signal to noise ratio but slows down image recording. A typical setting, as used throughout this work, is about $70\, \text{ms}$ to record a single image $\approx 14\, \text{images/s}$. This implies a dwell time of $267\, \text{ns}$ and 333 electrons per cell.

3.1.2 Transients in Image Recording

If the magnetic flux distribution in the lens varies over time, then the pattern $B_z(t)$ can be recognized within the image. Fig. 3.1 shows images recorded during a sine wave and during a step in the lens current. Since most, if not all, defocus estimation algorithms assume a constant $\Delta f$ within a single image variation of $B_z(t)$ is highly unwanted. In Fig. 3.1 the deformations of the image are a direct effect of a transient lens current. However, in Fig. 3.2 the step response of the lens system is captured. At $t = t_0$ the current rapidly rises to a constant level which is reached at $t = t_1$. A pure step can not be made because of a limited voltage range driving the coil, but the current reaches steady state after $\approx 20\, \text{ms}$. The magnetic flux density in the electron optic volume lags behind, which is illustrated with the slowly rising trajectory $B(t)$. From the image it is observed that at $t = t_2$ the lens system reaches steady state. The transition time $t_2$ is around $1.5\, \text{s}$. The lens and microscope system and settings used
Figure 3.1: Scan pattern and transient effects observed with imaging when not in steady state. The amplitude of the sine wave is about 0.01% of the total current range ($\pm 2.2 A$). The step is applied at $\approx 10 ms$ and has an amplitude of 10% of the total current range.

for this experiment are different from the ones in Fig. 3.1. The lens in this experiment is a so-called immersion lens as discussed in section 2.5. In chapter 4 an extended analysis of transient effects that occurring in SEM applications is provided.

### 3.2 Transient Effects in Electromagnetic Actuation

In this section the time domain response for magnetization of a simple geometry is derived from the Maxwell equations. The presented example involves the analysis of a coil with a ferromagnetic electrically conducting core. A similar example is presented in [87] for a coil with a non conducting core. Further background can be found in e.g. [76]. Fig. 3.3 shows two geometries, one is the coil considered in this example, the other one is a c-core. Both are much simpler than the electromagnetic lens as shown in Fig. 2.2. However, the control objective can be the same for any electromagnetic actuator: Given the geometry and the materials, how to switch as fast as possible between two non-zero constant states of magnetization? This objective is further formalized in section 3.4. The result of the derivation of the transfer functions relating current, magnetic flux density and voltage is wrapped up in section 3.2.4.

#### 3.2.1 Coil in Air

For a coil in air ($\mu = \mu_0$) the relation between current, voltage, magnetic field strength and magnetic flux density is defined by:

\[
U = L \frac{dI}{dt} + RI \tag{3.1a}
\]

\[
\oint H(a) dl = nI \tag{3.1b}
\]

\[
B(r) = \mu H(r). \tag{3.1c}
\]

Here the coil has $n$ windings, length $2b$ and radius $r = a$. The resistance of the wire is $R[\Omega]$. The only dynamics are found in the relation between current and voltage. There are no dynamics between the flux density and the current. The current through the coil is obtained by integration of the voltage divided by the coil value $L[H]$. The only reason for a transition time is therefore a limited voltage range of the driving circuit to generate the coil current. A constant current implies a constant magnetic field.
3.2. Transient Effects in Electromagnetic Actuation

Figure 3.2: Step response of the electromagnetic field observed from a recorded image. The trajectories of $I$ and $B$ are conceptual. The recorded image illustrates that the magnetic flux density lag behind the lens current.

3.2.2 Coil with a Ferromagnetic Conducting Core

The reason that the electromagnetic lens does show transient effects is found in the electrical conductivity of the material. A change in applied magnetic field strength $H [A/m]$ generates eddy currents. Due to this effect the inductance, the so-called $L$, can no longer be considered constant. The core of the coil is now made of isotropic material with a constant conductivity $\sigma$ and constant permeability $\mu = \mu_r \mu_0$. This example does not take into account a nonlinear $H,B$ curve, nor does it deal with hysteresis. Nonlinear effects present in the actual electromagnetic lens system influence the transient behavior, but this example for a simple linear system will show that the nonlinearities do not cause the transients.

The partial differential equations describing the system are the Maxwell equations:

$$\nabla \times E + \frac{\partial B}{\partial t} = 0 \quad (3.2a)$$
$$\nabla \times H - \frac{\partial D}{\partial t} = J. \quad (3.2b)$$

The constant permeability allows for substituting $B = \mu H$. The electric flux density is equal to the permittivity times the electric field strength $D = \varepsilon_0 E$. For a coil with a conducting core the permittivity is considered $\varepsilon_0 = \frac{1}{36\pi} 10^{-9} A s V^{-1}$. The conductivity is in the order of $\sigma \approx 10^7 V A^{-1} m^{-1}$. Further the current density is set equal to the conductivity of the core material times the electrical field strength $J = \sigma E$.

Due to this specific geometry a cylindrical coordinate frame $(r, \phi, z)$ is used where $\vec{u}$ represents the unit vector. The rotation for the electric field in cylindrical coordinates is defined as:

$$\nabla \times E = \vec{u}_r \left[ \frac{1}{r} \frac{\partial E_r}{\partial \phi} - \frac{\partial E_\phi}{\partial z} \right] + \vec{u}_\phi \left[ \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right] + \vec{u}_z \frac{1}{r} \left[ \frac{\partial (r E_\phi)}{\partial r} - \frac{\partial E_r}{\partial \phi} \right]. \quad (3.3)$$
Chapter 3. Dynamics and Hysteresis in Electromagnetic Lenses

\[ b - b_r = 0 \]
\[ r = a \]

Figure 3.3: Left.) Conceptual illustration of a coil driving the magnetic flux in the airgap of the c-core. Right.) Illustration of a long cylindrical coil.

It follows from the geometry that \( H_r = 0, H_\phi = 0, \frac{\partial H_z}{\partial r} = 0, E_r = 0, E_z = 0, \frac{\partial E_\phi}{\partial z} = 0 \).

\[ \nabla \times E = \frac{1}{r} \frac{\partial (r E_\phi)}{\partial r} = \frac{\partial E_\phi}{\partial r} + \frac{1}{r} E_\phi \quad (3.4a) \]
\[ \nabla \times H = -\frac{\partial H_z}{\partial r}. \quad (3.4b) \]

Note that the magnetic field has only a component in the \( z \)-direction and that the electric field only has a component in the \( \phi \) direction. Neglecting the other components is allowed due to the specific choice of geometry in which the radius \( a \) is much smaller than the length of the coil \( b \). Still \( b \) has finite length too make it possible to calculate the voltage over the coil (from \(-b\) to \(b\)). The Maxwell equations for the long coil with ferromagnetic conducting core are now given by:

\[ 0 = \frac{\partial E_\phi}{\partial r} + \frac{1}{r} E_\phi + \frac{\mu}{\sigma} \frac{\partial H_z}{\partial t} \quad (3.5a) \]
\[ 0 = \frac{\partial H_z}{\partial r} + \epsilon \frac{\partial E_\phi}{\partial t} + \sigma E_\phi. \quad (3.5b) \]

For a linear dynamical system the time derivative \( \frac{\partial}{\partial t} \) may be substituted by the Laplace operator \( s = j\omega \) which results in a description in the frequency domain. If the coil is considered in the frequency range from \( f \in [1 mHz, 10 kHz] \) and \( \omega = 2\pi f \) then \( j\omega E_\phi << \sigma E_\phi \). The contribution of \( j\omega E_\phi \) is further neglected from which follows that:

\[ E_\phi = -\frac{1}{\sigma} \frac{\partial H_z}{\partial r}. \quad (3.6) \]

Substitution of \( E_\phi \) in (3.5a) results in a partial differential equation that is only dependent on \( H_z(r, t) \):

\[ -\frac{1}{\sigma} \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{1}{\sigma} \frac{\partial H_z}{\partial r} + \frac{\mu}{\sigma} \frac{\partial H_z}{\partial t} = 0 \quad (3.7a) \]
\[ \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} - \frac{\sigma}{\mu} \frac{\partial H_z}{\partial t} = 0 \quad (3.7b) \]
\[ r^2 \frac{\partial^2 H_z}{\partial r^2} + r \frac{\partial H_z}{\partial r} - j\omega \sigma \mu r^2 H_z = 0. \quad (3.7c) \]
3.2. Transient Effects in Electromagnetic Actuation

The resulting equation is an example of Bessel’s differential equation:

\[ x^2 \frac{d^2 f(x)}{dx^2} + x \frac{df(x)}{dx} + (x^2 - \nu^2)f(x) = 0. \]  
(3.8)

The solution is \( f(x) = J_\nu(x) \) is a Bessel function of order \( \nu \):

\[ J_\nu(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{m+n}m!(\nu + m)!}. \]  
(3.9)

Now the wave number \( k = \sqrt{-j\omega\sigma\mu} = j\sqrt{\omega\sigma\mu} \). Substitution of \( x = kr \) and using \( \frac{df(r)}{dkr} = \frac{1}{k} \frac{df(r)}{dr} \) results in:

\[ (kr)^2 \frac{d^2 H_z}{d(kr)^2} + kr \frac{dH_z}{dkr} + ((kr)^2 - \nu^2)H_z = 0 \]  
(3.10)

From this result it follows that the solution is \( H_z(kr) = C_{bc}J_0(ka) \), a 0th-order Bessel function in which \( C_{bc} \) is a constant which can be obtained from the boundary conditions. The wires of the coil are considered infinitely thin and there is no space in between the windings. At the location of the wires at \( r = a \) the magnetic field strength is \( \oint H_z(ka)dl = nI \) where \( n \) is the number of windings. From this boundary condition it follows that:

\[ H_z(ka) = C_{bc}J_0(ka), \Rightarrow C_{bc} = \frac{nI}{J_0(ka)}. \]  
(3.11)

The resulting equations for \( H_z(r, j\omega) \) and \( B_z(r, j\omega) \) are:

\[ H_z(r, j\omega) = n \frac{J_0(kr)}{J_0(ka)} I(j\omega), \quad k = j\sqrt{\omega\sigma\mu} \]  
(3.12)

\[ B_z(r, j\omega) = \mu n \frac{J_0(kr)}{J_0(ka)} I(j\omega). \]  
(3.13)

Expression (3.13) is frequency domain description of the relation between the current running through the coil and the magnetic flux density at a point in the ferromagnetic core. Note that the length of the coil does not influence the relation and that the expression is only a function of \( r \) and not of \( \phi \) or \( z \). This is a consequence of the geometry in which \( a << b \).

3.2.3 Impedance of the Coil

To calculate the voltage over the coil, the frequency dependent impedance of the coil with ferromagnetic electrically conducting core is derived. For the derivation of \( E_\phi(r) \) the relation \( dJ_0(x)/dx = -J_1(x) \) is used, [87]:

\[ E_\phi(r, j\omega) = -\frac{1}{\sigma} \frac{\partial H_z}{\partial r} = \frac{n}{\sigma} \frac{kJ_1(kr)}{J_0(ka)} I(j\omega). \]  
(3.14)

The impedance \( Z(j\omega) \) can be calculated using:

\[ Z = -\frac{1}{|I|^2} \iint_S (E \times H^*) \cdot n'dA, \]  
(3.15a)

\[ E \times H^* = (u_\phi \times u_z)E_\phi H_z^* = \overline{u}_z E_\phi H_z^*. \]  
(3.15b)
Chapter 3. Dynamics and Hysteresis in Electromagnetic Lenses

\[ L_0 = 16 \text{mH}, \sigma = 1 \times 10^7 \text{A/(Vm)}, \mu = 5 \times 10^3, a = 2 \times 10^{-3} \text{m} \]

Figure 3.4: Frequency dependent impedance \( Z(j\omega) = R(j\omega) + j\omega L(j\omega) \) of the coil with ferromagnetic electrically conducting core. The upper plot shows the resistive part \( R(j\omega) \) and the lower plot shows the inductive part \( L(j\omega) \) as a function of frequency.

The normal \( n' \) points towards the symmetry axis of the cylinder. Only at \( r = a \) there is a contribution. Here \( n' = -u_r \) from which follows that:

\[
Z(j\omega) = \frac{U(j\omega)}{I(j\omega)} = \frac{1}{|I|^2} \int_0^{2\pi} r \int_{-b}^{b} (E_\phi H_z u_r) \cdot (-u_r) dz d\phi \bigg|_{r=a} \\
= -\frac{1}{|I|^2} \Omega (E_\phi H_z)|_{r=a} (2\pi)(2b) \\
= -\frac{4\pi abn^2 k J_1(ka)}{\sigma J_0(ka)}
\]

(3.16)

Fig. 3.4 shows the impedance of the coil split in a resistive and an inductive part \( Z(j\omega) = R(j\omega) + j\omega L(j\omega) \). From (3.16) the resistive part \( R \) is obtained by taking the real values and the inductance \( L \) by dividing the imaginary values by \( \omega \). Fig. 3.4 shows that the inductance decreases and the resistance increases as a function of frequency. Note that the DC-resistance is 0 since the wires of the coil are considered ideal and have no resistance. This neglected series resistance is about 4\( \Omega \) for the wires in the lens coil.

At low frequencies the eddy currents in the core have no significant contribution. At high frequencies they are dominant which cause the increase resistance along with frequency. In the \( \lim_{\omega \to 0} \) the magnetostatic inductance \( L_0 \) can be calculated. \( L_0 = 2\pi a^2 b n^2 \mu \) which is equal to 2 times the volume of the cylinder times the permeability times the squared number of coil windings. In the derivation stating from (3.16) it is used that \( Z_0 = \lim_{\omega \to 0} Z = \lim_{\omega \to 0} j\omega L_0, \lim_{\omega \to 0} J_0(ka) = 1 \)
3.2. Transient Effects in Electromagnetic Actuation

and \( \lim_{\omega \rightarrow 0} J_1(ka) = \frac{1}{2} ka \) from which follows that:

\[
Z_0 = \lim_{\omega \rightarrow 0} -\frac{4\pi abn^2}{\sigma} \frac{k J_1(ka)}{J_0(ka)} = \lim_{\omega \rightarrow 0} -\frac{4\pi abn^2}{\sigma} k^2 a = \lim_{\omega \rightarrow 0} 2\pi a^2 bn^2 \mu j \omega. \tag{3.17}
\]

3.2.4 Frequency domain behavior

The equations relating the input current to the magnetic flux density (3.13) and the current voltage relation (3.16) are evaluated in the frequency domain. The three quantities of interest are \( I, B(r), U \). Their relation is provided by (3.18) and (3.19):

\[
\frac{B_z(r, j \omega)}{I(j \omega)} = \mu n \frac{J_0(kr)}{J_0(ka)}, \quad k = j \sqrt{j \omega \mu \sigma}, \tag{3.18}
\]

\[
\frac{U(j \omega)}{I(j \omega)} = -\frac{4\pi abn^2}{\sigma} \frac{k J_1(ka)}{J_0(ka)} \tag{3.19}
\]

Fig. 3.5 shows the bode plots. The magnetic flux density is observed at \( r = 0 \) m. These bode-plots are obtained using a matlab implementation of the Bessel functions, [2]. For low frequencies \( f < 0.1 \) Hz in combination with the chosen geometry and the material parameters, the influence of dynamics is insignificant. The voltage over the coil is then equal to \( L_0 dI/dt \) and a constant current implies a constant magnetic flux density and a coil voltage equal to zero (the series resistance of the wires is
neglected). In the bode plot of $U(s)/I(s)$ a pure differentiator is found: An increase of $20\text{dB/decade}$ in the amplitude plot and a start at a phase shift of $+90^\circ$.

For high frequencies the asymptotic behavior of the current voltage relation is a so-called fractional order differentiator. That is the behavior approaches that of $\sqrt{s} = s^{0.5}$. In the bode plot this implies that the magnitudes increases with $0.5\cdot20\text{dB/decade}$ and that the phase is $0.5\cdot90 = 45\text{degrees}$. The frequency $f_c$ at which the conductance of the core material becomes significant is found when the skin depth equals the radius of the core, [56]:

$$a = \frac{1}{\sqrt{\pi f_c \mu \sigma}}, \quad \Rightarrow f_c = \frac{1}{\pi \mu \sigma a^2}. \quad (3.20)$$

In Fig. 3.5 this frequency is indicated with the dashed vertical line.

The behavior of the $I, B$ relation is low pass, illustrating that it takes time to magnetize the complete core. The amplitude plot of Fig. 3.5 stops at $-100\text{dB}$. From $\lim_{\omega \to \infty}$ of (3.18) it follows that the attenuation is infinite for this limit. From the phase plot in Fig. 3.5 it is observed that the phase keeps on decreasing with the frequency. In the plot the phase is wrapped between $\pm180\text{degrees}$. Such a behavior is also recognized with delays $e^{-s/\alpha}$. From the step response in time domain this will become more clear.

The bode plots for five different points at the radius $0 \leq r \leq a$ are shown in Fig. 3.6. This plot clearly shows the spatial dependence of the transfer between $I$ and $B_z(r)$. The phase plot is not wrapped between $-180$ and $180$ degrees to show that the phase keeps on decreasing.

![Bode diagram of $B$ as function of $I$ (3.18) for observation of the magnetic flux density at different positions $0 \leq r \leq a$. In this plot the phase is not wrapped between and keeps decreasing. This behavior approximates a delay.](image-url)
3.2. Transient Effects in Electromagnetic Actuation

3.2.5 Time Domain Behavior

The derived transfer functions represent linear dynamical systems. Simulation of the time domain response of a linear system can be carried out by multiplication of the transfer function with the Fourier transform of the input data. (A convolution in the time domain is a multiplication in the frequency domain, e.g. [32].) The inverse Fourier transform then provides the response of the model in time. Fig. 3.7 shows the response of $B(t)$ and $U(t)$ to a pre-filtered square-wave input current. Only the response to a rising edge is shown. Low pass pre-filtering ($1^{st}$ order, cutoff at $1kHz$) is used to limit the voltage which is proportional to $dI/dt$. The current reaches a constant level after a few $ms$, whereas the magnetic flux density and the voltage over the coil require times in the order of seconds. The discussed lag and delay of the magnetic field is clearly visible in the zoomed version of the plot. The magnetic flux density as a function of the applied current is presented for different positions on the radius $0 \leq r \leq a$. The different time domain responses of the individual lumped approaches from input current to the magnetic flux density at a single position in the core illustrates the spatially distributed nature of magnetization of electrically conducting materials.

![Figure 3.7: Time domain response to a pre-filtered square wave input current $I$ of the magnetic flux density $B$ at different radial positions $0 \leq r \leq a$ in the core and the voltage over the coil $U$. The pre-filter was first order lowpass with a cut-off frequency of $1kHz$ such that the current level is constant after a few $ms$. The zoomed version of $B(t)$ illustrates the approximation of a delay due to the diffusion of the electromagnetic field: for $r = 0$ about $100ms$ are required before $B(t)$ starts rising significantly.](image)

The main reason to carry out simulation using the frequency domain is that the transfer functions (3.18) and (3.19) are infinite dimensional as a result of a lumped description of a spatially distributed dynamical system. Without approximation such systems cannot be simulated using the well-known and convenient solvers for instance implemented in matlab/simulink.

The Maxwell equations are two coupled partial differential equations (PDE). The solution of the PDE represents a distribution of physical quantities, in this case $B(r)$ and $E(\phi)$. The obtained transfer
function in the frequency domain relating the input current and one point in the geometry is infinite dimensional. That is it cannot be fully described by descriptions with a limited number of states. Or with the presented transfer function description it cannot be fully described by the division of two polynomials containing only integer powers of the Laplace operator \( s \), e.g. \( s, s^2, \cdots s^n \). The transfer functions derived for the coil (3.18) and (3.19) contain non-integer or so-called fractional orders in this case \( s^{0.5} \). Any integer order realization will only be an approximation. A tutorial on this topic is found in [17]. In [53] an approach to modeling and simulation and control of such systems is provided. In [36] high integer order approximation of the infinite dimensional model is used for modeling the spatially distributed effects in an electromagnetic molding machine.

### 3.2.6 Dynamics of the Electromagnetic Lens

Using finite element software insight is obtained in magnetization of the electromagnetic lens. The specific package used [52] cannot provide the transient response in the time domain, but it can calculate the time-harmonic response to input currents of the form \( I(t) = \sin(2\pi ft) \). For the point at the optical axis that has the largest amplitude \( B_z(r = 0, z = 43\text{mm}) \) (Fig. 2.3, chapter 2) the bode-plot is obtained using this method, Fig. 3.8. The DC-gain is normalized to 0 dB. The plot representing the magnetic flux density at the optical axis shows the amplitudes when exciting the lens using sinusoidal currents with different frequencies. However, a step response is more of interest. By taking the time equal to \( t = 1/f \) the step response of \( B_z \) in time is approximated by the responses for different frequencies. The same approximation is presented in [86].

The snapshots in Fig. 3.9 present the magnetic flux distribution as an effect of a time-harmonic input with one specific frequency component. Also here insight of the step response can be obtained by replacing \( 1/f \) by the time from \( t = 0 \). This approximation is only used for illustration and understanding of the process of magnetization. The use of a more advanced finite element software package that can deal with transient simulation besides time harmonic simulations, e.g. [4], [13], [14] does not require such approximation.

### 3.2.7 Transient Effects: Eddy Currents

In the presented derivation of the response of \( B(I) \) from the Maxwell equations it was not explicitly mentioned what physical effect is causing the magnetization to lag behind the variation of the input...
3.3. Hysteresis Effects in Electromagnetic Actuation

In (3.2b) the current density $J$ was substituted with $J = \sigma E$. Due to the variation of the magnetic field in time electrical fields are induced which results in currents in the core material. The currents are called eddy current or Foucault’s currents, [65]. Since the conductivity of the core material is limited, eddy currents cause losses since their presence involves energy consumption. From the energy point of view it is intuitive that magnetization takes time since the energy applied to the lens coil is partly consumed by side-effects. One can also reason that the induced currents themselves produce a magnetic field that counteracts the one caused by the actuator. Next to eddy current effects, magnetization of ferromagnetic materials also involves hysteresis effects. In first instance hysteresis does not limit the transition time but it does influence the steady state flux distribution. The effect of hysteresis is explained in the next section.

### 3.3.1 Hysteresis in Image Recording

Complaints about hysteresis in the magnetic lens system appear in scientific literature around 1970, [75], [81]. In [7] the influence of hysteresis during autofocus algorithms is expressed in deviation of sharpness and an heuristic correction algorithm is described. Although that problems with hysteresis
are detected decades ago, a satisfying general solution is not available.

In Fig. 3.10 the effect of ferromagnetic hysteresis is illustrated by switching of operating point. The figure shows snapshots of the image sequence along with the applied input current and the estimated level of sharpness of the image $S$. Sharpness is highly related to the level of defocus. Optimal focus provides a high number, whereas a blurred image results in a lower number. Implementation of the algorithm is presented in chapter 4. The experiment presented in Fig. 3.10 is analyzed by dividing it into 5 modes:

1. **Operating point 1:**
   \[ M_1 : t < t_0 \]
   The initial situation. The current applied to the lens coil is constant. The system is in steady state since the input is constant and the sharpness is constant and equal to $S_1$. All images recorded in mode $M_1$ are considered in focus as observed from the sample image. The sharpness signal $S$ has frequency of $14Hz$ estimated from the recorded images $14images/s$.

2. **Transition of operating point:**
   \[ M_2 : t_0 \leq t < t_1 \]
   The lens current is changed over $-10\%$ of its total range. The input current is pre-filtered using a $3^{rd}$-order lowpass filter. Transient effects occur during the change of settings.

3. **Operating point 2:**
   \[ M_3 : t_1 \leq t < t_2 \]
   Besides the lens current no microscope settings have changed. The level of defocus is so high that the images recorded contain only noise. $S = S_2$ and is very low. The system stays in mode $M_3$ for $2s$ which is considered enough to converge.

4. **Transition of operating point:**
   \[ M_4 : t_2 \leq t < t_3 \]
   The initial operating point is reproduced by applying the same constant input as in mode $M_1$. Transient effects are observed from the change in $S$.

5. **Operating point 1:**
   \[ M_5 : t_3 < t \]
   After the transients have decayed, the sharpness signal $S$ converges to a constant level $S_3$. However, there is a significant difference between $S_1$ and $S_3$. Since the system is in both situations in steady state and no settings other than the lens current have changed, transient effects cannot explain this effect. The difference $\Delta S = S_1 - S_3$ is the result of an effective difference in magnetic flux density as a result of hysteresis.

In Fig. 3.11 a conceptual representation of the relation between applied current and magnetic flux density in both the time domain and the input-output plot is presented. Of interest is the steady state situation after decay of transient effects, which is provided in the phase plane. Before describing the effects of hysteresis on the control error, the physics of hysteresis is discussed.

### 3.3.2 Hysteresis and Dynamics

The effect of hysteresis is often observed and described for periodic inputs and low frequencies. In Fig. 3.12 a comparison of the response of a linear dynamical system and a hysteretic system with similar dynamics is presented for sinusoidal inputs. The representation of two time dependent signals in the phase plane is called an orbit, as is common in nonlinear dynamics literature, e.g. [77]. In magnetics literature an orbit is called a loop. The fundamental difference in the response of the two systems is observed for low frequencies. The area enclosed by the orbit approaches zero for linear dynamical systems. This means that there is no phase difference between the input and the output. For the system
3.3. Hysteresis Effects in Electromagnetic Actuation

Figure 3.10: The hysteresis effect observed when switching between operating points. In both mode $M_1$ and $M_5$ the same input current is applied while the observed images have a significant different level of defocus. The level of defocus is related to sharpness $S$. A high sharpness indicates a low level of defocus. $S$ is calculated for each individual image. 14 images/s are recorded.

with hysteresis the *multi-valued input-output mapping* also maintains for low frequencies $\lim_{f \to 0}$. For high frequencies the attenuation, high frequent roll-off, ensures that the area approaches zero for the described systems. In most magnetic literature the applied magnetic field strength $H[A/m]$ is plotted versus $B[T]$. However, also $H$ is a 3D distribution. Since $I$ is the input to the lens system and the total applied $H$ is a result of $nI$, in the presentation of $I$ vs. $B$ is used. $n$ represents the number of coil windings.

3.3.3 State-of-the-art Spatial Models

The point and quantity of interest is $B_z$ at the optical axis. In a lumped approach a description is required relating the input current to this *signal*. Then a scalar model hysteresis model is sufficient. However, actually the defined problem is a spatially distributed one. To calculate the magnetic flux density at every point in the lens geometry 3D vector hysteresis models are required. In the analysis of the transient behavior, extracting the lumped frequency domain transfer function from the coupled PDE’s did not cause problems. There is the problem of finding a good rational approximation of the infinite dimensional result, but the derived transfer function is lumped and valid.

It sounds attractive to include the hysteresis effect in the Maxwell equations and extract a lumped
Chapter 3. Dynamics and Hysteresis in Electromagnetic Lenses

Figure 3.11: Illustration of the influence of hysteresis for the measurement presented in Fig. 3.10. The signals in time $I(t), B(t)$ are both linked to the representation in the phase plane.

model description for the relation between lens current and magnetic flux density at a point in the geometry. However, for this approach to work, it should at least be possible to implement and simulate both the hysteresis and transient effects for the lens geometry. At time of writing, there is no commercially available software package that can predict the effects with the desired accuracy.

An accurate simulation model is one that can predict over a time scale of $0.1 ms$ to $60 s$ for a 3d distribution of size $1 dm^3$. The allowed prediction error for electron microscopy applications is, as derived in section 2.3, $\Delta B/B < 10^{-5}$. In e.g. [46] specifications for FEM for (static) lens design are provided. State-of-the-art software for 3d transient FEM including hysteresis is considered to be [14], [51]. In June 2010 hysteresis occurring in reluctance actuators was predicted using this software [100]. The author of [100] concludes that the results predicted by the simulation could never be the result of actual physics of magnetization. Other renowned software like [13] and [4] can also not deal with this type of problem. Modeling of spatial-temporal electromagnetic systems is a major activity in the magnetics society, e.g. [26]. Most applications are involved with prediction of the losses in electrical machines and transformers. The observation that the simulation software is not mature is the main reason not to continue with spatially distributed hysteresis models.

3.3.4 The Cause of Hysteresis

The actual cause of hysteresis occurring in magnetization is found at the atomic scale, [41], [40]. The question why magnetization of a material shows hysteresis? belongs to the set of questions that also contains why is a material electrically conducting? or why is a material thermally conducting?. The material characteristics vary as a function of the elements used in the resulting alloy e.g. Ni, Fe, Cr, Co, etc. and the method of processing the alloy, e.g. heating, milling. The question How to obtain a lens material with a high permeability and low amount of hysteresis? is then equivalent to How to produce high quality materials for semiconductors?

The choice for NiFe in electron microscopy applications is among other properties based on the obtained relative permeability $\mu_r$, which is $> 5000$. In combination with the geometry this material enables to obtain a magnetic field distribution with sufficient magnitude and a low level of optical aberrations. There are materials available with less dominant hysteresis behavior having a similar permeability. Similar reasoning holds for the eddy current effects. If the material is less conducting, $\sigma$ is lower, then transitions take less time. However, the complete set of design requirements, including the cost and availability have to be explicit to make an optimal selection. Material selection is not further considered in this research.
3.3. Hysteresis Effects in Electromagnetic Actuation

![Comparison of the response of a dynamical with and without hysteresis to sinusoidal excitation with different frequencies represented in the phase plane. The difference between the systems is observed for low frequencies: The area enclosed by the orbit for the system without hysteresis approaches zero while the orbit described by the hysteretic system approaches a multi-valued mapping for low frequencies.](image)

Building up models starting at the scale of atoms is another possibility. However, also this approach does not provide the required models (yet). Hysteresis models used in this research are phenomenological models which are identified from and validated using experiments. However, although a phenomenological model is not extracted from physical laws there prediction still has to fulfill physical properties. A similar discussion is presented in [23]. In e.g. [95] energy considerations in a micromagnetic hysteresis model are compared to a Preisach model which is a phenomenological model. An overview of existing modeling techniques is found in *The Science of Hysteresis* (2005) [10]. This reference work consists of 3 volumes with ≈ 700 pages each discussing the different aspects of hysteresis, mainly devoted to magnetization. However, hysteresis is natural phenomena occurring in various relations between physical quantities:

- The relation between voltage and displacement in **piezo electric actuators**.
- The relation between displacement and expansion in **magnetostrictive actuators**.
- The relation between water content in the soil and capillary pressure (**terrestrial hydrology**), [5].
- **Hysteresis in phase transitions** (liquids to solids, etc.), [12].

Functional analysis of the mathematics describing the models is treated in e.g. [42], [98], [12]. The models used in this work are phenomenological models taking into account a lumped relation between two signals in time. The models are defined in chapter 7.
3.4 Formulation of Control Objective

In this section the control objective, consisting of minimization of the maximum transition time and minimization of the maximum transition error, is explicitly defined. The lens system $L$ is interconnected with a controller $C$, as illustrated in Fig. 3.13. The controller is a conceptual controller needed to formulate the objective. The choice for a specific feedback or feed forward structure is made in chapter 6. Electron microscopy applications are not involved with periodic excitation of the electromagnetic lens. The purpose of control is fast and accurate switching between operating points. Therefore, the reference signal $R_B$ consists of a sequence of consecutive steps with varying amplitude and varying time in between steps.

For electron optics the magnetic flux distribution observed by the electrons is of importance. It is assumed that control of the amplitude of $B_z(t, r, z)$ at a single point within the electron optic volume is sufficient to control the distribution in the optical volume, for instance the point on the optical axis that has the maximum amplitude of $B_z$. In the presented simulations this was $(r = 0, z = 43\text{mm})$. For convenience the notation $B(t)$ replaces $B_z(t, r, \phi, z)$ and represents the magnetic flux density at this point.

Figure 3.13: Schematic representation of the lens system. $R_B$ is the step wise varying reference signal, $I$ the current for the lens coil, and $B$ the magnetic flux density at a specific point in the lens geometry.

3.4.1 Performance Indicators

Switching operating points of electromagnetic lenses is the transition from one constant magnetic flux density distribution to another. However, convergence to $dB/dt = 0$ is only defined in the limit of $t$ to infinity. However, as derived, a small variation does not influence the image quality. From this it follows that transitions can be considered finished if $\Delta B < \epsilon_{tr}$ over time period $\Gamma$ which is chosen equal to the time involved with the recording of e.g. 10 images. The difference in quality (sharpness/defocus) between the 10 images should be insignificant, quantified with the variable $\epsilon_{tr}$.

The maximum variation $\psi$ of $B(t)$ over a window length $\Gamma$ is now defined as:

Definition 3.4.1.1. Maximum variation $\psi(t)$

Given the signal $B(t)$ for $t_0 \leq t \leq t_e$, the maximum variation $\psi(t)$ of $B(t)$ over a window of length $\Gamma$ is defined by:

$$\psi(t) = \max_{\nu_1} B(t + \nu_1) - \min_{\nu_2} B(t + \nu_2),$$

$$0 \leq \nu_1 \leq \Gamma, \quad 0 \leq \nu_2 \leq \Gamma, \quad t_0 \leq t \leq t_e - \Gamma.$$  \hspace{1cm} (3.21)

The maximum variation $\psi(t)$ is a signal over time that indicates whether $B(t)$ can be used for imaging purposes. If $\psi > \epsilon_{tr}$ the influence of transients in still too high. Fig. 3.14 shows that
estimating $\psi$ can be considered as estimating the height of a rectangle with fixed width $\Gamma$ required to enclose the signal $B(t)$ from $t$ to $t + \Gamma$.

The set point transition is initiated by a change of $R_B$ at $t = t_0$. The transition time is defined as $\tau_{tr}$:

**Definition 3.4.1.2. Transition time $\tau_{tr}$**

*Given a change of operating point initiated at $t = t_0$. Then the time instance at which the transition is finished is defined as $t_{tr}$:*

\[
t_{tr} = \min_t \psi(t) \leq \epsilon_{tr}, \quad t_0 \leq t \leq t_e - \Gamma,
\]

(3.22)

*the total duration of the transition is then:*

\[
\tau_{tr} = t_{tr} - t_0.
\]

(3.23)

Online estimation of $t_{tr}$ will always lag $\Gamma s$ behind. To test and compare controller performance, estimation can actually be carried out offline. When a specific controller structure is chosen, an online (model based) estimator can be formulated in which a similar measure is based on observation of the states. In this work estimation of $\tau_{tr}$ takes place offline as is explained in chapter 6. The formulation of transition error and transition time is suitable for experiments of finite durations. It can be argued that the objective is to determine whether $dB/dt, d^2 B/dt^2, \ldots$ are very small, but that would require a bound for each derivative and more important this derivative based approach is not robust for dealing with high frequent noise with small amplitudes. On the low frequency side, drift can also be dealt with using the proposed method.

The definition of the transition time is exclusively based on the signal $B(t)$. The reference value $R_B$ is not included in the definition. The resulting $t_{tr} = t_0 + \tau_{tr}$ only tells when the system is ready for imaging, not if there is any error between the desired value and the obtained value. The reason for this approach is explained next when dealing with the hysteresis effect. The set point transition starts at $t = t_0$ and is considered finished $\tau_{tr}$ later. The transition error $\epsilon_{tr}$ is now given by:

![Image](image.png)

Figure 3.14: Illustration of transition time $\tau_{tr}$ and the maximum variation $\psi(t)$ of a duration of $\Gamma s$. 

43
Chapter 3. Dynamics and Hysteresis in Electromagnetic Lenses

**Definition 3.4.1.3. Transition error $e_{tr}(t)$**
Given the reference value $R_B(t) = R_B(t_0)$ for $t_0 \leq t \leq t_e$ and the signal $B(t)$ for $t_0 + \tau_{tr} \leq t \leq t_0 + \tau_{tr} + \Gamma$, the obtained magnetic flux density after the set point transition is defined as:

$$B_{tr} = \frac{1}{\Gamma} \int_{t_0 + \tau_{tr} + \Gamma}^{t_0 + \tau_{tr}} B(t) dt,$$

from which the transition error follows:

$$e_{tr} = R_B(t_0) - B_{tr}.$$  \hfill (3.25)

The average value of $B$ is calculated using the integral over time $\Gamma$. Fig. 3.14 illustrates the concept of transition error related to the transition time.

The performance indicators $\tau_{tr}$ and $e_{tr}$ defined in (3.23) and (3.25) depend on the initial conditions before switching and the new desired set point. The range of desired set points $R_B \in \mathbb{R}$ is continuous and bounded between the minimum $\hat{R}_B$ and the maximum $\check{R}_B$ such that $\hat{R}_B \leq R_B \leq \check{R}_B$. The range of the input current is also limited: $\hat{I} \leq I \leq \check{I}$. With this current range it is ensured that all required $B$-values can be obtained: $B(\check{I}) > \hat{R}_B$ and $B(\hat{I}) < \check{R}_B$.

**controller performance map**

Figure 3.15: Illustration of the multi-objective performance map showing the performance in terms of transition time $\tau_{tr}$ and transition error $e_{tr}$. The axis for the transition error is normalized such that an error $e_{tr} = 100\%$ corresponds to the maximum possible error: $e_{tr} = \check{R}_B - \hat{R}_B$.
Minimization of both \( \hat{\tau}_{tr} \) and \( \hat{e}_{tr} \) is a multi-objective criterion that does not have a single solution, but has a set of non-inferior solutions providing the optimal balance between error and speed. Fig. 3.15 shows the multi-objective performance map relating the transition time and transition error. It is very likely that the minimization of the transition error conflicts with the minimization of the transition time. The multi-objective criterion is defined as:

\[
\min_{\theta} [\hat{e}_{tr}(\theta), \hat{\tau}_{tr}(\theta)].
\]  

(3.26)

In a certain controller structure \( C(\theta) \) \( \theta \) represents the parameterization of the controller. As an example if \( C(\theta) \) is a PID feedback controller then \( \theta = [k_p, k_i, k_d] \). The set of non-inferior solutions are those solutions for which an improvement in one objective always requires a degradation of at least one of the other objectives, [49]. Therefore, an inferior solution is a parameter combination of no value, since improvements can be obtained in all objectives. The non-inferior solutions are also called Pareto optima, [20]. Now a non-inferior solution \( \theta^* \) is defined as:

**Definition 3.4.1.4. Non-inferior solution \( \theta^* \), [49]:**

If the admissible set of parameters is defined as \( \theta \in \Theta \), then point \( \theta^* \in \Theta \) is a non-inferior solution of the multi-objective criterion (3.26) if for some neighborhood of \( \theta^* \) there does not exist a \( \Delta \theta \) such that \( \theta^* + \Delta \theta \in \Theta \) and

\[
\hat{e}_{tr}(\theta^* + \Delta \theta) \leq \hat{e}_{tr}(\theta^*) \text{ and } \hat{\tau}_{tr}(\theta^* + \Delta \theta) < \hat{\tau}_{tr}(\theta^*).
\]

The objective is to investigate different controller structures, e.g. feed forward, feedback, and their parameterization such that \( \hat{e}_{tr} \) and \( \hat{\tau}_{tr} \) are minimized. The three relevant bounds important for microscopy applications are presented in the performance map, Fig. 3.15. A transition error smaller than lower bound is not of interest because this difference is not visible in image quality. A transition error larger than the upper bound for image based defocus is also not of interest because it then may happen (dependent on initial conditions) that the error is that large that no defocus information can be extracted from the images which will result in a significantly increased overall transition time.

Given a certain controller implementation then the maximum transition time \( \hat{\tau}_{tr} \) and the maximum transition error \( \hat{e}_{tr} \) determine the controller performance. An experimental procedure for estimation of these quantities is presented in chapter 6.

In section 2.4 it was explained that the controlled electromagnetic lens system is a part of defocus control. A lens controller which is not capable of controlling \( e_{tr} < 10^{-3}B \) is not necessarily a bad controller. The image based defocus control loop can take over once the transition is finished. Therefore, a low transition time is always beneficial. In section 2.4 it was also mentioned that the reference value \( R_B \) itself can be uncertain, which implies that a low error is of less importance. However, the maximum error (as derived in section 2.4.3) was estimated \( \approx 1\% \) of \( R_B \).

- **It is always beneficial to decrease the transition time of the lens system, since no high quality images can be recorded if the transition is not finished.**
- **It is beneficial to have a low error of the lens controller if this low error results in faster convergence of the overall procedure. That is including the image based focus optimization.**

Both transition time and transition error are involved with the overall time needed for switching operating points: from the moment the change is required until the image quality in the new operating point is sufficient for the application. For example, if the defocus procedure involves about 30 images to find the optimum, [71], this brings along a time of \( 30 \cdot 70\text{ms} = 2.1\text{s} \). If a reduction of the error in the lens setting costs an extra \( 1\text{s} \) but the number of images in the optimum search is brought back to \( 10, 1 + 10 \cdot 70\text{ms} = 1.7\text{s} \) which gains \( 0.4\text{s} \). The transition time and processing in between two images is neglected in this comparison.
Chapter 3. Dynamics and Hysteresis in Electromagnetic Lenses

3.5 Beyond the Scope

3.5.1 Lens Design Properties

The hysteresis effect is a direct consequence of the choice of the lens material. The significance of the effect can be influenced by the geometry and the applied excitation. Only the influence of the applied excitation is studied using the existing design of SEM objective lenses (Helios machine type, produced by FEI Company).

Geometric Properties

Electron optic requirements are translated to requirements on static magnetic flux density distributions in the electron optic volume. A change of geometry, e.g. thickness of the yoke, size and shape of the lens gap, influences the optical properties. Less material or lamination can make the difference between response times of seconds and tenths of seconds. E.g. in [55] a cut was made in the lens cooling material. This limited the effects of eddy currents and resulted in decreased response times. In this research the geometric properties are considered to be given and fixed based on the existing lenses.

Material

The lenses studied in this work are made of NiFe alloys. The choice for a specific material results among other things from the required amplitudes of the magnetic flux density distributions. With the use of different materials or different production processes of the materials the impact of hysteresis can be influenced. This line of research is not further considered in this thesis.

Number of Actuators

The magnetic flux density distribution in the electron optic volume is controlled using a limited number of actuators. Most of the time only one actuator, one coil, is present. This actuator is capable of influencing the flux distributions over the whole required range. The distributions are dependent on the geometry, materials and level of excitation (and previous excitations). However, the amplitude in steady state is again a requirement on static behavior. For control of transients, multiple actuators, more coils with different shapes at different locations, can be beneficial. This is not studied further.
Chapter 4

Scanning Electron Microscopy Experiments

In this chapter experimental characterization of electromagnetic effects during transitions of operating points is obtained by analyzing the resulting image series. For this purpose a commercially available state-of-the-art scanning electron microscope is extended with a rapid prototyping and data acquisition system. All resulting images are stored and are offline synchronized to the applied lens current profiles. By designing experiments that capture the essence of switching operating points of the electromagnetic lens, insight is obtained in the sensitivity of the machine and the significance of dynamical and hysteretic effects.

In chapter 2 the requirements on variation of the electromagnetic flux density for electron optics was discussed on basis of first principle models. In chapter 3 transients effects were coupled to the material properties and the lens geometry using simplified models. Hysteresis effects were illustrated using phenomenological models that will be further described in chapter 7. In this chapter hysteresis and transient effects as observed in electron microscopy applications are illustrated and analyzed.

4.1 Sharpness

In the introduction to scanning electron microscopy, chapter 2, it is shown that the level of defocus or the projected spot size are the relevant quantities to observe and control. However, neither of both can be measured online. Some methods derive the level of defocus from image series [82], but their application is limited to a specific range of specimen and magnifications. Instead, image sharpness, as a related quantity to defocus, is derived from each individual image. The sharpness detection takes into account that a low level of defocus corresponds with a sharp image, whereas an image recorded with a higher level of defocus appears more blurred as has a lower sharpness. Therefore, optimizing the level of sharpness is in first degree similar to minimizing the spot size or minimizing the absolute level of defocus.

In scanning electron microscopy the intensity of the signal corresponding to the scan pattern of the electron beam on the specimen is represented as a gray value. White pixels implies a high intensity, black a very weak detected signal. Whenever the projected spot size is too large, which means that the level of defocus is high, the image will look gray and blurred. The pixel intensities \( p \) of unsharp images have a smaller deviation from the average pixel intensity \( \bar{p} \). This is what variance as a sharpness measure \( S \) is based on:

\[
S = \frac{1}{n} \sum (p - \bar{p})^2
\]
Chapter 4. SEM Experiments

\[ \bar{p} = \frac{1}{n \cdot m} \sum_{i=1}^{n} \sum_{j=1}^{m} p(i, j), \quad p \in [0, 1] \quad (4.1) \]

\[ S = \frac{1}{n \cdot m - 1} \sum_{i=1}^{n} \sum_{j=1}^{m} (p(i, j) - \bar{p})^2, \quad S \in [0, 0.25] \quad (4.2) \]

\( S \) is normalized with respect to the number of pixels \((n \cdot m)\), but not for the image content (the composition of the specimen is considered unknown). In the experiments presented in section 4.3 the maximum obtained sharpness is about 0.025. The maximum \( S \) is a function of the image content and is maximally equal to \( S = 0.25 \) for an image with \( \bar{p} = 0.5 \) containing only black and white pixels. The theoretical lower bound on sharpness is zero since \( S \) takes into account squared differences. \( S = 0 \) is only obtained with a uniform pixel intensity. However, since there is always some noise present, the minimum \( S \) will be slightly higher than 0.

Variance belongs to the class of functions that does only take into account the average pixel variation, e.g. [78]. These functions do not take into account any information about image contents (the relation among neighboring pixels). The level to which the assumption about black and white variation is satisfied will highly depend on the specimen under study. Therefore, the method does not hold for all possible sample types.

It will be illustrated that (4.2) is not valid for highly unsharp images. This is due to the relation between the projected spot size and the dimensions of a cell of the scan pattern. If the projected spot radius has the same order of dimensions of the complete scan area the detected signal does not contain enough information about the specimen, as discussed in chapter 2. However, the complete analysis is carried out offline and all images are available. Any abnormalities can be checked by studying the corresponding image series. For online implementation of for instance autofocus algorithms, variance may not be the best choice. Other sharpness methods for scanning microscopy applications are topic of current research [71], but are not further considered here.

### 4.2 SEM setup

![Figure 4.1: Feed forward control setting based on a magnetic field reference signal and the estimated image sharpness as output.](image)

Fig. 4.1 shows the lens system in a feed forward control setting. The quantities that can be measured are the input current and the recorded images. The magnetic flux density cannot be measured during online operation of the machine. In all the presented experiments, the feed forward controller consists of a static linear scaling. The reference signal \( R_B \) is conceptual. \( R_B \) is mapped to the full input current scale of \(-2.2 \leq R_I \leq 2.2A\). The conceptual feed forward control only indicates that it is the magnetic flux density distribution that is controlled using the lens current. The input current is presented along with the results.

The settings of all other subsystems of the microscope are not changed during the experiments, e.g. the stage is not moving, the specimen scan area is fixed by the deflection system and the electron
acceleration voltage is set. The electron optical process is then only influenced by variation of the lens settings. After every scan an image is generated (e.g. 14\textit{images/s}). Each image is evaluated concerning the level of defocus.

All measurements are carried out on an extended version of a commercial scanning electron microscope (SEM, type: Helios by FEI Company) which is extended with a rapid prototyping and data acquisition system (dSPACE), Fig. 4.2. Using the normal microscope interface, magnification, acceleration voltage, detector settings and the stage-position can be controlled. The current profiles are designed offline in matlab. All images (about 14\textit{images/s}, depending on the settings) are stored on the experiment PC. The sharpness of each image is evaluated afterwards by a matlab implementation of the sharpness measure (4.2). All settings except the lens current are kept constant during the presented experiments.

![Diagram of the setup consisting of a scanning electron microscope extended with a rapid prototyping and data acquisition system.](image)

Figure 4.2: left.) Block diagram of the setup consisting of a scanning electron microscope extended with a rapid prototyping and data acquisition system. The input current of the objective lens can be controlled. All resulting images are stored and synchronized to the applied current profiles. The sharpness of each individual image is calculated offline using matlab. right.) Photograph of the Helios-type scanning electron microscope produced by FEI Company.

The total input range of the lens current $I[A] = \pm 2.2\text{A}$. Relative input variation is expressed as a percentage of the maximal input variation $(\Delta I/4.4) \cdot 100\%$. Table 4.1 shows the typical machine settings used. Since, the presented experiments are carried out over several months the settings for each separate test can be different. This table provides typical values. The acceleration voltage of $U = 1\text{kV}$ on a possible range of 0.5$kV$ to 30$kV$ is chosen because the lower $U$ the higher the demands on the absolute maximum error in magnetic flux density $\Delta B$, as discussed in chapter 2. The maximum variation scales with the operating point: $\Delta B < \epsilon_{rel} B$ where $\epsilon_{rel} \approx 10^{-5}$. Image quality is not significantly influenced by a smaller variation of $B$.

4.3 SEM Experiment: Sensitivity in an Operating Point

Fig. 4.3 presents the variation of the image-sharpness a result of a step-wise quasi-static variation of the lens current in an operating point. Snapshots of the image sequence are presented in Fig. 4.3. The sensitivity around an operating point (4.3) is obtained from the $I$ vs. $S$-graph resulting from step-wise quasi-static excitation, Fig. 4.3. $\frac{\Delta S}{\Delta I} |_I$ is expressed in percentage sharpness deviation with respect to
Chapter 4. SEM Experiments

<table>
<thead>
<tr>
<th>machine type</th>
<th>FEI Helios</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage position</td>
<td>4mm</td>
</tr>
<tr>
<td>beam current</td>
<td>$\in [0.1, 0.7]nA$</td>
</tr>
<tr>
<td>beam radius</td>
<td>$\in [200, 300]\mu m$</td>
</tr>
<tr>
<td>image rate</td>
<td>14[$Hz$]</td>
</tr>
<tr>
<td>number of pixels</td>
<td>512$^2$</td>
</tr>
<tr>
<td>vertical scan time</td>
<td>70$ms$, $1/14Hz$</td>
</tr>
<tr>
<td>horizontal scan time</td>
<td>0.14$ms$, $1/7.3kHz$</td>
</tr>
<tr>
<td>specimen size</td>
<td>15$\mu m$</td>
</tr>
<tr>
<td>magnification</td>
<td>$\approx 7500$</td>
</tr>
<tr>
<td>acceleration voltage</td>
<td>1$kV$</td>
</tr>
</tbody>
</table>

Table 4.1: Microscope settings

the sharpness in the operating point per ampere:

$$\left| \frac{\Delta S}{\Delta I} \right| = \left| \frac{S_1 - S_2}{S_{max}(I_1 - I_2)} \right|_{I_{dc}}$$

(4.3)

Due to quasi-static variation (low $dI/dt$) and limited amplitude $\Delta I$ the influence of hysteresis and dynamical effects is negligible. $\Delta S/\Delta t$ should be so small that $S$ can be considered constant within a single image. With the settings used, an image is constructed by a scanning procedure starting in the upper left corner, line by line down to the right corner. This takes $\approx 70ms$ including the time to set the electron beam back to the upper left corner.

The $I$ vs. $S$ graph shows the result for two periods of the excitation. The specific wave form, shown in Fig. 4.3 is used in order to save measurement time. By running an initial experiment it is known in what region the maximum and minimum sharpness is expected. In the regions with less sharpness less images are recorded. The total variation of the lens current is about 6%. An equal projected spot-size in overfocus and underfocus (chapter 2) results in a curve that is approximately symmetric with respect to the offset that corresponds to optimal focus.

That the sensitivity of sharpness to a variation in the lens current is extremely high is validated by studying the sharpness difference between images $e_1$ and $f_1$ in Fig. 4.3. The sharpness $S_{f_1}$ represents the global maximum:

$$100\% \cdot \left| \frac{S_{f_1} - S_{e_1}}{S_{f_1}(I_{f_1} - I_{e_1})} \right| = 23[\%]/0.8[mA] = 28.8[\%]/mA$$

The difference in sharpness $23\%$ as observed from images $e_1$ and $f_1$ in Fig. 4.4 is caused by an variation in lens current of $0.8mA$ which is $0.018\%$ of the total range.

The current range covering the whole range in which any image features are observed and sharpness is a valid measure is about $-0.47mA + 0.44mA = 30mA$. This corresponds to $0.7\%$ of the full range. This value corresponds well to the estimate of $1\%$ based on first principles calculations discussed in chapter 2. The estimate of $1\%$ corresponded to a projected spot size so large that it covers the complete scan area.

The local maxima ($LM$) observed near zero sharpness (Fig. 4.3) illustrate the limited validity of (4.2) for highly unsharp images. From construction of the experiments it is clear these maxima are false: Under the condition that the quasi-static input current variation is monotonically increasing (or decreasing), the magnetic field will also be monotonically increasing (or decreasing). The only reason for the sharpness to change in opposite direction is the switch from overfocus to underfocus or vice
4.4 SEM Experiment: Hysteresis and Switching Effects

Figure 4.3: Sharpness $S$ variation as a response to a quasi-static variation of the input current over $\approx 6\%$. The corresponding images are presented in Fig. 4.4. The width of the images as shown represents $\approx 15\mu m$. The distance between lens and specimen is $z \approx 4mm$, electron acceleration voltage, $1kV$. False local maxima in $S$ are indicated with LM.

versa. The local maxima cause no problems in the presented set of experiments; As long as the images are within the sharpness range of between optimal $f_1$ and the level indicated by image $b_1$, variance is considered a valid sharpness measure.

4.4 SEM Experiment: Hysteresis and Switching Effects

The main way hysteresis expresses itself in electron microscopy is the fact that a single input current value $I_{dc}$ can result in a range of possible outputs $S$ depending on the magnetization history. $B(I)$ is thus not uniquely defined but is a function of both the input current and the magnetization history.

The experiment presented in Fig. 4.5 is carried out twice. The corresponding images are shown in Fig. 4.6. In both cases, the exact same transient lens current is applied, the offset $I_{dc}$ has a difference of a few mA. The experiment starts with a quasi-static sine wave. As shown in the previous section, this provides the quasi-static $I - S$ map. The images $(a, b, c, d)$ illustrate the sharpness variation during the small-signal quasi-static variation. After the quasi-static small-signal variation, two switching events are applied (pulse $N1$ in negative direction and pulse $P1$ in positive direction).

Image $f1$ shows the noisy image during $N1$, having low $S$ and not showing any image features. Image $f2$ illustrates the effect of scanning in a raster pattern in combination with a switching event.
Figure 4.4: Images corresponding to the experiment presented in Fig. 4.3. With each image the sharpness level and the corresponding lens current are provided.

Since it takes about 70 ms from top left to bottom right and N1 is applied in the middle of the image f2, the upper part contains information about the specimen and the lower part looks like the noise as in f2. The very high Sf2 results from the difference between specimen view and noise and is therefore an artifact.

When comparing images and S of e1 (the situation before N1) and g1 (the result after N1), the influence of hysteresis seems negligible in this situation. During the second time the experiment is carried out, the sharpness before (e2) and after (g2) pulse N1 is significantly different, ∆S ≈ 25%. The opposite happens during pulse P1 (compare images g and h). These are not temporal effects; Both current and sharpness reach steady state after about a second after the pulses. If there is no further input excitation S remains the same. The observed effect is summarized as follows: Temporary changes of the input in one direction have minor influence, while changes in the other direction result in a significantly decreased sharpness level. The same system can be sensitive to changes in negative direction and not in positive direction and vice versa.

4.4.1 Modeling Hysteresis during Switching

With the help of an interconnected model including hysteresis, the results obtained with the switching experiment, Fig. 4.5 are analyzed. The model consists of a phenomenological rate-independent hysteresis model representing the electromagnetic effects in the lens interconnected with a positive unimodal curve that takes into account the relation between the amplitude of magnetic flux density distribution in the electron optic volume, Fig. 4.7. It will be shown that with just these building blocks it is possible to reconstruct the scenario obtained with the experiments. The difference in response with the pulses in positive and negative direction are a consequence of different initial conditions.

From the definition of S (4.2) it is known that S ≥ 0. Due to the fact that under and overfocus
4.4. SEM Experiment: Hysteresis and Switching Effects

Figure 4.5: Sharpness and applied lens current for 2 experiments. The corresponding images are shown in Fig. 4.6. At the start of the measurement the image is focused manually. As the response to the quasi-static sine wave shows, it is near to the optimum. The applied transient lens current is the same for the 2 experiments. The offset current $I_{dc}$ is slightly different. The response in $S$ after the first switching event $N_1$ is completely different.

...can result in similar $S$ a positive unimodal function with a single extremum (maximum) is defined:

$$y = a_1 \left(1 + \left(\frac{w - a_2}{a_3}\right)^2\right)^{-1}. \quad (4.4)$$

Parameter $a_1$ represents the amplitude, $a_2$ the offset from the origin and $a_3$ the width of the bell-shaped curve. Amplitude $a_1$ will differ with the specimen morphology, specimen material, electron beam current, etc. The offset $a_2$ differs with the sample position, and the electron acceleration voltage. The half-width of the curve is here controlled by $a_3$, it is dependent on the specimen and the electron acceleration voltage. The hysteresis model used is a differential equation implementation of the Coleman-Hodgdon model [15] which is a member of the Duhem class, [57]:

$$\dot{w} = h_3 \cdot |\dot{v}| \cdot [h_2 v - w] + h_1 \dot{v}, \quad w(0) = w_0. \quad (4.5)$$

The time derivative of $dw/dt$ is denoted as $\dot{w}$. The parameters are bounded by $h_3 > 0, h_1 < h_2 < 2h_1$. More details about this specific model are provided in chapter 7. In the presented simulations the
parameters of the model are denoted below the different graphs in Fig. 4.8. The main reason to use this model is that it has a single parameter $w_0[T]$ that represents the initial condition of magnetization. It is the influence of initial conditions of hysteresis that is causing the difference between the two experiments.

A simulation of the model is carried out in which the 2 pulses ($N_1$) in negative direction and ($P_1$) in positive direction are applied as an input to the model. Fig. 4.8 and 4.9 show all signals of the interconnected model. Besides the pulses, three time instances ($\alpha$), ($\beta$), ($\gamma$) are indicated at which all signals ($I, B, S$) are constant and $I = 0$. In order to obtain similar behavior as in the experiment, the initial value of the hysteresis model $w_0$ and the offset $a_2$ of the nonlinear static curve are the most important parameters to tune. In this case $w_0 = a_2 \rightarrow S(0) = 1$.

As in the experiments the same transient is applied in two situations, both have $S(0) = 1$. The difference is in the initial condition $w_0$ of the hysteresis operator and with that the offset of the sharpness function $a_2$ in order to start at $S = 1$. In the first situation Fig. 4.8 a.), $N_1$ results in a closed minor loop defined as that the begin situation is the same as the situation after the pulse. In the input-output plot this is observed as a closed curve/loop. It is not that hysteresis is not present, it is that the combination of amplitude of $N_1$ and the initial condition are such that $B(\alpha) = B(\beta)$.
4.5. SEM Experiment: Feed Forward Initialization

In situation b.) the loop is not closed and $B(\beta) < B(\alpha)$. In both a.) and b.) the positive pulse results in a decreased $S$ because $B(\gamma) > B(\alpha)$. Note that in situation b.) $S(\beta) \approx S(\gamma)$, but that $S(\beta)$ is the result of overfocus ($B(\beta) < B(\alpha)$) and $S(\gamma)$ is the result of underfocus ($B(\gamma) > B(\alpha)$), Fig. 4.9.

The time plot of the output b.) shows a peak at 30s which need some explanation. What happens is that during the transition from ($\beta$) to ($P1$) the maximum of the sharpness curve is passed. In the experiments this effect is not observed. The most plausible reason is that the time involved with capturing a single image is about 70ms while the time that the sharpness is at maximum is only ms (due to the high $dI/dt$ during transitions). The effect is averaged out in the sharpness estimation. Time involved with imaging is not taken into account in the model. The cause of the peak is thus not similar to the previously explained artifact in Fig. 4.5, image f2.

Excitation of hysteresis observed in the input-output plane shows the loop structure $H[A/m]$ or $I[A]$ vs. $B[T]$ known from magnetics. If another quantity is observed, in this case $S$ instead of $B$, and the relation between $B$ and $S$ is a unimodal function, the mapping $I - S$ is transformed into a so-called 2-to-1 map [28]. The combination of hysteresis, a unimodal function and a periodic excitation can result in an input-output graph that looks like a butterfly. This is named butterfly hysteresis.

The approach of reconstruction the behavior of intermediate quantities that can not be measured during the experiment is highly useful to gain understanding in the process. The presented interconnected model structure cannot be used for (inverse model based) control but it can be used to confirm hypotheses about hysteresis behavior. With this approach assumptions on trajectories in time are made explicit. The results form the basis for future experiments or control strategies.

This model based analysis of the experimentally obtained observations presented in Fig. 4.5 and 4.6 clearly indicates that the memory effect of hysteresis causes the in first instance unpredictable behavior. In the hysteresis model that is used for the analysis (4.5), the memory is the state of the integrator. In chapter 7 representation of hysteretic memory within phenomenological models is further discussed. In the next chapter a first step towards a solution for resetting the state is presented.

4.5   SEM Experiment: Feed Forward Initialization

This section shows the first result of feed forward initialization, a technique to reset the states of systems with hysteresis, introduced and implemented in chapters 7 and 8. In open loop (feed forward) a specific input current trajectory is applied to the electromagnetic lens. The purpose of this trajectory is to wipe out the influence of previous excitation on the magnetic flux density distribution.

In Fig. 4.10 two experiments subject to the exact same excitation are presented. The image sharpness at the start of the experiments is significantly different. The feed forward initialization trajectory is a low frequent sine wave excitation (2.5 periods in 10s, in chapter 8 the initialization time will be

![Interconnected model consisting of an phenomenological hysteresis model representing the relation between lens current and magnetic flux density within the electron optic volume and a positive static unimodal curve representing the relation between magnetic flux density and sharpness.](image-url)
Chapter 4. SEM Experiments

Figure 4.8: Simulations of the interconnected model for two different initial conditions. The results are similar to the measured responses shown in Fig. 4.5. The results in the phase plane are shown in Fig. 4.9.

reduced to 0.1 to 0.5s) with an amplitude of 15% of full scale. This amplitude is significantly larger than the range in which images are recorded (image range < 1% as estimated in section 4.3). After the transients have decayed the difference in sharpness levels for the two experiments is insignificant. Note that the sharpness level after initialization is less than before. The lines for $S$ in Fig. 4.10 are indistinguishable. This result is the experimental proof of concept of feed forward initialization. This technique is a feed forward technique that can be used to reset the state of the system. In this way a reproducible combination of input current and magnetic flux density is obtained. Correction for the level of defocus is then the next step carried out using image based focus optimization. As discussed in chapter 2, image based control only works in the region where images have sufficient quality. In section 4.3 it was shown that this region is 1% of the full range of operating point values. If hysteresis effects can cause errors larger than this range performance of image based defocus is no longer guaranteed. In chapter 6 it will be shown that using feed forward without initialization the maximum error is about 5%.

During the time that the initialization trajectory is applied imaging applications have to wait. The images show transient effects as illustrated in chapter 3. In the experiment presented in Fig. 4.10 this is observed from the low sharpness during the initialization trajectory. For the electron microscopy experiments presented in this chapter feed forward initialization is used to make experiments reproducible. The initialization procedure for scientific experiments has no strict timing constraints. In the presented experiments the initialization trajectory lasts up to 10s. If this technique is applied in online microscope applications the time involved with initialization should be as less as possible. Further details about (time-optimal) initialization of hysteresis are provided in chapters 7 and 8. In chapter 8 the initialization time will be reduced to 0.1 to 0.5s.
Figure 4.9: Simulations of the interconnected model for two different initial conditions. The results are similar to the measured responses shown in Fig. 4.5. The corresponding time domain response is found in Fig. 4.8
Chapter 4. SEM Experiments

Figure 4.10: Initialization profile used: large (15% of total range) but slowly varying sine waves ($I = I_{dc} + 0.35 \sin(0.5\pi t)$). The 2.5 periods of the sine wave put the magnetic system in the same state (same input $I_{dc}$ and same sharpness $S$).

4.5.1 Underfocus vs Overfocus

The variation in sharpness before and after initialization is the result of a small signal (amplitude < 0.1%) quasi-static excitation, in the plot denoted with stairs. In chapter 2 it was discussed that the level of defocus for a lens system in overfocus or underfocus can be equal. In order to check the focal situation the quasi-static stairs excitation is introduced. Only one stairs period is applied, first increasing than decreasing. A lens system in underfocus has a smaller absolute current input and, therefore, a lower magnetic flux density than a system in overfocus. In the experiments of Fig. 4.10 (x) is in optimal focus since there is a decrease in $S$ for both the positive and negative part of the stairs. For the case (o) a decrease in $|I|$ results in a higher $S$. Therefore, the begin situation is overfocus. After the initialization sequence both the $S$ trajectories are the same and both are in underfocus.

4.6 SEM Experiment: Transients

Magnetization of the lens yoke also deals with transient effects; as discussed in chapter 3 it takes time for the magnetic field to penetrate the yoke. Fig. 4.11 and Fig. 4.12 show the response of $S$ during a rising edge of a switching event. The corresponding images are provided to interpret the sharpness number. The microscope settings used in this experiment result in $85\mathrm{ms/image}, \approx 11.7\text{images/s}$. By trial and error the magnetization history and the amplitude of the pulse where chosen such that the end situation is close to optimal sharpness, which is not trivial and just for illustration purposes.

When studying the $S$-curve the transition time is $700\mathrm{ms}$. However, studying the images and taking an application in mind, a lower sharpness may be sufficient for the application to work. If image (e) is compared with image (i) similar features can be recognized and it is up to the type of
application if \( (e) \) is acceptable. If so, the time to obtain \( (e) \) is only \( 340\, ms \rightarrow 50\% \) of the transition time. The effect of a threshold value on the estimation of the transition time is presented in chapters 6 and 8. Experimental estimation of the transition time based on measurements of the magnetic flux density within the lens are also discussed in chapter 6.

### 4.7 SEM Experiment: Hysteresis and Circular Symmetry

The experiments presented so far have shown that hysteresis and transient effects have a significant effect on the sharpness level of the images. In chapter 2 it is shown that a change in defocus is an effect of variation of the magnetic flux density \( B_z \). In first instance the electromagnetic lens is considered circular symmetric. However, after switching operating points, it is possible that the electron beam has moved slightly. Movement \( (xy) \) of the electron beam results in a slightly shifted view of the specimen. An experiment has been formulated in which this effect is analyzed.

Fig. 4.13 shows a quasi-static variation over 20s followed by 11 pulses. The corresponding images are shown in Fig. 4.14. The presented images represent a specimen area of about \( 10\, \mu m^2 \). Images \((a2, b2, c2, d2)\) represent snapshots of the image series recorded during quasi-static excitation. The image contents in all four is the same despite the variation in sharpness. From this it is concluded that electron beam shifts are not presented with quasi-static excitation (proper alignment of the microscope is required).

However, the images recorded after the pulses \((e2, f2, g2)\) show a huge difference in image contents. The initial condition before the first pulse at 30s is image \(d2\). The displacement \((xy)\) of the electron beam, obtained from comparison of \(d2\) and \(e2\) is over \(2\, \mu m\) in both \(x\) and \(y\) direction. The circular symmetry of the electromagnetic field distributions is disturbed by the high frequency, high amplitude pulses. Although that no conclusions about the cause of this effect can be drawn from this experiment alone, it is plausible that the effects are hysteresis effects in the \(xy\)-direction. So far the hysteresis effect on \(B_z\) was studied. \(B_z\) controls the focal distance and with that the image sharpness. The magnetic flux density in \(xy\) was considered circular symmetric and described in \(r\phi\) coordinates. However, pulsed excitation (similar to fast switching of operating points) disturbs the symmetry. By comparison of the image content of all images after the different pulses, e.g. \((e2, f2, g2)\), it is observed that the beam converges back to the original position as a consequence of the pulses. Further
study is required to fully understand the effect, but if the beam position can be put back close to the original position by means of further excitation, also here feed forward initialization can form a solution.

Note that the observed displacement is not a function of magnification. The effect will be much more disturbing at high magnification. The electron beam displacement due to high frequency, high amplitude variation of the electromagnetic field is not further investigated in this thesis.

4.8 Conclusions

All experiments have been carried out on a state-of-the-art scanning electron microscope extended with a rapid prototyping and data acquisition system. The range in which image based focus optimization can operate was experimentally derived. Information about the specimen is available in only 0.7% of the full input current range. A model based reconstruction of hysteresis effects occurring while switching operating points showed that the unpredictable behavior of hysteresis is caused by a difference in initial conditions at the time of switching. The concept of feed forward initialization was introduced which is shown to be capable of wiping out the history of previous excitations (in a limited range). This concept will be further investigated in chapters 7 and 8.

Transients during switching were analyzed by estimating the sharpness values of the resulting images after a step wise change of the input current. About 14 images/s were recorded. The open loop transition time was estimated at 0.7s. By means of switching experiments it was shown that the circular symmetry of the magnetic flux density distribution can be disturbed by high frequency, high amplitude switching operation. Although further experiments are required for confirmation, this is assumed to be caused by the effects of hysteresis in the $xy$-direction.
4.8. Conclusions

Figure 4.13: Experiment to isolate the effect of focus change and displacement during small-signal quasi-static variation and after high-frequent pulses.

Figure 4.14: Experiment to isolate the effect of focus change and displacement during small-signal quasi-static variation and after high-frequent pulses.
Chapter 5

The Electromagnetic Lens Setup

In this chapter the electromagnetic lens setup is introduced. The setup contains a similar lens as is integrated in various types of scanning electron microscopes from FEI Company. An example is the FEI Helios machine which was also used for the image measurements presented in chapter 4, Fig. 4.2. In online electron microscopy applications no magnetic flux sensing is integrated. In the setup presented here, high accuracy, high bandwidth sensors are integrated. However, the electromagnetic lens is tested standalone which implies no images can be recorded.

In online operation of the microscope there are various restrictions on the sensors:

- Sensor operation may not disturb the electron imaging process. Positioning within the electron optical volume is not allowed since that is where the electrons travel through. Positioning near the electron optic volume puts very high restrictions on any imposed magnetic and electric fields introduced by the sensing mechanism.
- The sensor (and wiring) should be placed in the lens gap, where there is only limited space due to various electron detectors and for instance scan electronics.
- The sensors should not disturb the vacuum and should be vacuum resistant.
- The electromagnetic field present for lens operation may not disturb the sensor operation.

Sensor integration in the offline setup presented in this chapter is not restricted. There is no vacuum, nor are there electron detection electronics present, just the lens in air. Two magnetic flux density sensors are placed at different positions in the lens geometry. The setup serves several purposes:

- **Observation.** The behavior of the electromagnetic field in the lens as a function of transient input currents can be measured. The transition times and error can now be quantized by interpreting measured magnetic flux density signals. Every microscopy application can be simulated using the lens as a hardware-in-the-loop component.
- **Modeling.** Experiments required for modeling can be performed.
- **Feed Forward Control.** Control strategies can be designed with the help of developed models and experiments carried out using the lens setup. The performance of feed forward strategies can be experimentally validated.
- **Feedback Control.** Control principles based on feedback of sensed magnetic flux density require integration of the sensor in vacuum in the lens gap. Using the lens setup the performance in terms of transition error and transition time can be obtained before actual implementation in an electron microscope. In the setup one is free to experiment with different sensors at different locations.
Chapter 5. The Electromagnetic Lens Setup

Figure 5.1: Schematic view of the electromagnetic lens and the two magnetic flux density sensors \( sB_1 \) and \( sB_2 \). The circles drawn on the sensor elements indicate the sensitive area, the arrows indicate the orientation of the sensor. The three images on the right hand side show the magnetic flux distribution in steady state for a constant current of 1A. All three represent the same simulation but use a different scaling of gray levels to indicate the flux distribution in the lens and in the iron.

In chapter 6 and 8 different controller structures will be experimentally validated. In this chapter the hardware that is required is discussed. First arguments are presented for the choice of the sensor principle. The hardware of the setup is presented next.

5.1 Magnetic Flux Sensing

In chapter 2 it was illustrated that the requirements on the ratio of maximum variation of the magnetic flux density and the absolute value of the magnetic flux density in the operating point is \( \Delta B/B \approx 10^{-5} \). A variation \( \Delta B \) larger than \( 10^{-5}B \) does significantly disturb the image quality. When implementing feedback control, this accuracy requirement also holds for the sensor. However, there are more requirements. Their combination makes that availability of a sensor suitable for feedback control is far from trivial.

- **Amplitude Range.** The maximum of the absolute magnetic flux density within the gap of the lens geometry is in the order of 0.2T.

- **Best Resolution.** The maximum \( B \) value is required for an electron acceleration voltage of \( U_{max} = 30kV \). The minimum acceleration voltage is \( U_{min} = 500V \). From (2.12) derived in chapter 2 it is found that \( B_{max} = \sqrt{U_{max}/U_{min}}B_{min} \), a factor 7.7. If the maximum value for a working point is 0.2T then the minimum is \( 0.2/7.7 = 26mT \). If the maximum variation in an operating point is \( \Delta B/B \approx 10^{-5} \) then the sensor should have a resolution of 0.26\( \mu T \).

- **Essential Resolution.** As explained in chapter 2, the magnetic flux controller is part of a larger scheme that controls the level of defocus. If the best resolution specification is not met by the magnetic lens controller it can be corrected using image based feedback control. Definition for the essential resolution bound is fuzzy since it highly depends on the application. At least the
5.1. Magnetic Flux Sensing

resolution should be better than 1000 times the best resolution required, as discussed in chapter 2. This comes down to $0.26mT$.

- **Positioning.** The size of the sensor should be such that it fits in the lens gap. Below the coil this is $\approx 20mm \times 40mm$. For online implementation it is even more strict since this space is already filled up with other components. Note that if the sensor is not placed at the position with maximum amplitude the requirements for the resolution expressed in absolute values become even more strict.

- **Upper Bound Bandwidth.** An estimate of the open loop transition time derived from the image experiment in chapter 4 is $0.7s$. With the help of feedback control the objective for the maximum transition time is taken $10ms$. A first estimate for the bandwidth of the sensor and corresponding electronics and signal processing should then at least be $1kHz$.

- **Lower Bound Bandwidth/Drift.** Open loop transition times are in the order of $1s$, but imaging applications can take from seconds to minutes to hours. Translated in the frequency domain $10^{-3}Hz$ is the desired lower bound.

- **Linearity, offset calibration.** The relation between actual magnetic flux and the sensor reading should be monotonically increasing. A linear relation is welcome but not strictly necessary. Also offsets and calibration errors are of minor concern.

- **Orientation.** The magnetic field is a 3D distribution of which $B_z$ within the electron optic volume is the variable to control. Measurement in one direction is sufficient. However, this direction should be chosen such that a good signal to noise ratio is obtained.

**Sensing Possibilities**

The following sensor principles are considered:

- **Sens coils.** A coil located near or around the electron optic volume is perfectly capable of sensing changes in the magnetic field. To obtain $B(t)$ integration is required. Drift and accuracy of the integrator implementation seems to be the main limiting factor.

- **Flux gate and Squids.** Several very accurate magnetic flux density sensor principles and either not deal with the absolute range of $\pm 0.2T$ or do not have the desired bandwidth of $5kHz$.

- **Hall effect.** Hall effect sensors are found most suitable for the application. The resolution for this sensors principle is the biggest issue.

Only a very specific combination of sensing based on multiple Hall effect elements and signal conditioning electronics meets the requirements. The sensor type of choice is thus a Hall effect sensor. The specific types as used in the setup are 2D ultra low noise magnetic-flux-density-to-analogue-voltage transducers, [73]. Up to the knowledge of the author this is the only sensor-type that combines the large dynamic range of $0.2T$ with a $\mu T$ resolution, a bandwidth of several $kHz$ and drift specifications $< 0.1\mu T$ for $f < 0.01Hz$. The relevant specifications are provided in table 5.1. In e.g. [64], [63] more information about this specific sensor design including the electronics for signal conditioning is provided. It is important to notice that the sensors come with electronics including a.o. the current sources driving the Hall elements, the signal amplifiers and including second order anti-aliasing filters (with a cut-off frequency of $5kHz$). Although it uses the same basic principle as conventional Hall elements used in e.g. encoders for measuring rotation speeds, etc. their performance is highly different (and so is their price).
Chapter 5. The Electromagnetic Lens Setup

<table>
<thead>
<tr>
<th>Name</th>
<th>SENIS 2D Ultra-Low Noise Transducer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>1.1µT</td>
</tr>
<tr>
<td>Broad-band noise</td>
<td>$B_{rms} &lt; 1.1µT$</td>
</tr>
<tr>
<td>Offset drift $f \in [0.01, 0.1] Hz$</td>
<td>$B_{off-rms} &lt; 0.1µT$</td>
</tr>
<tr>
<td>Linear range</td>
<td>±0.2T</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>5kHz</td>
</tr>
<tr>
<td>Probe dimensions</td>
<td>1.5 × 3 × 30mm$^3$</td>
</tr>
<tr>
<td>Orientation</td>
<td>2D yz</td>
</tr>
<tr>
<td>Spinning frequency</td>
<td>31.25kHz</td>
</tr>
</tbody>
</table>

Table 5.1: Main sensor specifications for the chosen SENIS transducer based on the Hall effect

5.1.1 Sensor Positioning

An electromagnetic lens consist of a cylinder shaped coil surrounded by a solid yoke made of ferromagnetic material, e.g. NiFe. Fig. 5.1 shows a schematic drawing of the electromagnetic lens which is part of the lens setup. The coil within the lens is water cooled, the water tube is drawn on top of the lens coil. Two Hall effect magnetic flux density sensors are positioned within the lens geometry. The drawing of the lens and the sensors is on scale. As shown the sensor length of 3cm is significant when compared to the dimensions of the gap. The sensitive area of the sensors is indicated with a circle. The wire of sensor 1 ($sB_1$) enters from the top of the lens. The wire for the second sensor ($sB_2$) leaves via an opening which is in microscopy operation used for electron detectors. From this it becomes clear that the lens is in first instance circular symmetric, but that this symmetry is disturbed by required openings in the geometry.

Fig. 5.1 shows an illustration of the magnetic flux density distribution in steady state. Since the flux density is significantly higher within the iron, a scaled version is provided which gives insight into the differences of the observed amplitude depending on the sensor location. The sensor $sB_1$ is not placed in the electron optical volume since the amplitude level is too low. By placing it closer to the yoke a better signal to noise ratio is obtained. Two identical sensors are used. Note that $sB_1$ measures in the z-direction and $sB_2$ in its y-direction, indicated by the arrows.

- **Reference sensor**: Sensor $sB_1$ serves as a reference sensor. It is placed near the electron optical volume and can, therefore, not be used at this position in online applications. *For observation of effects during transitions of operating point the readings of $sB_1$ are considered to represent $B_z$ important for optics.*

- **Sensor for feedback control**: Sensor $sB_2$ is placed at a position that is considered allowed in online microscopy applications. Comparison between the measured $B_1(t)$ and $B_2(t)$ will show (chapter 6) if magnetic flux sensing at different positions is sufficient to observe the properties of $B_1$ at the reference position.

5.1.2 Hardware

The lens is a prototype from a FEI Company scanning electron microscope e.g. FEI Helios. The coil current is controlled by a Prodrive High Linearity Power Amp which is a switching amplifier. The resolution of the current is 1µA versus a range of ±2.2A which requires 22bits to quantize it. The drift specifications are 1ppm/600s. Both lens and amplifier use active water cooling.

The rapid prototyping and data acquisition system is a modular dSPACE system using a DS2004 high speed A/D board and a dSPACE DS1005 processor board. The matlab/simulink interface uses the matlab real-time workshop and dSPACE’s mlib library. The sampling frequency is set to 16kHz.
5.2 Controller Configurations

Figure 5.2: Photograph of the electromagnetic lens setup at TU/e.

However, the A/D conversion of the magnetic flux density measurements uses an oversampling scheme of 32 times resulting at a sampling frequency of $512kHz$. Downsampling uses 7th-order Chebyshev type2 filtering with a cutoff around $2kHz$ and a stop band attenuation of $-100dB$ from $7.5kHz$.

5.2 Controller Configurations

In chapter 6 the performance of different controllers is experimentally validated. In this section the controller structure is presented. In Fig. 5.3 three different structures are defined.

- **Feed Forward Control of $B_1$:** The first controller configuration is feed forward control of $B_1$. The controller $C_{ff}$ relates a desired magnetic flux density at the position of sensor $sB_1$, the reference signal $R_{B_1}$, to the input current $I$ of the lens. This is the current situation for the scanning electron microscopes discussed in chapter 4. The error signal is the difference between reference and magnetic flux density $e(t) = R_{B_1}(t) - B_1(t)$.

- **Feedback Control of $B_2$:** The second controller configuration is feedback control of the magnetic flux density measured with sensor $sB_2$. The reference signal for the magnetic flux density at this position is explicitly mentioned as $R_{B_2}$ and the error as $e_2 = R_{B_2}(t) - B_2(t)$. This controller is an intermediate step towards control of $B_1$ using information measured at a different position in the geometry. This configuration is used to obtain performance specifications for the case that feedback control based on magnetic flux density measurements is available.

- **Feed Forward Control of $B_1$ using Feedback Control of $B_2$:** The third configuration $C_{fb1}$ adds a feed forward part to the feedback controller $C_{fb2}$. The feed forward part takes reference $R_B$ as input and $R_{B2}$ as output. The error of this structure is $e(t) = R_B(t) - B_1(t)$. The performance of this configuration will show if the magnetic flux density within the electron optic volume can be accurately controlled using measurements at other positions in the geometry. This evaluation is carried out in chapter 6. First the implementation of the feedback controller is presented.
ff: feed forward $B_1$

\[ R_B \xrightarrow{C_{ff}} I \xrightarrow{L} B_1 \]

$e$ is the error term.

fb2: feedback $B_2$

\[ R_{B2} \xrightarrow{C_{fb2}} I \xrightarrow{L} B_2 \]

$e_2$ is the error term.

fb1: feedback $B_2$ and feed forward $B_1$

\[ R_B \xrightarrow{C_{fb1}} R_{B2} \xrightarrow{C_{fb2}} I \xrightarrow{L} B_2 \]

Due to the nonlinearities in the relation between current and magnetic flux density a DC-gain cannot be defined. However, a rough approximation between the magnetic flux density in steady state observed with sensor $sB_1$ is $B_1/I \approx 1/30[T/A]$. For sensor $sB_2$ to come down to $B_2/I \approx 1/45[T/A]$.

5.2.1 Signal Range

The signals involved are bounded and in a continuous range:

\[
\begin{align*}
\hat{R}_B & \leq R_B \leq \tilde{R}_B, & R_B & \in \mathcal{R}_B = [-20mT, 50mT] \\
\hat{R}_{B2} & \leq R_{B2} \leq \tilde{R}_{B2}, & R_{B2} & \in \mathcal{R}_{B2} = [-14mT, 35mT] \\
\hat{I} & \leq I \leq \tilde{I}, & I & \in \mathcal{I} = [-2.2A, 2.2A] \\
\hat{B} & \leq B \leq \tilde{B}, & B & \in \mathcal{B} = [-60mT, 60mT] \\
\hat{B}_2 & \leq B_2 \leq \tilde{B}_2, & B_2 & \in \mathcal{B}_2 = [-45mT, 45mT]
\end{align*}
\]

Due to the nonlinearities in the relation between current and magnetic flux density a DC-gain cannot be defined. However, a rough approximation between the magnetic flux density in steady state observed with sensor $sB_1$ is $B_1/I \approx 1/30[T/A]$. For sensor $sB_2$ to come down to $B_2/I \approx 1/45[T/A]$.

5.2.2 Design of the Feedback Controller

The purpose of the feedback controller designed in this section is testing whether the effect of hysteresis in steady state can be sufficiently suppressed, experiments will be presented in chapter 6. Integral action needs to be included in the control loop such that the difference between the reference $R_{B2}$ and the sensor reading $sB_2$ converges to zero for constant reference signals.
5.2. Controller Configurations

Hysteresis effects in a feedback loop complicate stability and performance analysis due to its nonlinear nature. In the literature several design strategies for hysteretic systems are found. Passivity and dissipativity of hysteresis operators are popular approaches, [33]. Inverse compensation such that feedback control only has to deal with the model error is an option for performance enhancement, e.g. [37]. However, in a major part of the presented works hysteresis is considered as a nonlinearity with a bounded gain. Hysteresis is here isolated from the dynamics. The dynamical part is considered linear. For stability analysis of the loop transfer, hysteresis is replaced by a bounded uncertainty on the gain. The offset introduced by hysteresis can be seen as an output disturbance, but has no further influence on the stability. These techniques deal with hysteresis in the same way as they deal with static nonlinearities, e.g. [61], [60], [43], [39]. From the latter approach, in which hysteresis is considered as a gain disturbance acting on a linear system, it becomes clear that if the difference between the largest possible gain disturbance and the smallest possible gain disturbance is small, hysteresis is a minor problem for stability issues of feedback control. In the lens system the gain variation $dB/dI$ is about 10%. By approximating the lens system as a linear system and developing a controller that has sufficiently large stability margins, a stable transfer with sufficient performance is obtained. By making this crude assumptions this controller is not likely to be optimal, but it is sufficient for further analysis.

A linear model $L_M$, relating the input current and the magnetic flux density at sensor position $sB_2$, is estimated from experimental data. The applied input excitation consisted of a stepwise varying input current such that the amplitude was uniformly distributed within the total input current range. The time period of this random excitation was 0.1s. The choice for stepwise variation was made such that the data has a close relation to the application of set-point regulation. The open loop transition time was estimated around 0.7s, chapter 4. Therefore, a time period of 0.1s is considered to sufficiently excite the dynamics.
Chapter 5. The Electromagnetic Lens Setup

An $8^{th}$-order discrete time ARX model is estimated. The sampling interval is $62.5\mu s$ corresponding to the $16kHz$ of the lens setup. This model takes into account the relation between the current set point $I$ and the measured magnetic flux density at sensor $sB_2$. Therefore, next to the transfer of magnetic flux density as a function of applied input current also the current amplifier, anti-aliasing filters and sensor electronics, delays of the data-acquisition etc. are taken into account in the model. By definition of the linear ARX structure, hysteresis is not explicitly dealt with in the modeling process. However, since the data is obtained on the real system hysteresis will disturb the obtained model. Validation results for prediction of the obtained model in the time domain for data of similar nature as used for identification result in a prediction error of $\approx 10\%$.

When comparing the obtained bode diagram of the model, Fig.5.4, with the example of the coil with ferromagnetic core Fig. 3.5 and the bode plot extracted from finite element analysis Fig. 3.8, chapter 3 the same characteristics are recognized. The phase shift as a function of frequency does not converge to a constant value, but decreases endlessly because of the diffusion term in the electromagnetic relations. This effect has a close correspondence with a delay. In the time domain, it will take a few $100\mu s$ for a change in lens current is observed at the position of the sensor. The second characteristic is that no clear pole or zero is visible which is due to the lumped approximation of a spatially distributed system, also described in chapter 3. In the amplitude bode plot, the attenuation as a function of frequency is gradually until $2kHz$ where the effect of implemented filters becomes visible.

The DC gain of the model $B_2[T]/I[A]$ is $\approx 1/45[T/A]$. Fig. 5.4 shows the bode plot of the model $LM$, the open loop transfer $LMC$ and the complementary sensitivity $T = LMC/(1 + LMC)$. The controller is derived by conventional loop shaping techniques is a discretized and balanced state space representation of a series connection of a first order lag and first order lead filter and second order roll-off filter.

5.3 Recommendations and Remarks

5.3.1 Interference

The spinning current frequency of the sensors is attenuated by the anti-aliasing filters. However, the order of the filters used is two and the spinning frequency is at $31.25kHz$ while the cut-off of the filters is at $5kHz$. This implies that the spinning current component is only attenuated by $\approx -40dB$. In combination with a sampling frequency of $16kHz$ this leads to aliasing. The desired signal quality of the setup has a dynamic range of about $10^5$, the ratio between amplitude range and resolution. Therefore, an oversampling scheme with an oversampling ratio of 32 is implemented such that the first 8 harmonics of the spinning current frequency can be filtered out digitally.

The combination of a switching amplifier and a switching sensor principle (spinning current) causes interference. The ratio between the spinning frequency of the Hall effect sensor electronics and the switching amplifier turns out to be nearly integer, which results in interference observed as a ripple of $\mu T$ readings positioned around $0.1$ to $2Hz$ which are physically not present. Note that this is due to a mix of two signals in the analog domain. Neither of both frequencies can be changed using the current implementation.

Note that the cause of interference between sensor and amplifier is the physical presence of frequency components within the time varying magnetic flux density. The frequencies correspond to the switching frequency of the amplifier. The switching amplifier used is normally not part of the electron microscope. It is used in this setup since it has an extreme good performance in terms of bandwidth, resolution and range. The amplifier is designed to control e.g. electrical machines in which any ripple at the switching frequency causes no problems at all. In electron microscopy care should be taken since the mass of an electron is so low that also high frequency components which have an extreme low amplitude can influence the trajectory.

The dSPACE environment allows to choose the combination of the sampling and the oversampling
5.3. Recommendations and Remarks

factor. With this degree of freedom any remaining high frequency disturbances above the Nyquist frequency $0.5 \cdot 16kHz$ can be shifted into a frequency region in which digital filtering can deal with them. The spinning current frequency for the hall sensors can be chosen (in range over several $kHz$) during the manufacturing and calibration process. With a proper trade-off of sampling frequency, oversampling ratio, anti-aliasing and digital filtering, the spinning current frequency and the switching frequency of the amplifier, interference can be dealt with.

5.3.2 Beyond the Scope

Alternatives Magnetic Flux Sensing

The sensor type used in the setup became commercially available around October 2009. Up till then no sensors were found that fulfilled the desired requirements. Developing a sensor for this setup based on a combination of different principles sounds like a possible solution. For instance the high bandwidth of a sense coil can be combined with a sensor that has low bandwidth and extremely less drift. Further research and development of sensor principles and/or integration of the sensor in the lens design can boost the performance but is beyond the scope of this research.
Chapter 6

Performance of Feed Forward and Feedback Control

This chapter presents the experimental performance evaluation of feed forward control, feedback control and feedback control with limitations on sensor positioning. An experimental procedure is defined which provides the transition times and transition errors for different controller structures implemented on the electromagnetic lens setup. The experimental procedure formulated in section 6.1 is used for controller evaluation throughout the complete thesis. The experiments are based on operating point transitions starting from random initial conditions. Performance of feed forward control, in which hysteresis and dynamics are not actively controlled, provides the open loop system performance and serves as the benchmark for controller comparison, section 6.2.

In chapter 5 a feedback controller taking into account the magnetic flux density measured by sensor $sB_2$ was presented. Evaluation of its performance will show whether feedback control based on magnetic flux sensing can decrease the transition error and/or the transition time, section 6.3. Sensor $sB_2$ is placed at a position that is considered possible for online microscopy operation. By controlling the magnetic flux density at the reference position $sB_1$ partly in feed forward, but also using the feedback controlled transition between the input current and $B_2$, conclusions can be drawn about the suitability of feedback control in combination with spatially distributed hysteresis effects and restrictions on the sensor positioning, section 6.4.

6.1 Experimental Procedure

The notation of dynamical systems is introduced to describe the experimental procedure providing the maximum transition time and the maximum transition error. The lens system $L$ is now represented by:

$$
\dot{x} = L_F(x, I, x_0) \quad (6.1a)
$$

$$
B = L_G(x). \quad (6.1b)
$$

Here $\dot{x}$ represents the time derivative of the state $x$. $I$ is the input current, $B$ the magnetic flux density at a point in the lens geometry. The function $L_F$ implies that the time derivative of the state is a function of the current state and the input. The initial condition is equal to $x(t_0) = x_0$. The physical interpretation of the state is found in the derivation of the transfer function for a coil with ferromagnetic electrically conducting core, section 3.2. The state of the spatially distributed system contains the distributions of electrical flux density $\vec{D}$, electrical field strength $\vec{E}$, magnetic flux density $\vec{B}$ and magnetic field strength $\vec{H}$. At this point no realization of $L$ is required, the lens is considered as a dynamical system with input $I$ output $B$ and state $x$.

Fig. 6.1 shows the interconnected structure of a feed forward controller $C$, the lens system $L$, and the estimation of $\psi(t)$ defined as the maximum variation of $B(t)$ over a duration of $\Gamma[s]$ (definition 3.4.1.1). When $\psi(t) \leq \epsilon_t$, the transition is finished. This implies that the influence of any remaining transients on the recorded image series is insignificant and that image based focus optimization can

73
Chapter 6. Performance of Feed Forward and Feedback Control

start. The function that estimates \( \psi(t) \) is in Fig. 6.1 denoted as \( \Psi(\Gamma) \). As discussed in section 3.4, the control objective is to minimize both the maximum transition error \( e_{tr} \) (definition 3.4.1.3) and the maximum transition time \( \tau_{tr} \) (definition 3.4.1.2).

\[
\begin{align*}
R_B(t) & \rightarrow C \rightarrow I(t) \\
\dot{x} &= L_F(x, I; x_0) \\
B &= L_C(x) \\
\Psi(\Gamma) &\rightarrow \psi(t) \\
\psi(t) &\leq e_{tr} \quad \text{finished?}
\end{align*}
\]

\( \Psi(\Gamma) \) estimates the maximum variation of \( B(t) \) over a time window \( \Gamma \). If the signal \( \psi(t) < e_{tr} \) the transition is finished.

Figure 6.1: Schematic representation of the lens system controlled in feed forward. \( R_B \) is the step wise varying reference signal, \( I \) the current for the lens coil, and \( B \) the magnetic flux density at a specific point in the lens geometry. \( \Psi \) estimates the maximum variation of \( B(t) \) over a time window \( \Gamma \). If the signal \( \psi(t) < e_{tr} \) the transition is finished.

The worst case situation, resulting in maximal transition error, depends on the current state, the initial conditions before changing the set point and the next set point initiated by a change of the reference value \( R_B \) at \( t = t_0 \). Since the structure of the lens system \( L \) including hysteresis is not known with sufficient detail, the worst case combination of \( x_0 \) and value of \( R_B \) is not known. Therefore, an experimental procedure is developed which evaluates random set point transitions starting from random initial conditions. A single evaluation is divided in three modes \( M_1, M_2, M_3 \) as shown in Fig. 6.2. The index \( j_1 \) indicates the number of the evaluation. The ranges of the different signals involved are defined in chapter 5, (5.1).

1. **Force random initial condition:**
   \( M_1(j_1) : t_{-2} \leq t < t_{-1} \)
   Force the system to an unknown initial state by random stepwise transient excitation of the reference signal. Transition errors or transition times are not taken into account in mode \( M_1 \). The goal is to obtain a random initial state. Amplitudes of the random generator are uniformly distributed over the range \( R_B \). The period time of the random generator is \( T_{\text{rand}}[s] \).
   \[
   R_B(t, j_1) = \text{random}(T_{\text{rand}}, R_B), T_{\text{rand}} = 0.2s
   \]

2. **Constant excitation:**
   \( M_2(j_1) : t_{-1} \leq t < t_0 \)
   In the application, the system is most likely in steady state when switching to a new set point. Therefore, the reference is held constant for a few seconds.
   \[
   R_B(t, j_1) = R_B(t_{-1})
   \]

3. **Set point change:**
   \( M_3(j_1) : t_0 \leq t < t_1 \)
   Set point change starting from a constant random initial condition. From this point on the controller performance is taken into account. 12 different set point levels are defined. The random generator randomly chooses one of the 12 levels. This time instance is defined as \( t_0 \).
   \[
   R_B(t_0, j_1) = \text{random}(R_B(12)) \\
   R_B(t, j_1) = R_B(t_0, j_1)
   \]
6.1. Experimental Procedure

In chapter 3.4 the maximum variation, the transition time and the transition error were formulated as:

\[ \psi(t) = \max_{\nu_1} B(t + \nu_1) - \min_{\nu_2} B(t + \nu_2), \]

where \( 0 \leq \nu_1 \leq \Gamma, \quad 0 \leq \nu_2 \leq \Gamma, \quad t_0 \leq t \leq t_e - \Gamma, \)

transition finished at \( t_{tr} = \min_t \psi(t) \leq \epsilon_{tr}, \quad t_0 \leq t \leq t_e - \Gamma, \)

transition time \( \tau_{tr} = t_0 - t_{tr}, \)

obtained set point \( B_{tr} = \frac{1}{\Gamma} \int_{t_0 + \tau_{tr} + \Gamma}^{t_0 + \tau_{tr}} B(t) \, dt, \)

transition error \( \epsilon_{tr} = R_B(t_0) - B_{tr}. \)

The transition time is obtained from the comparison of the estimated maximal variation \( \psi(t) \) over a time window \( \Gamma = 1[s] \) and the threshold \( \psi(t) \leq \epsilon_{tr} = 12\mu T \) which is 0.017% of the full set point range of 70mT. The threshold value of 12\mu T for the maximal variation of \( \psi(t) \) was estimated as the smallest value that can accurately estimated on the electromagnetic lens setup. To be sure that one set point value is evaluated from multiple different initial conditions 12 fixed set point values are chosen instead of a uniform distribution of the required set points over the interval \( R_B. \) An overview of the parameter values for the experimental procedure is shown in table 6.1. Next to the statistics providing the number of evaluations required there are practical reasons that put a limit. As will be illustrated, evaluating performance for a single combination of \( R_B \) and \( x_0 \) costs \( \approx 25s. \) That includes the time of the measurement and the time involved with saving the data set. For 100 different evaluations that is 40min per controller. As will be shown further in this thesis at least 4 different controllers are compared. All different controllers are tested on the same work day which is limited to 10h minus the 2h required to reached the thermal equal equilibrium of the electronics. The number of evaluations is thus limited to a few 100 per controller. For each controller compared in this chapter and in chapter 8 the number of function evaluations per controller is set to 100 for identification of \( \zeta \) and 100 for validation.
Chapter 6. Performance of Feed Forward and Feedback Control

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>set point range</td>
<td>$-20mT \leq R_B \leq 50mT$</td>
</tr>
<tr>
<td>full range</td>
<td>$70mT = 100%$</td>
</tr>
<tr>
<td>current range</td>
<td>$-2.2 \leq I \leq 2.2A$</td>
</tr>
<tr>
<td>window length</td>
<td>$1s$</td>
</tr>
<tr>
<td>threshold variation</td>
<td>$12\mu T = 0.017%$ of full range</td>
</tr>
<tr>
<td>number of set point levels</td>
<td>12</td>
</tr>
<tr>
<td>number of evaluation (identification)</td>
<td>100</td>
</tr>
<tr>
<td>number of evaluation (validation)</td>
<td>100</td>
</tr>
<tr>
<td>duration of overall procedure</td>
<td>$\approx 80min$</td>
</tr>
</tbody>
</table>

Table 6.1: Parameter values for the experimental procedure used for controller evaluation.

6.2 Feed Forward Control

In this section the results for a feed forward controller subject to the experimental procedure providing $\tau_{tr}$ and $\epsilon_{tr}$ are presented. The feed forward controller $C_{ff}$ consists of a static nonlinear function. There is no feed forward compensation for transient effects, only a function $I[A] = \zeta_{ff}(R_B[T])$ that can correct for static nonlinear relations in the steady state equilibrium map. A 5th-order polynomial is chosen to represent $\zeta_{ff}$. The polynomial coefficients are tuned using an initial run of the experimental procedure discussed in section 6.1. Fig. 6.3 shows the concept of identification and validation.

First an initial controller consisting of a gain only is used to obtain the relation between the constant lens current and the obtained magnetic flux density. After 100 function evaluations the polynomial coefficients are estimated. The experiment is repeated for validation. From this data set the performance parameters are calculated.

Validation is carried out with 100 evaluations. Fig. 6.4 presents the signals $B(t)$, the absolute error in time and the estimated $\psi$ for $t \geq t_0$. Since the input current is set to constant at $t = t_0$ the variation of the actual magnetic flux density is expected to converge to zero over time. However, due to for instance sensor noise and interference, as discussed in section 5.3.1, estimation of $\psi$ has a limited resolution. This resolution is estimated at $12\mu T$ which implies that evaluation for transition times for thresholds $\epsilon_{tr} \leq 12\mu T$ is not possible. The full scale of the reference signal $R_B = [-20mT, 50mT]$.
6.2. Feed Forward Control

Figure 6.4: \( C_{ff} \), feed forward control. The maximal obtained transition time is indicated with a vertical dashed line. top.) \( B(t) \) for 12 desired set point values starting from random initial conditions. middle.) Absolute error \( |e(t)| = |R_B(t) - B(t)| \) on a logarithmic scale in mT and as a percentage of the full range of 70mT. bottom.) the maximal variation \( \psi(t) \) over a window \( \Gamma = 1s \).
Chapter 6. Performance of Feed Forward and Feedback Control

Figure 6.5: Steady state equilibrium map for feed forward control. Left, the mapping of the reference versus the measured value and right the reference versus the absolute transition error.

is 70mT. The relative $\epsilon_{tr}$ is 100% · 12μT/70mT = 0.017% of full range. The desired resolution of $\psi(t)$ with $\Gamma = 1s$ is $10^{-5}$, as derived in chapter 2. The evaluation of $\psi(t)$ for feed forward control shows that this specification is not met. Estimation of threshold values lower than 12μT can possibly be obtained by improving filtering procedures, but this improvement is left as a recommendation.

Both the variation $\psi(t)$ and the absolute error $|e(t)| = |R_B(t) - B(t)|$ are in Fig. 6.4 shown on a logarithmic time scale such that convergence towards operating points is clearly visible. As observed (and expected) the errors do not converge to zero due to the hysteresis effect. The maximum transition time $\hat{\tau}_{tr}$ and maximum transition error $\hat{\epsilon}_{tr}$ are found to be:

\[
\hat{\tau}_{tr} = 0.48s \\
\hat{\epsilon}_{tr} = 3.4mT \\
\frac{3.4mT}{70mT} \cdot 100\% = 4.8\%.
\]

In the plot of the magnetic flux density, the difference between the 12 desired operating points can be hardly discriminated due to the large errors of about 5%. In chapter 2 it is stated that the upper bound on the error above which image based focus optimization methods fail is equal to about 1% of the full range. With feed forward control the error is more than 5 times higher.

The threshold used to determine when the transition is finished is set to $\epsilon_{tr} = 12μT$. The obtained transition time is found when $\psi(t) \leq \epsilon_{tr}$ for the first time after the moment of switching $t = t_0$. For feed forward control this moment is indicated with vertical dashed lines in Fig. 6.4. The maximum obtained transition time measured with feed forward control is $\hat{\tau}_{tr} = 0.48s$.

Since the lens is controlled in feed forward with a static correction $\zeta_{ff}$ only, the effect of hysteresis is expected to result in a multi-valued steady state equilibrium map of the reference $R_B$ versus the measured values $B$. This mapping and the error of each function evaluation are shown in Fig. 6.5. The effect of hysteresis is clearly visible.
6.3 Feedback Control

6.2.1 Conclusions Feed Forward Control

The proposed feed forward controller layout is not capable of reducing the transition error below the 1% upper bound. The maximum transition error $\hat{e}_{tr}$ obtained in the test was 4.8% of the total range of 70mT. The corresponding maximum feed forward transition time $\hat{\tau}_{tr}$ was estimated to be 0.48s. This implies that the use of feed forward lens control in electron microscopy application can possibly result in an error so large that performance of the imaging based defocus controller is no longer guaranteed. The controller $C_{ff}$ only consists of a correction for static nonlinearities, $\zeta_{ff}$. Hysteresis introduces a multi-valued steady state equilibrium map and the correction can only deal with single valued nonlinear steady correction.

6.3 Feedback Control

In this section the performance of the feedback controller $C_{fb2}$ designed in chapter 5 is evaluated. Fig. 6.6 shows the controller layout. Note, that the position of the sensor used in this configuration is the second sensor that is placed in the lens gap, not the one near the electron optic volume. Fig. 5.1 in chapter 5 shows the positioning of both sensors in the lens geometry. In the next section controller $C_{fb2}$ is extended with a feed forward part to control the difference in magnetic flux density between the two different positions in the lens geometry. Here, the performance of the feedback part on itself is under test. The reference, error and magnetic flux density are not indicated with $R_{B2}$, $e_2$ and $B_2$ to indicate the different sensor position used.

\[
\begin{align*}
R_{B2} & \quad \rightarrow \quad C_{fb2} \quad \rightarrow \quad I \quad \rightarrow \quad L \quad \rightarrow \quad B_2 \\
\end{align*}
\]

Figure 6.6: $C_{fb2}$ Layout of feedback controller taking into account measurements of the magnetic flux density at the position of $sB_2$.

Integral action added to the controller design and the improved bandwidth of the closed loop system, as presented in chapter 5, have a very beneficial effect on both the transition time as the transition error, Fig. 6.7. The maximum transition time $\hat{\tau}_{tr}$ and maximum transition error $\hat{e}_{tr}$ are found to be:

\[
\begin{align*}
\hat{\tau}_{tr} &= 54ms \\
\hat{e}_{tr} &= 0.43\mu T \\
\frac{0.43\mu T}{70mT} \cdot 100\% &= 0.00062\%.
\end{align*}
\]

The reduction in transition time is over a factor 10 when compared to feed forward, while the error reduction is a few 1000 times. The obtained steady state equilibrium map, Fig. 6.8 shows that hysteresis has no measurable influence on the transition error. However, great care should be taken with interpretation of these values. As discussed in section 6.2, the measurement resolution of the setup is limited and very close to the desired error level of $10^{-5}70mT = 0.7\mu T$. It was also shown that the estimation of maximum variation $\psi(t)$ was limited to $12\mu T$ over $1s$. With the feedback implementation much smaller values are obtained. Note for instance the error presented
Chapter 6. Performance of Feed Forward and Feedback Control

Figure 6.7: $C_{fb2}$, feedback control using sensor 2. The maximal obtained transition time is indicated with a vertical dashed line. top.) $B_2(t)$ for 12 desired set point values starting from random initial conditions. middle.) Absolute error $|e_2(t)| = |R_{B2}(t) - B_2(t)|$ on a logarithmic scale in $mT$ and as a percentage of the full range of 70$mT$. bottom.) the maximal variation $\psi(t)$ over a window $\Gamma = 1s$. 

absolute error, $\max(|e_2|_{tr}) = 0.00043mT = 0.00062\%$ of 70$mT$

feedback sensor 2, measured magnetic flux density, $\max(\tau_{tr}) = 0.054s$

$\psi$ $[mT]$

$|e_2|$ $[mT]$

$B_2$ $[mT]$

$|e_2|$ $[]$

$\psi$ $[mT]$

$|e_2|$ $[mT]$

$\tau_{tr} = 0.012 mT$

$\psi$ $[mT]$

$|e_2|$ $[mT]$

$\psi$ $[mT]$

$|e_2|$ $[mT]$

$\tau_{tr} = 0.012 mT$
6.4. Feed Forward Control Combined with Feedback at a Different Position

The feedback controller $C_{fb2}$ of section 6.3 is in this section extended with feed forward compensation $C_{ff}$ in order to control the magnetic flux density at the location of sensor $sB_1$ using feedback control at the position at $sB_2$. Instead of the transfer between $I$ and $B$, now the transfer between $B_2$ and $B$ is controlled in feed forward. The controller layout is shown in Fig. 6.10. In a similar way as with the feed forward controller, a static compensation $\zeta_{fb1}$ is used. However, now the $5^{th}$-order polynomial relates $R_{B2} = \zeta_{fb1}(R_B)$. Using this control scheme it becomes clear whether the hysteresis effect at one position in the lens geometry can be reduced using feedback information obtained at another position.

The coefficients of the function $\zeta_{fb1}$ are again identified on an initial run of 100 evaluations. Validation, as presented here, is also carried out using 100 evaluations. Fig. 6.11 presents the estimated variation $\psi(t)$ and the error over time. It becomes immediately clear that the error is much larger than with pure feedback. It turns out that the transition error when compared to pure feed forward is only reduced by a factor 5. The maximum error $\hat{e}_{tr} = 0.68mT$ related to a full range of $70mT$ this

with the steady state equilibrium map, Fig. 6.8. The values are all under $1\mu T$. Explanation is found in the definition of the error: It represents the difference between reference and sensor output. The sensor and measurement noise are thus compensated by the feedback controller which results in low error. However, the effect is that the current in steady state will show a significant ripple (peak-to-peak amplitude $\approx 100\mu A$). The effect of attenuation of sensor noise near in steady state is highly unwanted, but hard to solve using a linear time invariant controller. However, a switching controller scheme, or a scheme that adapts the bandwidth can be designed which is left as a recommendation.
Figure 6.9: Validation of the transition error for feed forward control combined with feedback at a different position.

Figure 6.10: $C_{fb}$, A combination of feed forward control of $B$ using feedback control of $B_2$.

corresponds to:

$$\hat{\tau}_{tr} = 47ms$$
$$\hat{e}_{tr} = 0.65mT$$

$$\frac{0.65mT}{70mT} \cdot 100\% = 0.93\%.$$

Hysteresis is still the performance limiting effect. From the distribution of the error in steady state, Fig. 6.9, it is observed that the relation between reference and absolute error is still multi-valued. Despite the fact that only limited error reduction is obtained using this controller scheme, the maximal transition time ($\hat{\tau}_{tr} = 47ms$) is approximately equal to the transition time of the feedback controller alone ($\hat{\tau}_{tr} = 54ms$).

The sensor positioning for feedback control, as discussed in chapter 5, is not optimized. It is placed at a position that is possible for online operation. It is expected that the error level varies along with the sensor position. A study to the possibilities for other positions is left as a recommendation.
6.4. Feed Forward Control Combined with Feedback at a Different Position

![Graph of feed forward combined with feedback control]

- **Feed forward combined with feedback control** $\max(\tau_{tr}) = 0.047s$
- Absolute error, $\max(|e|_{\text{tr}}) = 0.65mT = 0.93\%$ of 70mT
- maximal variation $\psi(t)$ over a window $\Gamma = 1s$

Figure 6.11: $C_{fb}$, Feed forward control of $B$ combined with feedback of $B_2$ at a different position. The maximal obtained transition time is indicated with a vertical dashed line. **Top.** $B(t)$ for 12 desired set point values starting from random initial conditions. **Middle.** Absolute error $|e(t)| = |R_B(t) - B(t)|$ on a logarithmic scale in mT and as a percentage of the full range of 70mT. **Bottom.** the maximal variation $\psi(t)$ over a window $\Gamma = 1s$. 

83
6.5 Comparison of Feed Forward with and without Feedback Control

Illustration of the performance in Fig. 6.12 shows the comparison between forward control based on the relation between I and B and feed forward control based on the relation between B₂ and B using magnetic flux density feedback at a different position in the lens. Since the transition time is highly sensitive to variation of the threshold \( \epsilon_{tr} \), the transition time is calculated for a range \( \epsilon_{tr} \in [10^{-5}, 10^{-3}] \). The sensitivity of the transition time on the threshold \( \epsilon_{tr} \) is significantly reduced when using feedback. For all relevant threshold values, a transition time below 50 ms is obtained while for the controller \( C_{ff} \) the transition time increases from about 50 ms to 0.5 s with a decrease of \( \epsilon_{tr} \). The transition time using feedback is improved by a factor of 10 which is highly relevant for the microscopy application. Using the feedback controller \( C_{fb2} \) as a part of the feed forward scheme also results in a decreased error by a factor 5. This brings the maximum transition error just below 1%, which is at the boundary for guaranteed performance of the image based defocus control as discussed in chapter 2.

![Figure 6.12: Performance map of the transition time \( \tau_{tr} \) as a function of the threshold \( \epsilon_{tr} \in [10^{-5}, 10^{-3}] \) for the maximum variation of \( B(t) \) over \( \Gamma[s] \) and the performance map of the transition error \( e_{tr} \) as a function of the transition time.](image)

6.6 Conclusions and Recommendations

6.6.1 Conclusions

Table 6.2 presents an overview of the comparison between the three controllers evaluated in this chapter. Fig. 6.13 shows the graphical interpretation in which the relevant bounds of the application are also indicated.

| \( C_{ff} \) | \( 3.4 mT \) | \( 4.8\% \) | 1x | \( 0.48s \) | 1x |
| \( C_{fb2} \) | \( \approx 12 \mu T \) | \( 0.01\% \) | 3000x | 54 ms | 10x |
| \( C_{fb} \) | \( 0.65 mT \) | \( 0.93\% \) | 5x | 47 ms | 10x |

Table 6.2: Overview of the transition time and transition error for the controllers presented so far.
6.6. Conclusions and Recommendations

The lowest threshold $\epsilon_{tr}$ on the maximum variation $\psi(t)$ over a time window $\Gamma$ is set to $\epsilon_{tr} = 12\mu T$. This lower bound is limited by the sensor and measurement noises as discussed in section 6.2. The ratio between the lower bound and the full reference range is $12\mu T/70mT = 1.4 \cdot 10^{-4}$. The desired value as stated in chapter 2 was $10^{-5}$.

The feed forward transition time was estimated to be $0.5s$. The transition error obtained from the test shows a maximum of 4.8% of the full scale. In chapter 2 it was illustrated that an error of about 1% is the upper bound on the error that guarantees performance of the image based defocus loop. Feed forward control based on a correction for any static nonlinearity only is thus not suitable for lens control. Hysteresis introduces a multi-valued steady state equilibrium mapping which cannot be sufficiently compensated for with a single valued (nonlinear) function.

The feedback controller improves the transition time by a factor 10 and the transition error a few 1000 times when compared to feed forward, table 6.2. As discussed in chapter 5 the feedback controller $C_{fb2}$ is designed using a linear model which was estimated on experimental data. Evaluation of its performance in section 6.3 shows that there is sufficient integral action, which results in a transition error below $1\mu T$. However, from the feed forward experiments the noise level was estimated around $12\mu T$. After the transition the feedback controller is amplifying the sensor and measurement noises. This is validated by checking the response of the system to a slow varying current ripple of about $100\mu A$ imposed on top of the offset current. Measured values below $12\mu T$ cannot be trusted.

Feedback control in online microscopy operation may only be applied if the sensor does not disturb the imaging process (discussed in chapter 5). Feedback control is therefore implemented on a sensor $sB_2$ placed at a position in the lens that is possible in online operation. The transfer between the magnetic flux density at the feedback sensor position and the magnetic flux density observed with the reference sensor $sB_1$ is controlled in feed forward. The resulting controller is thus the feedback controller $C_{fb2}$ extended with a static single value correction function.

The results clearly show that the factor 10 in decreased transition is maintained. Time involved with switching the operating point can be decreased from $0.5s$ down to $50ms$ by investing in feedback control based on magnetic flux density sensing. The performance of feedback control is only investigated using a single sensor position. It is expected that the maximum transition error can be further decreased by a more optimal sensor placement. The arrows in Fig. 6.13 indicate that the performance in terms of minimizing the transition error can be influenced by moving the sensor.

The maximum transition error obtained using this controller scheme shows a factor 5 improvement when compared to feed forward $C_{ff}$, table 6.2. However, a multi-valued steady state equilibrium map ($R_B$ vs $B_{tr}$) is still obtained as an effect of hysteresis. The performance in terms of maximum transition error for feedback control combined with feed forward is right at the critical boundary for image based focus optimization.

Hysteresis is involved in spatially distributed electromagnetic fields. From feedback it had already been observed that hysteresis is present in the relation between applied current and the magnetic flux density at a point. Now, it is also concluded that hysteresis introduces a multi-valued relation between the magnetic flux densities at two points in the lens geometry.

6.6.2 Recommendations

Optimal Sensor Position

The sensor positioning for feedback control, as discussed in chapter 5, is not optimized. It is placed at a position that is possible for online operation, but no study is carried out for other positions. It is expected that the error level varies along with the sensor position. This is indicated with the arrows in the performance map Fig. 6.13. The sensor type used is large compared to the available space. It may be possible to construct a sensor having a shape that better fits the electromagnetic lens.
Chapter 6. Performance of Feed Forward and Feedback Control

Dynamic Feed Forward Compensation

The feed forward controller evaluated in this chapter consisted of a single valued nonlinear function for correction of the steady state equilibrium map only. A dynamical system as a feed forward compensator can be used to further decrease the transition time. Due to hysteresis the transition error will remain of the same order. From inspection of the feed forward step response it can be reasoned that some overshoot in the current trajectory is required. In [56], [55], [85], [86] dynamic compensation for electromagnetic lenses is discussed. However, the influence of hysteresis on the dynamics during the step response is not taken into account in these articles. Dynamic feed forward compensation is left as a recommendation.

Bandwidth Feedback Controller

Further improvements on the transition time using magnetic flux density feedback are still possible. The bandwidth of the current amplifier is about 5kHz. The maximal slew rate of the amplifier is 1A/ms resulting in 4.4ms for a step over the full range of the lens. Based on these numbers a transition time in the order of 5ms is considered possible. The bandwidth of the controller should therefore be increased to a few 100Hz. However, a more advanced controller design procedure is required to achieve this. Hysteresis should explicitly be taken account in the controller design procedure. The
anti-aliasing filters and digital filter procedure should be adapted to fit the controller scheme. How to increase the bandwidth of the closed loop system is not further investigated in this thesis.

Sensor Noise and Feedback Control

In the presented analysis the influence of the sensor noise becomes significant with feedback control, section 6.3. The controller gain at low frequencies is high due to the integral action. This implies an amplification of the sensor noise which results in a noisy input current. During the transition this is not important, but during image recording the maximum broad band input noise is specified around $1\mu A$. To meet this specification the controller should \textit{fade out}; if the set point is reached the feedback should gradually be disabled resulting in a constant input current set point. This can be achieved in the frequency domain by designing a bandpass-controller or by a nonlinear scheme that e.g. adapts the bandwidth of the controller in time.
Chapter 7

Feed Forward Initialization of Hysteretic Systems

7.1 Introduction

In this chapter alternative methods for feed forward set point control of systems that are both dynamic and hysteretic are investigated using phenomenological models. In chapter 6 it was shown that conventional time-invariant feed forward is not accurate enough. The problem is caused by the property that systems with hysteresis can have multiple steady state equilibria corresponding to a single constant input. The obtained set point after switching operating point depends on the initial condition at the time of switching. This problem can be overcome by feedback control using a highly accurate sensor. However, if the sensor cannot be placed at the position of interest, in the electron optic volume, the performance in terms of error reduction is not guaranteed. In chapter 6 it was experimentally validated that feedback control including integral action based on a sensor placed outside the electron optic volume has poor performance in terms of error reduction within the electron optic volume.

The input of the hysteretic electromagnetic lens system can be controlled very accurately (a resolution of $1\mu A$ over a range of $\pm 2.2A$) and with a high bandwidth ($\approx 5kHz$) and low drift ($< 10\mu A/10min$). From electron microscopy experiments the conclusion is that there is almost no influence of disturbances; series of images recorded in steady state at a rate of $14images/s$ over e.g. $3min$ do not show any sign of electromagnetic disturbances. The reason that feed forward control did not work was caused by the hysteretic system behavior itself, not by influences from outside.

The feed forward solution proposed in this chapter is based on a forced reset of the state at each switching event. With the use of an applied feed forward input trajectory, the state of the system is actively forced close to a unique value. The new initial condition, the state after the initializing input is applied, is nearly equal for all situations. The duration of initialization should be as short as possible in order to have a low transition time. By means of observations on the behavior of hysteresis models and the behavior of the lens system, directions for the design of initialization trajectories are obtained.

To make an optimal trade-off between minimal duration and maximum error reduction the design of an initialization trajectory requires further study to the memory structure of systems with hysteresis. For existing models, like the rate-independent Preisach or Duham hysteresis models, basic requirements are derived. However, an electromagnetic lens system with spatially distributed eddy current effects is not a system that can be considered rate-independent. It will be illustrated that an explicit coupling between hysteresis and dynamics is required. This coupling takes into account that magnetization takes time, which is an effect that is often neglected in hysteresis modeling.

---

1 Part of this chapter is published in Bree, P.J. van, Lierop, C.M.M. van, Bosch, P.P.J. van den (2009). Control-oriented hysteresis models for magnetic electron lenses, IEEE Transactions on Magnetics, vol. 45, no. 11, pp. 5235-5238, [90]

2 Part of this chapter is published in Bree, P.J. van, Lierop, C.M.M. van, Bosch, P.P.J. van den (2010). Feed Forward Initialization of Hysteretic Systems, Proceedings of the 49th IEEE Conference on Decision and Control, Atlanta USA, [92]
Chapter 7. Feed Forward Initialization of Hysteretic Systems

The study to what excitation wipes out contributions of previous excitation has a very close link to the study of nonlinear dynamical systems. Compare the often discussed *hysteresis loops* with representation of trajectories (orbits) of nonlinear dynamical systems in the phase plane. Convergence of nonlinear systems is connected to system properties like causality, passivity and dissipativity. Since convergence to a unique trajectory independent on the initial conditions is the first step of feed forward initialization as proposed in this chapter a comparison between dynamic behavior and hysteretic behavior is presented.

It will be shown that convergence of the system for periodic inputs provides a handle to influence the initial conditions. The desired behavior of an hysteresis model is extracted from experiments on the electromagnetic lens system. The proposed *feed forward initialization* technique leads to a controller implementation that is evaluated in the chapter 8.

7.1.1 Feed Forward Possibilities

A conventional feed forward approach is inverse modeling. The allowed transition error is defined by the error range in which image-based focus optimization methods work. In chapter 4 this range was estimated $< 1\%$ of the total range. The error in an inverse model based controller design is determined by the accuracy of the model. The obtained state-of-the-art modeling error as reported in literature is in the range of 1 to 10\%, e.g. [90]. The maximum error of 1\% of the lens system is too demanding. Inverse based compensators for transition error reduction will, therefore, not be investigated further.

The idea which is analyzed further is initiated by further thoughts on a concept used in magnetic recording [68] and with demagnetization of ferromagnetic materials. For demagnetization, e.g. [50], [22], the techniques described in [50] can be summarized as "Apply a slowly oscillating signal with a large magnitude that is slowly decreasing in time". If the applied frequency and the decrease of the amplitude of the envelope is sufficiently slow then the origin is reached and the material is demagnetized. The relevant questions are how "sufficiently low frequency" and "sufficiently slow decrease of the amplitude of the envelope" are defined for the lens system. Without any doubts the maximum frequency for accurate demagnetization processes is related to the depth of penetration of the electromagnetic field for a specific sinusoidal excitation. The skin depth and the eddy current effects were discussed in chapter 3.2. For the lens system ($\approx 1\, dm^3$) the maximum frequency will be orders lower than for the film-layer on a tape that is about $10\, \mu m$ thick, [68].

When analyzing magnetic tape recording it is found that a high frequency carrier wave is added to a ramp signal, [22]. When the amplitude of the envelope approaches zero, the ramp stops and the input is set to a constant level. The system is then magnetized to a specific value independent of the previous history. This technique is known as *anhysteretic (de-)magnetization*. Such magnetization techniques deal with hysteresis by applying specific input profiles. The input-output combination that is obtained after that the profiles have finished are *reproducible*; no matter from what initial conditions are when the profile starts, the end result is the same (within a very small range). It will be shown that these techniques are also useful for set point control of the electromagnetic lens. However, there are two fundamental differences: magnetic saturation and the zero-input after demagnetization.

After recording, the tape is removed from the recording device. The input is thus set to zero. A tape reader measures the amount of remanent magnetization. Therefore, the material properties of the tape are chosen such that there is a large hysteresis effect such that the amount of remanent magnetization is large. In magnetic storage applications the hysteresis effect is thus used as a beneficial memory effect.

In demagnetization the objective is to reach the origin which also implies that the signal ends at zero. The origin is a hard to reach state. To reach this point the system is first magnetically saturated in order to remove the influence of initial conditions. In the electromagnetic lens there is no need to reach the origin or to work with zero excitation. A set point is obtained with constant excitation. The removal of the influence of initial conditions is the only requirement. Therefore, magnetic saturation
is discussed next.

If the amplitude of the magnetization source is increased further and further, the system will eventually reach a so-called saturation state, e.g. [22]. From this point the hysteresis effect does not hold, meaning that the relation between input and output is single-valued until the magnetization is again under the saturation threshold. The reset obtained by saturation is a widely used technique which is required before almost all known hysteresis characterization and identification techniques, e.g. [62].

The coil of the lens system in combination with the material and the geometry is not designed to deliver the magnetic field strength required to saturate the magnetic material entirely. However, although magnetic saturation is an unwanted effect for lens optics, for future designs this possibility would be very beneficial to reset the initial conditions.

Despite that the saturation state cannot be reached the idea of a reset sounds attractive. Reformulation of the definition of a reset is reducing the difference between trajectories that are subject to the same input but start from different initial conditions. The design of such an initializing input is discussed in section 7.2.3.

7.2 Dynamical Systems with Hysteresis

This section provides the definitions required to describe the behavior of the electromagnetic lens system from a systems and control point of view.

7.2.1 Definitions

Let \( \mathbb{R} \) denote the set of real numbers. \( u \) represents the input, \( x \) the state, \( x_0 \) the initial condition, \( y \) the output of the system and \( r \) the reference value. \( \dot{x} \) represents the time derivative \( \frac{dx}{dt} \). The steady state values of \( u(t), x(t), y(t) \) are represented by \( u_c, x_c, y_c \). \( U, X, Y, R \) represent the input, state, output and reference space respectively. All spaces are defined as bounded continuous sets such that \( U : \underline{u} \leq u \leq \bar{u}, \ X : \underline{x} \leq x \leq \bar{x}, \ Y : \underline{y} \leq y \leq \bar{y}, \ R : \underline{r} \leq r \leq \bar{r} \). The notation \( \bar{y} \) is the maximum of the set and \( \underline{y} \) the minimum. In the notation \( (u(t); x_0) \) an input \( u(t) \) is applied and \( x(t) \) starts from the initial state \( x_0 \).

Representation of all possible steady state equilibrium input-output combinations \( (u_c, y_c) \) in the phase plane is called the steady state equilibrium map. This mapping in the space \( \mathbb{R}^2 \) is indicated with symbol \( \Upsilon \). A trajectory \( u(t) \) versus \( y(t) \) in the phase plane is further denoted as the orbit \( \phi(t) \).

A periodic signal with period time \( T \) is indicated with subscript \( T \). A periodic trajectory \( u_T(t) \) versus \( y_T(t) \) in the phase plane is indicated with \( \phi_T(t) \), a closed orbit.

7.2.2 Step Convergence and Fading Memory

As a starting point it is assumed that the behavior of the electromagnetic lens including hysteresis and all transient effects can be described as a dynamical system in state space notation.

Definition 7.2.2.1. Dynamical systems in state space notation:

Let \( \dot{x} = F(x, u; x_0) \), \( y = G(x) \) (7.1) be a causal (non-)linear stable, time invariant dynamical system in state-space notation for which the function \( G \) is a monotonically increasing function for which holds that \( dG(x)/dx > 0 \):

\[ \dot{x} = F(x, u; x_0) \]
\[ y = G(x) \]
\[ u(t) \in \mathbb{R}^{d_u}, x \in \mathbb{R}^{d_x}, y \in \mathbb{R}^{d_y} \]

Stability implies that the output and state trajectory of the system as a response to a periodic input will in the \( \lim_{t \to \infty} \) converge to be periodic. Next to that the output and the state will in the \( \lim_{t \to \infty} \) converge to constant for any constant input. If this constant input is applied from \( t = t_0 \) than the input can be seen as a step. For this property the term step convergence is defined in [57].
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Definition 7.2.2.2. **Step convergence:**
A system of the class defined by 7.2.2.1 is said to be step convergent if the state converges to be constant \( x_c \) for any allowed constant input \( u_c \) in combination with any initial condition \( x_0 \).

If step convergence holds then there exists a state \( x_c \) such that:

\[
0 = F(x_c, u_c; x_0), \quad \forall x_0 \in \mathbb{R}^{d_x}, \forall u_c \in \mathbb{R}^{d_u}
\]

\[
y_c = G(x_c).
\]

(7.2)

\( x_c \) does not have to be unique for a specific constant input \( u_c \).

The specific definitions for stability of step convergent systems are discussed in [44], [57], [58].

Step convergence has to hold for both hysteretic and non-hysteretic stable systems. However, for set point regulation of systems with dynamics and hysteresis the problem is that multiple equilibria correspond to one single constant input \( u_c \). The specific equilibrium reached depends on both \( u_c \) and the initial condition at the time of switching. A stable dynamical system without hysteresis knows a unique combination \( u_c, y_c \) that is independent of the initial condition. This equilibrium can be obtained by applying \( u_c \) and waiting. This property in which the system "forgets" its initial condition is called **fading memory**. The term fading memory is already used in e.g. [11, p.74].

Definition 7.2.2.3. **Fading memory:**
A system of the class defined by 7.2.2.1 and 7.2.2.2 has fading memory if, the difference between two trajectories driven by an equal input \( u(t) \), starting from different initial conditions \( x_0 \) and \( x_0 + \delta \), vanishes in time for all perturbations \( \delta \in \mathbb{R}^{d_x} \) and all possible input trajectories \( u(t) \in \mathbb{R}^{d_u} \):

\[
\lim_{t \to \infty} \{ F(x(t), u(t); x_0) - F(x(t), u(t); x_0 + \delta) \} = 0,
\]

\[
\forall x_0 \in \mathbb{R}^{d_x}, \delta \in \mathbb{R}^{d_x}, u(t) \in \mathbb{R}^{d_u}.
\]

(7.3)

Fig. 7.1 shows an illustration of two step convergent systems, one with fading memory and one without with fading memory. The behavior of an electromagnetic lens system fits the description of step convergent but has no fading memory. Next to an illustration of the time domain response also the steady state equilibrium map \( \Upsilon \) is presented. From the illustration of \( \Upsilon \) it follows that a range of possible values \( y_c(u_c) \) is possible for the system without fading memory.

A steady state equilibrium \( (u_c, x_c, y_c) \) for step convergent fading memory systems only depends on the applied constant input \( u_c \) and not on the initial condition. Therefore, the combination \( u_c, x_c, y_c \) is unique. This implies that a steady state mapping of a constant input to steady state output, for all possible initial conditions, is a possibly nonlinear, but single-valued relation \( y_c = G_c(u_c) \).

For linear dynamical systems described by transfer function \( G(s) \), \( G_c \) is a constant called the DC-gain, [32, p.104]. The steady state equilibrium map for linear systems is a line of which the slope is determined by the DC-gain. For systems with a static input or output nonlinearity this map is a single valued nonlinear function.

Fig. 7.2 shows a comparison of the steady state equilibrium map for different types of systems. When hysteresis is involved, the relation between \( u_c \) and \( y_c \) is multi-valued. Hysteretic systems can be step convergent but have no fading memory. However, also hybrid or switched systems can have a multi-valued equilibrium map. These systems do not fulfill the fading memory property since they do remember in which switching state or mode they are in. However, as indicated in Fig. 7.2 the steady state equilibrium map \( \Upsilon \) can be multi-valued but the equilibria corresponding to any single constant input are isolated. Fig. 7.3 presents a brief classification of systems on the basis of the step convergence and fading memory property.
7.2. Dynamical Systems with Hysteresis

Step convergent and fading memory

Step convergent but no fading memory

Figure 7.1: Illustration of the fading memory property for two stable step convergent systems. The steady state equilibrium for fading memory is unique and depends only on \( u_{c1} \). If fading memory does not hold the resulting equilibrium also depends on the initial condition of the system.

Transversal Type Hysteresis

For ferromagnetic hysteresis, the steady state mapping is not only multi-valued but the number of equilibria for one single constant input is infinite since all equilibria are in a continuous but bounded range. This property is used to further specify the hysteretic behavior as observed in the lens system.

**Definition 7.2.2.4.** Systems with transversal type hysteresis [58]:

*If a step convergent, time invariant, dynamical system as defined in 7.2.2.1 does not have fading memory, and the resulting steady state equilibria map \( \Upsilon \) is a continuous surface with a non-zero area the system is said to have transversal type hysteresis.*

If a system of the class defined in 7.2.2.1 has multiple isolated equilibria then it is of the bifurcation-type, [58]. The number of steady state output values of systems with bifurcation type hysteresis corresponding to a single constant input is \( u_c \) is countable for all possible \( u_c \). An example of a model for bifurcation type hysteresis is the relay operator which is the building block of the discrete Preisach model, [50].

In [58] it is shown that a feedback interconnection of linear systems with a static nonlinearity can result in both bifurcation type as well as transversal type hysteresis. Bifurcation type will not be discussed further, the focus is on transversal type hysteretic systems. In section 7.5.2 an example of a such a nonlinear feedback model with transversal type hysteresis is presented. In the remaining part
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Figure 7.2: Representation of the steady state input output equilibrium map $\Upsilon$ for a linear system, a nonlinear system, a hybrid or switching system, bifurcation type hysteresis and transversal type hysteresis.

of this research the term hysteresis implies systems with transversal type hysteresis. Next the response of such systems to quasi-static and periodic inputs is analyzed.

7.2.3 Quasi-static and Rate-independence

The steady state equilibrium map $\Upsilon$ is a continuous surface in which each point is an equilibrium point. Further insight of the memory structure is obtained by studying the trajectories $\phi$ for quasi-static excitation. The dissipativity property of hysteresis can be linked to the observed quasi-static trajectory.

The notion of quasi static excitation is illustrated using the response for linear systems. Given an example first order linear system $H(s) = \frac{\alpha}{s+\pi}$. Since $H$ is a low pass system it holds that the higher the applied frequency component, the more attenuation. The phase shift of the output when compared to the input starts at zero for low frequencies and approaches -90 degrees for high frequencies. Non-sinusoidal excitation can be decomposed into a summation of sinusoidal components using the Fourier transform. The energy of the input signal for different frequencies compared to the phase shift and amplitude attenuation as derived from the frequency response of the system $H$ indicate whether the dynamics of $H$ are significantly excited or not. From the bode plot of the system an upper bound on the frequency can be indicated for which it holds that if all energy of the input $U(j2\pi f)$ is in the region $f < f_{qs}$ then amplitude attenuation and phase shifts are insignificant. $f_{qs}$ is the upper bound on the quasi-static region. This is a subjective measure since attenuation and phase shifts are always present.
7.2. Dynamical Systems with Hysteresis

Figure 7.3: Classification of systems with and without the step convergence and fading memory properties. Hysteresis can be further classified as transversal type and bifurcation type on basis of the properties of the steady state equilibrium map.

For quasi-static excitation it should always hold that if the input stops varying \( (du/dt = 0) \), then the steady state is immediately obtained. Therefore, all quasi-static trajectories \( \phi_{qs} \) represented in the phase plane provide the structure of the steady state equilibrium map \( \Upsilon \). Quasi-static excitation is closely related to the often used system property of rate-independence.

**Definition 7.2.3.1. Rate-independence:**

*Given a time varying input-output combination \((u(t), y(t))\) represented in the phase plane as \(\phi(t)\). Then the system is rate-independent if the trajectory \(\phi(t)\) is invariant with respect to time scaling. That is, if the pair \((u(t), y(t))\) is an admissible input-output pair; then so is \((u(at), y(at))\) for any \(a > 0\). This implies that the continuous set of points described by \(\phi(t)\) resulting from \(u(t), y(t)\) is equal to the set of points described by \((u(at), y(at))\).*

The system under study, an electromagnetic lens system, is not rate-independent. However, for an input signal \(u(t)\) of which all energy is in the frequency region \(f < f_{qs}\) it holds that a time scaling \((u(at), y(at))\) for \(0 < a \leq 1\) is still valid. *The class of dynamical systems under study can thus be considered rate-independent for quasi-static excitation.* The excitation varies so slow that the effect of dynamics can be neglected. It can be checked if excitation may be considered quasi-static by applying a slowed down version \((0 < a < 1)\) to the system and comparing the resulting trajectories \(\phi\) in the phase plane. The trajectories should be equal.

Hysteresis models which have the rate-independent property are made insensitive to time scalings. They can provide a good approximation of the quasi-static behavior of the system. For any quasi-static excitation of dynamical systems with or without hysteresis it should hold that:

\[
\text{sign}(\dot{u}) = \text{sign}(\dot{y}).
\]

Consider that if this property would not hold then it would imply that there exists a quasi-statically monotonically increasing input for which the output is decreasing or vice versa. In Fig. 7.4 the
effect of this condition of the map $\Upsilon$ is illustrated. Note that the condition only holds for quasi-static excitation since for higher frequency excitation a phase shift is introduced. A difference in phase between two sinusoidal signals is observed as an ellipsoid in the phase plane. An ellipsoidal orbit implies that $\text{sign}(\dot{u})$ is not always equal to $\text{sign}(\dot{y})$. Already in chapter 3 the comparison of the response of dynamical system and hysteretic systems to sinusoids with different frequencies was provided. The corresponding figure is repeated here, Fig 7.5. However, at the point where $du/dt$ changes sign the second derivative of the output $d^2y/dt^2$ is discontinuous. Fig. 7.6 illustrates this. A sinusoidal excitation is applied and in first instance it appears that the output lags behind. Inspection of the first and second order derivative shows the discontinuous behavior.

Figure 7.4: from left to right: Clockwise hysteresis, Counter clockwise hysteresis, $\text{sign}(dy/dt) = \text{sign}(du/dt)$, $\text{sign}(dy/dt) \neq \text{sign}(du/dt)$ for all cases.

For a linear dynamical low pass system the output lags behind the input which implies a negative phase shift. In the same way a high pass system will introduce a phase lead. The direction of travel of a trajectory $\phi(t)$ as a result of two sinusoidal signals represented in the phase plane will be counter clockwise if the output lags behind the input and clockwise if the output leads, 7.4.

For strictly proper systems with hysteresis trajectories $\phi$ for quasi-static periodic excitation also have a clockwise or counter clockwise orientation indicating a phase lead or lag. If a quasi-static periodic excitation is applied and the response has converged to periodic, then the trajectory $\phi_T(t)$ is a closed trajectory. The area enclosed by the trajectory $\phi_T(t)$ for one single period has the interpretation of energy if the multiplication of the input and the output signal in time represents power. The integral of power over time is energy. Now if the orientation of $\phi_T(t)$ is counter-clockwise it means that the enclosed surface is the energy that is dissipated. A counter clockwise quasi-static trajectory implies a dissipative system. Similar statements are made in [3]. This result makes it more intuitive to reason about the physical interpretation of observed orbits in the phase plane. Of course one is free to study the orbits relating other quantities not directly related to power, but interpretation is more difficult or even confusing. The dissipativity property of dynamical systems enables to make use of controller design based on these properties. Examples are found in e.g. [33], [38].

In the case of the electromagnetic lens system the current applied to the lens coil is observed in combination with the magnetic flux density at a point in the geometry. In the electrical domain current should be studied in combination with voltage. Next to that, the spatially distributed nature of the system is not taken into account. From observations at a single point it is not possible to observe the state of the complete lens system. For these reasons no energy related interpretations of the experiments carried out on the lens system will be provided.
7.2. Dynamical Systems with Hysteresis

Figure 7.5: Comparison of the response of a dynamical with and without hysteresis to sinusoidal excitation with different frequencies represented in the phase plane. The difference between the systems is observed for low frequencies: The area enclosed by the orbit for the system without hysteresis approaches zero while the orbit described by the hysteretic system approaches a multi-valued mapping for low frequencies.

Figure 7.6: Response of a rate-independent hysteresis model to a sinusoidal excitation. Since the quasi-static property \( \text{sign}(\dot{u}) = \text{sign}(\dot{y}) \) holds and counter-clockwise hysteresis introduces a phase lag, the 2\(^{nd}\)-order derivative is discontinuous at each change of sign(\(\dot{u}\)). The model is defined in (7.8)
Chapter 7. Feed Forward Initialization of Hysteretic Systems

7.2.4 Periodic Excitation

The next step is to study the convergence of orbits to a periodic orbit. The goal is to investigate whether there exists a periodic excitation that results in a periodic orbit that contains the same set of points independent of the initial conditions of the system at the time the periodic excitation started. If such an excitation exists than it means that the influence of initial conditions can be wiped out by applying this excitation.

Definition 7.2.4.1. The class of periodic signals:

\[ u_T(t) = u_T(t + kT) = o_u + a_u \Phi_u \left( \frac{2 \pi}{T} t + \varsigma_u \right), k \in \mathbb{Z}. \]  

(7.4)

\( u_T \) is defined by a real valued function \( \Phi \) which satisfies \( ||\Phi||_\infty = 1 \), with a period \( T \), a phase \( \varsigma \), and where \( o_u \) denotes the offset and \( 2a_u \) denotes the peak to peak amplitude. The combination of these parameters is denoted as the parameter vector \( \theta = [o_u, a_u, \Phi, T] \). The set of all possible parameters is defined as \( \Theta \). Similar notation holds for \( x_T, y_T \).

It can be checked if a signal \( y(t) \) is periodic with period time \( \kappa \) if:

\[ (y(t + m\kappa) - y(t)) = 0, \quad \forall t \in \mathbb{R}, m \in \mathbb{Z}. \]  

(7.5)

When the smallest possible \( \kappa \) for which (7.5) holds equals \( T \), the period of the output is equal to the period of the source, \( y_\kappa = y_T \). For the set of hysteretic systems that is under study it holds that \( y_\kappa = y_T \). From now on the subscript \( T \) is used to indicate a periodic response.

For dynamical systems with fading memory the resulting periodic output does not depend on the initial conditions \( y(t) = y_T(u_T(t)) \). For systems with hysteresis \( y(t) = y_T(u_T(t); x_0) \) is dependent on the initial condition. The trajectory of the input versus the output is often called a loop in hysteresis literature. If the input and the output are both periodic then the orbit is closed, a closed minor loop in hysteresis terminology.

Definition 7.2.4.2. Equal orbits

An orbit is considered as a continuous periodic set of points irrespective of its phase. Therefore, two orbits \( \phi_1(t_1; u_T, x_{01}), \phi_2(t_2; u_T, x_{02}) \) are considered being equal if there exists a time shift \( 0 \leq |\tau| \leq T \) such that the set of points described by both is the same and is evaluated in the same order:

\[ \{y_T(t_1, u_T(t_1); x_{01}) - y_T(t_2 + \tau, u_T(t_2 + \tau); x_{02}) \} = 0. \]

As observed from the notation, the orbit \( \phi(u_T; x_0) \) can depend on the initial conditions. With this defined result the concept of feed forward initialization can be further defined. If a certain periodic excitation is an initializing input then the influence of the initial conditions has to vanish over time under the requirement that the input is applied.

Definition 7.2.4.3. Periodic initialization trajectory:

A periodic input \( u_T(t, \theta) \) is a periodic initialization trajectory if

\[ \phi(u_T(t, \theta); x_0 + \delta) \rightarrow \phi(u_T(t, \theta); x_0), \quad \forall x_0 \in \mathcal{X}, \forall \delta \in \mathcal{X}, \forall (x_0 + \delta) \in \mathcal{X}. \]  

(7.6)

The set of parameters for which (7.6) holds is defined as \( \theta \in \Theta \).

The requirements on parameters \( \Theta_i \) depend on the memory structure of the hysteretic system.
7.3. Quasi-Static Initialization of Various Hysteresis Models

7.2.5 Memory Structure of Hysteretic Systems

If all possible periodic excitations drive the response of the hysteretic system to a periodic orbit that is independent of \( x_0 \) then the system has local memory and any periodic input is an initializing input:

**Definition 7.2.5.1.** Hysteretic systems with local memory:

Let \( \delta \) be a perturbation on the initial condition \( x_0, \delta \in \mathbb{R} \), then a hysteretic system has local memory if

\[
\phi_T (u_T(t); x_0) = \phi_T (u_T(t); x_0 + \delta), \quad \forall u_T(t), \forall x_0 \in \mathcal{X}, \forall \delta \in \mathcal{X}, \forall (x_0 + \delta) \in \mathcal{X} \tag{7.7}
\]

**Definition 7.2.5.2.** Hysteretic systems with non-local memory [50]:

Let \( \delta \) be a perturbation on the initial condition \( x_0, \delta \in \mathbb{R} \), then a hysteretic system has non-local memory if there exists any combination of \( u_T(t), x_0, \delta \) such that

\[
\phi_T (u_T(t); x_0) \neq \phi (u_T(t); x_0 + \delta)
\]

The definition of non-local memory states that there is at least one combination of \( u_T(t), x_0, \delta \) for which the resulting orbit is unique. However, there may still exist a specific \( u_T \in \mathcal{U} \) which ensures an equal orbit for all possible \( x_0 \in \mathcal{X} \). The difference between local and non-local memory will be explained by analysis of examples with local memory (Duhem class hysteresis models) and models with non-local memory (Preisach class hysteresis models)

7.3 Quasi-Static Initialization of Various Hysteresis Models

In this section examples from two hysteresis model classes are compared: the Duhem model class and the Preisach model class. Both are rate-independent models which makes them independent of time scaling. In relation to so-called rate-dependent physical systems, like the electromagnetic lens system, rate-independent behavior is considered valid in the limited frequency range where the quasi-static approximation holds as discussed in section 7.2.3. As a consequence the resulting trajectories \( \phi(t) \) in the phase plane are equal for all wave forms with an equal amplitude trajectory, e.g. triangular, sine or square waves.

7.3.1 Duhem Hysteresis Model

As an example, the response of the rate-independent Duhem hysteresis model to periodic inputs is analyzed. This model is often used because of its simplicity. It can be described with a single nonlinear ordinary differential equation that can be decomposed as a combination of two linear systems with a switching rule on the change of sign of the time derivative of the input. Next to that, certain model representations out of the class allow for an analytical expression for the solution of the differential equation. This model is widely used and analyzed, e.g. [16], [98], [47], [6], [57], [59].

This specific implementation of a Duhem model is also considered in [16], [6]:

\[
\begin{align*}
\dot{x} &= h_1 |\dot{u}| [h_2 u - x] + h_3 \dot{u}, \\
\dot{y} &= (h_1 \text{sign}(\dot{u}) [h_2 u - x] + h_3), \quad x(t_0) = x_0, \\
y &= x.
\end{align*}
\tag{7.8}
\]

The three parameters are all constants \( h_1 > 0, 0 < h_3 < h_2 < 2h_3 \). Equation (7.8) can be rewritten such that it only depends on \( \text{sign}(\dot{u}) \) and not on \( \dot{u} \) itself:

\[
\frac{dx}{du} = h_1 \text{sign}(\dot{u}) [h_2 u - x] + h_3, \quad x(t_0) = x_0.
\tag{7.9}
\]

Equation (7.9) is providing the evolution of the input-output trajectory instead of the evolution in time. The resulting vector field \( \frac{dx}{du} \) in a point \((u, x)\) has two possibilities, one for increasing input
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Figure 7.7: Simulation of the Duhem model (7.9) for different initial conditions. a. The input-output plot \((u, y)\). The dashed line shows the closed orbit \(\phi(u_T)\). b. The response for \(u_T = \sin(2\pi/T), T = 10s\) in the time-domain. c. The difference between trajectories \(|y_1(t) - y_2(t)|\) on a logarithmic scale.

Figure 7.8: Simulation of the Duhem model (7.9) for different initial conditions. a. The input-output plot \((u, y)\). The dashed line shows the closed orbit \(\phi(u_T)\). b. The response for \(u_T = 0.3 \sin(2\pi/T), T = 10s\) in the time-domain. c. The difference between trajectories \(|y_1(t) - y_2(t)|\) on a logarithmic scale.
and one for decreasing input. The set of possible initial conditions $x_0 \in \mathcal{X}$ is bounded by $x_{llim} = (h_2 u - \frac{h_2 - h_3}{h_1}) \leq x_0 \leq x_{ulim} = (h_2 u + \frac{h_2 - h_3}{h_1})$.

Differential equation (7.9) becomes linear for piecewise monotonically increasing or monotonically decreasing input segments, since $\text{sign}(\dot{u})$ is then equal to either $\pm 1$. This specific Duhem model can thus be represented as a switching system with two linear dynamical modes. The switching rule is acting on a change of $\text{sign}(\dot{u})$.

The analytical solution is formulated as follows:

$$ y = h_2 u - \frac{\text{sign}(\dot{u})}{h_1} \left(1 - \exp\left(-\text{sign}(\dot{u}) h_1 (u - u_0)\right)\right) $$

$$ \left(x_0 - h_2 u_0\right) \exp\left(-\text{sign}(\dot{u}) h_1 (u - u_0)\right). $$

(7.10a)

(7.10b)

(7.10c)

The main term of the response is a line (7.10a). The larger the amplitude of the increasing (or decreasing) segment $(u - u_0)$ (7.10b), the more $y$ approaches the lower asymptote $x_{llim}$. The third component (7.10c), takes into account the initial condition which vanishes exponentially with the amount of excitation. For example, the difference between two trajectories starting from different initial conditions, $x_1(t_0) = x_{01}, x_2(t_0) = x_{02}$ subject to the same increasing input $u_\uparrow$ is:

$$ |y_1 - y_2| = |x_{01} - x_{02}| \exp\left(-\text{sign}(\dot{u}) h_1 (u_\uparrow - u_0)\right). $$

(7.11)

The larger the magnitude $|u - u_0|$, the smaller the difference between $y_1$ and $y_2$ becomes. However, since the input range is bounded, it is not possible to keep increasing the input in order to make the difference between the trajectories $y_1$ and $y_2$ smaller. Therefore, the input is set to be periodic. From analysis of the analytic expression for periodic inputs it can be derived (as is done in [6]) that the solution converges to a periodic orbit $\phi_T$ that is independent of $x_0$. As a consequence, the Duhem model has local memory. Inspection of (7.11) shows that the largest possible amplitude $u_\uparrow - u_0$ is most beneficial.

A resulting periodic orbit is always positioned around the line $y = h_2 u$, where $h_2$ is one of the model parameters. This line is the anhysteretic curve of this model. Any periodic orbit will be positioned around the point $(o_u, h_2 \cdot o_u)$ where $o_u$ is the offset of the applied periodic signal. All possible $u_T$ (7.4) can serve as initializing trajectories. This is a general property of models with local memory. From the notion of a quasi-static input and a rate-independent model structure, it makes no sense to express the rate of convergence in time. However, from the analysis it follows that the higher the amplitude $a_1$, the lower the number of periods that is required to initialize the model to a certain degree. The period time $T$ has no influence on the resulting orbit, where the offset $o_0$ controls its position. The number and sequence of extrema in the function $\Phi$ influences the set of points described by the orbit. The phase $\theta$ is such that the trajectory starts at $u_0$.

As an illustration, Fig. 7.7 shows the response to $u_T = \sin(2\pi/T)$ and Fig. 7.8 shows the response to $u_T = 0.3 \sin(2\pi/T)$ for three different initial conditions. For both inputs, the trajectories converge to an orbit $\phi_T(u_T)$ that is independent of the initial conditions. The difference $|y_1 - y_2|$ in time is shown on a logarithmic scale in subplot $c$. Note that at the points where $\dot{u}$ changes sign, convergence towards $\phi_T(u_T)$ is slower. Depending on the amplitude the rate of convergence per period can be controlled. The larger the amplitude, the higher the rate of convergence. This relation is derived and expressed in (7.11).

The fact that even the smallest periodic perturbation can reset the state of the Duhem model does not make sense from a physical point of view: It is not likely that a low frequent excitation with an amplitude of a few $\mu A$ has a significant effect on the state.

Whether a model is good or bad depends on the application. In a practical application in which a measured signal is applied to the model the measurement noise will significantly disturb the behavior.
Chapter 7. Feed Forward Initialization of Hysteretic Systems

of the model. However, for large periodic signals with zero offset, both the qualitative difference between response of physical systems and the Duhem models is minor. Quantitative differences between model output and measured system output can probably be tuned to be smaller than 1%.

7.3.2 Hysteresis Models with Deletion Property

Another common way of modeling rate-independent hysteresis is found in the Preisach model class [50]. This model class knows many representations such as the classical Preisach model and the multiplay model that is also known as the Prandl or Prandl-Ishlinskii model [42], [98] [12]. All these models have the deletion or wiping out property. This property, mathematically defined in e.g. [12], implies that initialization in these models is maximally a two step procedure: switching to the maximum input, followed by the minimum or vice versa is sufficient to delete all history. All models in this class are formulated such that they fulfill this property. As a consequence these models have nonlocal memory. In [94] it was shown that the wiping out property is also obtained in a 3D micromagnetic hysteresis model for iron grains with a dimension of about $1\mu m^3$. That the micromagnetic model has similar memory properties as the phenomenological Preisach model is not trivial since micromagnetic models are formulated at the smallest scale and hysteresis follows from dislocations in the material.

![Figure 7.9: Illustration of the deletion property for a Preisach model.](image)

Figure 7.9: Illustration of the deletion property for a Preisach model. The obtained situation $i$ is not influenced by the excitation $b, \cdots, h$: The exact same situation $i$ would be obtained when switching directly from $a$ to $i$. The orbits $b, \cdots, h$ are immediately closed. The model "forgets" previous excitation when the memory it is overwritten by a larger excitation.

Fig. 7.9 shows an simulation of a Preisach model implementation that illustrates the deletion property. Since the Preisach model is a rate-independent hysteresis model the duration in time (number of seconds) is irrelevant and not shown. The simulation starts in the origin, representing the demagnetized state. When switching from $a$ to $i$, a small amplitude (0.2 to 0.7) periodic sequence is applied.
This excitation has no effect on the obtained level \(i\): The same memory status would be obtained if the system was switched directly from \(a\) to \(i\). This can be checked by inspecting the slope of the trajectory from \(a\) to \(i\) that is not influenced by the small amplitude excitation. The orbit (minor loop) \(b, \ldots, h\) seems to be attached without any consequence. The model “forgets” previous excitation when the memory it is overwritten by a larger excitation. This simulation is based on observations already described by Ewing in 1894 [29] (recently discussed in [37]), first formalized by Madelung in 1905 [48] first modeled by Prandtl and Berliner in 1906 [9] and modeled by Preisach in 1935 independently from previous research [66] (as discussed in e.g. [12, p.27,72]).

The Duhem model showed exponential convergence towards a periodic orbit. From Fig. 7.9 it is observed that two steps are sufficient for convergence of the response for models with the deletion property: consider the sequences \(b, \ldots, h\) and \(j, \ldots, p\) which form a closed orbit from the first period on. This behavior is further studied to derive the conditions for initialization.

One possible implementation structure of a Preisach model is a multiplay approach consisting of a parallel connection of \(N\) weighted play (or backlash) operators \(p(u, x_0, \Delta)\) where \(2\Delta\) is the width of the play (e.g. [27], [1]). A single play operator is defined as:

\[
 x_i = p(u, x_{0i}, \Delta_i) = \max(\min(x_{0i}, u + \Delta_i), u - \Delta_i). \tag{7.12}
\]

The parallel connection of play operators is defined as:

\[
y = P(u, x_0, \Delta) = \frac{1}{N} \sum_{i=1}^{N} \beta_i p(u, x_{0i}, \Delta_i), \Delta_i \geq 0, \beta_i \geq 0. \tag{7.13}
\]

The weighting vector \(\beta\) is normally used to tune the response of the output to fit the experimental
Figure 7.11: Simulation of a single play operator from 20 different initial conditions. The input is switched to $u = 0.2$. a.) Trajectories in the phase plane. b.) The input in time. c.) The output trajectories starting from the different initial conditions. d.) The absolute difference between the first trajectory $y_1$ and all others on a logarithmic scale.

Figure 7.12: Simulation of a multiplay hysteresis model from a set point transition starting from 20 different initial conditions. 10 play operators are place in parallel. The play width ranges from $\Delta_1 = 0$ to $\Delta_{10} = 0.9$. 
7.3. Quasi-Static Initialization of Various Hysteresis Models

Figure 7.13: Simulation of a single play operator from 20 different initial conditions including initialization by a square wave. Note that initialization is finished after a single period. The same play operator as in Fig. 7.11 is evaluated.

Figure 7.14: Simulation of a multiplay hysteresis model from a set point transition starting from 20 different initial conditions including initialization by a square wave. Note that initialization is finished after a single period. 10 play operators are in parallel. The play width ranges from $\Delta_1 = 0$ to $\Delta_{10} = 0.9$. 
data. Since $\beta$ does not influence the memory of the model it is set to one for further analysis. Fig. 7.11 shows the response of a single play element for 20 different initial conditions to a set point change. The play operator has a width $\Delta = 0.35$. The input is switched to $u = 0.2$. As observed from the time domain behavior, the system is step convergent but has no fading memory. The difference in output after switching is large as represented in subplot (c) that represents the difference between the first trajectory and all others. In Fig. 7.12 a similar simulation is carried out for a multiplay model with 10 parallel play operators in which $\beta = 1$. The play width ranges from $\Delta_1 = 0$ to $\Delta_{10} = 0.9$. Also here the transition error is large.

Since this model contains a parallel connection there is no interaction among the different operators. To establish the parameter conditions of a periodic input (7.4), such that it is an initialization trajectory of the multiplay model, it is sufficient to study the properties for a single element. The input is bounded between $\pm u_m$. The play operator is defined symmetric around the origin. The set $X$ is bounded by $u - \Delta \leq x_0 \leq u + \Delta$ for $|u| \leq u_m$. Fig. 7.10 presents the two possible situations: $a$ and $b$ it holds that if $(x_0, u_0) \in I$ then $x$ stays in $I$, $\forall |u| \leq u_m$. Consider $p_1(u, x_{01} \in II, \Delta_1)$, and $p_2(u, x_{02} \in III, \Delta_2)$. For $\Delta < u_m$ it follows from (7.13) that $u_{ini} = [u_0, u_m, u_m - 2\Delta]$ or $u_{ini} = [u_0, -u_m, -u_m + 2\Delta]$ is the minimal sequence of points that initializes both $p_1$ and $p_2$. For the situation that $u_m \leq \Delta$ (case $b$) the initialization sequences becomes $u_{ini} = [u_0, u_m, -u_m]$ or $u_{ini} = [u_0, -u_m, u_m]$. This result implies that only a trajectory containing both $u_m$ and $-u_m$ initializes the multiplay model. However, a single period is sufficient. This condition is known as the 1st-order wiping out property.

The resulting orbit $\phi$ is the sum of the orbits of the individual elements. If $\max u_T = u_m$, $\min u_T = -u_m$ and $\Phi$ has only 1 maximum and 1 minimum, then for case $a$ $\phi_i$ is the boundary of region $I$. For case $b$ $\phi$ is a line with offset $x_0$ if $x_0 \in I$, offset $u_m$ if $x_0 \in III$ and offset $-u_m$ if $x_0 \in II$. The phase $\theta$ is again such that the trajectory starts at $u_0$.

Fig. 7.13 shows the response of a single element to an initialization sequence consisting of three periods with maximal amplitude. Fig. 7.14 shows the response for the multiplay model that was also discussed in the situation without initialization. The closed orbits are reached after a single period. Three periods are shown to illustrate that this model does not know exponential convergence.

### 7.3.3 Discussion Initialization for Quasi-static inputs

Analysis of two very common rate-independent hysteresis models has shown that there is a fundamental difference in the initialization behavior of the two models. Preisach-type models are based on the deletion property which results in a two-step initialization requiring both the maximal and minimal input. The difference between trajectories for Duhem models vanished exponentially with the amount of excitation. This requires a number of initializing periods. However, initialization for Duhem models works with all periodic signals independent of the amplitude. Still maximum amplitude guarantees fastest convergence.
7.4 The Accommodation Effect

As discussed in chapter 3 the reason to work with phenomenological models is that the "true" hysteresis behavior extracted from physical laws is not available. The electromagnetic lens behavior is extracted from experiments in which periodic signals with different offsets and amplitudes are applied. The figures presented in this section were also published in [90].

7.4.1 Accommodation Experiment

An uninterrupted trajectory containing 5 periodic sequences (cycles) is applied to the lens setup, [90]. The input sequence is shown in Fig. 7.15, the experiment takes about 35 min. This experiment is derived from [102] in which only simulation data was presented and experimental data was requested. The input consists of low pass filtered steps \((f_{cut-off} = 1Hz)\) with a period time of \(10s(0.1Hz)\). All the energy of the applied input is considered to be in the quasi-static region. As a result of the quasi-static excitation the total measurement takes over \(30\) min.

The corresponding input-output trajectory is shown in Fig. 7.16. Zoomed versions of all the individual orbits of all 5 cycles are provided in Fig. 7.17. The obtained orbits are not periodic within a single period as with Preisach type models. Instead exponential convergence towards periodic is observed. This exponential convergence can be checked by inspecting the time responses of orbit number (4) as provided in Fig. 7.18. To make sure that this effect is not mixed up with transients due to switching of the offset current, the current is set to the constant offset values for a few 100s before applying the periodic inputs. This is shown in Fig. 7.15 and a zoomed version is presented in Fig. 7.18. The variation of the output over time after switching to the constant offset of (4) for \(1150s < t < 1300s\) is about 4 times higher than the variation of the extrema as a response to the periodic signals for \(1300s < t < 1450s\).

The direction of convergence of the orbits is indicated in Fig. 7.16 using arrows and can also be observed from the zoomed version in Fig. 7.17. In the phase plane up and down are used to indicate the directions. This direction possibly depends on the applied electromagnetic field of the previous cycles, higher or lower. However, in other works in literature, e.g. [8], it is often discussed that the direction of travel is always towards the anhysteretic curve which serves as an attractor. As discussed the memory structure of the Duhem model is constructed such that orbits not only travel in the direction of the anhysteretic curve, but are eventually positioned around a point on the curve. This is not the case in the electromagnetic lens. Although on a limited number of 15 periods is applied, it is concluded that the position of the loop (the offset of input and output) does not change anymore. The observed effect is in a sense a combination between Duhem and Preisach models. The lens behavior knows both exponential convergence and stable periodic orbits with an offset unequal to the anhysteretic curve. In literature about magnetics this effect is called accommodation or reptation.

7.4.2 Available Accommodation Models

The phenomenon accommodation is often described as a drift of minor loops when varying the input between two fixed values. This description is equal to the convergence of the orbit \(\phi(t)\) towards a periodic orbit \(\phi_T(t)\) as used here. In [24] accommodation is defined as: A rate-independent drift of successive minor loops towards a limit cycle. In the input-output plane it is observed that a trajectory will converge to a closed trajectory.

Models that describe/predict accommodation effects are still under development. As stated in the previous section, models with local memory show accommodation that always converges to the anhysteretic curve. Models with non-local memory that show the deletion property [22] show no accommodation at all. In [102] these two models are combined to capture accommodation. An extension to Preisach models is developed in [24], [21], [25], [101].

Measured data illustrating accommodation is published in [8]. Here accommodation is observed as the drift of minor loops as a response to oscillations. The minor loops are attached to the descending
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Figure 7.15: Measured accommodation behavior: Input sequence used to illustrate the accommodation effect. Lower, complete sequence. Upper, zoomed in. The complete experiment takes over 200s (30min).

Figure 7.16: Measured accommodation behavior: Input-output plane corresponding to the accommodation experiment. Responses measured using the electromagnetic lens setup. The units are normalized. In order to better illustrate the hysteresis effect a linear term \(0.75 \cdot \text{input}\) is subtracted from the output.
7.4. The Accommodation Effect

Figure 7.17: Measured accommodation behavior: Zoomed version of separate minor loops (orbits). The response for the first 2 periods of the input is shown in red. The response for the remaining periods fades from blue to gray. This illustration clearly shows if the orbit travels up or down.

Figure 7.18: Measured accommodation behavior: Upper left. Observed accommodation in time of minor loop (4). As expected an exponential convergence to a stable situation is observed. Upper right. zoomed version (top) of accommodation cycles. Lower left. response to constant input. Lower right. zoomed in to lower part of accommodation cycles. Note that the output varies a factor 4 more with accommodation compared to the time response for a constant input.
branch of the major loop. Model validation on this data is carried out in [102], [101] and [80]. The presented results in literature show that models with a different basis are capable of describing the presented measurements. What is concluded in those works is that a curve-fit (model parameter tuning) of the presented models to the data results in a small (sum of squares) error. However, only the fit of the model to the identification data is presented. Validation data containing sufficiently rich trajectories seems not available in literature. Although that the presented models in [102], [101] and [80] have a completely different memory structure, the output response to this single data set appears the same. The only valid conclusion based on this result is that validation on different data sets is required.

7.4.3 Initialization for Hysteretic Systems with Accommodation

The combination (e.g. parallel connection) of models with local memory such as the Duhem model and models with the deletion property such as the Preisach model does show accommodation behavior. Since, no better description for accommodation than phenomenological models is known the requirements for initialization are derived from adding up the requirements as derived for the Duhem model and the Preisach model.

Initialization of hysteretic systems with accommodation:

- If the input range is bounded maximum amplitude is necessary. Orbits as a response to periodic inputs with an amplitude \( a_u < u_{\text{max}} \) do not guarantee convergence towards the same orbit. All those minor orbits are enclosed by the one with maximal amplitude.

- Multiple periods are required. Due to accommodation effects convergence towards a periodic response is exponential. The number of periods depends on the required accuracy of the procedure.

7.4.4 Quasi-static vs. the Time Scale

As discussed with the explanation of the quasi-static excitation it was mentioned that experiments are required for validation. The presented experiment should be repeated at lower rates in order to check if the response of the orbits is equal. In chapter 3 it was shown, by means of an analytical example and with the help of finite element simulations of the lens geometry, that because of the spatially distributed nature of the system the lumped transfer functions are infinite dimensional. For systems that can be accurately described using a finite number of pole and zero locations, distinct frequency regions in which transients express themselves can be indicated. For systems with a fractional nature, the transient behavior is significant over a time range of many decades (e.g. \( \mu s \) to hours). Therefore, it is not trivial to indicate a quasi-static region.

In the literature describing accommodation effects the relation to time domain effects is mentioned. Terminology used is e.g. creep, aftereffect and viscosity. The more accurate that the effects are described, the more the relation between spatially distributed dynamical effects and hysteresis effects at different time scales mix up. Accommodation is defined as a rate-independent effect but it is still caused because electromagnetic fields are changing over time. To take this into account the memory of the hysteresis effects should be coupled with the memory for dynamical effects. In the next section a conceptual model is introduced for a system that shows both dynamical and hysteretic effects. The model contains only one type of memory for both effects.

7.5 Coupled Hysteresis and Dynamics

The proposed coupling of hysteresis effects and dynamical effects using phenomenological models is often an interconnection of a rate-independent hysteresis model and linear dynamical systems. Fig. 7.19 shows three possible forms, all combinations of these are also valid. If the nonlinearity is in between two linear dynamical systems it is often referred to as a sandwich system. These model
structures are capable of providing simulation errors (sum of squared difference between model output and measurement) in the order of $1$ to $10\%$. However, in the context of initialization the goal is different.

Consider the case where the rate-independent hysteresis model is in front of the dynamics. For this situation the previously presented analyses for rate-independent models is still completely valid. The dynamics only influence the output signal, but have no influence on the memory of hysteresis. For the case in which the dynamical system precedes hysteresis, the initializing input can be derived from the response of the dynamical system to a periodic input. Again a rate-independent analysis will hold.

Under study is the influence of the input excitation on the memory state of hysteresis. In these interconnected structures, based on rate-independent hysteresis models, the memory of dynamics and hysteresis is still isolated. This can be overcome by the introduction of nonlinear feedback models.

### 7.5.1 Nonlinear Feedback Models

The class of nonlinear feedback models of hysteresis is extensively discussed in [58]. Fig 7.20 shows the most general description. Here $G(s)$ is a linear time-invariant multiple input multiple output (MIMO) dynamical system and $\Lambda$ represents a static nonlinearity. The properties of the dynamics and the specific nonlinearity $\Lambda$ determine whether the overall system relating $y(t)$ to $u(t)$ for each specific constant input has either one unique equilibrium (no hysteresis), multiple isolated equilibria (bifurcation type hysteresis) or a continuous set of equilibria (transversal type hysteresis).

The memory of the nonlinear feedback models is found in the states of the linear dynamical system $G(s)$. Under specific conditions on both the dynamics and the static nonlinearity, transversal type hysteresis can be described using feedback interconnections of two basic building blocks (linear systems and static nonlinearities). Hysteresis arises from the nonlinearity and the result is that the system has both hysteresis and dynamics of which the memory (state) is the same and, therefore, also shared.

This model structure, which enables to work in the state space modeling framework, has the desired coupling between hysteresis and dynamics and next to that, the required framework for stability analysis is well developed. In this case the interconnection of linear dynamics with a static nonlinearity is under question, which is less complicated than stability analysis of dynamical systems interconnected with rate-independent hysteresis models. An example of a framework for analysis is...
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Figure 7.20: Block diagram of a nonlinear feedback model. $G(s)$ is a linear system and $\Lambda$ a static nonlinearity.

found in [60].

7.5.2 Nonlinear Feedback Model with Dead-zone

Transversal type hysteresis is obtained by a specific choice of the properties of both the linear dynamics and the static nonlinearity. In previous chapters it was argued how to interpret the initial conditions and the memory of a system with hysteresis. Using the framework of nonlinear feedback models, initial conditions are the values of the states of the dynamics at $t = t_0$. Hysteresis is the effect of multiple steady state equilibria for one single input resulting from the combination of the model’s interconnection structure, the specific static nonlinearity and the linear dynamics.

Figure 7.21: Block diagram of a nonlinear feedback model. Left. The specific implementation of the first order nonlinear feedback model with dead-zone. Right. When the dead-zone width $\Delta$ is set to zero, a first order linear low pass system $H(s) = \beta \alpha / (s + \alpha)$ is obtained.

With the specific choice of a dead-zone representing the nonlinearity $\Lambda$ and $G(s) = \alpha \frac{1}{s}$ the nonlinear feedback model with dead-zone results in a system with transversal type hysteresis. A dead-zone is defined as:

$$w = d_2 \Delta(v) = \max(\min(0, v + \Delta), v - \Delta).$$

(7.14)

Here, $2\Delta \geq 0$ is the width of the dead-zone. The most simple case which the linear dynamical system $G(s)$ is represented by a gain $\alpha$ interconnected with an integrator $\frac{1}{s}$ is considered. As illustrated in Fig. 7.21 a zero dead-zone width results in a linear system $H(s) = \beta \alpha / (s + \alpha)$. This is a first order low pass filter with a pole location at $f = \alpha / (2\pi)$ and DC-gain $\beta$, (7.15a), (7.15b). The specific structure of a feedback interconnection of an integrator with a dead-zone approaches the behavior of a single play operator for quasi-static excitation, [27]. The frequency indicating the upper bound on the applied frequencies is now found in the time constant $\alpha_{ij}$. For quasi-static excitation the nonlinear feedback model with dead-zone approximates a (multi-) play operator. If $f < \alpha_{ij} / (2\pi 10)$ the behavior of the nonlinear feedback model with dead-zone can be considered equal to the behavior of a play operator.
The factor 10 is a rough rule of thumb. Therefore, $\Delta$ for the play operator and for the nonlinear feedback model are described by the same symbol. The memory of the play operator is now captured by the memory of the dynamical system $G(s)$.

Each section indicated with index $(i,j)$ has its own dead-zone width $2\Delta_i$ and its own time constant $\alpha_j$. The gain of each section is controlled by $\beta_{ij}$. The summation of all parallel sections with a normalization for the number of elements $1/(n_\Delta n_\alpha)$ results in the output $z(t)$. An conceptual representation of this structure is shown in Fig. 7.22. When all $\Delta$'s are set to zero, a linear model in the following form is obtained:

$$H(s) = \frac{\beta_1}{s + \alpha_1} + \frac{\beta_2}{s + \alpha_2} + \cdots + \frac{\beta_{n_\alpha}}{s + \alpha_{n_\alpha}}$$

The time constants $\alpha$ have to be chosen such that the time domain behavior fits the application under study. With the example of the coil with ferromagnetic conducting core the lumped transfer functions for the relation of the magnetic flux density at different locations on the core radius is available. As discussed in chapter 3 this is an infinite dimensional transfer function.
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Figure 7.23: Convergence of the first order nonlinear feedback model to a closed orbit. Four different dead-zone widths are used: $\Delta \in [0, 0.1, 0.45, 0.7]$. The initial condition for all is set to 0. The response is shown for sine-wave excitation with three different frequencies. Note that the case for $\Delta = 0$ represents a linear first-order low pass system.

Fig. 7.23 and Fig. 7.24 show the convergence of single sections of the model to an orbit starting from initial condition $x_0 = 0$ for three different sine wave frequencies. Fig. 7.23 uses an equal time constant and different $\Delta$ and Fig. 7.24 an equal $\Delta$ with different $\alpha$. Due to the low pass nature of the system, the area enclosed by the orbit displayed in the phase plane gets smaller with higher frequency. For quasi-static signals the behavior of a single play operator is approached.

7.5.3 Initialization Requirements

The results presented here, concerning initialization of the nonlinear feedback model with dead-zone, are an extension of the results which were already presented in [92]. All models presented so far agree on two points: Optimal initialization trajectories should have maximum amplitude, and the initialization sequence should contain both the maximum and the minimum. This also holds for the proposed nonlinear feedback structure, since in [92] it was shown that in the quasi static limit it has a similar behavior as the multiplay hysteresis model. This implies that it can be initialized using one single period of a square wave if the period time is so large that the steady state is reached for both the maximum and minimum input. The next question is what the optimal strategy is when only a limited amount of time is available, such that steady state cannot be reached.

In Fig. 7.25 and 7.26 feed forward initialization is shown for a model that contains $n_{\alpha}n_{\Delta} = 40$ parallel sections. Since, each section has model order 1 the model order is 40. However, if only the linear component is required (no hysteresis) all $\Delta$’s are zero and a model order 4 (the number of $\alpha$’s) is sufficient. In a similar way, quasi-static simulation can be obtained with a model order 10, all the
different $\Delta$’s where the 10 $\alpha$’s are set to a very high number (such that the cut-off frequency is about 10 times higher than the maximum frequency in the applied excitation). The time constants are chosen $\alpha \in 2\pi[4, 10, 30, 50] \text{rad/s}$. All gains $\beta$ are set to one and the 4 $\Delta$’s are equally distributed in the range 0 to 1.

The initialization time for both simulations is set to 0.5s. In combination with the chosen time constants steady state cannot be reached. The error level after initialization is defined as the difference between the first output trajectory and all others $|z_1 - z_i|$. In the situation of Fig. 7.25 a single period is used 2Hz and for the situation of Fig. 7.26 5 periods at 10Hz. A lower error level is obtained with multiple periods, as indicated in subplot $d$) of both figures. Due to the distribution of the time constants, the higher frequency resets the fastest first order sections whereas the slower require more time to converge completely. When only a single period is used, the fast sections are “waiting” as can be observed from the error plot. The results of all different sections are added. Since only limited time is available more effect can be obtained by making sure that all the fasts sections have converged. This is done using multiple periods at a higher frequency.

It turns out that this model can be initialized using arbitrary high frequencies: Although that the output as a response to a high frequency input is attenuated, the memory can still be reset. If the applied amplitude is larger than the dead-zone width there is a nonzero signal applied to the integrator. The larger the signal, the more power is applied to the system and the faster the integrator will reach a certain level. If the input is limited one just has to wait until the integrator has converged to the maximum. Initialization in this context is nothing more than forcing the system to an equal energy level. It turns out that the maximum is much easier to reach than the zero state. It is thus easier to
Chapter 7. Feed Forward Initialization of Hysteretic Systems

Figure 7.25: Initialization of the nonlinear feedback model with dead-zone consisting of 40 parallel first order sections with different dead-zone width and different time constants. A single period of a square wave at \( 2Hz \) is applied resulting in an initialization time of \( 0.5s \). \( \alpha \in 2\pi[4, 10, 30, 50] \text{rad/s}, \) \( \beta = 1, \Delta \) equally distributed between 0 and 1. The simulation is carried out for 15 different initial conditions.

Figure 7.26: Similar to Fig. 7.25 except now 5 periods of a square wave at \( 10Hz \) are applied.
completely fill the buffers then to empty them. Emptying requires a demagnetization strategy, which as explained in the beginning of this chapter, requires a oscillating signal with decreasing envelope.

The fact that high frequency excitation can reset the memory of the model does not comply with what happens in the electromagnetic lens. In chapter 3 it was shown that a high frequency electromagnetic field cannot "reach" all material but only that close to the coil. This effect was caused by eddy current and mentioned as the skin effect. Although the model consisting of a parallel connection of multiple nonlinear feedback models with dead-zone has coupled hysteresis and dynamics, the model structure does not correspond to the spatially distributed problem of the lens. Note that the low pass behavior can be included by placing several of the defined parallel models in series. Although such a phenomenological modeling approach can be helpful to obtain insight and understanding further investigation is required to take into account the requirements for a lumped approximation of spatially distributed hysteresis and dynamics.

7.6 Conclusions

In this chapter several phenomenological models of hysteresis have been introduced, analyzed and discussed. On the basis of simulations and observations of the obtained trajectories, the significant parameters for feed forward initialization were derived. Using periodic input trajectories, the influence of previous excitation can be wiped out. A proper model that represents the electromagnetic lens behavior for quasi-static excitation was not found. The rate-independent Duhem model has so-called local memory and as a result will converge to the anhysteretic curve for periodic inputs with arbitrary small amplitude. The memory of the electromagnetic lens cannot be reset using this small signal excitation. The Preisach class hysteresis models have the deletion property. This property implies that a full period with maximum amplitude is required to initialize the system. Quasi-static excitation applied to the lens setup shows that the lens needs multiple periods for the output to converge to a periodic trajectory. In magnetism literature this effect is called accommodation. Analysis of systems with accommodation for quasi-static excitation does show that multiple periods with maximum amplitude are required to initialize hysteretic systems with accommodation.

When modeling dynamical systems in the state space framework, the memory of the system is represented by the state vector of the model. It has been shown that hysteresis can be modeled using this framework. A linear dynamical system interconnected with a static nonlinearity can result in a system with a multi-valued steady state equilibrium map that is characteristic for systems with hysteresis. In this approach hysteresis and dynamics share the same memory operator. Now the parameters of the nonlinearities can be tuned to fit the quasi-static hysteresis behavior and the time constants can be tuned to fit the desired time domain response. Analysis of possible periodic initialization trajectories for single elements of the nonlinear feedback model with dead-zone shows that the state of the model can be reset independently of the frequency of the excitation. This observation does not comply with the eddy current and skin effects discussed in chapter 3. Further investigation is required to take into account the requirements for a lumped approximation of spatially distributed hysteresis and dynamics.

In the next chapter different initialization parameter combinations are evaluated using the electromagnetic lens setup. Although no hard conclusions can be drawn from the hysteresis models discussed in this chapter, the following observations are taken into account for the choice of the parameter ranges:

- \( n_i \), number of periods: Depending on the memory-structure of the system, at least 1 full period is required. Only switching to a single extremum (0.5 period) has been shown to be insufficient for models that show the deletion property. In Duhem models the error decreases exponentially with the number of periods. The electromagnetic lens shows accommodation which can be modeled by combining both model classes. Also the parallel connection of several first order nonlinear feedback models with dead-zone showed a faster convergence with multiple periods than with a single period (equal initialization time). It is thus expected that multiple periods are
required. The number of periods depends on the required accuracy of the application.

- **$T_i$, period time**: The analysis of nonlinear feedback models showed that the effective time that the maximum input is applied should be long enough to reset the state of the buffers. However, this can be achieved by multiple periods and a small $T_i$ or a single period and a large $T_i$. Simulations indicate that when the initialization time is limited, multiple periods are beneficial. The minimum $T_i$ (maximum frequency) is, however, limited by the low pass behavior of this spatial problem: The input does not reach all iron equally but depends on magnetic diffusion phenomena. High frequency initialization generates eddy currents. Due to the resulting skin depth only a limited part of the lens iron is effectively initialized.

- **$o_i$, $a_i$, offset and amplitude**: Maximum amplitude results in the fastest reset of the state. From the analysis of hysteresis models, the peak-to-peak range of the initialization profile should at least be equal to the range of the application.

- **$\Phi$, waveform**: Analysis of the nonlinear feedback model shows that sufficient time with both the maximum and minimum input amplitudes is required to equalize the level of the states in the model. A square wave results in more power than a sine wave with equal frequency. Therefore, square waves are preferred.
Chapter 8

Feed Forward Initialization Performance

In this chapter the performance of controllers based on feed forward initialization is evaluated and compared with the controller performance without initialization as obtained in chapter 6. Parameters of the initialization trajectories are the period time (frequency) and the number of periods of a square wave. Choice for this signal type was justified in chapter 7 on the basis of hysteresis model properties, observations from lens behavior and observations on the spatial-temporal nature of the electromagnetic lens. First the controller performance in terms of transition time and transition error for one single parameter setting is evaluated. In the second part of this chapter the performance of initialization trajectories that last from 0.05s up till 0.5s is compared. The same initiation duration can be obtained with multiple combinations of the number of periods versus the frequency of the square wave initialization trajectory. With each initialization time 4 different combinations are tested such that conclusions can be drawn about whether to prefer a higher frequency and more periods or if a single period is enough, as discussed in chapter 7.

8.1 Initialization Controller Structure

Fig. 8.1 presents an overview of the four controller configurations under study: Two initialization controllers $C_{ffi}$ and $C_{fbi}$ are added to the previously discussed feed forward $C_{ff}$ and feed forward combined with feedback $C_{fb}$ scheme. The initialization signal in the feed forward setting is the lens current. Initialization in the feedback implementation is obtained by switching the reference signal $R_{B2}$ instead of directly using the lens input current. After that initialization has finished the controller switches to a constant reference determined by the static mapping $\zeta$. 

119
Chapter 8. Feed Forward Initialization Performance

\[ C_{ff}: \text{Feed Forward} \]
\[ R_B \rightarrow \zeta_{ff} \rightarrow I \rightarrow L \rightarrow B \]

\[ C_{fb}: \text{Feed Forward combined with Feedback} \]
\[ R_B \rightarrow \zeta_{fb} \rightarrow R_{B2} \rightarrow C_{f_{b2}} \rightarrow I \rightarrow L \rightarrow B \]

\[ C_{ffi}: \text{Feed Forward Initialization} \]
\[ R_B \rightarrow dR_B/dt \ \downarrow \ \text{enbl} \rightarrow R_i \rightarrow \zeta_{ffi} \rightarrow I \rightarrow L \rightarrow B \]

\[ C_{fbi}: \text{Feed Forward Initialization combined with Feedback} \]
\[ R_B \rightarrow dR_B/dt \ \downarrow \ \text{enbl} \rightarrow R_i \rightarrow \zeta_{fbi} \rightarrow R_{B2} \rightarrow C_{f_{b2}} \rightarrow I \rightarrow L \rightarrow B \]

Figure 8.1: Overview of the different controller structures.
An initialization trajectory $u_i(t, \theta)$ is parameterized by $\theta$ and has duration $\tau_i$. The parameters that define the maximum variation $\psi(t)$ over a time window $\Gamma$, the allowed variation in an operating point $\epsilon_{tr}$, the duration of a transition $\tau_{tr}$, and the transition error $e_{tr}$ are defined in chapter 3, section 3.4. The modes $M$ of the switching procedure are illustrated in Fig. 8.2. The concept of set point regulation based on feed forward initialization is defined as:

1. **Initial condition before switching**
   
   \[ M_1 : \lim_{t \to t_0} \]
   
   \[ r(t) = r_0 \]
   
   \[ u(t) = u_0 \]

2. **Feed forward initialization**
   
   \[ M_2 : t_0 \leq t < t_0 + \tau_i \]
   
   At $t = t_0$ the reference set point changes. The controller detects the change in $dr/dt$ and applies the initialization trajectory $u(t) = u_i(t, \theta)$ to the system. Initialization takes $\tau_i[s]$.
   
   \[ r(t) = r_1 \]
   
   \[ u(t) = u_i(t - t_0, \theta) \]

3. **Convergence**
   
   \[ M_3 : t_0 + \tau_i \leq t < t_0 + \tau_i + \hat{\tau}_{tr} \]
   
   The initialization is finished and the controller switches to the set point dependent constant input value $u_{c1}$. In this implementation this value is determined by the mapping $u_{c} = \zeta(r)$. The output starts to convergence towards the steady state.
   
   \[ r(t) = r_1 \]
   
   \[ u_{c1} = \zeta(r_1) \]

4. **Transition finished**
   
   \[ M_4 : t_0 + \tau_{tr} \leq t \]
   
   If the maximum variation $\psi(t)$ of the output over a time window $\Gamma$ is less than or equal to $\epsilon_{tr}$ then the transition is finished.

   \[ \psi(t) = \max_{\nu_2} y(t + \nu_1) - \min_{\nu_2} y(t + \nu_2), \]
   
   \[ 0 \leq \nu_1 \leq \Gamma, \quad 0 \leq \nu_2 \leq \Gamma, \quad t_0 + \tau_i \leq t \leq t_c - \Gamma \]
   
   \[ t_{tr} = \min_{t} \psi(t) \leq \epsilon_{tr} \]
   
   \[ \tau_{tr} = t_0 - t_{tr} \]
   
   \[ y_{tr} = \frac{1}{\Gamma} \int_{t_0 + \tau_{tr} + \Gamma}^{t_0 + \tau_{tr}} y(t) dt \]
   
   \[ e_{tr} = r_1 - y_{tr} \]
Chapter 8. Feed Forward Initialization Performance

Figure 8.2: Schematic representation of feed forward initialization. The applied initialization trajectory $u_i(t, \theta)$ consists of 3 periods of a square wave and has a duration $\tau_i$. The transition is finished if the maximal variation $\psi(t)$ over a time window $\Gamma$ is less than or equal to $\epsilon_{tr}$. The total transition time $\tau_{tr}$ includes initialization. The transition error $e_{tr}$ is defined as the reference $r_i$ minus the obtained value $y_{tr}$.

8.2 Results of Feed Forward Initialization

As discussed in chapter 7 the initialization profile is defined as a square wave function:

$$\tau_i = n_i \cdot T_i = n_i/F_i, \quad 2n_i \in \mathbb{N}^+$$  \hfill (8.1a)

$$u_{T_i}(t) = \alpha_i + a_i \text{sgn}\left\{\sin\left(\frac{2\pi}{T_i} t\right)\right\}, \quad t_0 \leq t \leq \tau_i$$  \hfill (8.1b)

The parameter vector is denoted as $\theta = [\alpha_i, a_i, T_i, n_i]$, the offset, the peak to peak amplitude, the period time and the number of periods. The time involved with the initialization trajectory itself is then $\tau_i = n_i \cdot T_i$. The parameter $F_i = 1/T_i$ is introduced since it can be more convenient to use the initialization frequency instead of the initialization period time. The initialization parameters are fixed and chosen $n_i = 5, T_i = 0.1s, a_i = 2.2A, \alpha_i = 0$ which results in an initialization time of $\tau_i = 0.5s$. This choice serves as a starting point. Further on in this chapter different parameterizations are tested. The results for both initialization controllers are again based on 100 evaluations of a random set point change starting from random initial conditions for identification of the static function $\zeta(R)$ and 100 evaluations for validation. The procedure was presented in section 6.1.

8.2.1 Performance of the Feed Forward Initialization Controller

In each function evaluation the same initialization profile is applied, which with the chosen parameters $n_i = 5, F_i = 10Hz$ results in a duration of $\tau_i = 0.5s$. The results for the maximum variation $\psi(t)$ over $\Gamma = 1s$ is shown in Fig. 8.3. Due to the duration of initialization the signal $\psi(t)$ starts decreasing after 0.5s. The resulting maximum transition error in the test is significantly smaller than without
8.2. Results of Feed Forward Initialization

Figure 8.3: $C_{ffi}$, feed forward initialization. The total initialization time $\tau_i = 0.5s$ consisting of $n_i = 5$ periods at $F_i = 10Hz$. The maximal obtained transition time is indicated with a vertical dashed line. top.) $B(t)$ for 12 desired set point values starting from random initial conditions. middle.) Absolute error $|e(t)| = |R_B(t) - B(t)|$ on a logarithmic scale in $mT$ and as a percentage of the full range of 70$mT$. bottom.) the maximal variation $\psi(t)$ over a window $\Gamma = 1s$. 
Chapter 8. Feed Forward Initialization Performance

Figure 8.4: $C_{fbi}$, feed forward initialization combined with feedback using sensor $sB_2$. The total initialization time $\tau_i = 0.5s$ consisting of $n_i = 5$ periods at $F_i = 10$Hz. The maximal obtained transition time is indicated with a vertical dashed line. **top.)** $B(t)$ for 12 desired set point values starting from random initial conditions. **middle.)** Absolute error $|e(t)| = |R_B(t) - B(t)|$ on a logarithmic scale in $mT$ and as a percentage of the full range of $70mT$. **bottom.)** the maximal variation $\psi(t)$ over a window $\Gamma = 1s$. 

$\text{initialization and feedback sensor 2, max}(\tau_{tr}) = 0.562s$

$\text{absolute error, max}(|e|_{tr}) = 0.013mT = 0.019\%$ of $70mT$

$\psi(t)$: maximal variation of $B(t)$ within $\Gamma = 1s$

$\epsilon_{tr} = 0.012mT$
8.3. Sensitivity of the Transition Time on the Threshold initialization:

\[ \hat{\tau}_{tr} = 1.03 \text{s} \]
\[ \hat{e}_{tr} = 24\mu T \]
\[ \frac{24\mu T}{70\mu T} \cdot 100\% = 0.03\%. \]

The steady state equilibrium map, Fig. 8.5, shows that the multi-valued relation of hysteresis has no significant influence on the distribution of the error any more.

8.2.2 Performance of the Feed Forward Initialization combined with Feedback

In chapter 6 the combination of feedback controller and a feed forward part \( C_{fb} \) showed to be very fast (\( \hat{\tau}_{tr} = 47\text{ms} \)), but not so accurate (\( \hat{e}_{tr} = 0.65\mu T \)). From Fig. 8.4 it becomes clear that the time involved with initialization is dominating the overall transition time:

\[ \hat{\tau}_{tr} = 0.56 \text{s} \]
\[ \hat{e}_{tr} = 13\mu T \]
\[ \frac{13\mu T}{70\mu T} \cdot 100\% = 0.02\%. \]

The maximum transition time is 0.56s of which 0.5s are used for initialization and 60ms to converge towards the operating point. However, when a low transition error is the objective, the investment in time is worth it since the maximum error is only \( 2 \cdot 10^{-5} \) of the total range, Fig. 8.6.
Chapter 8. Feed Forward Initialization Performance

Figure 8.6: Steady state equilibrium map for feed forward initialization combined feedback using sensor $sB_2$. Left, the mapping of the reference versus the measured value and right the reference versus the absolute transition error.

8.3 Sensitivity of the Transition Time on the Threshold

The maximal transition time is highly sensitive to a change in the threshold $\epsilon_{tr}$. To provide insight into this dependence Fig. 8.7 shows $\tau_{tr}$ for different threshold values in the range for $\epsilon_{tr} \in [12 \mu T, 0.1 mT]$. Next to that Fig. 8.7 provides the experimentally obtained performance map: the maximal transition time versus the maximal transition error for the different threshold values. The large sensitivity to $\epsilon_{tr}$ in feed forward makes that the corresponding transition times vary from 50ms to over a second. With the help of feedback control based on sensor $sB_2$ this problem is solved. The performance map clearly illustrates that initialization significantly improves the performance in terms of transition error. However, the price is paid in an increase of the transition time. The next step is to investigate the sensitivity of the performance to variation of the initialization parameters.

8.4 Optimal Initialization Trajectories

In the previous section is has been shown that feed forward initialization can reduce the error over 100 times when compared to feed forward without initialization. However, the specific initialization profile used takes 0.5s which is disadvantageous for the transition time. In this section, experiments are carried out in which different parameterizations are tested. With the results it is possible to draw conclusions on the bound of the maximum initialization frequency, and whether it is better to use more periods and a high frequency or less periods and a low frequency.

To obtain an even higher accuracy than with the procedure defined in chapter 6 that was used to compare the 4 controller strategies, instead of 12 set point values, only one set point is evaluated from 100 different initial conditions. Using this slightly different test it is not necessary to run an extra initialization test to estimate the parameters of the static correction $\zeta$. This makes it possible to evaluate the 17 parameter combinations on a single working day. Since only a single set point is
8.4. Optimal Initialization Trajectories

Figure 8.7: *left.* The maximal transition time as a function of a range of different threshold values $\epsilon_{tr}$. *right.* Experimentally obtained performance map of the transition error $e_{tr}$ versus the transition time $\tau_{tr}$ for a range of different threshold values of $\epsilon_{tr}$. Four different controller structures are evaluated. This figure is the experimental realization of the conceptual performance map as presented in chapter 3, Fig. 3.15.

| $\tau_{tr} [s]$ | $|e_{tr}| [%]$ | $|e_{tr}| [mT]$ |
|-----------------|-------------|--------------|
| 0.25            | 0.001%      | 0.007        |
| 0.5             | 0.01%       | 0.07         |
| 0.75            | 0.1%        | 0.7          |
| 1               | 1%          | 7            |
| 1.25            | 10%         | 70           |

This table shows the performance transition time and error for different control structures.

where $i$ is the index of the number of function evaluations. As a reference also the feed forward situation without initialization is tested in this way.

The optimal parameters for initialization trajectories are those that represent the non-inferior solutions for the multi-objective criterion which consists of transition time minimization and transition error minimization. For non-inferior solutions, an improvement in one objective always requires a degradation of at least one of the other objectives. In this case this implies that given a range of maximum transition errors (e.g. $e_{tr} \in [0.01\%, 1\%]$) only those parameters that result in the minimum possible transition time corresponding to each error value in the set are of interest. Therefore, an inferior solution is a parameter combination of no value, since improvements can be obtained in all objectives. The non-inferior solutions is also called Pareto optima, and the goal here is to estimate the Pareto front for initialization sequences. The optimal initialization parameters are thus in a set of non-inferior values which provides the least time consuming initialization trajectory with a defined
maximum transition error. The feasible parameter space $\Theta_i$ is defined as $\theta \in \Theta_i$:

$$
\Theta = \left\{ \begin{array}{l}
o_i : \hat{I}_i \leq a_0 \leq \hat{I}_i \\
a_i : 0 \leq a_1 \leq \hat{I}_i - \hat{I}_i \\
n_i \in [0, 0.5, 1, 1.5, \ldots, 10] \\
T_i \in [10^{-3}, 10] \text{ s} \Rightarrow F_i \in [500, 1] \text{ Hz}
\end{array} \right. 
$$

subject to:

$$
o_i + a_i \leq \hat{I}_i, \\
o_i - a_i \geq \hat{I}_i, \\
n_i \cdot T_i \leq \hat{\tau}_i
$$

Here $\hat{I}_i$ denotes the maximum current value for initialization. The current amplitude range used for set point regulation itself can be smaller than the current amplitude range used for initialization.

Since it takes about 40 min to evaluate one parameter setting via the procedure defined in section 8.1 only 17 parameter combinations are evaluated. Fig. 8.8 shows the initialization time $\tau_i$ versus the obtained set point deviation $\Delta B$. Four different initialization times are defined next to the situation without initialization: $\tau_i \in [0, 0.05, 0.1, 0.2, 0.5] \text{ s}$. Each initialization time is realized with 4 different combinations of the number of periods $n_i \in [0.5, 1, 2, 5]$ periods and the initialization frequency $F_i$. 

Figure 8.8: Performance map of the initialization time $\tau_i$ versus the deviation $\Delta B$ for one single set point from 100 different initial conditions. Next to the situation without initialization, 16 different parameter combinations are tested. Four different initialization times are defined: $\tau_i \in [0, 0.05, 0.1, 0.2, 0.5] \text{ s}$ in combination with $n_i \in [0.5, 1, 2, 5]$ periods. The resulting initialization frequency of the initializing square wave with an amplitude of $2.2 A$ ranges from 1 Hz to 100 Hz.
8.4. Optimal Initialization Trajectories

Figure 8.9: Response of feed forward control without initialization for a single set point starting from 100 different initial conditions. The set point change is applied at $t_0 = 0s$. top.) overview of the response $B(t)$ for 3s. middle.) Deviation $\Delta B$ from the average value. bottom.) zoomed version of initialization directly after switching.

max($\Delta B$) = 2.5 mT = 3.5% of 70 mT
Chapter 8. Feed Forward Initialization Performance

Figure 8.10: Response of feed forward initialization for a single set point starting from 100 different initial conditions. The initialization parameters are \( n_i = 0.5 \) period and \( F_i = 1 \text{Hz} \) resulting in \( \tau_i = 0.5 \text{s} \). The set point change is applied at \( t_0 = 0 \text{s}. \) top.) overview of the response \( B(t) \) for 3s. middle.) Deviation \( \Delta B \) from the average value. bottom.) zoomed version of of initialization directly after switching.
8.4. Optimal Initialization Trajectories

Figure 8.11: Response of feed forward initialization for a single set point starting from 100 different initial conditions. The initialization parameters are \( n_i = 5 \) period and \( F_i = 100\,\text{Hz} \) resulting in \( \tau_i = 0.05\,\text{s} \). The set point change is applied at \( t_0 = 0\,\text{s} \). 

- top.) overview of the response \( B(t) \) for \( 3\,\text{s} \).
- middle.) Deviation \( \Delta B \) from the average value.
- bottom.) zoomed version of initialization directly after switching.

\[ \tau_i = 0.05\,\text{s}; \quad F_i = 100\,\text{Hz}, \quad n_i = 5\,\text{periods} \]
Figure 8.12: Response of feed forward initialization for a single set point starting from 100 different initial conditions. The initialization parameters are $n_i = 5$ periods and $F_i = 10\text{Hz}$ resulting in $\tau_i = 0.5\text{s}$. The set point change is applied at $t_0 = 0\text{s}$. top.) overview of the response $B(t)$ for $3\text{s}$. middle.) Deviation $\Delta B$ from the average value. bottom.) zoomed version of the response directly after switching.
8.5. Conclusions and Recommendations

<table>
<thead>
<tr>
<th>Controller type</th>
<th>( \hat{\epsilon}_{tr} )</th>
<th>% of 70 mT</th>
<th>( x )</th>
<th>( \hat{\tau}_{tr} )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{ff} )</td>
<td>3.4 mT</td>
<td>4.8%</td>
<td>1x</td>
<td>0.48 s</td>
<td>1x</td>
</tr>
<tr>
<td>( C_{ff,i} )</td>
<td>24 µT</td>
<td>0.03%</td>
<td>141x</td>
<td>1.03 s</td>
<td>0.47x</td>
</tr>
<tr>
<td>( C_{fb} )</td>
<td>0.65 mT</td>
<td>0.93%</td>
<td>5x</td>
<td>47 ms</td>
<td>10x</td>
</tr>
<tr>
<td>( C_{fb,i} )</td>
<td>13 µT</td>
<td>0.02%</td>
<td>262x</td>
<td>0.56 s</td>
<td>0.86x</td>
</tr>
</tbody>
</table>

Table 8.1: Overview and comparison of the maximum transition time and maximum transition error for the four controllers evaluated in chapter 6 and this chapter. Improvement introduced by the controllers is related to the feed forward controller \( C_{ff} \) which is taken as a benchmark.

The amplitude of the initializing square wave is 2.2 A. There is no offset.

In the performance map, Fig. 8.8, the results with and without initialization are presented. On one hand there is the case without initialization which involves a maximum transition error of about 5% but does not involve extra time. On the other hand, there is the result of initialization with 5 periods at 10 Hz which requires 0.5 s but brings down the error below 0.01% of the total range of 70 mT. The results of the experiment clearly indicate that multiple periods are beneficial for the performance of initialization. Fig. 8.8 shows that initialization with 0.5 periods does decrease the maximum error more than five times when compared to the situation without initialization. However both \( n_i = 0.5 \) and \( n_i = 1 \) are inferior values since a smaller maximum error can be obtained in the same time by increasing both the frequency and the number of periods.

For 4 parameter combinations the response of \( B(t) \) for the 100 function evaluations is shown in Fig. 8.9 (no initialization), Fig. 8.10 (half a period at 1 Hz, \( \tau_i = 0.5 \) s), Fig. 8.11 (5 periods at 100 Hz, \( \tau_i = 0.05 \) s), and Fig. 8.12 (5 periods at 10 Hz, \( \tau_i = 0.5 \) s).

For initialization times of 0.2 and 0.5 s the performance is approximately equal. For \( \tau_i = 0.05 \) or 0.1 s the performance becomes worse. This is due to the fact that high frequency electromagnetic waves cannot penetrate deep enough into the lens yoke, as discussed in chapter 3. The frequency at which this skin effect (eddy currents) becomes dominant is located in between 15 Hz and 30 Hz. This conclusion is based on the obtained performance for \( \tau_i = 0.05 \) and 0.1 s: although that the difference in obtained error is very small, the performance of \( n_i = 3 \) periods at 30 Hz and 60 Hz is better than 5 periods at 50 or 100 Hz.

The performance seems to increase exponentially with the initialization time and exponentially with the number of periods. Only 16 parameter combinations are evaluated but from the experiments it can be concluded that the non-inferior initialization parameters can be found using:

1. **Choose the initialization time to** \( \tau_i = [0.2] s \).

   Larger initialization times do not result in a significant error improvement. For initialization times less than 0.2 s the performance of initialization will exponentially decrease.

2. **Choose the number of periods to** \( n_i = 5 \) **which in combination with the chosen initialization time provides the frequency** \( F_i = \tau_i / n_i \).

No experiments were carried out with \( n_i = 4 \). The performance is expected to be in between obtained results for \( n_i = 3 \) or 5 periods which are very close to each other. Since \( n_i = 5 \) performs better for \( F_i \leq 25 \) Hz and the difference for higher frequencies between 3 or 5 periods is insignificant it is proposed to set the number of initialization periods to 5.

8.5 Conclusions and Recommendations

The performance of the 4 controllers \( C_{ff} \) and \( C_{fb} \) in chapter 6 and \( C_{ff,i} \) and \( C_{fb,i} \) in this chapter, is evaluated on basis of the obtained maximal transition time \( \hat{\tau}_{tr} \) and maximal transition error \( \hat{\epsilon}_{tr} \). Table
Figure 8.13: Performance map of the transition time versus the transition error for the four different controller structures. The dashed lines represent the trade-off that can be made between speed and accuracy by choosing the initialization parameters.

8.1 and Fig. 8.13 show the comparison between the different controller structures. The comparison holds for a threshold value $\epsilon_{tr} = 12 \mu T$ on the maximum variation $\psi(t)$ where the time window $\Gamma$ is equal to one second. With a fixed initialization parameter setting $n_i = 5, F_i = 10Hz$, $\tau_i = 0.5s$ an error reduction of more than 100 times is obtained in feed forward and more than 250 times with the feedback scheme when compared with the situation of feed forward without initialization. The price being paid is an enlarged transition time which is in this case increased by $\tau_i = 0.5s$. Fig. 8.13 shows the performance map of the maximum transition time versus the maximum transition error.

The influence of the initialization parameters on the performance has been investigated using a separate test in which a single set point is reproduced starting from 100 different initial conditions. 17 parameter combinations have been evaluated. The initialization times are chosen as 50ms, 0.1, 0.2 and 0.5s. All cases are evaluated with 0.5, 1.3 and 5 periods. The performance of initialization increases exponentially with the initialization time and with the number of periods. The differences between 3 and 5 periods and between the initialization times of 0.2 and 0.5s are insignificantly small. The frequency at which the skin effects (eddy currents) become dominant and decrease the performance is found between 15 and 30Hz. As an engineering rule 5 periods are prescribed. An initialization time of 0.2s is sufficient for the specific lens under test. The influence of the initialization
parameters is represented by the dashed lines in Fig. 8.13. Evaluation of the feed forward initialization strategy shows that the transition error can be brought well below the upper bound on the transition error. The experiment presented in Fig 8.8 shows that an error reduction of about 50 times can already be obtained with 5 periods at $100Hz$ resulting in an initialization time of $50ms$. For both the feed forward controller scheme and the scheme with feedback control this investment in extra transition time brings the maximal transition error well below the critical bound of $1\%$ error.

The evaluation of the influence of the initialization parameters has been carried out using a test in which only a single set point was reproduced instead of 12 set points as was done with the controller comparison. Due to a protocol change more parameter combinations could be tested in a single day with a higher accuracy. Now that the results indicate that 5 periods are preferred and $0.2s$ is sufficient for initialization of the lens system, it is recommended to repeat the experiments using the original protocol with 12 set points such that their performance can be added to the performance graph of Fig. 8.13.
Conclusions and Recommendations

Different control strategies for fast and highly accurate switching of the operating point of electromagnetic lenses have been presented. It has been derived that the highest image quality in scanning electron microscopy applications can only be obtained if the error in the magnetic flux density is smaller than $10^{-5}$ times the full range of possible operating point values. A transition of the operating point is a two step procedure: First, the lens controller brings the magnetic flux density as fast as possible to a steady level close to the required value. Second, the level of defocus is further optimized using image based feedback techniques. The first step is topic of control. The control objective is to minimize both the transition error and the transition time. By means of first principle derivations and validation using microscopy experiments, it has been shown that the upper bound on the transition error made by the lens controller is $1\%$ of the full range. For larger errors, the overall system performance is no longer guaranteed.

A protocol for evaluation of controller performance has been implemented on a developed setup consisting of an electromagnetic lens extended with magnetic flux density sensors, a data acquisition system and a rapid prototyping system. Experimental validation of feed forward control showed that the transition error in feed forward is 5 times larger than the upper bound. This large error level is caused by ferromagnetic hysteresis effects present in the relation between the lens current and the magnetic flux density in the electron optic volume. The transition time, due to magnetization dynamics like eddy currents, has been estimated at 0.5s.

Feedback control based on magnetic flux density sensing is capable of decreasing the transition time 10 times from 0.5s down to 50ms. An unrestricted sensor position would also solve the problems caused by hysteresis. However, since the sensor may not be placed in or very close to the electron optic volume, it is placed at a certain distance in the lens gap. The relation between the magnetic flux density at the position of the sensor and the magnetic flux density at the position important for elec-

<table>
<thead>
<tr>
<th>Controller type</th>
<th>error $\hat{e}_{tr}$</th>
<th>%of 70 mT</th>
<th>time $\hat{\tau}_{tr}$</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{ff}$</td>
<td>3.4mT</td>
<td>4.8%</td>
<td>0.48s</td>
<td>1x</td>
</tr>
<tr>
<td>$C_{fI}$</td>
<td>24µT</td>
<td>0.03%</td>
<td>1.03s</td>
<td>0.47x</td>
</tr>
<tr>
<td>$C_{fb}$</td>
<td>0.65mT</td>
<td>0.93%</td>
<td>47ms</td>
<td>10x</td>
</tr>
<tr>
<td>$C_{fbi}$</td>
<td>13µT</td>
<td>0.02%</td>
<td>0.56s</td>
<td>0.86x</td>
</tr>
</tbody>
</table>

Table 9.1: Overview and comparison of the transition time and transition error for the four controller strategies. $C_{ff}$: feed forward no sensor, $C_{fI}$: feed forward initialization, $C_{fb}$ feedback with a constrained sensor position, $C_{fbi}$: initialization and feedback. The full range of operating point values equals 70mT which is set to 100%.
Figure 9.1: Experimentally obtained performance map of the transition error $e_{tr}$ versus the transition time $\tau_{tr}$. Different controller structures have been evaluated: Feed forward without any magnetic flux sensing, feedback control with an optimal sensor placement, feedback control with a constrained sensor position, and feed forward initialization applied to both the feed forward and feedback scheme. The dashed lines indicate the performance of feed forward initialization as a function of the initialization parameters. The arrows presented with feedback control with a constrained sensor position indicate that moving the sensor influences the performance in terms of transition error but that the low transition time is maintained.
tron optics has to be controlled in feed forward. In this situation the low transition error is no longer guaranteed. The measured maximum transition error of a developed controller scheme, using the sensor information at the constrained position, is 1% of the full range. The position of the sensor within the lens geometry combined with spatially distributed hysteresis effects cause the problem. Again the multi-valued steady state equilibrium map introduced by hysteresis limits the performance. However, the transition time using feedback control is not sensitive to the sensor position. The decreased level of 50ms is maintained with the restricted sensor position. The arrows in Fig. 9.1 indicate that moving the sensor has an influence on the maximum transition time but not on the maximum transition error.

To guarantee a transition error well below the critical level of 1% of the full range, feed forward initialization is introduced. A forced reset of the memory of the ferromagnetic material is obtained by applying a feed forward initialization trajectory consisting of a square wave at maximum amplitude. The transition error can be decreased over 250 times by combining feed forward initialization and the feedback scheme. The price that is paid is the duration of initialization. For the performance mentioned, initialization enlarges the transition time by 0.5s. In feed forward without any online sensing, the transition error can still be made 100 times smaller using feed forward initialization. Table 9.1 and Fig. 9.1 present an overview of the multi-objective controller performance.

The reasons why feed forward strategies can work are that the system is an open loop stable system, the influence of any external disturbances is insignificantly low and the input can be controlled with a high bandwidth, a large dynamic range and an extreme high accuracy. Feed forward initialization forces the state of the system to a reproducible value by means of multiple periods of an excitation with an amplitude covering the full range. The behavior of the electromagnetic lens has been compared to the behavior of hysteresis models. The requirements for initialization trajectories of the electromagnetic lens have been established by analyzing the requirements for the models that have similar behavior as the lens.

Comparison of the performance for different initialization parameter settings showed that an initialization time of 0.2s is sufficient. The dashed lines in Fig. 9.1 that connect the situation with and without initialization represent the performance as a function of the initialization parameters. Analysis showed that it is best to choose the maximum available initialization time and apply 5 periods of a square wave at maximum amplitude. For an initialization time less than 0.2s, the performance decreases exponentially as a result of a limited depth of penetration of the initializing electromagnetic field with higher frequencies.
Bibliography


Bibliography


Curriculum Vitae

Patrick van Bree was born on 12 July 1981 in Weert, The Netherlands. He received his M.Sc. degree in Electrical Engineering from Eindhoven University of Technology, The Netherlands in 2006. The topic of his Master thesis was *Online Prediction of Battery Behavior for Hybrid Electrical Vehicles*. During 2006 he continued research on this topic within the Control Systems group of the Department of Electrical Engineering. In February 2007 he started working on the PhD project *Control of Dynamics and Hysteresis in Electromagnetic Lenses*, the results of which are presented in this dissertation.