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Analysis of stochastic resonances in electromagnetic couplings to transmission lines

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Abstract—Resonances present in coupling phenomena between a randomly varying thin-wire transmission-line, and an electromagnetic field are stochastically characterized. This is achieved by using the first 4 statistical moments in order to appreciate the intensity of the resonance phenomena. The stochastic method proposed is applied to a thin-wire transmission line connected to a variable impedance, and, undergoing random geometrically localized perturbations.

I. INTRODUCTION

With the advent of numerical computation power, the use of simulations based on theoretical electromagnetic models represents a useful and economical tool when compared to the cost of actual experiments. The range of applicability of these models is assessed by their versatility, i.e. their ability to accurately depict the reality of the electromagnetic interaction in a variety of possible configurations. In EMC, the accurate modeling of the internal or external environment of devices can give rise to problems of prohibitive complexity. More generally, these issues are particularly relevant for ageing and fatigue analyzes of electronic equipment, as well as for the design of moving systems such as conformal antennas.

In all these cases, a systematic study of each of the possible configurations can be numerically intractable given the computations that it requires. When the parameters through which the electromagnetic interaction is observed, also known as observables, vary smoothly as a function of the changes of configurations, the study of a few sample situations already grants a satisfactory picture of the overall interaction. In the more general case, a sensitivity analysis provides some insight into the behavior of the observable, but based on local-variation hypotheses.

Instead, a stochastic approach offers an appealing alternative to both of the previous methods, by handling the global variations of the configurations, and by using probability theory to characterize the variations of the observable.

Such stochastic rationales are commonly used in Mode-Stirred-Chamber theory to depict the properties of the power distribution in the chamber [1],[2]. In this framework, the assumption of an ideal chamber, with a uniform and isotropic power distribution, is often made. Scattering phenomena in-

volving rough surfaces of very large extent are also popular candidates for stochastic methods: The field scattered by surfaces similar to the sea is best described randomly [3]. In these cases the infinite extent of the rough surface allows for asymptotic assumptions which ease the computations.

In EMC problems involving devices of finite extent affected by geometrical variations, stochastic methods are also more and more praised for the uncertainty quantification they yield. In this case however, assuming an ideal geometry or invoking asymptotic relations is usually not advisable. A random model is instead associated to the geometry of the scatterer. Probability theory is then used to propagate the randomness of the inputs through the model, and to characterize the induced randomness of the output parameters.

An example of particular importance, in this framework, is the coupling of an electromagnetic field to a wire structure. These problems occur when designing the wiring of an electronic medical or military device, of a building or of aircrafts. The stochastic methods applied to this class of interactions are often based on transmission-line theory, which provides analytical solutions for the electromagnetic response parameters of interest such as the induced voltage or the current induced at some port of the devices [4]. We have studied such wiring structures by resorting to an integral-equation approach [5],[6]. The equations are solved numerically and at a certain cost which makes it necessary to resort to a computationally efficient uncertainty quantification.

This approach is pursued further in the present article where a stochastic method is employed to characterize resonant electromagnetic couplings involving thin wires. Local stochastic deformations are assumed to affect the geometry of the wire, which is terminated by a varying load. The resonances that may occur are characterized by efficiently computing and jointly analyzing the variance and kurtosis of the voltage induced at the port of the wire. The variance will yield a quantitative measure of spread of the observable, whereas the kurtosis, or 4th order moment, will indicate the presence of extreme values of the response parameters. It is worth mentioning that kurtosis-based analyzes have become common in financial risk analysis [7] since the 1998 crisis which caused

the collapse of Long-Term Capital Management: This hedge fund had been conducting financial projections solely based on second order statistical moments [8], hereby misjudging potential risks which could have been foreseen via the kurtosis.

The outline of this paper is as follows. The scattering setup is first deterministically described in Section II, before being stochastically parameterized in Section III. This parametrization allows for the definition of the variance and the kurtosis, which are computed by quadrature rules. These moments are finally illustrated in Section IV through the study of a locally perturbed transmission-line which is terminated by a varying impedance and illuminated by a plane-wave.

II. DETERMINISTIC PARAMETRIZATION

The electromagnetic interaction involves a transmission line S illuminated by an incident field $\mathbf{E}^i$, as shown in Figure 1.

Fig. 1. Interaction configuration

The structure S is a perfectly electrically conducting (PEC) thin-wire which is horizontal and located 5 cm above a PEC ground plane. The height of the axis of S is parameterized in a Cartesian coordinate system as follows

$$z(y) = 5 + w(y - y_*) \quad \text{in cm} \quad (1)$$

where

$$w(t) = \delta_z \left(\left(\frac{t}{\tau} \right)^2 - 1 \right) \exp \left(- \left(\frac{t}{\tau} \right)^2 \right), \tau = 10 \text{ cm}. \quad (2)$$

The deformation represent a local perturbation which belongs to the family of so-called "Mexican-hat" wavelets [9]. It is centered around y_* , and spans over a range $[y_* - \delta_y/2; y_* + \delta_y/2]$ with $\delta_y \approx 4$ cm. The amplitude of this deformation is given by $\delta_z = 4$ cm. This type of deformation is for instance representative of a local defect in the manufacturing process of the transmission-line. More generally, this geometrical model is a type of multi-resolution representation of the shape of the wire, rather than the Fourier decomposition used in [5].

The transmission-line is terminated at one end by an impedance $Z = Z_r + jZ_i$. The other end of the line is in an open-circuit state via a vertical thin wire which contains a port region denoted P . All the parameters of the transmission line are gathered in the vector $\mathbf{a} = (y_*, Z_r, Z_i)$, and the wire is denoted $S(\mathbf{a})$ to mark its dependence on $\mathbf{a}$.

The parameters of the incident field $\mathbf{E}_b^i$, such as its direction of propagation, its polarization or its amplitude, form the vector $\mathbf{b}$. Therefore the vector $\boldsymbol{\psi} = \mathbf{a} \oplus \mathbf{b}$ contains all the information necessary to define the configuration.

The electromagnetic coupling between $S(\mathbf{a})$ and $\mathbf{E}_b^i$ is observed through the electromotive force $V_e(\boldsymbol{\psi})$ induced at the port P and defined as [10]

$$V_e(\boldsymbol{\psi}) = -\frac{1}{I_T} \int_{S(\mathbf{a})} \mathbf{j}\mathbf{a} \cdot \mathbf{E}_b^i. \quad (3)$$

where $\mathbf{j}\mathbf{a}$ is the current induced on the device in a transmitting state, i.e. in absence of $\mathbf{E}_b^i$, when a current source $I_T = 1$ A is applied at P . This current follows by solving a frequency-domain electric-field integral equation (EFIE) corresponding to the transmitting state [6]. This equation is solved by the method of moments by using quadratic-segment basis functions [11], together with a reduced kernel in Pocklington's integral equation. This numerical strategy has a certain cost stemming from the need to fill a full impedance matrix, and to solve the subsequent linear system.

Using transmission-line theory, the impedance $Z_l(f, \mathbf{a})$ seen from the load Z can be shown to depend on the frequency f , and on the shape of $S(\mathbf{a})$ [13]. Whenever $Z_l(f, \mathbf{a}) = -Z$, resonances will occur and translate into extreme values of V_e .

III. STOCHASTIC PARAMETRIZATION

Considering uncertainties in the deterministic model presented above is equivalent to assuming that the actual configuration $\boldsymbol{\psi}$ is unknown, or varies among an ensemble of possible configurations $\Omega_{\boldsymbol{\psi}}$.

In the stochastic approach, rather than computing $V_e(\boldsymbol{\psi})$ for each value of $\boldsymbol{\psi}$ in $\Omega_{\boldsymbol{\psi}}$, the variations of $\boldsymbol{\psi}$ in $\Omega_{\boldsymbol{\psi}}$ are viewed as random according to a *known* distribution $p_{\boldsymbol{\psi}}$. The probability function $p_{\boldsymbol{\psi}}$ can be chosen according to the knowledge available on the distribution of $\boldsymbol{\psi}$ in $\Omega_{\boldsymbol{\psi}}$.

Given its definition in Equation (3), the voltage $V_e(\boldsymbol{\psi})$ thus becomes a random variable. Statistical moments such as its mean $\mathbb{E}[V_e]$ and its standard deviation $\sigma[V_e]$ are defined as

$$\mathbb{E}[V_e] = \int_{\Omega_{\boldsymbol{\psi}}} V_e(\boldsymbol{\psi}') p_{\boldsymbol{\psi}}(\boldsymbol{\psi}') d\boldsymbol{\psi}', \quad (4)$$

$$\sigma[V_e] = \sqrt{\mathbb{E}[|V_e|^2] - |\mathbb{E}[V_e]|^2}, \quad (5)$$

with the standard deviation $\sigma[V_e]$ quantifying the spread of V_e around $\mathbb{E}[V_e]$. In some very particular cases (exponential, uniform or Gaussian distributions), $\mathbb{E}[V_e]$ and $\sigma[V_e]$ suffice to fully define the probability distribution p_{V_e} of V_e . It is however not granted that V_e has any of these special distributions.

In the more general case, it is possible to define a normalized random voltage V_n which is centered and has a unit standard deviation

$$V_n = \frac{V_e - \mathbb{E}[V_e]}{\sigma[V_e]}. \quad (6)$$

Chebychev's inequality [12] then yields more general bounds as it states that

$$\Pr[|V_n| > m] \leq \frac{1}{m^2}, \text{ for } m \geq 1 \quad (7)$$

highlighting the local nature of the information provided by $\sigma[V_e]$ on the distribution of the samples of V_e .

The third and fourth order moments, respectively known as the skewness and kurtosis can be defined for $|V_e|$ in order to obtain qualitative information on the probability distribution of $|V_e|$. The skewness measures the degree of symmetry of $p_{|V_e|}$, whereas the kurtosis evaluates the peakedness of $p_{|V_e|}$ around $\mathbb{E}[|V_e|]$ [12]. Rather than focusing on the behavior of $p_{|V_e|}$ around $\mathbb{E}[|V_e|]$, it is possible to use the kurtosis as a means of weighing the tail of $p_{|V_e|}$ [7],[12], hereby measuring the likelihood of presence of extreme values of $|V_e|$. The kurtosis $\kappa[|V_e|]$ is defined as

$$\kappa[|V_e|] = \mathbb{E} \left[\left(\frac{|V_e| - \mathbb{E}[|V_e|]}{\sigma[|V_e|]} \right)^4 \right] \geq 0. \quad (8)$$

This dimensionless parameter is positive and equal to 3 if V_e has a Gaussian distribution. Gaussian random variables are usually low-risk random variables as they take 97% of their values within 2σ of their average. Therefore, the higher the value of $\kappa[|V_e|]$ above 3, the higher the risk contained in the p_{V_e} which translates in the presence of extreme values of V_e .

In EMC, the occurrence of such extreme values of V_e is typical of resonance phenomena. The kurtosis-based analysis can therefore be used to appreciate the intensity of the resonances present in stochastic interactions.

All the statistical moments, defined as in Equation (4), consist of integrals over a computable integrand depending on $V_e(\psi)$, and the same support Ω_ψ . They can be evaluated by quadrature rules which are cautiously chosen, to efficiently handle the dimension of Ω_ψ . Moreover given the common support of these integrals, the same samples of V_e can be re-used to simultaneously evaluate all the statistical moments that are being computed.

IV. RESULTS

Using the notations of figure 1, the position y_* of the geometrical deformation is assumed to be unknown and uniformly distributed along the axis of the wire, between the abscissae $y \in \Omega_* = [y_m, y_M] = [0.1; 0.9]$ m. This wire is meshed into 224 segments, 200 of which are devoted to the horizontal portion of $S(\mathbf{a})$.

The impedance has a value of $Z = Z_r + jZ_i$ where $Z_r = 50\Omega$ and Z_i belongs to the domain $\Omega_i = [0; 100]\Omega$. However, unlike y_* , Z_i is not assumed to be random but is taken instead as a conditioning parameter, to obtain parametric results based on the different values of Z_i . The statistics computed can therefore be interpreted as conditional statistics, knowing Z_i . The incident field is a plane-wave propagating in the direction $\theta_i = 45^\circ$, $\phi_i = 45^\circ$, and such that $\mathbf{E}_b^i$ lies in the plane of incidence, with $|\mathbf{E}_b^i| = 1 \text{ V.m}^{-1}$.

The variance and the kurtosis of V_e are computed by a trapezoidal quadrature rule, for several values of the reactance $Z_i \in \Omega_i$, and for frequencies f ranging from 100 to 500 MHz. The results hence highlight the joint influence of Z_i and f on the random distribution of V_e induced by the randomness of y_* .

The standard deviation depicted in figure 2 shows that $\sigma[V_e]$ varies of 5 orders of magnitude in the range of parameters considered. For $Z_i \in [45; 100]\Omega$, and in the vicinity of $f_0 = 100 \text{ MHz}$, $f_1 = 233 \text{ MHz}$ and $f_2 = 366 \text{ MHz}$, $\sigma[V_e]$ increases, signaling resonances.

Fig. 2. $\sigma[V_e]$ as a function of f and X_β , on a logarithmic scale

Although these peaks quantitatively indicate a rise in the variability of the samples of V_e , $\sigma[V_e]$ does not inform on the nature of the spread of the values of V_e . This spread can either be generalized, or due to the presence of some extreme values of V_e coexisting with a cluster of values around $\mathbb{E}[V_e]$.

This limitation is circumvented by the analysis of the kurtosis, shown in figure 3, which generally confirms the trends of $\sigma[V_e]$. The relatively low value of $\kappa[|V_e|]$, which remains below 1, highlights the limited risk in p_{V_e} .

Fig. 3. $\kappa[|V_e|]$ as a function of f and X_β , on a logarithmic scale

Comparing Figures 2 and 3, for $Z_i \in [45; 100]\Omega$, it is worth noting that $\sigma[V_e]_{f_1}$ is higher than $\sigma[V_e]_{f_2}$, but that conversely $\kappa[|V_e|]_{f_1}$ is lower than $\kappa[|V_e|]_{f_2}$. The consequence of this result

	f	Z_i	$\sigma[V_e]$	$\kappa[V_e]$
f_1	233 MHz	66 Ω	3.148 V	7.410^{-4}
f_2	336 MHz	66 Ω	1.801 V	4.610^{-2}

TABLE I
RESULTS AT GIVEN f AND X_β

is twofold. First, when reasoning physically with the values of $\sigma[V_e]$, the samples of V_e will be distributed on a larger domain of the complex plane, measured in volts, at f_1 than at f_2 . However, from a statistical perspective, the values of $\kappa[|V_e|]$ indicate that the normalized samples of V_e will be located in a narrower domain, measured in terms of $\sigma[V_e]$, at f_1 than at f_2 .

To confirm these observations, 1000 deterministic samples have been computed for each of the configurations specified in table I. These samples are first centered as follows $V_c = V_e - \mathbb{E}[V_e]$, then plotted in figure 4.

Fig. 4. Centered Samples V_c for f_1 and f_2 . Higher spread at f_1

After division by the standard deviation, the normalized samples are shown in figure 5, where the concentric circles correspond to Chebychev circles derived from equation 7.

Fig. 5. Normalized Samples V_n for f_1 and f_2 . Higher spread at f_2

These graphs confirm the predictions based on the analysis

of $\sigma[V_e]$ and $\kappa[|V_e|]$. All the samples are grouped within $2\sigma[V_e]$ of the average, and the statistical spread is slightly more accentuated at $f_2 = 336$ MHz than at $f_1 = 233$ MHz.

V. CONCLUSION

The method presented in this paper shows how the efficient computation of the average, the standard deviation, and the kurtosis can yield a valuable stochastic characterization of random electromagnetic interactions. The variance measures physically the spread of the random observable, whereas the kurtosis yields complementary qualitative information on the statistical spread of the observable. This distinction has shown that in some cases, what appears as a highly dispersed random voltage, through a high standard deviation, may still correspond to a local distribution of the normalized samples, thus with a low kurtosis.

The applications discussed herein focused on thin-wire setups affected by localized random deformations, and connected to a varying load. A conditional probabilistic approach allowed to capture both the effects of the frequency, and of the load on the distribution of the voltage.

At the conference, this method will be further illustrated by the study of rougher geometries with more pronounced resonances, and interacting with random incident fields.

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