Multilayer array antennas with integrated frequency selective surfaces conformal to a circular cylindrical surface

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Abstract—In this paper, we present the analysis of periodic arrays on cylindrical surfaces using open-ended waveguide radiators loaded with radomes and frequency selective surfaces (FSS). The multilayer structure can be used to obtain a filtering behavior by properly choosing the radomes and the size of the FSS apertures. The effect of the gradual cutoff properties of the cylindrical waves is also addressed and the design of a number of filtering structures including one or two FSS is presented.

Index Terms—Conformal arrays, Floquet’s theorem, frequency selective surfaces (FSS), gradual cutoff, radome.

I. INTRODUCTION

A variety of military systems employ multiple antenna apertures on a single platform such as a ship or an aircraft. In order to reduce cost and improve performance characteristics such as radar cross section (RCS) and impedance matching, it is desirable to combine multiple functions into a single aperture and/or address stringent electromagnetic interference (EMI) problems. Wide bandwidth, multipolarization phased arrays with frequency selectivity properties are needed to accomplish this goal. Another key issue of modern array systems is the use of structurally integrated antennas. In most of these cases, the antenna should fit on a platform whose dimensions and shape are dictated by aerodynamic and/or structural constraints. In addition to the evident structural benefits, the adoption of conformal antennas provides also a number of operational benefits. For example: elimination of moving parts, potential increase of the available aperture (providing narrower beam-width and possibly higher antenna gain), wider scan angles and reduced RCS (if the antenna follows low-RCS shapes). Furthermore, a new paradigm for designing modern multifunction structurally integrated array radars is the use of multilayer structures with integrated radomes and frequency selective surfaces (FSSs) [1], [2], like the example shown in Fig. 1. It is evident that in general merely placing an FSS in front of an antenna does not necessarily reduce the RCS of the structure, on the contrary the FSS should be properly placed and shaped around the antenna itself. In this work we present an efficient and accurate modeling approach for the study of periodic arrays of open-ended waveguides conformal to circular cylinders and integrated in a multilayer layout with conformal FSS panels. In this case, the use of the FSS is focused on the possibility of shaping (enlarging or reducing) the array bandwidth, while the analysis and improvement of the RCS will not be specifically addressed. For what concerns the radiating elements of the array, waveguide radiators may not always appear as the most obvious choice for lightweight, conformal, wide-band arrays, as microstrip patch antennas. Nevertheless, in recent years, technology has matured to the point where the realization of very compact and light conformal arrays, using open-ended waveguide radiators integrated with T/R modules, has become realistic and cost-effective [3]. In addition, waveguide radiators are known for their inherent wide band characteristics and they have the unique feature of high-pass filtering behavior, due to the cutoff frequencies of the waveguide modes. Furthermore, they have very well predictable characteristics, good element impedance matching over a large bandwidth, and it is still possible to have small dimensions employing a proper dielectric filling.

The analysis of conformal cylindrical arrays was initially developed in the early seventies, both for axial slits [4], [5] and open-ended waveguides [6], [7]. In the same period, Hessel and Sureau [8] considered, for these structures, the effect of a conformal dielectric sheet cover. More recently, both accurate and approximate approaches have been used to study the effect of various radome configurations [9]. In [10], the authors studied the effect of a circular cylindrical radome consisting of an array of thin metal strips parallel to the axis (metal grating) or, similarly, of alternating metal strips and dielectric shells forming a cylindrical surface (metal-dielectric grating). But none of the mentioned works considered the waveguide array, radomes and metal gratings at the same time. In this paper, we present an efficient full-wave approach for the analysis of multilayer periodic cylindrical arrays of open-ended waveguides, where an arbitrary number of dielectric layers and thick metal gratings can be included in the analysis, as well as any filtering structure or tuning element inside the waveguides. The developed tool gives the possibility to design the array antenna as a whole, taking full advantage of the integrated structure and giving to the designer a number of different options and different degrees of freedom in the design process. An efficient approach, based on a multi-mode equivalent network (MEN) formulation, for the analysis of planar multilayer radiating structures was proposed for the first time in [11]. This approach was originally developed for
the study of waveguide cavity based filtering structures [12]. In [13], [14], we applied the MEN formulation, with a new integral equation approach [15], to the study of cylindrical arrays of open-ended waveguides. In this paper, we fully exploit the modularity and generality of the MEN formulation, to analyze and design cylindrical multilayer periodic arrays, addressing specific design problems in order to enhance the filtering performances of the structure.

II. UNIT CELL APPROACH AND MULTIMODE EQUIVALENT NETWORK FORMULATION APPLIED TO CYLINDRICAL ARRAYS

The structure under study is reported in Fig. 1. It consists of an array of rectangular open-ended waveguides symmetrically developed along an infinite metallic circular cylinder. Dielectric radomes and/or frequency selective surfaces (FSS) can be put in front of the apertures, in order to enhance some characteristics of the array, such as return loss, radiation pattern. The unit cell approach [16] can be applied to the proposed structure under the hypothesis that all the apertures are placed symmetrically in both directions $z$ and $\varphi$ and are fed with the same amplitude, but with a progressive phase shift between two successive apertures. This assumption implies that radomes and/or FSS placed in front of the array must maintain the periodicity of the structure. Within each unit cell, as illustrated in a three-dimensional (3-D) view in Fig. 2, we can identify the exciting rectangular waveguide, the radial phase shift wall waveguides (RPSWW), representing dielectric radomes or free space, and the metallic radial waveguides, representing the apertures in the screen of
the FSSs, as shown in Fig. 3. Assuming this representation, the problem is reduced to the analysis of a cascade of transitions between adjacent waveguides. For example, referring to Fig. 2, we can recognize:

- the transition rectangular waveguide—RPSWW followed by a RPSWW of length $L_1$ loaded with dielectric $\varepsilon_{r1}$, representing the first radome;
- the FSS of thickness $t_1$ (RPSWW-metallic radial waveguide transition + metallic radial waveguide of length $t_1$ + metallic radial waveguide-RPSWW transition loaded with dielectric $\varepsilon_{r2}$);
- • • •
- a RPSWW of length $L_2$ loaded with dielectric $\varepsilon_{r3}$, representing the third radome, followed by a transition with the RPSWW representing the free space ($\varepsilon_r = 1$).

In order to analyze these transitions, we need the complete spectrum of each guiding structure. These spectra can be found in canonical literature [17]. Nevertheless, it is necessary to investigate in some details the propagation characteristics of the RPSWW and of the metallic radial waveguide, in order to discuss the results which will be shown in the next section.

In general, in a radial waveguide, the electromagnetic field cannot be represented in terms of transverse (to $\rho$) vector modes. The transverse field representation must consequently be effected on a scalar basis as a superposition of $TE_\varphi$ and $TM_\varphi$ modes w.r.t. $z$ axis [17]

$$E_\varphi(\rho, \varphi, z) = \sum_{i=1}^{\infty} V_i(\rho)e_{\varphi i}(\varphi, z)$$

(1)

$$\rho E_\varphi(\rho, \varphi, z) = \sum_{i=1}^{\infty} V_i(\rho)e_{\varphi i}(\varphi, z)$$

(2)

$$H_\varphi(\rho, \varphi, z) = \sum_{i=1}^{\infty} I_i(\rho)h_{\varphi i}(\varphi, z)$$

(3)

$$\rho H_\varphi(\rho, \varphi, z) = \sum_{i=1}^{\infty} I_i(\rho)h_{\varphi i}(\varphi, z).$$

(4)

The index $i$ refers to a combination of transverse indexes and $V_i(\rho)$, $I_i(\rho)$ the voltage and current amplitudes of the modal expansion. $e_{\varphi i}$, $h_{\varphi i}$, $\rho e_{\varphi i}$, $\rho h_{\varphi i}$ represent the transverse modal components of the RPSWW or the metallic radial waveguide. By introducing (1)-(4) in Maxwell equations and applying proper boundary conditions, we can write the modal components as follows:

$$e_{\varphi i} = N_{e_{\varphi i}} \Psi_i(\varphi, z)$$

(5)

$$h_{\varphi i} = N_{h_{\varphi i}} \Psi_i(\varphi, z)$$

(6)

$$e_{\varphi i} = N_{e_{\varphi i}} \Phi_i(\varphi, z)$$

(7)

$$h_{\varphi i} = N_{h_{\varphi i}} \Phi_i(\varphi, z).$$

(8)

For the RPSWW, (5)-(8) assume the following expressions (Fig. 2):

$$\Phi_i(\varphi, z) = \psi_i(\varphi, z) = e^{-j\nu_m(z+\Delta\varphi)} e^{-j\kappa_{mn}(z+\Delta\varphi)}$$

(9)

$$N_{e_{\varphi i}} = \begin{cases} \frac{1}{12\varphi} \kappa_{mn} k_{mn} & TE_z \\ -\frac{h_{\varphi i}}{12\varphi} & TM_z \end{cases}$$

(10)

$$N_{e_{\varphi i}} = \begin{cases} 0 & TE_z \\ -\frac{1}{12\varphi} & TM_z \end{cases}$$

(11)

$$N_{h_{\varphi i}} = \begin{cases} \frac{h_{\varphi i}}{12\varphi} & TE_z \\ -\frac{1}{12\varphi} & TM_z \end{cases}$$

(12)

$$\Delta\varphi = \frac{2\pi}{N}$$

(13)

$$k_{\varphi i} = k_{\varphi 0} + \frac{2\pi n}{\Delta z}$$

(14)

$$n = 0, \pm 1, \pm 2, \ldots$$

(15)

$$\nu_m = \nu_0 + m\Delta$$

(16)

[Diagram of FSS in unit cell shown in Fig. 1.]

Fig. 3. 3-D view of the first FSS in the unit cell whose transverse-to-$z$ section is shown in Fig. 1.
\[ N_{h_{m, i}} = \left\{ \begin{array}{ll} 2\frac{r_m}{h} & \text{TE}_z \\ \frac{2\sqrt{2}}{\sqrt{\Delta \varphi}} & \text{TM}_z \end{array} \right. \]

\[ N_{h_{z, i}} = \left\{ \begin{array}{ll} 2\frac{r_m}{h} & \text{TE}_z \\ 0 & \text{TM}_z \end{array} \right. \]

\[ k_{z n} = \frac{n\pi}{h} r_m = \frac{m\pi}{\Delta \varphi} \]

\[ \delta_k = \left\{ \begin{array}{ll} 1 & k = 0 \\ 2 & k \neq 0 \end{array} \right. \]

with \( h \) and \( \rho \Delta \varphi \) being the height and the arc width of the metallic radial waveguide representing the aperture of the FSS, as shown in Fig. 3.

Finally, it is worth to repeat that the modal index \( i \) refers to a combination of transverse indexes \( m \) and \( n \) either in the RPSWW or in the metallic radial waveguide.

While in the transverse directions \( \varphi \) and \( z \) the RPSWW and the metallic radial waveguide have the different modal functions previously reported, in the radial coordinate they show the same behavior. For example, the \( V_i(\rho) \) voltage of the \( i \)th TM\(_ m \) mode must satisfy the following differential equation:

\[ \frac{1}{\rho} \frac{d}{d\rho} \left[ \rho \frac{dV_i(\rho)}{d\rho} \right] + \left[ \frac{\omega^2 \mu \varepsilon_0 \varepsilon_r - k_{z n}^2 - \frac{\nu_m^2}{\rho^2} \right] V_i(\rho) = 0 \]

\[ \frac{d}{d\rho} \left[ \frac{dV_i(\rho)}{d\rho} \right] + \frac{2\sqrt{2\pi \varepsilon_r^2}}{\epsilon^2} \left[ f^2 - f_{c,i}^2(\rho) \right] V_i(\rho) = 0 \]

(19)

being

\[ f_{c,i}(\rho) = \frac{c}{2\sqrt{\pi \varepsilon_r}} \sqrt{\frac{\nu_m}{\Delta \varphi}} + k_{z n}^2 \]

(20)

the cutoff frequency of the \( i \)th mode and \( c = 1/\sqrt{\mu_0 \varepsilon_0} \). Similar expression holds for the TE\(_ z \) modes, replacing \( V_i(\rho) \) with the modal current \( I_i(\rho) \). Obviously, \( \nu_m \) and \( k_{z n} \) must be replaced in (19), (20) by (12), (13) for the RPSWW or by (18) for the metallic radial waveguide. The solution of (19) is expressed in terms of Hankel functions or modified Bessel functions, or their derivatives

\[ V_i(\rho) = A_{mm} V_{m, \nu_m, i}^+(\rho) + B_{mm} V_{n, \nu_m, i}^-(\rho) \]

(21)

\[ I_i(\rho) = A_{mm} V_{m, \nu_m, i}^+(\rho) - B_{mm} V_{n, \nu_m, i}^-(\rho) \]

(22)

\[ V_{m, \nu_m, i}^+(\rho) = \left\{ \begin{array}{ll} j_{\nu_m} N_{\nu_m, \alpha_m}(k_{z n} \rho) & \text{TE}_z k_{z n}^2 > 0 \\ -j_{\nu_m} N_{\nu_m, \alpha_m}(k_{z n} \rho) & \text{TM}_z k_{z n}^2 < 0 \end{array} \right. \]

(23)

\[ V_{n, \nu_m, i}^-(\rho) = \left\{ \begin{array}{ll} j_{\nu_m} N_{\nu_m, \alpha_m}(k_{z n} \rho) & \text{TE}_z k_{z n}^2 > 0 \\ -j_{\nu_m} N_{\nu_m, \alpha_m}(k_{z n} \rho) & \text{TM}_z k_{z n}^2 < 0 \end{array} \right. \]

(24)

The Hankel functions \( H_{\nu_m}^{(1)}(k_{z n} \rho) \) (or the modified Bessel function \( K_{\nu_m}(\alpha_m \rho) \)) and \( H_{\nu_m}^{(2)}(k_{z n} \rho) \) (\( I_{\nu_m}(\alpha_m \rho) \)) represent forward and backward propagating (nonpropagating) modes, respectively. Forward waves “see” an enlarging radial waveguide, while the backward waves “see” a reducing radial waveguide. Hence, their characteristic impedances (25), (26) have different expressions and they are function of the radial coordinate [17].

\[ Z_{m, \nu_m, i}^+(\rho) = \left\{ \begin{array}{ll} \frac{k_{z n} N_{\nu_m, \alpha_m}(k_{z n} \rho)}{\alpha_m N_{\nu_m, \alpha_m}(k_{z n} \rho)} & \text{TE}_z k_{z n}^2 > 0 \\ \frac{k_{z n} N_{\nu_m, \alpha_m}(k_{z n} \rho)}{\alpha_m N_{\nu_m, \alpha_m}(k_{z n} \rho)} & \text{TM}_z k_{z n}^2 < 0 \end{array} \right. \]

(25)

\[ Z_{n, \nu_m, i}^-(\rho) = \left\{ \begin{array}{ll} \frac{k_{z n} N_{\nu_m, \alpha_m}(k_{z n} \rho)}{\alpha_m N_{\nu_m, \alpha_m}(k_{z n} \rho)} & \text{TE}_z k_{z n}^2 > 0 \\ \frac{k_{z n} N_{\nu_m, \alpha_m}(k_{z n} \rho)}{\alpha_m N_{\nu_m, \alpha_m}(k_{z n} \rho)} & \text{TM}_z k_{z n}^2 < 0 \end{array} \right. \]

(26)

where

\[ k_{z n} = \sqrt{2 \mu \varepsilon_0 \varepsilon_r} \]

\[ = \sqrt{\frac{2 \mu \varepsilon_0 \varepsilon_r}{k_{z n}^2 + \frac{2\pi \nu_m}{\Delta \varphi}}} \]

\[ = \sqrt{(k_{z n}^2 + \frac{2\pi \nu_m}{\Delta \varphi}) - \omega^2 \mu \varepsilon_0 \varepsilon_r} \]

(27)

(28)

\( A_{mm}, B_{mm} \) are modal amplitudes. The definition of \( Z^\pm \) needs some further comments.

In the hypothesis that (19) and (20) refer to the RPSWW, we can classify the modes as follows.

- The mode is propagating for every value of \( \rho \) if \( k_{z n} = 0 \)
- The mode is not propagating for any value of a combination of \( k_{z n} \) and \( \nu_m \)

\[ f \geq \frac{c}{2\pi \sqrt{\varepsilon_r}} k_{z n} = \frac{c}{2\pi \sqrt{\varepsilon_r}} \left( k_{z n} + \frac{2\pi \nu_m}{\Delta \varphi} \right) \]

\[ f \geq f_{c,i} \forall \rho \]

(29)

In this case, from (27), \( k_{z n}^2 > 0 \) and (21)–(26) are expressed in terms of Hankel functions.

- The mode is not propagating for any value of \( \rho \) in presence of a combination of \( k_{z n} \) and \( \nu_m \)

\[ f \leq \frac{c}{2\pi \sqrt{\varepsilon_r}} k_{z n} = \frac{c}{2\pi \sqrt{\varepsilon_r}} \left( k_{z n} + \frac{2\pi \nu_m}{\Delta \varphi} \right) \]

\[ f \leq f_{c,i} \forall \rho \]

(30)

In this case the values of \( k_{z n} \) and \( \nu_m \) do not change the propagation characteristic and, from (27), \( k_{z n}^2 < 0 \). (21)–(26) are expressed in terms of modified Bessel functions.

- The mode could be propagating or nonpropagating, depending on the combination of all the transverse indexes. If \( k_{z n} = 0 \) or \( \nu_m = 0 \), setting \( k_{z n} \) and \( \nu_m \) such that

\[ f \geq \frac{c}{2\pi \sqrt{\varepsilon_r}} k_{z n} = \frac{c}{2\pi \sqrt{\varepsilon_r}} \left( k_{z n} + \frac{2\pi \nu_m}{\Delta \varphi} \right) \]

(31)
(21)-(26) are expressed in terms of Hankel functions but there is a value of the radial coordinate, \( \rho_{c,i} \), for which \( f = f_{c,i}(\rho_{c,i}) \). Hence, the mode is not propagating on the left side of \( \rho_{c,i} \) and is propagating on the right side. Obviously, this transition is not abrupt but it occurs gradually. This behavior is known as “gradual cutoff.”

From (19), the value of the “gradual cutoff radius” is

\[
\rho_{c,i} = \left| \frac{\nu_m}{k_m} \right| = \left| \frac{\nu_0 + mN}{\sqrt{\omega^2 \mu \varepsilon_0 \varepsilon_r - \left(k_{z,0} + \frac{2\pi}{\lambda} \right)^2}} \right|. \tag{32}
\]

This effect can be clarified if we set \( \nu_m \) and \( k_{z,0} \) such that (31) is verified and if we choose a radial coordinate \( \rho \) such that

\[
\omega^2 \mu \varepsilon_0 \varepsilon_r - k_{z,0}^2 - \left( \frac{\nu_m}{\rho} \right)^2 = k_i^2 - \left( \frac{\nu_m}{\rho} \right)^2 = -\frac{1}{\rho^2} \left[ k_{z,0}^2 - (\nu_m)^2 \right] = -\frac{k_i^2}{\rho^2} \left[ k_{c,i}^2 - \rho^2 \right] \ll 0. \tag{33}
\]

Equation (33) implies that \( \rho \ll \rho_{c,i} \). The asymptotic expansion for \( H_{\nu_m}^2(k_{z,0}) = j \nu_{m}(k_{z,0}) - j \nu_{m}(k_{z,0}) \) for \( k_{z,0} \ll \nu_{m}(\rightarrow \rho \ll \rho_{c,i}) \) is [18]

\[
H_{\nu_m}^2(k_{z,0}) \sim j \sqrt{\frac{\pi}{2}} \frac{\nu_m + \sqrt{\nu_m^2 - (k_{z,0})^2}}{\nu_m \nu_m^\prime \left[ k_{z,0}^2 - (k_{z,0})^2 \right]^\dagger}. \tag{34}
\]

Equation (34) represents a typical behavior of a forward wave below cutoff.

On the contrary, maintaining the same value of transverse indexes \( \nu_m \) and \( k_{z,0} \), but choosing a radial coordinate such that

\[
-\frac{1}{\rho^2} \left[ k_{z,0}^2 - (\nu_m)^2 \right] \gg 0 \tag{35}
\]

we can write

\[
H_{\nu_m}^2(k_{z,0}) \sim \sqrt{\frac{\pi}{2}} e^{-\frac{j}{\nu_m^\prime} \left[ \sqrt{(k_{z,0})^2 - (\nu_m)^2} \right]^\dagger} \tag{36}
\]

which represents a typical behavior for a forward wave above cutoff. For all the radial coordinates in the neighborhood of \( \rho_{c,i} \) (32), the wave changes its propagation characteristic from (34) to (36). Similar considerations can be made for the metallic radial waveguide.

Once derived the spectra of the RPSWW and the metallic radial waveguide, the MEN formulation [11]-[15] is used to obtain the scattering parameters of the overall structure, shown in Figs. 1 and 2, by cascading the impedance matrices of the constituting blocks.

In [13] and [14], we have already presented the derivation of the impedance matrix of the transition between the rectangular waveguide and the RPSWW. Hence, in the next subsections of this paper we will analyze only the matrices of the other blocks: the metallic Frequency Selective Surfaces and the dielectric layers.

We can reduce the problem of finding the impedance matrix of the FSS included in the unit cell, shown in Fig. 3, to the cascade of three matrices corresponding to the following blocks:

1) RPSWW—metallic radial waveguide discontinuity at \( \rho_1 = R + L_1 \);
2) metallic radial waveguide of length \( L_1 \);
3) metallic radial waveguide—RPSWW discontinuity at \( \rho = \rho + L_1 \).

It is to be noted that the two discontinuities are represented by two different impedance matrices, because of the variation of the cross section with radius.

The impedance matrix representing the RPSWW—metallic radial waveguide discontinuity can be obtained by the Multimode Equivalent Network approach in radial coordinates. For sake of clarity, we will summarize here the relevant equations. The problem under investigation can be seen, in general terms, as the junction between two arbitrary waveguides in radial coordinates, as shown in Fig. 4.

The first step in the Integral Equation formulation is the imposition of the continuity condition of the magnetic field at the discontinuity plane. The next step consists in the introduction of the “accessible” and “localized” modes concept [19]. The accessible modes are the first modes excited by the discontinuity (all the propagating modes plus first few nonpropagating modes). These modes are responsible for the interaction between adjacent discontinuities. The localized modes are the infinite remaining modes localized in the neighborhood of the discontinuity.

The final multimode equivalent network will present as many input and output ports as the number of accessible modes, without neglecting the higher order modes that are kept in the...
kernel of the integral equation. By separating the accessible from the localized modes, we can write the continuity of the magnetic field at the surface discontinuity as [14]

\[ \sum_{i=1}^{N^C} I_i^C \mathbf{h}_i^C(s) - \sum_{q=N^C+1}^{\infty} V_q^C \frac{Z_q^C}{Z_q^L} \mathbf{h}_q^L(s) = \sum_{i=1}^{N^R} I_i^R \mathbf{h}_i^R(s) \]

\[ + \sum_{q=N^R+1}^{\infty} V_q^R \frac{Z_q^R}{Z_q^L} \mathbf{h}_q^L(s) \]

where the indices \( i \) and \( q \) refer to a combination of transverse indexes \( n \) and \( m \) for TE_{n} and TM_{m} modes. The superscripts \( L \), \( R \) refer to the left side and to the right side of the discontinuity, respectively, and \( s \) represents the coordinates of the surface discontinuity. \( Z^+, \ Z^- \) are the modal admittances relative to the backward \( (−) \) and to the forward waves \( (+) \) as in (25) and (26) and \( N^C, N^R \) are the number of the accessible modes in the two regions. In (37) we have introduced the modal voltage amplitudes, which are defined by the following equation, \( \rho \) being the direction of propagation:

\[ V_q^\delta = \int_{c,s} [\hat{\rho} \times \mathbf{E}(s')] \cdot \mathbf{h}_q^*(s') ds' \quad \forall q \quad \delta = L, R. \]  

(38)

The unknowns of the problem, namely the transverse electric field in the aperture \( \hat{\rho} \times \mathbf{E} \), can now be expanded in terms of proper sets of vector expanding functions \( \mathbf{M}_i^C, \mathbf{M}_i^R \) weighted by the amplitudes of the accessible modes

\[ \hat{\rho} \times \mathbf{E}(s) = \sum_{i=1}^{N^C} I_i^C \mathbf{M}_i^C(s) - \sum_{i=1}^{N^R} I_i^R \mathbf{M}_i^R(s). \]  

(39)

We can expect, in fact, that the resulting electric field will be dependent on the amplitudes of the exciting modes. This expression can now be used in (37) and (38) so that, equating term by term, we can write

\[ \mathbf{h}_i^C(s) = \int_{c,s} \mathbf{M}_i^C(s') \cdot \frac{\sum_{q=N^C+1}^{\infty} \mathbf{h}_q^C(s') \mathbf{h}_q^L(s')}{Z_q^C} ds' \]

\[ + \sum_{q=N^R+1}^{\infty} \frac{\mathbf{h}_q^L(s) \mathbf{h}_q^R(s')}{Z_q^R} ds' \]

\[ \delta = L, R. \]  

(40)

Equation (40) defines a set of integral equations, where the unknowns are the expanding functions \( \mathbf{M}_i^C \). The integral equations can now be solved by using the method of moments. The unknown vector functions are expanded as linear combinations of orthonormal functions \( \mathbf{F}_i(s) \), consisting of the complete orthonormal set of the metallic radial waveguide modes

\[ \mathbf{M}_i^C(s) = \sum_{i=1}^{L_{\infty}} \alpha_i^C \mathbf{F}_i(s). \]  

(41)

The final step for the solution of the resulting equations is the application of the Galerkin’s procedure that leads to the following matrix equation system

\[ [\mathbf{L}]^{\delta i} \mathbf{a}^{\delta i} = [\mathbf{K}]^{\delta i} \]  

(42)

where

\[ U_{ij} = \sum_{q=0}^{\infty} \sigma_{i j}^{C} \frac{\partial C_{q}}{Z_d^L} + \sum_{q=0}^{\infty} \sigma_{i j}^{R} \frac{\partial C_{q}}{Z_d^R} \]

(43)

\[ \sigma_{i j}^{C} = \int_{c,s} \mathbf{h}_i^C(s) \cdot \mathbf{F}_i(s) ds' \quad \delta = L, R, \]  

(44)

\[ \sigma_{i j}^{R} = \int_{c,s} \mathbf{h}_i^R(s) \cdot \mathbf{F}_i(s) ds' \quad \delta = L, R. \]  

(45)

Finally, recalling (38) and (39), we can write

\[ V_q^\gamma = \sum_{i=1}^{N^C} I_i^C \mathbf{Z}_{q i}^C \mathbf{L} - \sum_{i=1}^{N^R} I_i^R \mathbf{Z}_{q i}^R \]  

(46)

where we have defined

\[ \mathbf{Z}_{q i}^\delta = \int \mathbf{M}_i^C(s') \cdot \mathbf{h}_q^*(s') ds' \]

\[ = \sum_{i=1}^{L_{\infty}} a_i^C \alpha_i^R \]  

(47)

and \( \gamma, \delta = L, R \). Equations (41) and (47) complete the formal solution of the generic discontinuity between two waveguides, in terms of the finite multimode equivalent network, represented by the impedance matrix (47).

### B. Dielectric Radomes

The TE_{\infty} and TM_{\infty} modes in radial coordinates, employed in the transverse representation of the RPSWW, are separable with respect to the z axis (the axis of the cylinder), but propagating in the radial direction. With ordinary TE and TM modes in the direction of propagation, the air-dielectric or dielectric-dielectric interface would represent a simple junction between transmission lines, and the TE and TM modes would not be coupled together at the interface. On the contrary, the TE_{\infty} and TM_{\infty} modes used in the RPSWW do couple at this interface. For the characterization of simple or multilayer dielectric radomes we use the equivalent circuit representation shown in Fig. 5, [20], [21], where

\[ N^2 = -\left( \frac{\Delta \rho}{\Delta z} \right)^2 \left[ \frac{\omega^2 \mu \varepsilon_0 (\varepsilon_{t 1} - \varepsilon_{t 2})}{(\omega^2 \mu \varepsilon_0 (\varepsilon_{t 1} - k^2 \varepsilon_{2 n}) - (\omega^2 \mu \varepsilon_0 (\varepsilon_{t 2} - k^2 \varepsilon_{2 n}))^2} \right] \]  

(48)

### III. Numerical Results

In the previous sections, we have shown that the MEN approach permits to obtain Z matrices representing lines and discontinuities which can be cascaded in order to obtain the scattering parameters of complex structures. In a previous work [14], we have analyzed a cylindrical periodic array of open ended waveguides without FSS panels adopting the unit cell approach. For this structure we have evaluated the reflection coefficient in the feeding waveguide, the coupling coefficient between adjacent apertures and the active element pattern, in the hypothesis of single element excitation. In order to derive
these quantities from the analysis of the structure based on the unit cell approach (where on the contrary all the elements are excited) we have used the technique of eigenexcitations expansion [4], [23]. This technique permits to model any excitation distribution in the azimuthal direction of the array. The \( N \) eigenexcitations are defined as the \( N \) spectral harmonics allowed by the periodicity of the structure (cylindrical array with \( N \) apertures on a ring and \( N \) permitted circular phase shifts between apertures). We have realized also a demonstrator to validate this approach, comparing experimental and theoretical results, obtaining a very good agreement.

In this paper, we analyze similar cylindrical periodic array structures but with an FSS loading. Referring to Fig. 3, we consider an array of 20 or 40 (\( N \)) horizontal WR90 waveguides (\( a = 22.86 \) mm, \( b = 10.16 \) mm) uniformly distributed on an infinite conducting cylinder of radius (\( R \)) 0.15 m and fed with equal amplitude and phase in \( \varphi \) and \( z \) directions (\( \theta_0 \Delta \varphi = 0 \), \( k_0 \Delta z = 0 \) → broadside condition), loaded with FSS panels. The height of the unit cell (\( \Delta z \)) is 20 mm and its angular inter-element array distance varies from

\[
\Delta \varphi = \frac{2\pi}{N} = 0.314 \text{ rad} \quad \left( R \Delta \varphi \approx 47.1 \text{ mm} \right)
\]

for \( N = 20 \) to

\[
0.157 \text{ rad} \quad \left( R \Delta \varphi \approx 23.55 \text{ mm} \right)
\]

for \( N = 40 \). A FSS is placed in front of the cylinder with the same number of apertures as the array. The apertures are 8.48 mm in height (\( t_h \)) and 0.1226 rad large (\( \Delta \varphi' \)) (corresponding to an arc \( ( R + L_1) \Delta \varphi' \approx 19 \text{ mm} \)). The FSS is 2.5 mm thick (\( t_f \)) and is placed at 5 mm (\( L_1 \)) from the cylinder, with \( \varepsilon_{r,1} = 1 \). The reflection coefficient \( S_{11} \) of the WR90 waveguide is shown in Fig. 6. The presence of the FSS introduces a resonance at about 9.15 GHz (\( N = 40 \), continuous line) that shifts down (9 GHz) (\( N = 20 \), continuous line with square), increasing the angular inter-element array distance. This effect can be explained by the shift of the modes cutoff to lower frequencies due to the increase of the unit cell section. An interesting effect is shown in the same figure if we increase \( R \) and \( N \), keeping constant the horizontal (angular)

![Fig. 5. Equivalent circuit representing the coupling between TE\(_z\) and TM\(_z\) modes at the interface between two RPSWW with different dielectrics. TE\(_z\) and TM\(_z\) modes have the same combination of transverse indexes \( m \) and \( n \).](image)

![Fig. 6. Reflection coefficient \( S_{11} \) seen from a horizontal WR90 waveguide (\( a = 22.86 \) mm, \( b = 10.16 \) mm) feeding the unit cell shown in Fig. 3, with \( \varepsilon_{r,1} = 1 \), \( L_1 = 5 \text{ mm} \), \( t_1 = 2.5 \text{ mm} \), \( \Delta \varphi' = 0.1225 \text{ rad} \), \( ( R + t_1) \Delta \varphi' \approx 19 \text{ mm} \), \( h = 8.48 \text{ mm} \), \( R \Delta \varphi \) from 47.1 mm (\( N = 20, 200 \)) to 23.55 mm (\( N = 40, 400 \)). The number of the radiating elements (\( N \)) and the radius of the cylinder (\( R \)) are shown in the legend. The “HFSS” curve (only dots) refers to the simulation of a uniform planar structure with the commercial software, as explained in the text.](image)

![Fig. 7. Reflection coefficient \( S_{11} \) seen from a horizontal WR90 waveguide (\( a = 22.86 \) mm, \( b = 10.16 \) mm) feeding the unit cell shown in Fig. 3, with \( N = 40 \), \( R = 0.15 \text{ mm} \), \( R \Delta \varphi = 23.55 \text{ mm} \), \( t_1 = 2.5 \text{ mm} \), \( \varepsilon_{r} = 2.0 \), \( L_1 = 1 \text{ mm} \), \( h = 8.48 \text{ mm} \). The dielectric constant \( \varepsilon_r \) and the layer length \( L_1 \) are shown in the legend.](image)
line). Comparing the reflection coefficients of these cases, we can observe a resonance shift toward higher frequencies (from 9.15 to 9.25 GHz): this effect is due to the $\text{TM}_{10}$ mode in the RPSWW, which is the first mode showing “gradual cutoff” (the fundamental mode $\text{TM}_{00}$ is not subject to “gradual” cutoff). The cutoff radius ($R$) varies from 0.208 m ($N = 400$) to 2.08 m ($N = 400$) at $f = 9.15$ GHz for the $\text{TM}_{10}$ mode in the RPSWW. We can observe that exponential decay in (34) becomes more and more effective as the cutoff radius increases. Hence, the $\text{TM}_{10}$ mode becomes less excited as $N$ increases. Increasing $N$, the cutoff radius becomes very large and we can consider only one mode propagating in the RPSWW, the $\text{TM}_{00}$ mode, being the $\text{TM}_{10}$ completely under cutoff. The same considerations hold for the other curves ($N = 200, R = 0.15, \text{continuous line with square}$) and ($N = 200, R = 1.5, \text{dashed line with square}$).

In order to validate our results, we present some comparisons with simulations performed with the commercial software package (HFSS [22]), for the limit case of very large radii of curvature of the cylinder (actually an infinite planar uniform structure for HFSS).

In Fig. 6, we report also the reflection coefficient, computed with the commercial software, relative to the uniform planar structure. The HFSS curve (only dots) is quite similar to the case $N = 400, R = 1.5 \text{ m}$ (dashed line), which already represents a very large cylinder, corroborating our results. It is also important to remind again that in [14], we have already validated experimentally the model of the conformal array without the FSS loading, obtaining very good agreement. This of course is another element that further strengthens our belief in the correctness of the results.

The effects of dielectric radome are shown in Fig. 7, where the previous case with $N = 40, \varepsilon_{\text{rad}} = 1$ is compared with the results obtained with $\varepsilon_{\text{rad}} = 2.0$ and various lengths $L_1$, keeping fixed all the other geometrical quantities. The radome lowers the frequency cutoff of the modes. Therefore, the resonance effect at 9 GHz with $\varepsilon_{\text{rad}} = 1$ disappears increasing the dielectric constant (Fig. 7), if the same length $L_1 = 5 \text{ mm}$ of the previous case is maintained.

We must recall that the structure under investigation (Fig. 3) can be considered as a “filter,” where $L_1$ represents the length of the cavity contained between the aperture of the rectangular waveguide and the FSS. The resonance frequency can be shifted by proper choosing the length of the cavity. This effect appears also in the cylindrical configuration, as shown in the three curves reported in Fig. 7 for $\varepsilon_{\text{rad}} = 2.0$: changing the length of the cavity, the resonance of the “filter” moves from frequencies out of band (lower than 8 GHz) ($L_1 = 5 \text{ mm}$) to 8.30 GHz ($L_1 = 3 \text{ mm}$) and 9.1 GHz ($L_1 = 1 \text{ mm}$). Moreover, the aperture size of the FSS affects the “filter” response, as actually shown in Fig. 8.

Recalling the microwave filter theory, in order to control the filtering behavior of the structure, a cascade of FSS can be used. Obviously, the sizes of their apertures and the lengths of the cavities between FSS must be properly chosen. We simulated the cascade of two FSS as illustrated in Fig. 2 and the results are shown in Figs. 9 and 10. We considered two different configurations, whose geometric parameters are reported in Table I (structures n. 1 and 2). The scattering parameter $S_{11}$ in Fig. 9 refers to the reflection coefficient “seen” from the horizontal WR90 rectangular waveguide. $S_{12}$ and $S_{13}$ represent the transmission coefficients to the fundamental $\text{TM}_{00}$ mode and to the first higher mode $\text{TM}_{01}$, respectively, both propagating after the
second FSS in the unit cell. Once again, TM_{01} is subject to “gradual” cutoff. The presence of two FSSs permits to obtain a good reflection coefficient for N = 100 and R = 0.386 m (S_{11}, continuous line), but the TM_{01} mode shows a non negligible amount of power (S_{13}, dashed line). If we increase R and N (R = 0.772 m, N = 200), while keeping constant the horizontal inter-element array distance to 24.247 mm, we can observe that the presence of the TM_{01} mode tends to disappear, showing a lower S_{13} (dashed line with square). This is due to the higher attenuation (34) of the TM_{01} mode as the cutoff radius (32) increases. For R = 1.929 m and N = 500, the structure shows better filtering characteristics since the effect of such mode becomes negligible in the band of interest. Once again, to validate our results, we report in the same Fig. 9 the reflection coefficient computed with HFSS, relative to the uniform planar structure obtained by increasing the radius of the cylinder to the infinite. The HFSS curves [only circles (S_{11}) or triangles (S_{12})] are quite similar to the case N = 500, R = 1.929 m, which already represents a very large cylinder, corroborating our results. Similar considerations can be applied to the filter of Fig. 10, which acts at higher frequencies.

The bandwidth of the “filter” can be changed by increasing/decreasing the number of the FSS, as shown in Fig. 11. Referring to a value of −15 dB, the bandwidth is about 3%, 5% and 7%, for an array with R = 1.5 m, N = 400, Δz = 20 mm, inter-element distance 23.55 mm, broadside condition (k_{θ}Δφ = 0, k_{z}Δz = 0), fed by WR90, and with the other dimensions as in Table I (structures n. 3, 4, and 5, respectively). The mode subject to “gradual” cutoff has a negligible amount of power for these structures. It should be noted that the structures 3, 4, and 5 have a FSS directly placed on the open-end of the WR90.

Finally, the effect of the horizontal inter-element phase shift (k_{θ}Δφ) is shown in Fig. 12 for the structure n. 4 in Table I. The main effect is a shift of the center frequency of the “filter.” This effect has also been shown in [1], [2] for a multilayer planar FSS. In this last case the effect was much less pronounced since the structure was specifically optimized in order to reduce the dependency of the FSS from the scanning angle. In the present work any particular optimization has been performed on the structure regarding this specific aspect.
TABLE I

<table>
<thead>
<tr>
<th>N.</th>
<th>FSS on the WR90 aprt.</th>
<th>Num. of FSS</th>
<th>FSS thickness</th>
<th>Unit Cell height $\Delta z$</th>
<th>%L</th>
<th>$\rho \Delta \phi_{r}$</th>
<th>%L</th>
<th>$\rho \Delta \phi_{h}$</th>
<th>%L</th>
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<th>%L</th>
<th>$\rho \Delta \phi_{h}$</th>
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Fig. 12. Reflection coefficient relative to the structure n. 4 of Table I with $N = 400$, $R = 1.5 \text{ m}$, horizontal inter-element distance at the surface of the cylinder equal to 23.55 mm, $k_{0 \Delta z} = 0$, varying the horizontal phase shift, $\nu D \Delta \phi_{r}$, between horizontal WR90 feeding the array: 0° (line), 30° (line with crosses), 60° (line with circles), 90° (line with dots).

IV. CONCLUSION

In this paper we have applied the MEM technique based on the unit cell approach to the analysis of cylindrical periodic arrays based on open-ended waveguide radiators loaded with radomes and FSS. The effect of the dielectric layers and of the FSS aperture size are discussed. The presence of FSS permits to obtain a structure showing a filtering behavior which can be tuned in frequency by a proper choice of the geometric parameters. The cylindrical waves subject to “gradual cutoff” influence such filtering behavior and they must be taken into account in the synthesis process for a correct evaluation of the scattering parameters.

The use of the unit cell approach necessarily restricts the analysis to cases where all the elements of the array are excited with the same amplitude and a linear phase shift. Nevertheless, the simulation of more generic excitation conditions, which are necessary in order to generate a directive beam with a cylindrical array, can still be performed with the proposed method by resorting to the technique of eigenexcitations expansion presented in [4], [23]. This aspect has not been addressed in the present paper, but the reader interested in more details could refer to [14]. In that work, we have adopted the eigenexcitations expansion technique in combination with the MEN approach for the study of a cylindrical array of open ended waveguides, with generic excitation.

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