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Electromagnetic modelling of large complex 3-D structures with LEGO and the eigencurrent expansion method

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Introduction

Linear embedding via Green's operators (LEGO) is a computational method in which the multiple scattering between adjacent objects — forming a large composite structure — is determined through the interaction of simple-shaped building domains, whose electromagnetic (EM) behavior is accounted for by means of *scattering* operators. This method has been successfully demonstrated for 2-D electromagnetic band-gaps (EBG) and other structures [1], and for very simple 3-D configurations [2]. In this communication we briefly report on the full extension of LEGO to large complex 3-D structures, which may be EBG-based but may also include finite antenna arrays as well as frequency selective surfaces, to name but a few applications. In particular, we shall outline two modifications that were crucial for scaling up the LEGO method, namely, the introduction of a *total inverse scattering operator* S^{-1} and the *eigencurrent expansion method* (EEM) [3].

In the LEGO concept (Fig. 1), we tackle the numerical solution to the EM problem by first characterizing an object or a set of objects — located within a bounded domain $\mathcal{D}_1$ — using a scattering operator S_{11} , which relates *equivalent* incident currents q_1^i (reproducing the incident field inside $\mathcal{D}_1$) to *equivalent* scattered currents q_1^s (radiating the scattered field outside $\mathcal{D}_1$). S_{11} is an integral operator which can be obtained upon posing proper boundary integral equations on $\partial\mathcal{D}_1$ and the object surface. Subsequent application of the Method of Moments (MoM) — in conjunction with Rao-Wilton-Glisson (RWG) basis functions to expand the currents — yields $[S_{11}]$, the matrix (algebraic) counterpart of S_{11} . The next step, in principle, consists of combining the scattering operators of N_D domains to obtain the (larger) operator S of the whole structure: we refer to this procedure as *embedding* [1]. To be specific, starting from, e.g., S_{11} and S_{22} , a new operator, twice as big, may be built: then, we have to iterate this step as many times as necessary in order to characterize N_D domains as a whole. Clearly, once S is known, the scattered currents simply ensue, with evident notation, through $q^s = S q^i$, where q^i (q^s) contains the incident (scattered) currents of all domains (see Eq. (1) below).

Although the strategy above was indeed followed for 2-D problems [1], we recognize that it is hardly viable for 3-D ones. In fact, as the size of the new $[S]$ matrix doubles at each step, a naïve application of the embedding would soon drain the memory of most computers — which ultimately poses serious limitations to the number of domains that can be handled at the same time. In order to circumvent this drawback, we have combined LEGO with EEM applied to S^{-1} , instead of S .

Figure 1: Modified LEGO concept: multiple scattering between adjacent objects is described by 1) defining elementary domains described via scattering operators and 2) building a total inverse scattering operator S^{-1} which accounts for all the EM interactions.

The operator S^{-1} and the eigencurrent expansion method

Since computing and storing the total $[S]$ turns out to be impracticable for large structures, we first observe that its inverse, S^{-1} , may be written analytically in a formal fashion, in terms of S_{kk}^{-1} and *transfer* operators T_{kl} , $k, l = 1, \dots, N_D$ (Fig. 1), where T_{kl} acts on q_l^s on ∂D_l to yield q_k^i on ∂D_k . Thus, it appears more convenient to formulate the EM problem as an equation to be solved for q^s , namely, $S^{-1}q^s = q^i$, where

$$S^{-1} = \begin{bmatrix} S_{11}^{-1} & -T_{12} & \cdots & -T_{1N_D} \\ -T_{21} & S_{22}^{-1} & \cdots & -T_{2N_D} \\ \vdots & \vdots & \ddots & \vdots \\ -T_{N_D 1} & -T_{N_D 2} & \cdots & S_{N_D N_D}^{-1} \end{bmatrix}, \quad q^{s,i} = \begin{bmatrix} q_1^{s,i} \\ q_2^{s,i} \\ \vdots \\ q_{N_D}^{s,i} \end{bmatrix}, \quad q_k^{s,i} = \begin{bmatrix} J^{s,i} \\ M^{s,i} \end{bmatrix}. \quad (1)$$

To solve $S^{-1}q^s = q^i$, however, we do not employ the MoM directly, for $[S]^{-1}$ could neither be stored nor inverted, but rather we use the MoM combined with the EEM. The very idea behind the EEM is to expand $q^{s,i}$ on a set of basis functions which are “approximations” to the eigenfunctions of S^{-1} , say s_n^k , $n \in \mathbb{N}$.

To begin with, by applying the MoM with N_F RWG basis functions per domain ∂D_k , we determine the eigenvalues λ_{kp} and eigenvectors u_p^k , $p = 1, \dots, N_F$, of $[S_{kk}]$. We note in passing that u_p^k are dubbed *eigencurrents*, since they indeed exhibit the physical dimension of a current density, owing to the meaning of S_{kk} . Then, we form a basis $E = \{e_p^k\}$ for $q^{s,i}$ upon juxtaposing u_p^k : for instance, the element $e_p^k = [0, \dots, 0, u_p^k, 0, \dots, 0]^t$, i.e., it vanishes over all domain boundaries but the k -th. We still name $\{e_p^k\}$ eigencurrents, even though, apparently, they are *not* the *true* eigencurrents of S . Besides, we speculate that only few elements of E , namely, those germane to the larger eigenvalues of each $[S_{kk}]$, will depart considerably from the corresponding s_n^k : we say that these eigencurrents are *coupled*. Conversely, we expect most elements of E , viz., those relative to the higher order eigenvalues, to constitute increasingly better approximations to s_n^k . The latter may properly be termed *uncoupled*.

In light of these observations, it appears advantageous to adopt E to represent $q^{s,i}$, for we may stipulate, *prior* to filling $[S]^{-1}$, that not all of its entries are equally meaningful. More precisely, on the one hand the off-diagonal entries corresponding to the interaction between uncoupled or coupled/uncoupled eigencurrents must be relatively small and may be neglected altogether: in this instance, we are left with a diagonal inverse (partial) scattering operator $[S_{\text{unc}}]^{-1} = \text{diag}\{\lambda_{kp,\text{unc}}^{-1}\}$, the inversion of which is manifestly formal. On the other hand, the entries pertaining to the interaction of the coupled eigencurrents do matter and constitute a *reduced* inverse scattering operator $[S_{\text{red}}]^{-1}$, whose size is *far smaller* than that of the full $[S]^{-1}$, i.e., if we had used the original RWG functions distributed over all of the N_D domains. Therefore, computing q^s boils down to just filling and inverting $[S_{\text{red}}]^{-1}$ and eventually reverting to the pristine basis of RWG functions, also taking into account the uncoupled eigencurrents.

To give a clue as to the benefits gained in terms of memory, for a problem with N_D domains and N_F RWG functions per domain and $N_\lambda \ll N_F$ coupled eigencurrents, the size of the system to be actually stored and inverted shrinks down to $N_D N_\lambda \times N_D N_\lambda$ from a humongous $N_D N_F \times N_D N_F$. As a further advantage of the proposed approach, we cursorily point out that the construction of $[S]^{-1}$ relies on $[T_{kl}]$, the calculation of which involves just pairs of domains at a time (see Fig. 1). Since these calculations can obviously be carried out in parallel, it is seen that LEGO along with the inverse scattering operator intrinsically lends itself to parallelization. As a final remark, we emphasize that $[S]^{-1}$ is *never* built as such nor are its matrix elements $[S_{kk}]^{-1}$, for their eigencurrents and eigenvalues are computed through $[S_{kk}]$.

Validation and results

We have implemented LEGO and EEM in a numerical (parallel) code and to validate it we have considered, among others, two dielectric spheres (Fig. 2a) illuminated by a plane wave. Superimposed in Fig. 2c are the radar cross sections (RCS) obtained solving the EM problem directly (—, —) with a PMCHW equation ($2 \times 2 \times 300$ RWG functions) and applying the LEGO/EEM concept (•) with $N_F = 2 \times 900$ RWG functions on each cubic domain boundary and only $N_\lambda = 10 \ll N_F$ coupled eigencurrents: it is seen that the two results are practically undistinguishable.

As an example of application, for an open defect cavity, made of 36 finite-length cylindrical holes in a homogeneous dielectric background and arranged in a hexagonal lattice (Fig. 2b), we have computed the RCS in response to a plane wave impinging from top (Fig. 2d). The number of RWG functions over each hexagonal domain boundary is $N_F = 2 \times 756$, whence the total $[S]^{-1}$ would virtually be size 54432×54432 , but upon using the EEM with only $N_\lambda = 10$ coupled eigencurrents, we just handle $[S_{\text{red}}]^{-1}$ size 360×360 .

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Figure 2: LEGO validation and results: (a) two cubic domains, which embed dielectric spheres, and triangular mesh; (b) 36-hole open defect cavity and triangular mesh (embedding hexagonal domains omitted for the sake of brevity); (c) RCS of the two dielectric spheres, (•) LEGO, (—, — —) direct solution with PMCWH equation; (d) RCS of the open defect cavity.

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