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Experiments on rapidly rotating turbulent flows

L. J. A. van Bokhoven, H. J. H. Clercx, G. J. F. van Heijst, and R. R. Trieling

*Department of Applied Physics, J.M. Burgers Centre and Fluid Dynamics Laboratory,
Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

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A novel laboratory experiment for investigating statistically steady rotating turbulence is presented. Turbulence is produced nonintrusively by means of electromagnetic forcing. Depending on the rotation rate the Taylor-based Reynolds number is found to be in the range of $90 \leq \text{Re}_\lambda \leq 240$. Relevant properties of the turbulence, both with and without rotation, have been quantified with stereoscopic particle image velocimetry (SPIV). This method enables instantaneous measurement of all three velocity components in horizontal planes at a distance H from the bottom. The root-mean-square turbulent velocity decreases inversely proportional to H in the nonrotating experiments and is approximately constant when background rotation is applied. The integral length scale shows a weak H -dependence in the nonrotating experiments which is presumably due to the spatial extent of the forcing. Based on the behavior of the principal invariants of the Reynolds stress anisotropy tensor, the rotating turbulence has been characterized as a three-dimensional two-component flow. Furthermore, these SPIV measurements provide supporting evidence for (i) reduction of the dissipation rate, (ii) suppression of the vertical velocity as compared to the horizontal velocity, and (iii) increased spatial and temporal correlation of the horizontal velocity components, with the temporal correlation growing ever stronger as the rotation rate is increased. A less commonly known feature of rotating turbulence, quantified here for the first time in a laboratory setting, is the reverse dependence on the rotation rate of the spatial horizontal velocity correlation functions. Another interesting result concerns the linear (anomalous) scaling of the longitudinal spatial structure function exponents in the presence of rotation, consistent with a study by Baroud *et al.* [Phys. Rev. Lett. **88**, 114501 (2002)]. © 2009 American Institute of Physics.
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I. INTRODUCTION

Many applications in engineering, geophysics, and astrophysics involve turbulence subjected to rotation. Among the numerous engineering examples are the turbulence in boundary layers and free shear layers in a centrifugal pump or an axial-flow compressor.¹ Geophysical turbulence and planetary waves are usually related to and dependent upon Earth's rotation. Rotating turbulence inside Earth's liquid core is responsible for stretching and twisting Earth's magnetic field lines, thus preventing it from being extinguished from natural decay.² The examples illustrate the diversity of rotating turbulent flows. The examples also ground the need for improving the fundamental understanding of rotating turbulence. Improving fundamental understanding is essential for further advancing existing engineering applications and improving models of geophysical and astrophysical phenomena.

This paper presents a novel laboratory experiment. Local electromagnetic forcing of fluid is applied to generate statistically steady three-dimensional (3D) turbulence. Moreover, the turbulence generated in this way is studied with background rotation. The turbulence is predominantly generated near the bottom of the container and decays with increasing (vertical) distance to the forcing region. The turbulence is therefore assumed to be statistically inhomogeneous in the vertical direction but it will be shown to be statistically

homogeneous in the horizontal plane. In this sense the present setup to investigate 3D rotating turbulence resembles the oscillating grid experiment by Hopfinger *et al.*³ The turbulence generation by electromagnetic forcing has previously been proven useful in various experiments on two-dimensional (2D) turbulence, in a few experiments on Lagrangian aspects of nonrotating 3D turbulence,^{4,5} and in actuation and control of boundary layers in turbulent channel flow.^{6–8} Stereoscopic particle image velocimetry (SPIV) measurements have been performed to obtain two-dimensional three-component (2D3C) velocity fields. Unlike conventional PIV measurements, which give 2D2C velocity fields, no stereoscopic PIV measurements have been reported for rotating turbulence. The added value in SPIV is that it enables one to reconstruct all three velocity components in a plane defined by the light sheet. Among the key quantities that have been studied in detail are the full Reynolds stress anisotropy tensor and its invariants, all accessible correlation length and time scales, and the probability distribution functions of the individual velocity components. Before going into detail, however, we first give an overview of relevant previous work.

Over the past 3 decades, a variety of laboratory configurations has proven useful in studying rotating turbulent flows (Table I). Ibbetson and Tritton⁹ used hot-wire measurements to quantify the decay of grid turbulence inside a rotating annular container. A principal result was that increasing the

TABLE I. Laboratory experiments on freely decaying (*D*) and statistically steady (*S*) turbulence subjected to steady rotation. The Rossby number based on the grid oscillation frequency f_g is given by $Ro_g \equiv f_g/(2\Omega)$, with Ω as the background rotation rate. The grid Reynolds number is defined by $Re_g \equiv u'M/\nu$, with u' as the rms velocity, M as the grid mesh size, and ν as the kinematic viscosity.

Reference	Forcing	Ro_g	Re_g
Ibbetson and Tritton ^a (<i>D</i>)	Single grid stroke	0.3–1.9	360
Wigeland and Nagib ^b (<i>S</i>)	Fixed grid	6.0–492	$(0.9–5.5) \times 10^3$
Hopfinger <i>et al.</i> ^c (<i>S</i>)	Oscillating grid	3–33	10^3
Dickinson and Long ^d (<i>S</i>)	Oscillating grid	1.4–11.9	$(3.8–5.9) \times 10^3$
Jacquin <i>et al.</i> ^e (<i>S</i>)	Fixed grid	4.0–95.4	$10–500^f$
Baroud <i>et al.</i> ^g (<i>S</i>)	Jets	0.06–1.1 ^h	360^f
Morize <i>et al.</i> ⁱ (<i>D</i>)	Single grid stroke	2.4–120	$(3.1–6.2) \times 10^4$
Davidson <i>et al.</i> ^j (<i>D</i>)	Single oscillation	1.5–3.5 ^h	600^k
Staplehurst <i>et al.</i> ^l (<i>D</i>)	Single grid stroke	1.0–2.7 ^h	$83–130^k$

^aReference 9.

^bReference 10.

^cReference 3.

^dReference 11.

^eReference 12.

^fTaylor-scale based Reynolds number $Re_\lambda \equiv u'\lambda/\nu$ instead of Re_g .

^gReferences 13 and 14.

^hMicro-Rossby number $Ro_\omega \equiv \omega'/(2\Omega)$ instead of Ro_g .

ⁱReference 15.

^jReference 16.

^k Re_g based on the grid bar width instead of the grid mesh size.

^lReference 17.

rotation rate caused faster decay of the turbulence. They also found that rotation produced a large increase in the length scale parallel to the rotation axis and a smaller increase in that perpendicular to it. Hopfinger *et al.*³ carried out an experiment on statistically steady rotating turbulence. A grid near the bottom of a vertically elongated cylindrical tank was used to produce and maintain inhomogeneous turbulence. Wood particle visualizations revealed vortical columns. A similar experiment by Dickinson and Long¹¹ resulted in beautiful visualizations of the “McEwan-type” vortices.¹⁸ Columnar structures are a characteristic feature of rotating flows reflecting their strong tendency toward two dimensionality. They are induced by inertial waves, a form of internal wave motion supported by any incompressible rotating fluid.^{19,20}

In the late 1980s, Jacquin *et al.*¹² performed extensive hot-wire measurements on rapidly rotating homogeneous turbulence, generated by passing an air flow through a rotating dense honeycomb structure and a grid. A similar but down-scaled experiment was carried out before by Wigeland and Nagib.¹⁰ Both studies revealed that the primary effect of rotation is to inhibit the energy cascade and so to reduce the dissipation rate of the turbulent kinetic energy. The experiments also showed that the velocity correlation length scales along the rotation axis grow more rapidly in the presence of rotation than they do in the absence of rotation. Numerical simulations have shown that the faster growth of certain integral length scales is also related to the inhibited energy cascade.^{21–23}

Among the most recent experimental studies are those by Baroud *et al.*,^{13,14} Morize and co-workers,^{15,24} and by Davidson and co-workers.^{16,17} Baroud and co-workers studied the flow in a rotating annulus with forcing created by pumping water into (out of) the annulus through an inner (outer)

ring of holes on the bottom of the annulus. At a macroscopic Rossby number $Ro \equiv u_{rms}/2\Omega\mathcal{L}=0.1$, with u_{rms} as the root-mean-square (rms) turbulent velocity, Ω as the rotation rate, and $\mathcal{L}$ as the integral length scale, this setup leads to a nearly 2D flow in which vortices are advected by a counter-rotating azimuthal jet. Velocity statistics were extracted with hot-film and two-dimensional two-component (2D2C) PIV techniques. In laboratory experiments reported in Ref. 15 initially isotropic homogeneous turbulence is generated by rapidly towing a grid in a rotating liquid-filled container. During the decay, strong cyclonic coherent vortices emerge as the result of enhanced stretching of the cyclonic vorticity by the background rotation and the selective instability of the anti-cyclonic vorticity by the Coriolis force. Velocity statistics were obtained with 2D2C PIV. Davidson and co-workers also used a single grid stroke for generating turbulence (inhomogeneous in Ref. 16, homogeneous in Ref. 17). They visualized the columnar structuring with pearlescence which highlights regions of strong shear, and also performed 2D2C PIV measurements.

Numerical investigations have pointed out that the primary effect of rotation is to inhibit the energy transfer toward smaller scales, and hence to reduce dissipation.^{25–29} Other effects of rotation are two dimensionalization, easily inferred from increased integral length scales along the axis of rotation, and a major reduction of the high wave number part of the radial energy spectrum. It should be emphasized here that most issues discussed so far by numerical studies mainly concern rotating statistically homogeneous turbulence. In a laboratory experiment, however, confinement often plays an important role, in particular, causing the turbulence to be statistically inhomogeneous. An appropriate example is the investigation by Hopfinger *et al.*³ Statistically steady turbulence was maintained by an oscillating grid near the bottom

FIG. 1. (Color online) Side and top views of the experimental setup. Turbulence is generated in the central compartment of the container using electromagnetic forcing. The electrodes are placed in compartments that are separated from the measurement domain by cotton membranes in order to avoid gas bubbles from disturbing the measurements. The tubes attached to the container provide exits to the gas bubbles. An array of magnets is placed directly underneath the thin bottom of the container. The vectors define a local coordinate system.

of a vertically elongated cylindrical tank. The grid Rossby number, $Ro_g \equiv f_g/(2\Omega)$, with f_g as the grid oscillation frequency, was kept large [$\mathcal{O}(3-33)$] so that the turbulence was locally unaffected by rotation. As the turbulence decayed away from the grid, its length scale increased while its rms velocity decreased, and rotation became increasingly important. At some distance from the grid, where the local $Ro \approx 0.20$, they observed the emergence of (mainly cyclonic) vortical columns, aligned with the rotation axis and extending throughout the remainder of the tank. They found that the total vortex population decreases with increasing Rossby number, while their size and average spacing increase. The experiment by Hopfinger and co-workers was mimicked numerically by Godeferd and Lollini.²⁹ They observed more cyclones than anticyclones for high Reynolds numbers and moderate local Rossby numbers [$\mathcal{O}(0.2)$]. More importantly, they showed that vertical confinement promotes the columnar structuring by rotation, especially when the rotation rate is high.

This paper is organized as follows. Section II describes the laboratory experiment and the adopted experimental procedures. The turbulence produced by the electromagnetic forcing is characterized in Sec. III. How this turbulence is affected by solid-body rotation is described in Sec. IV. Both the nonrotating and rotating turbulences are investigated for signs of self-similarity in Sec. V. The most important results of our study are summarized and discussed in Sec. VI.

II. EXPERIMENTAL PROCEDURES

The experimental setup is shown in Fig. 1. A rectangular Perspex container (inner dimensions $L_x \times L_y \times L_z = 700 \times 500 \times 250$ mm³) is placed on a specially designed, dedicated rotating table facility. The rotating table can adopt rotational speeds $\Omega \in [0.01, 10]$ rad/s with an accuracy of 0.005Ω . 3D turbulence with a moderate Taylor-based Reynolds number [$Re_\lambda = \mathcal{O}(90-240)$] is generated in the

FIG. 2. Schematic top view of the electromagnetic forcing. Coated electrodes are placed on opposite sides of an array of axially magnetized neodymium magnets. Small elongated bar magnets (north pole up) are placed in between large flat bar magnets (poles indicated by capital N and S, respectively). The dashed rectangle indicates the common field of view of the SPIV camera system. The vectors define a local coordinate system.

bottom part of the fluid bulk using electromagnetic forcing. In order to generate turbulence electromagnetically, the tank is filled with a concentrated sodium chloride solution ($\rho = 1.16$ g/cm³; $\nu = 1.3$ mm²/s), and two ceramic coated electrodes are placed near the bottom at the shorter end walls of the container. A remote-controlled power supply (KEPCO, type BOP 50-8P) is connected to the electrodes and provides a constant electrical current I from time $t = t_0$. An array of axially magnetized permanent (neodymium) magnets is placed directly underneath the bulk fluid. The magnets have a magnetic field strength of approximately 1.4 T at the center of the magnet surface. The magnets, kept in position by a polyvinyl chloride (PVC) frame, are fixed on a 10 mm thick steel plate to increase the density of the magnetic field lines in the fluid bulk. A range of flow scales is forced by using two differently sized magnets, viz., (i) elongated bar magnets, $10 \times 10 \times 20$ mm³ in size; and (ii) flat bar magnets, $40 \times 40 \times 20$ mm³ in size. The exact arrangement of the magnets is shown in Fig. 2: The orientations of the neighboring flat bar magnets vary in angle and alternate in magnet pole, the elongated bar magnets do not vary in angle and all are placed with the north pole upward. The purpose of the small elongated bar magnets is to avoid large stagnation zones in between the large flat bar magnets. The magnetic

TABLE II. Relevant Ekman spin-up time scales, $T_E \equiv L_z / \sqrt{\nu \Omega}$, with L_z as the inner vertical dimension of the fluid container, $\nu = 1.3$ mm²/s as the kinematic viscosity, and Ω as the rotation rate. The corresponding Ekman number, $Ek \equiv \nu / (\Omega L_z^2)$, and Ekman boundary layer thickness, $d_E \equiv L_z \sqrt{Ek}$, are also given.

Ω (rad/s)	T_E (s)	Ek (-)	d_E (mm)
1	2.2×10^2	2.1×10^{-5}	1.14
5	9.8×10^1	4.2×10^{-6}	0.51
10	6.9×10^1	2.1×10^{-6}	0.36

TABLE III. Several time-averaged global quantities for a selection of parameter triplets $\{H, \Omega, I\}$. The spatially averaged mean and rms fluctuating velocity are defined by $U' \equiv \sqrt{\langle U_i^2(\mathbf{x}) \rangle_{\text{SA}}}$ and $u'(t) \equiv \sqrt{\langle u_i^2(\mathbf{x}, t) \rangle_{\text{SA}}}$, respectively. The kinetic energy is given by $e(t) \equiv u'^2(t)/2$. The dissipation rate $\epsilon(t) \equiv 2\nu \langle s_{ij} s_{ij} \rangle_{\text{SA}}$, with ν as the kinematic viscosity ($\nu = 1.3 \text{ mm}^2/\text{s}$) and s_{ij} as the strain rate tensor, is approximated by the surrogate $\epsilon \approx 15\nu \langle (\partial u_i / \partial x_j)^2 \rangle_{\text{SA}}$ (because gradients in the direction perpendicular to the measurement plane are inaccessible with SPIV). The Kolmogorov velocity scale is defined by $u_\eta \equiv (\nu \langle \epsilon \rangle_{\text{TA}})^{1/4}$. The spatially averaged vertical vorticity is $\omega'_3(t) \equiv \sqrt{\langle \omega_3^2(\mathbf{x}, t) \rangle_{\text{SA}}}$. The vertical vorticity at a given node $(\mathbf{x}, t)$ is computed from a local eight-point circulation calculation around that node.

$\{H, \Omega, I\}$	$U' / \langle u' \rangle_{\text{TA}}$	$\langle u' \rangle_{\text{TA}}$ (mm s ⁻¹)	u_η (mm s ⁻¹)	$\langle \epsilon \rangle_{\text{TA}}$ (mm ² s ⁻²)	$\langle \epsilon \rangle_{\text{TA}}$ (mm ² s ⁻²)	$\langle \omega'_3 \rangle_{\text{TA}}$ (s ⁻¹)
{20,0,4}	0.76	12.6	1.97	79	12	1.21
{50,0,4}	0.34	9.3	1.71	43	6.8	0.90
{100,0,4}	0.30	3.5	1.02	6.3	0.89	0.30
{20,1,4}	0.48	9.7	1.30	47	2.3	0.57
{20,5,4}	0.41	6.3	1.12	20	1.2	0.39
{20,10,4}	0.65	4.9	1.14	12.2	1.3	0.36
{50,1,4}	0.40	9.5	1.22	45	1.8	0.53
{50,5,4}	0.42	6.4	1.07	21	1.0	0.38
{50,10,4}	0.65	4.8	1.01	11.5	0.82	0.32
{100,1,4}	0.30	10.5	1.27	53	2.1	0.56
{100,5,4}	0.43	6.6	1.07	22	1.0	0.38
{100,10,4}	0.56	4.9	1.03	11.9	0.92	0.32

field strength rapidly decreases with increasing distance to the magnet surface, and reaches the order of 0 T approximately 50 mm above the magnet array. The above-described electromagnetic forcing is therefore highly localized in space, and only the lowest part of fluid bulk is effectively stirred. Evidently, the generated turbulence will decay along the direction of the rotation vector, and so rotation will become relatively more important with increasing distance normal to the tank bottom.

In order to investigate the turbulence with SPIV, the following steps are taken. The solution is seeded with polymethyl methacrylate tracer particles (density of 1.18 g/cm³; average size of 83 μm). A 2 mm thick light sheet is horizontally traversed through the fluid bulk at a vertical height H as measured from the tank bottom. The illumination source is a Q -switch triggered, doubly pulsed Nd:YAG (yttrium aluminum garnet) laser system (Quantel Twins CFR400, wavelength 532 nm, pulse energy 200 mJ), corotating with the fluid container to avoid variance in light sheet position. Digital images are acquired with two identical CCD (charge coupled device) cameras (Roper Scientific Megaplus ES1-10, sensor resolution 1008 $\times$ 1019 pixels), each equipped with a flat 28 mm lens (Nikon Nikkor, $f_\# = 4.0$) and mounted onto Scheimpflug adapters (Dantec Dynamics A/S) to satisfy the Scheimpflug criterion stating that object, lens, and image plane must be collinear in order to obtain good focus over the entire plane (at the cost of stronger perspective distortion). The adapters are mounted vertically to a rigid camera frame which itself is fixed to the tabletop. The

optical axes of the two cameras define a vertical plane containing the rotation axis (along Ω). Each camera has a mean viewing angle of 37.5° in air giving a stereoscopic angle $2\theta \approx 56^\circ$ in the electrolyte. The common field of view (FOV) is $173.6 \times 150.5 \text{ mm}^2$, corresponding to a mean optical magnification $\bar{M} \approx 0.05$. The laser system and both cameras are synchronized by a digital delay generator (Stanford Research systems DG535, delay resolution 5 ps), programed for multiframe single exposure SPIV at a sampling frequency $f_s = 15 \text{ Hz}$. The exposure time delay is set according to the forcing strength I and the measurement height H .

Two types of experiments have been performed: (i) reference experiments ($\Omega = 0 \text{ rad/s}$) and (ii) rotating experiments with $\Omega = 1, 5$, or 10 rad/s . In both cases SPIV measurements were performed at different heights, viz., $H = 20, 50$, and 100 mm . The forcing current I was either 4 or 8 A. In the remainder of this paper we shall use the parameter triplet $\{H, \Omega, I\}$ to identify individual experiments.

The experimental procedures were slightly different for reference and rotating experiments. In the reference experiments a still, homogeneously seeded sodium chloride solution formed the starting point. At a time $t = t_0$ say, a current I was supplied to the cathode, and the cameras started recording images. The current was maintained at a constant level for the remainder of the experiment using a feedback loop. The recordings were stopped after approximately ten minutes. The procedure was the same in the rotating experiments except prior to applying the current, the still fluid bulk was

FIG. 3. Normalized turbulent kinetic energy $e/\langle e \rangle_{TA}$ as a function of normalized time t/T_e (with $T_e = D_m / \langle u' \rangle_{TA}$, using the width of the flat bar magnets as the characteristic forcing length scale). From top to bottom the panels correspond to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively.

spun up to solid-body rotation at a (constant) rate Ω . To ensure that the fluid was in solid-body rotation at $t=t_0$, spin-up times amounted to typically $5T_E$, with the Ekman time scale T_E defined as $T_E = L_z / \sqrt{\nu \Omega}$. For reference, all relevant Ekman time scales are summarized in Table II. The corresponding values of the Ekman number, $Ek = \nu / (\Omega L_z^2)$, and the thickness of the Ekman boundary layers, $d_E \equiv L_z \sqrt{Ek} = \sqrt{\nu / \Omega}$, are also listed in Table II.

The principle of stereoscopy enables one to reconstruct a single 3C displacement vector from two 2C displacement vectors. The 2C displacement vectors are obtained from two different PIV calculations, which in the present algorithm are performed in physical space. This requires prior backprojection of the original images to physical space, also referred to as “dewarping.” After each experiment, the raw image sequences were dewarped with sample sizes optimized per height. The present implementation of SPIV uses mapping functions based on multiplane polynomials, with the necessary coefficients obtained from a straightforward calibration procedure that consists of recording a well-defined grid pattern at several out-of-plane positions resolving the light sheet volume. The advantage of this empirical approach is its ability to correct (depending on the polynomial order) for various optical distortions resulting from perspective projection, lens aberrations, and index of refraction changes that are not due to turbulence or sharp optical interfaces. Dewarped images forming a PIV image pair were subsequently evaluated with PIV using square interrogation windows of size 32×32 pixels and a 50% window overlap in both in-plane directions. The recovered 2D2C displacement maps thus consist of roughly 50^2 PIV nodes. The 2D2C maps were slightly conditioned with a vector median filter³⁰ before reconstructing the out-of-plane velocity component by triangulation³¹—this technique was also used to correct for light sheet misalignment. After 3C reconstruction, each

FIG. 4. (Color) Turbulent velocity field at normalized time $t/T_e = 20$. From top to bottom, the panels correspond to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively. Vectors show normalized in-plane turbulent velocity $(u_1, u_2)(\mathbf{x}) / \langle u' \rangle_{TA}$. Colors show normalized out-of-plane velocity $u_3(\mathbf{x}) / \langle u' \rangle_{TA}$.

displacement vector was normalized by the PIV time delay in order to obtain the corresponding velocity vector. Eventually, we recovered a time series of almost 10 000 succeeding 2D3C velocity fields, each consisting of approximately 2500 3C velocity vectors—verified to be more than sufficient to derive reliable spatial and temporal statistics from.

FIG. 5. Panels on the left show the normalized time evolution of the principal invariants II (solid line) and III (dashed line) of the Reynolds stress anisotropy tensor **b**. Panels on the right show the invariant map (II, III). From top to bottom, the panels correspond to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively.

III. REFERENCE EXPERIMENTS

A. Statistical steadiness and homogeneity of the turbulent flow

Experiments without background rotation have been carried out for parameter values $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$ to investigate the vertical decay of turbulence in our container. The mean flow in these experiments is defined according to $U_i(\mathbf{x}) \equiv \langle \tilde{u}_i(\mathbf{x}, t) \rangle_{\text{TA}}$, where the instantaneous flow field $\tilde{u}_i(\mathbf{x}, t)$ is measured with SPIV and $\langle \cdot \rangle_{\text{TA}}$ denotes the time average over the statistically steady part (see below) of the full time series of the experiment. The spatially averaged mean flow is defined as $U' \equiv \sqrt{\langle U_i^2(\mathbf{x}) \rangle_{\text{SA}}}$ (here $\langle \cdot \rangle_{\text{SA}}$ denotes spatial averaging). The instantaneous fluctuating part of the flow field is $u_i(\mathbf{x}, t) \equiv \tilde{u}_i(\mathbf{x}, t) - U_i(\mathbf{x})$ and the spatially averaged rms of the fluctuating velocity

field is denoted by $u'(t) \equiv \sqrt{\langle u_i^2(\mathbf{x}, t) \rangle_{\text{SA}}}$. When we speak of the time-averaged rms fluctuating velocity we refer to $\langle u' \rangle_{\text{TA}} \equiv \langle \sqrt{\langle u_i^2(\mathbf{x}, t) \rangle_{\text{SA}}} \rangle_{\text{TA}}$.

A mean flow pattern is obtained in the horizontal plane at $H=20$ mm (just above the magnet array) reflecting the magnet arrangement (shown in Fig. 2). Further away from the forcing ($H=50$ mm) the pattern is less pronounced. At some point ($H=100$ mm) no pattern is seen at all. The relative importance of mean flow and fluctuating flow can be inferred from the ratio $U'/\langle u' \rangle_{\text{TA}}$, see Table III. Although stronger close to the forcing the mean flow remains smaller than the fluctuating flow for any of the heights considered.

The kinetic energy and dissipation rate provide important global measures of the state of the turbulence. Let the turbulent kinetic energy at a given time t be defined by

FIG. 6. (a) Time-averaged total temporal velocity correlation coefficient $\langle \rho_{ii} \rangle_{TA}$ as a function of normalized correlation time τ/T_e . (b) Like panel (a) but without normalization of the correlation time. [(c) and (d)] Like panel (a) but now for the temporal velocity correlation coefficients $\langle \rho_{aa} \rangle_{TA}$ with $\alpha=1$ and 3, respectively. The solid, dashed, and dotted lines correspond to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively.

$e \equiv \frac{1}{2} u'^2$ and the dissipation rate by $\epsilon \equiv 2\nu \langle s_{ij} s_{ij} \rangle_{SA}$ (adopting Einstein's index convention), with $s_{ij}(\mathbf{x}, t)$ as a component of the (symmetric) strain rate tensor.

The temporal evolution of e and ϵ reveal that statistically steady turbulence is achieved. As an example, we show in Fig. 3 the time evolutions of e at several heights. The statistically steady state is established within roughly ten eddy turn-over times for $H=20$ and 50 mm. For $H=100$ mm, only two additional eddy turn-over times are required to establish a statistically steady state. These data support the use of the temporal average $\langle \cdot \rangle_{TA}$ to approximate the ensemble average $\langle \cdot \rangle$.

In an attempt to gain insight into the homogeneity of the turbulent flow, we have extracted velocity time series at several locations and generated three different scatter plots, viz., (u_2, u_1) , (u_3, u_1) , and (u_3, u_2) . These scatter plots should be point symmetric with respect to the origin if the turbulence is 3D and isotropic. Moreover, homogeneity implies independence of the scatter plots of the sampling location. The measured scatter plots show that the turbulence is approximately isotropic for each of the considered sampling points and, moreover, suggest that the turbulence is within good approximation spatially homogeneous, within each measurement

plane at least. For the remainder of this section, it is therefore assumed that the ensemble average $\langle \cdot \rangle$ may also be approximated by the spatial average $\langle \cdot \rangle_{SA}$.

B. Vertical decay

We have shown that statistically steady turbulence is established after merely ten eddy turn-over times (Sec. III A). A reliable qualitative impression of the turbulent velocity field at a given height H may therefore be derived from a single velocity field snapshot, provided it is taken during the statistically steady state. Figure 4 shows velocity field snapshots at normalized time $t/T_e=20$ for $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$. We define the eddy turn-over time as $T_e = D_m / \langle u' \rangle_{TA}$, using the width of the flat bar magnets ($D_m = 40$ mm) as the characteristic forcing length scale $\mathcal{L}$. Figure 4 suggests that in-plane motion is most abundant and least localized in space near the forcing. Furthermore, vertical motion in the positive $\mathbf{e}_3$ -direction seems more likely than it does in the negative $\mathbf{e}_3$ -direction. This behavior is probably strongly related to the electromagnetic forcing which mainly injects energy in the lower part of the fluid bulk. The background colors reveal that the vertical velocity

TABLE IV. Characteristic time scales for a selection of parameter triplets $\{H, \Omega, l\}$. The eddy turn-over time is defined as $T_e \equiv D_m / \langle u' \rangle_{TA}$, with D_m the width of the flat bar magnets and $\langle u' \rangle_{TA}$ the time-averaged rms fluctuating velocity. The rotational time scale is defined as $T_\Omega \equiv 2\pi / \Omega$, with Ω as the rotation rate. The integral correlation time scale is defined by $T \equiv \int_0^\infty \langle \rho_{ii} \rangle_{TA} d\tau$. The Taylor-based correlation time scale $\tau_{\lambda, \rho}$ has been derived from a parabolic fit of the initial behavior of the time-averaged temporal velocity correlation coefficient $\langle \rho_{ii} \rangle_{TA}(\tau)$. The Kolmogorov time scale is defined by $\tau_\eta \equiv \sqrt{\nu / \langle \epsilon \rangle_{TA}}$, with ν as the kinematic viscosity and ϵ as the dissipation rate (see caption Table III).

$\{H, \Omega, l\}$	T_e (s)	T_Ω (s)	$\frac{T}{T_e}$	$\frac{T}{T_\Omega}$	$\frac{\tau_{\lambda, \rho}}{T_e}$	$\frac{\tau_{\lambda, \rho}}{T_\Omega}$	$\frac{\tau_\eta}{T_e}$	$\frac{\tau_\eta}{T_\Omega}$
$\{20, 0, 4\}$	3.2	...	1.1 ± 0.1^a	...	0.80 ± 0.05^b	...	0.11	...
$\{50, 0, 4\}$	4.3	...	1.1 ± 0.1	...	0.60 ± 0.05	...	0.11	...
$\{100, 0, 4\}$	11.3	...	1.4 ± 0.1	...	0.40 ± 0.03	...	0.12	...
$\{50, 1, 4\}$	4.2	6.28 ± 0.03^c	7.2 ± 0.3	4.8 ± 0.2	2.00 ± 0.05	1.3 ± 0.03	0.21	0.14
$\{50, 5, 4\}$	6.2	1.257 ± 0.006	3.6 ± 0.3	18 ± 1.5	2.4 ± 0.1	12 ± 0.5	0.19	0.92
$\{50, 10, 4\}$	8.3	0.628 ± 0.003	4.4 ± 0.3	58 ± 4.2	3.0 ± 0.15	40 ± 2	0.15	2.0

^aError margins based on the estimated discretization error.

^bError margins based on the sensitivity of the fitting parameter.

^cError margins based on the rotation rate error $\Delta\Omega = 0.005 \Omega$.

u_3 deviates more from the time-averaged rms fluctuating velocity $\langle u' \rangle_{TA}$ further away from the forcing. These observations suggest that the probability distribution function $p(u_3)$ becomes more positively skewed further away from the forcing, i.e., extreme events in u_3 become more likely as the turbulence decays in the positive e_3 -direction. The latter inference is indeed confirmed by pdfs (probability distribution function) of the velocity field (discussed in Sec. III D).

Next, we consider the evolution of the kinetic energy e . Figure 3 shows this quantity (normalized by their time average) for $\{H, \Omega, l\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$. Further away from the forcing, strong deviations from the time average occur for short periods of time. Combining this result with the earlier observation of spatially localized regions of strong velocity fluctuations, particularly for $H=50$ mm and $H=100$ mm, we conclude that the turbulence shows more extreme velocity fluctuations with increasing distance to the forcing.

The Reynolds stress anisotropy tensor allows us to quantify the degree of anisotropy of a turbulent flow. It is defined as the deviatoric part of the Reynolds stress tensor, i.e., $b_{ij} \equiv (R_{ij}/R_{ii}) - (1/3)\delta_{ij}$, with δ_{ij} as the second-order Kronecker tensor and R_{ij} as the components of the Reynolds stress tensor. Obviously, $\mathbf{b}$ is symmetric and it has zero trace ($b_{ii}=0$). Furthermore, the off-diagonal components of $\mathbf{b}$ are all zero in case of isotropy so that any nonzero off-diagonal components of $\mathbf{b}$ may serve as an anisotropy measure. Insight into the degree of anisotropy of the turbulence is gained by plotting the principal invariants of the Reynolds stress anisotropy tensor,

$$\text{II} \equiv \frac{1}{2}b_{ij}b_{ji}, \quad \text{III} \equiv \frac{1}{6}b_{ij}b_{jk}b_{ki}, \quad (1)$$

in accordance with Ref. 32. Consider the time evolution of II and III in the left column of panels in Fig. 5. From top to bottom, the panels correspond to $\{H, \Omega, l\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$. In all cases, both invariants fluctuate randomly about a time-averaged value that lies close to zero: $(\langle \text{II} \rangle_{TA}, \langle \text{III} \rangle_{TA}) = (0.01, 1 \times 10^{-4})$, $(0.01, 1 \times 10^{-4})$ and

$(0.02, 2 \times 10^{-4})$ for $H=20, 50$, and 100 mm, respectively. The smallest fluctuations, but also the smallest values for $\langle \text{II} \rangle_{TA}$ and $\langle \text{III} \rangle_{TA}$, are found for $H=50$ mm. The anisotropy invariant map (III, II) is shown in the right column of panels in Fig. 5. This map describes the shape of the Reynolds stress tensor and one can infer from it the anisotropy of a turbulent flow. The range of each axis matches the range of the invariants. The limited ranges of the invariants define a triangle (dashed-dotted line) within which all realizable Reynolds stress invariants must lie.^{33,34} Of importance here is to know that the origin corresponds to 3C isotropic turbulence. In each reference case, the invariant coordinates (III, II) adopt values that lie close to the origin. Especially at $H=50$ mm the turbulence state is best described as 3C isotropic.

C. Temporal and spatial correlation functions

Next, we study the height dependence of the time-averaged temporal velocity correlation coefficients

$$\langle \rho_{ij} \rangle_{TA}(\tau) = \left\langle \frac{\langle u_i(\mathbf{x}, t) u_j(\mathbf{x}, t + \tau) \rangle_{SA}}{\langle u_i(\mathbf{x}, t) u_j(\mathbf{x}, t) \rangle_{SA}} \right\rangle_{TA}, \quad (2)$$

where τ is the correlation time. Here, we assume that the turbulence is approximately homogeneous and statistically steady. The diagonal components $\langle \rho_{\alpha\alpha} \rangle_{TA}$ and the trace $\langle \rho_{ii} \rangle_{TA}$ have been computed for a range of correlation times, $\tau \in [0, \tau_{\text{mx}}]$, with $\tau_{\text{mx}} > 15T_e$.

Figure 6(a) shows $\langle \rho_{ii} \rangle_{TA}$ as a function of the normalized correlation time τ/T_e for $\{H, \Omega, l\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively [see also Fig. 6(b) where these functions are plotted as function of τ]. The curves for $H=50$ and 100 mm show a reasonable collapse, suggesting approximately self-similar vertical decay away from the forcing. The curve for $H=20$ mm behaves differently for $\tau \geq 3T_e$ which is probably due to the continuous forcing of the lowest part of the fluid bulk ($H < 50$ mm). Similar trends are found for the coefficients $\langle \rho_{\alpha\alpha} \rangle_{TA}$, see Figs. 6(c) and 6(d) ($\langle \rho_{22} \rangle_{TA}$ not

FIG. 7. Time-averaged spatial correlation coefficients $\langle \rho_{ij}^k \rangle_{TA}$ as a function of normalized correlation length r_k/D_m , with $k=1, 2$. Left, center, and right column of panels display longitudinal correlation coefficients $\langle \rho_{11}^1 \rangle_{TA}$ and $\langle \rho_{22}^2 \rangle_{TA}$; lateral correlation coefficients $\langle \rho_{11}^1 \rangle_{TA}$ and $\langle \rho_{22}^2 \rangle_{TA}$; and lateral correlation coefficients $\langle \rho_{33}^1 \rangle_{TA}$ and $\langle \rho_{33}^2 \rangle_{TA}$, respectively. The solid and dashed lines represent fit results as explained in the text. The first, second, and third row of panels correspond to $\{H, \Omega, l\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively.

shown but similar to $\langle \rho_{11} \rangle_{TA}$). Additionally, they reveal that the out-of-plane velocity is somewhat weaker correlated in time than the in-plane velocities, meaning that slight anisotropy is part of our forcing.

One can define an integral time scale according to $T \equiv \int_0^\infty \langle \rho_{ii} \rangle_{TA} d\tau$. Table IV shows that the numerical values of T and T_e are almost equal, confirming T_e as a meaningful integral time scale. Furthermore, we report in the same table a Taylor-based time scale $\tau_{\lambda, \rho}$ (derived from $\langle \rho_{ii} \rangle_{TA}$) and the Kolmogorov time scale $\tau_\eta \equiv \sqrt{\nu / \langle \epsilon \rangle_{TA}}$. The Taylor-based time scale decreases with increasing H , not in support of a self-similar vertical decay. The Kolmogorov time scale is nearly independent of H .

We have also studied the velocity correlation in space to gain more insight into the (an)isotropy of the flow. For as far

as accessible through SPIV, we have computed the time-averaged spatial velocity correlation coefficients

$$\langle \rho_{ij}^k \rangle_{TA}(\mathbf{r}\mathbf{e}_k) = \left\langle \frac{\langle u_i(\mathbf{x}, t) u_j(\mathbf{x} + \mathbf{r}\mathbf{e}_k, t) \rangle_{SA}}{\langle u_i(\mathbf{x}, t) u_j(\mathbf{x}, t) \rangle_{SA}} \right\rangle_{TA} \quad (3)$$

exploiting the earlier observation that the turbulence can, in good approximation, be considered as approximately homogeneous and statistically steady. The following discussion is restricted to the diagonal components $\langle \rho_{\alpha\alpha}^1 \rangle_{TA}$ and $\langle \rho_{\alpha\alpha}^2 \rangle_{TA}$.

Figure 7 shows the longitudinal and lateral autocorrelations $\langle \rho_{\alpha\alpha}^k \rangle_{TA}$ and $\langle \rho_{\alpha\alpha}^k \rangle_{TA}$ ($k \neq \alpha$). The correlation distance, $r_k \equiv |\mathbf{r}\mathbf{e}_k|$, has been normalized by the characteristic forcing

TABLE V. Relevant dimensionless numbers for a selection of parameter triplets $\{H, \Omega, I\}$. The integral and Taylor-scale based Reynolds numbers are defined as $\text{Re} = \langle u' \rangle_{\text{TA}} \mathcal{L} / \nu$ and $\text{Re}_\lambda = \langle u' \rangle_{\text{TA}} \langle \lambda \rangle_{\text{TA}} / \nu$, respectively, with $u' \equiv \sqrt{\langle u_i^2 \rangle_{\text{SA}}}$ as the spatially averaged rms of the fluctuating velocity field and ν the kinematic viscosity. The integral length scale is $\mathcal{L} = D_m$, with D_m as the width of the flat bar magnets. The Taylor microscale is $\lambda \equiv u' \sqrt{15 \nu / \epsilon}$, with ϵ as the dissipation rate (see caption Table III). The macroscopic and Taylor-scale based Rossby number are $\text{Ro} = \langle u' \rangle_{\text{TA}} / (2\Omega \mathcal{L})$ and $\text{Ro}_\lambda = \langle u' \rangle_{\text{TA}} / (2\Omega \langle \lambda \rangle_{\text{TA}})$, respectively, with Ω as the rotation rate.

$\{H, \Omega, I\}$	Re	Re_λ	Ro	Ro_λ
{20,0,4}	387	158	...	...
{50,0,4}	285	113	...	...
{100,0,4}	109	46	...	...
{50,1,4}	298	239	0.12	0.15
{50,5,4}	202	141	0.02	0.02
{50,10,4}	150	88	0.01	0.01

length scale D_m . From top to bottom, the rows of panels correspond to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively. For each height, the longitudinal autocorrelations (left column of panels) collapse. The lateral in-plane autocorrelations (center column of panels) and the lateral out-of-plane autocorrelations (right column of panels) do so only for $H=20$ and 50 mm. Furthermore, the lateral autocorrelations fall off more rapidly with increasing correlation distance than the longitudinal autocorrelations. The latter fall off exponentially even for small r , reflecting the relatively low-Reynolds number of the turbulence (Table V). In isotropic turbulence, the lateral autocorrelation function $g(r)$ is related to the longitudinal autocorrelation function $f(r)$ according to $g(r) = f(r) + \frac{1}{2} r f'(r)$, see Ref. 35. This relationship has been verified for each parameter triplet by fitting the function $f(r) = \exp(ar)$, with a as some fitting constant, through the data points describing $\langle \rho_{11}^1 \rangle_{\text{TA}}(r)$. The function $g(r)$ immediately follows by substituting the resulting fit in the above relationship. Only the turbulence at $H=50$ mm shows a convincing collapse between data points and fit results for any of the investigated correlation coefficients. This provides further support that the turbulence at this particular vertical height is approximately 3D isotropic.

Numerical values of various relevant length scales are summarized in Table VI. The Kolmogorov length scale $\eta \equiv (\nu^3 / \epsilon)^{1/4}$ is of the order of $\langle \eta \rangle_{\text{TA}} \approx 1$ mm, and increases with increasing distance to the forcing due to the vertical decay. The Taylor length scale $\langle \lambda \rangle_{\text{TA}}$ (with $\lambda \equiv u' \sqrt{15 \nu / \epsilon}$) seems almost constant for $H \leq 100$ mm. The integral length scales, defined according to $\langle L_{\alpha\alpha}^k(\mathbf{x}, t) \rangle_{\text{TA}} \equiv \int_0^\infty \langle \rho_{\alpha\alpha}^k(\mathbf{x}, \mathbf{r}\mathbf{e}_k, t) \rangle_{\text{TA}} dr$, have been computed by numerical integration of $\langle \rho_{\alpha\alpha}^k \rangle_{\text{TA}}$ over the available range of correlation distances r_k . The numerical values indicate that the turbulence is not fully developed with a well-established energy cascade over several decades.

D. Probability distribution functions of velocity

We conclude our investigation on the vertical decay of the nonrotating turbulence with a discussion on time-averaged pdfs of the velocity field, $\langle p(u_i) \rangle_{\text{TA}}$. Each pdf is normalized by its time-averaged standard deviation $\langle \sigma(u_i) \rangle_{\text{TA}}$, with $i=1, 2, 3$. A single time-averaged pdf is obtained by dividing the velocity data of 9000 (successive) velocity field samples over 101 equally sized bins, with the width of a single bin equal to $10\sigma(u_i)$ divided by the total number of bins. Only the statistically steady state is considered. We have verified that the resulting pdfs had sufficiently converged. The time-averaged pdfs for parameter triplets $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, are presented in Fig. 8. From left to right, the abscissa and ordinate show $u_i / \langle \sigma(u_i) \rangle_{\text{TA}}$ and $\langle p(u_i) \rangle_{\text{TA}} / \langle \sigma(u_i) \rangle_{\text{TA}}$, respectively, for $i=1, 2, 3$. None of the pdfs have Gaussian tails. In particular, exponential tails are found for $H=100$ mm. Comparison of the normalized pdfs of a given triplet reveals that the out-of-plane velocity behaves somewhat differently than the in-plane velocities, indicating slight anisotropy. The normalized pdfs are affected by the vertical decay, which for that reason cannot be considered self-similar. The normalized pdf of u_3 becomes flatter and more positively skewed as the vertical decay progresses. The nonskewed pdfs of the different velocity components collapse best in the case $\{50, 0, 4\}$, supporting the earlier classification of the corresponding turbulence state

TABLE VI. Relevant time-averaged correlation length scales for a selection of parameter triplets $\{H, \Omega, I\}$. The integral length scales $L_{\alpha\alpha}^k$ are defined in Sec. III C. The Kolmogorov length scale and the Taylor microscale are defined as $\eta \equiv (\nu^3 / \epsilon)^{1/4}$ and $\lambda \equiv u' \sqrt{15 \nu / \epsilon}$, respectively, with ν as the kinematic viscosity, ϵ as the dissipation rate (see caption Table III), and $u' \equiv \sqrt{\langle u_i^2 \rangle_{\text{SA}}}$ as the spatially averaged rms of the fluctuating velocity field.

$\{H, \Omega, I\}$	$\langle L_{11}^1 \rangle_{\text{TA}}$ (mm)	$\langle L_{22}^2 \rangle_{\text{TA}}$ (mm)	$\langle L_{22}^1 \rangle_{\text{TA}}$ (mm)	$\langle L_{11}^2 \rangle_{\text{TA}}$ (mm)	$\langle L_{33}^1 \rangle_{\text{TA}}$ (mm)	$\langle L_{33}^2 \rangle_{\text{TA}}$ (mm)	$\langle \lambda \rangle_{\text{TA}}$ (mm)	$\langle \eta \rangle_{\text{TA}}$ (mm)
{20,0,4}	21	21	10.6	11.0	9.1	10.2	16.4	0.66
{50,0,4}	19	20	12.4	11.9	12.0	10.5	16.1	0.76
{100,0,4}	22	23	21	14.2	14.9	9.0	16.9	1.29
{50,1,4}	53	48	16.7	23	11.3	11.6	32.0	1.07
{50,5,4}	42	36	18.9	20	6.8	6.3	28.0	1.22
{50,10,4}	35	39	13.5	13.8	5.5	5.2	23.5	1.29

FIG. 8. From left to right are shown the normalized time-averaged pdfs of velocity $\langle p(u_i) \rangle_{TA} / \langle \sigma(u_i) \rangle_{TA}$, for $i=1,2,3$, with $\langle \sigma(u_i) \rangle_{TA}$ as the standard deviation of the time-averaged pdf $\langle p(u_i) \rangle_{TA}$. The abscissa displays $u_i / \langle \sigma(u_i) \rangle_{TA}$. The symbols $+$, $\times$, and $\square$ correspond to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively. The solid line shows a Gaussian fit for the data corresponding to $\{H, \Omega, I\} = \{20, 0, 4\}$.

as 3D isotropic. For a clearer visualization of the turbulence state—at least for as far as (an)isotropy is concerned—we computed the 2D pdfs of velocity, $\langle p(u_i, u_j) \rangle_{TA} / \langle u' \rangle_{TA}^2$. The results for the triplet $\{50, 0, 4\}$ are shown on the top row of Fig. 15 (to facilitate comparison with the results from rotating turbulence). These 2D pdfs clearly indicate that the turbulence is 3D isotropic.

E. Summary of the reference experiments

The reference experiments have shown that the electromagnetically generated turbulent flow is statistically steady, isotropic, and homogeneous, especially at $H=50$ mm where an approximate balance is found between forcing and dissipation. Near the bottom plate the anisotropic forcing dominates whereas far from the forcing region, dissipation dominates. The velocities are found to be non-Gaussian distributed. Moreover, contradicting with Long³⁶ the integral length scale remains approximately constant with increasing height and Re_λ decreases with increasing distance to the bottom plate.

IV. ROTATING EXPERIMENTS

We now study the effects of rotation on the turbulent flow generated in our setup. We have shown in Sec. III that in the absence of rotation the turbulence at $H=50$ mm is almost 3D isotropic. The following discussion is therefore mainly concerned with parameter triplets $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$.

Statistical steadiness, homogeneity, and reproducibility have been studied for the rotating experiments in the same way as for the reference experiments. If time is expressed in the number of eddy turn-over times, the following results are obtained. A statistically steady state is reached after 15–20 eddy turn-over times (depending on the rotation rate), considerably longer than in the nonrotating case. It must be noted, however, that the turbulent kinetic energy shows stronger deviations from the temporal mean for longer periods of time. The outcome of scatter diagrams (u_2, u_1) and (u_3, u_1) favors spatial homogeneity (in each of the considered measurement planes). In the rotating case, however, the

(u_3, u_1) -plots reveal strong suppression of the out-of-plane velocity. This reflects the anticipated anisotropy that is characteristic of rotating turbulence. More details can be found elsewhere.³⁷ The reproducibility of the rotating experiments, finally, is similar to that of the reference experiments.

A. Characterization of parameter triplets $\{50, \Omega, 4\}$

The rotating experiments with $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$ reveal that the mean flow is affected by rotation. Higher rotation rates imply stronger suppression of the out-of-plane velocity. The in-plane mean flow for rotating case $\{50, 10, 4\}$ is highly organized such as reference case $\{20, 0, 4\}$. For triplets $\{50, 1, 4\}$ and $\{50, 5, 4\}$, the mean flow seems less well organized. The degree of organization of the mean flow might be related to the inertial wave motion. After all, inertial waves transport more energy along the rotation vector in the same amount of time as the rotation rate increases. The impact of the Coriolis force is thus more pronounced at higher rotation rates, and accordingly the organized pattern of vortices is expected to be better preserved. (Note that the group velocity of the fastest inertial waves is $2\Omega/k_f$ with the forcing wave number $k_f \approx 2\pi/D_m$. When $\Omega=10$ rad/s, we obtain a speed of 127 mm/s so it takes about 4 s for these waves to travel up and down the tank.)

Figure 9 shows snapshots of the turbulent velocity field at time $t/T_e=20.0$. The in-plane velocity field is now much smoother than without rotation (compare case $\{50, 0, 4\}$ in Fig. 4). The out-of-plane velocities are (strongly) damped by the rotation. Moreover, in the cases $\{50, 5, 4\}$ and $\{50, 10, 4\}$ the out-of-plane velocities appear highly localized in space. The analysis of the time-evolving velocity fields reveals that the out-of-plane velocities rapidly decorrelate, both in space and time. The in-plane velocity field, on the other hand, is characterized by vortical structures that retain their shape for more than ten eddy turn-over times while being slowly advected by the surrounding structures.

Figure 10 displays the time evolution of the normalized kinetic energy for $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$. The physical time is normalized by the eddy turn-over time to enable comparison with the reference experi-

FIG. 9. (Color) Turbulent velocity field at normalized time $t/T_e = 20$. From top to bottom, the panels correspond to $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively. Vectors show normalized in-plane turbulent velocity $(u_1, u_2)(\mathbf{x})/\langle u' \rangle_{TA}$. Colors show normalized out-of-plane turbulent velocity $u_3(\mathbf{x})/\langle u' \rangle_{TA}$.

ment $\{50, 0, 4\}$ (shown once more in the bottom panel). The signals smoothen with increasing rotational speed. The latter could be better illustrated by normalizing the physical time by the rotational time scale $T_\Omega \equiv 2\pi/\Omega$ instead of T_e . This alternative scaling is not used at this time though because it complicates the comparison with nonrotating experiments where such a scaling is not possible. The

FIG. 10. Normalized turbulent kinetic energy $e/\langle e \rangle_{TA}$ as a function of normalized time t/T_e . The first three panels correspond to rotating experiments $\{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively. The bottom panel shows reference experiment $\{50, 0, 4\}$.

numerical values for $\langle e \rangle_{TA}$ and the temporal averages of $e_\alpha(t) \equiv \langle u_\alpha^2(\mathbf{x}, t) \rangle_{SA}/2$, with $\alpha = 1, 2, 3$, are reported in Table VII and those for $\langle e \rangle_{TA}$ are reported in Table III. The values for $\langle e_\alpha \rangle_{TA}$ (with $\alpha = 1, 2, 3$) show that especially the out-of-plane velocity is reduced by rotation. Higher rotation rates imply stronger reduction in the dissipation rate, a well-known feature of rotating turbulent flows. In order to further quantify this anisotropy it is necessary to consider other quantities, e.g., the Reynolds stress anisotropy tensor, or correlation coefficients.

The temporal evolution of the Reynolds stress anisotropy tensor $\mathbf{b}$ for $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$ reveals a strong correlation between the in-plane velocity components u_1 and u_2 . The observed behavior of $\mathbf{b}$ corresponds to an approximately 2C turbulent flow, i.e., $0 \leq |u_3| \ll |u_1|, |u_2|$. The principal invariants of $\mathbf{b}$ allow for a

TABLE VII. Time-averaged turbulent kinetic energy for a selection of parameter triplets $\{H, \Omega, I\}$. The instantaneous turbulent kinetic energies are defined by $e_\alpha(t) \equiv \langle u_\alpha^2(\mathbf{x}, t) \rangle_{SA}/2$, with $\alpha = 1, 2, 3$ and $e(t) \equiv \langle u^2(\mathbf{x}, t) \rangle_{SA}/2$, respectively.

$\{H, \Omega, I\}$	$\langle e_1 \rangle_{TA}$ (mm ² s ⁻²)	$\langle e_2 \rangle_{TA}$ (mm ² s ⁻²)	$\langle e_3 \rangle_{TA}$ (mm ² s ⁻²)	$\langle e \rangle_{TA}$ (mm ² s ⁻²)
$\{20, 0, 4\}$	29	27	22	79
$\{50, 0, 4\}$	13.6	14.9	14.2	43
$\{100, 0, 4\}$	1.53	2.1	2.6	6.3
$\{50, 1, 4\}$	17.4	20	6.6	45
$\{50, 5, 4\}$	9.3	8.9	1.9	21
$\{50, 10, 4\}$	4.6	5.4	1.40	11.5

FIG. 11. Panels on the left display the evolution in normalized time t/T_e of the principal invariants II (solid line) and III (dashed line) of the Reynolds stress anisotropy tensor **b**. Panels on the right show scatter diagrams (II, III). From top to bottom, the rows of panels correspond to $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively.

better classification of the state of the rotating turbulence. Figure 11 displays the evolution of invariants II and III in invariant coordinate space for parameter triplets $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$. Consider the left panels first. The above-described behavior of **b** implies $\langle \text{II} \rangle_{\text{TA}} \geq 0$ and $\langle \text{III} \rangle_{\text{TA}} \approx 0$, consistent with Fig. 11. The numerical values for the nonrotating and three rotating cases are $(\langle \text{II} \rangle_{\text{TA}}, \langle \text{III} \rangle_{\text{TA}}) = (0.01, 0.1 \times 10^{-3})$, $(0.06, 1.7 \times 10^{-3})$, $(0.08, 1.9 \times 10^{-3})$, and $(0.06, 0.7 \times 10^{-3})$ for $\Omega = 0, 1, 5$, and 10 rad/s, respectively. A similar approach is followed to characterize the anisotropy of turbulent rotating convection, see Fig. 3 in Ref. 38. However, the trajectory in the Lumley map in the present investigation is found to be distinctly different compared to the experimentally obtained data from Kunnen *et al.*³⁸

Interestingly, both invariants start to oscillate in time as

rotation increases—the oscillations are clearest for the experiment with $\{H, \Omega, I\} = \{50, 5, 4\}$. The frequency spectra of II and III have been determined to quantify the period of the oscillations. For $\{H, \Omega, I\} = \{50, 5, 4\}$, with $\Omega/(2\pi) \approx 0.8$ Hz, clear peaks are observed at 0.8 and 1.6 Hz. For the $\{H, \Omega, I\} = \{50, 10, 4\}$, with $\Omega/(2\pi) \approx 1.6$ Hz, we found peaks at 1.6 and 3.2 Hz. No clear peaks were observed for the triplet $\{50, 1, 4\}$. The period of the oscillations is thus proportional to the Rossby number $\text{Ro} = \langle u' \rangle_{\text{TA}} / (2\Omega\mathcal{L})$, with $\mathcal{L} = D_m$ so these slow dynamics are essentially governed by resonant triads.²³ Now consider the right panels in Fig. 11. The rotating turbulence is still homogeneous (within the horizontal plane) but no longer 3C isotropic. Instead, based on the invariant maps one tends to classify the rotating turbulence as a 3D2C flow, consistent with other numerical simulations^{22,29,39} and laboratory experiments.^{3,16}

FIG. 12. (a) Time-averaged total temporal velocity correlation coefficient $\langle \rho_{ii} \rangle_{TA}$ as a function of normalized correlation time τ/T_Ω , with $T_\Omega = 2\pi/\Omega$. (b) Like panel (a) but without normalization of the correlation time. [(c) and (d)] Like panel (a) but now for the temporal velocity correlation coefficients $\langle \rho_{aa} \rangle_{TA}$ with $\alpha=1$ and 3, respectively. The solid, dashed, and dotted lines correspond to $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively.

B. Temporal and spatial correlation functions

Figure 12 shows the time-averaged temporal velocity correlation coefficients $\langle \rho_{ii} \rangle_{TA}$ and $\langle \rho_{aa} \rangle_{TA}$ (with $\alpha=1$ and 3) for $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$. The correlation time τ is now scaled by the rotational time scale T_Ω instead of the eddy turn-over time scale T_e . Clearly, higher rotation rates imply (relatively) stronger temporal velocity correlation $\langle \rho_{ii} \rangle_{TA}$. Moreover, the horizontal correlation is much stronger than the vertical correlation, in agreement with the qualitative results discussed earlier in this section. We note that the kink in $\langle \rho_{33} \rangle_{TA}$ at $\tau/T_\Omega \approx 4$ is related to the limited temporal resolution of the currently available SPIV data, insufficient for resolving the behavior of $\langle \rho_{33} \rangle_{TA}$ for correlation times $\tau \lesssim 4T_\Omega$.

As with the reference experiments, Taylor-based time scales and integral time scales have been derived from $\langle \rho_{ii} \rangle_{TA}$, see Table IV. The time scales are significantly larger in the rotating experiments than in the reference experiment. This is in agreement with the qualitative observation that flow structures live longer in the presence of rotation than in absence of rotation.

Figure 13 shows the longitudinal and lateral correlation coefficients. From top to bottom, the rows of panels correspond to parameter triplets $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively. Comparison between rotating case $\{50, 1, 4\}$ and reference case $\{50, 0, 4\}$ (Fig. 7, second row) reveals that background rotation considerably increases the autocorrelations. Moreover, the autocorrelations no longer collapse, indicating rotation-induced anisotropy. Comparison of cases $\{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$ shows that the spatial velocity correlation (in particular, $\langle \rho_{33}^k \rangle_{TA}$) decreases as the rotation rate increases. (We note that $\langle \rho_{33}^k \rangle_{TA}$ is not entirely resolved in cases $\{50, 5, 4\}$ and $\{50, 10, 4\}$ for correlation distances smaller than $D_m/4$.) This behavior is related to the fact that the vortex population grows as Ω increases,³ as already mentioned in Sec. I. The behavior of the longitudinal autocorrelations $\langle \rho_{11} \rangle_{TA}$ and $\langle \rho_{22} \rangle_{TA}$ is consistent with the numerical studies by Godeferd and Lollini²⁹ and by Yeung and Zhou²⁸ on forced rotating turbulence with and without vertical confinement, respectively.

C. Probability distribution functions of velocity

The effects of rotation on the (time-averaged) normalized pdfs of the velocity field are shown in Fig. 14 for $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$. The pdfs have been computed exactly the same way as those for the reference experiments. Unlike the reference experiments, however, the pdfs of the in-plane velocities do not fully converge—further convergence cannot be obtained with the current data set. The lack of convergence must be related to the stronger spatial (velocity) correlation in the presence of rotation. Consequently, less *independent* samples are involved in computing the pdfs. The lack of convergence expresses itself by unexpected wiggles in the pdfs of u_1 and u_2 , and any observation derived from these pdfs should be considered with care. Despite the lack of convergence, one might still infer from Fig. 14 that the normalized pdfs of velocity are only weakly affected by the rotation rate.

Before concluding this section, the rotation-induced anisotropy among the Reynolds stresses is quantified in terms of the 2D pdfs of velocity, which is briefly mentioned at the end of Sec. III. Contour plots of $\langle p(u_i, u_j) \rangle_{TA} \langle u'_i \rangle_{TA}^2$ are depicted in Fig. 15. From left to right, the panels correspond to $(i, j) = (1, 2)$, $(1, 3)$, and $(2, 3)$, respectively. From top to bottom, the panels correspond to cases $\{50, 0, 4\}$, $\{50, 1, 4\}$, and $\{50, 5, 4\}$, respectively. Based on these 2D pdfs, the turbulence state was classified as nearly isotropic for reference case $\{50, 0, 4\}$ (Sec. III). Direct comparison with the rotating cases enables one to easily identify the rotation-induced anisotropy.

FIG. 13. Time-averaged spatial correlation coefficients $\langle \rho_{ij}^k \rangle_{TA}$ as a function of normalized correlation length r_k/D_m , with $k=1, 2$. Left, center, and right columns of panels display longitudinal correlation coefficients $\langle \rho_{11}^1 \rangle_{TA}$ and $\langle \rho_{22}^2 \rangle_{TA}$; lateral correlation coefficients $\langle \rho_{22}^1 \rangle_{TA}$ and $\langle \rho_{11}^2 \rangle_{TA}$; and lateral correlation coefficients $\langle \rho_{33}^1 \rangle_{TA}$ and $\langle \rho_{33}^2 \rangle_{TA}$, respectively. From top to bottom, the rows of panels correspond to $\{H, \Omega, l\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively.

D. Vertical dependence in case of rotation

Rotating experiments at $H=20$ and 100 mm were performed to study the vertical decay of the turbulence in the presence of rotation. The results are very similar to those found for $H=50$ mm. It is therefore sufficient to merely report the numerical values for $\langle u' \rangle_{TA}$, $\langle \epsilon \rangle_{TA}$, and other characteristic quantities, see Table III. Comparison of rotating experiments with the same parameter settings except for the height shows that the rotating turbulence is almost independent of the vertical height H . This suggests that the vertical decay observed in the reference case is almost entirely inhibited by rotation. This observation fits well within the context of “cigar-shaped” turbulence. Note that Hopfinger *et al.*³ reported a similar observation in their experiment on rotating grid-generated turbulence, viz., that the turbulent flow field becomes independent of depth for local Rossby numbers smaller than 0.2 .

V. SELF-SIMILARITY ANALYSIS

An important question regarding turbulence is whether or not the statistics are self-similar for a wide range of spatial scales. Self-similarity may be inferred from the scaling of the longitudinal spatial velocity structure functions,

$$S_i^p(l) \equiv \langle |u_i(\mathbf{x} + l\mathbf{e}_i, t) - u_i(\mathbf{x}, t)|^p \rangle, \quad (4)$$

with p as the order of the structure function. The orientation of the spatial separation vector $\mathbf{l}$ has been chosen along the base vectors $\mathbf{e}_i$. It is important to distinguish the presence of power-law scaling of the structure functions from self-similarity. If the turbulence is self-similar then $S_i^p(l) \sim l^{\zeta_p}$, with ζ_p as the varying linearly with p . In particular, if 3D homogeneous isotropic turbulence is self-similar then theories predict the existence of an inertial range with $\zeta_p = p/3$.⁴⁰ When the 3D turbulence is not self-similar power-law scal-

FIG. 14. From left to right are shown the normalized time-averaged pdfs of velocity $\langle p(u_i) \rangle_{TA} \langle \sigma(u_i) \rangle_{TA}$ for $i=1,2,3$, with $\langle \sigma(u_i) \rangle_{TA}$ the standard deviation of the time-averaged pdf $\langle p(u_i) \rangle_{TA}$. The abscissa displays $u_i / \langle \sigma(u_i) \rangle_{TA}$. The symbols $+$, $\times$, and $\square$ correspond to $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively. The solid line shows a Gaussian fit for the data corresponding to $\{H, \Omega, I\} = \{50, 10, 4\}$.

ing is still observed in the inertial range but the scaling exponent ζ_p does not vary linearly with p .

Interestingly, recent experiments on low Rossby number forced turbulence have revealed a self-similar flow with anomalous scaling of the structure functions, $\zeta_p = p/2$.¹³ This implies, with $\zeta_2 = 1$, that the 1D spectrum computed from the

horizontal velocity has the following form: $E(k) \sim k^{-2}$, with k the wave number in a plane perpendicular to the rotation vector. In direct numerical simulation of rotating turbulence with large-scale isotropic forcing, Müller and Thiele⁴¹ found $\zeta_2 \approx 1$ and higher order exponents ζ_p intermediate between $p/2$ and the values usually found in (intermittent) 3D turbu-

FIG. 15. Contour plots of the normalized time-averaged 2D pdfs of velocity, $\langle p(u_i, u_j) \rangle_{TA} \langle u' \rangle_{TA}^2$. From left to right, the panels correspond to $(i, j) = (1, 2)$, $(1, 3)$, and $(2, 3)$, respectively. From top to bottom, the rows of panels correspond to $\{H, \Omega, I\} = \{50, 0, 4\}$, $\{50, 1, 4\}$, and $\{50, 5, 4\}$, respectively.

FIG. 16. Spatial structure functions $\langle S_1^p \rangle_{TA}$ as a function of separation distance r_1/D_m . The symbols +, $\times$, $\square$, and $\circ$ correspond to structure function order $p=1, 2, 3$, and 4 , respectively. From left to right, the top row of panels corresponds to $\{H, \Omega, I\} = \{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively; the bottom row of panels corresponds to $\{H, \Omega, I\} = \{50, 1, 4\}$, $\{50, 5, 4\}$, and $\{50, 10, 4\}$, respectively.

lence. In decaying rotating turbulence experiments by Seiwert *et al.*²⁴ the scaling of the longitudinal velocity structure function is analyzed up to order $p=8$. The exponents at the end of the decay are somewhat larger than those reported by Baroud *et al.*:¹³ $\zeta_p \approx 0.7p$. Intrigued by these results, we have subjected our velocity field data (both reference and rotating experiments) to a self-similarity analysis. We have computed the longitudinal structure functions S_1^p and S_2^p for $p \leq 4$ in the same way as we computed the temporal and spatial velocity correlation functions (Sec. III B). That is, for improving convergence of statistics the ensemble average in Eq. (4) has been approximated by a subsequent spatial and temporal average. S_1^p and S_2^p are found to collapse in all experiments, so that the following discussion can be restricted to S_1^p .

A cautionary remark should be made here when interpreting data from structure function measurements. Depending on the rotation rate or the measurement height in the fluid the Taylor-based Reynolds number is found to be in the range of $90 \leq \text{Re}_\lambda \leq 240$ in the present experiments. This range of Re_λ might be too small to observe descent power-law behavior of the structure functions. As we will see it seems to hamper the presence of structure function scaling in the nonrotating experiments only. It should be noted, however, that the lack of an observed power law does not necessarily preclude self-similarity.

Figure 16 shows S_1^p as a function of l for various parameter triplets $\{H, \Omega, I\}$. The top row of panels corresponds to reference experiments at distinct heights, viz., $\{20, 0, 4\}$, $\{50, 0, 4\}$, and $\{100, 0, 4\}$, respectively. No clear power-law re-

FIG. 17. (a) Scaling of structure function exponents ζ_p with order p for rotating experiments $\{50, 1, 4\}$, $\{50, 5, 4\}$ and $\{50, 10, 4\}$. (b) Like (a) except now ζ_p is scaled by ζ_3 . (c) Like (a) but for higher forcing current $I=8$. All results concern $\langle S_1^p \rangle_{TA}$. The error bars are of the size of the symbols and therefore omitted.

TABLE VIII. Structure function scaling in terms of slope C_ζ for all the rotating experiments.

H (mm)	$I=4$ A			$I=8$ A		
	Ω (rad/s)			Ω (rad/s)		
	1	5	10	1	5	10
20	0.64	0.68	0.54	...	0.75	...
50	0.64	0.70	0.65	...	0.76	0.76
100	0.65	0.68	0.64	...	0.76	0.73

game is observed for any of these triplets for resolved scales smaller than the forcing scale D_m . This is most likely due to the limited Taylor-based Reynolds numbers in the nonrotating experiments. Nevertheless, one may conclude that the behavior of the structure functions obtained for the triplets $\{20,0,4\}$ and $\{50,0,4\}$ does not contradict with the assumption of self-similarity as $\zeta_p = p/3$ seems to be a reasonable approximation for a (very limited) range $r_1/D_m \in [0.4, 0.8]$.

The bottom row of panels in Fig. 16 shows S_1^p as a function of l for the rotating experiments $\{50,1,4\}$, $\{50,5,4\}$, and $\{50,10,4\}$, respectively. A clear power-law region is readily identified for each order p , viz., $r_1/D_m \in [0.375, 1.0]$. The scaling of the corresponding power-law exponents ζ_p with order p has been studied. The result is shown in Fig. 17(a) and numerical values of the estimated slopes are given in Table VIII. A linear (anomalous) scaling of the structure function exponents, i.e., $\zeta_p = C_\zeta p$, is observed for each triplet. This indicates that the rotating turbulence is spatially self-similar within the mentioned range of separation distances. Comparison of the various triplets reveals that C_ζ weakly depends on H , viz., $C_\zeta \in [0.64, 0.70]$ for most triplets except for $\{20,10,4\}$ (an explanation for this discrepancy is lacking at the moment, but it may be related to a coupling between the rotation and the forcing). Basically, we find $C_\zeta \approx 0.64$, 0.70, and 0.64 for $\Omega = 1, 5$, and 10 rad/s, respectively. The slope C_ζ appears to depend weakly on the rotation rate Ω for $I=4$ A, which might be attributed to the limited range of Re_λ , and the measured values are close to the one observed by Seiwert *et al.*,²⁴ $C_\zeta \approx 0.7$ (or $E(k) \sim k^{-2.4}$).

The measured slopes are obviously steeper than the Kolmogorov prediction $C_\zeta = 1/3$. This prediction is based on the assumptions that the statistical properties of the 3D velocity field are locally homogeneous and isotropic and that there exists a constant-energy cascade from large to small scales. Coriolis forces, however, alter the energy transfer, ultimately leading to, for example, anisotropy among the Reynolds stresses, a strong reduction in the dissipation rate and the emergence of columnar vortical structures. The energy flux from large to small scales is obviously reduced in the rotating case as compared to the nonrotating case. In other words it reflects the existence of spatially coherent Taylor columns. The anomalous scaling is a direct consequence of these phenomena in rotating flows. Moreover, the measured slopes are steeper than the slope reported by Baroud *et al.*¹³ ($C_\zeta = 0.5$). In their experiment turbulence is generated by continuously pumping fluid in and out of holes at the

bottom of a rapidly rotating annulus. Although their Rossby number is similar to the Rossby numbers considered here, their Taylor-based Reynolds number is several times larger (depending on the parameter triplet). Taking into consideration the different experimental conditions, there is no reason to expect the same slope for both experiments. What is remarkable here is that both experiments capture self-similar behavior of turbulence that is subjected to background rotation.

For low-Reynolds-number turbulence, as in the present case, it is of interest to consider also the relative structure function scaling that can be obtained with the extended self-similarity (ESS) technique by Benzi *et al.*⁴² The ESS approach amounts to deducing the exponent ratios ζ_p/ζ_3 from the slopes of a doubly logarithmic plot of S_1^p versus S_1^3 . The ESS procedure has been carried out for all the rotating experiments and the resulting ESS plots reveal a power-law regime, see Fig. 17(b). Similar results have been obtained for $\{H, \Omega, I\} = \{20, \Omega, 4\}$ and $\{100, \Omega, 4\}$.

VI. SUMMARY AND DISCUSSION

A laboratory experiment for studying effects of rotation on nonintrusively forced turbulent flow has been designed and shown to be fruitful. The time-evolving 2D3C velocity field has been measured with SPIV. We have found that the turbulence reaches a statistically steady state within approximately ten eddy turn-over times, spatially homogeneous, and well reproducible. Vertical decay of the turbulence in the absence of rotation is inherent to the experiment, and has been characterized both qualitatively and quantitatively (Sec. III B). The results show that the decay is not self-similar. The turbulence has been found to be spatially anisotropic both close to and far away from the forcing. In between these extremes the turbulence is closely 3C isotropic. It must be remarked that the temporal out-of-plane velocity correlations are slightly anisotropic for each of the considered H . Also, it must be noted that the turbulence is not fully developed in the typical sense of a well-established energy cascade over several decades.

Effects of rotation have been most extensively studied for height $H=50$ mm because the nonrotating turbulence is almost isotropic at this level. Three rotation rates have been considered: $\Omega=1, 5$, and 10 rad/s. Based on the principal coordinates of the Reynolds stress anisotropy tensor the rotating turbulence has been classified as a 3D2C flow. This result is consistent with numerous earlier numerical and experimental studies on rotating turbulent flows, e.g., Refs. 3, 16, 22, 29, and 39. The following common features of rotating turbulent flows have been quantified: (i) the reduction of the dissipation rate, (ii) suppression of the vertical velocity as compared to the horizontal velocity, and (iii) stronger spatial and temporal correlation of the horizontal velocities, with the temporal correlation growing ever stronger as the rotation rate is increased. A less common feature of rotating turbulence is the reverse dependence of the spatial horizontal velocity correlation functions on the rotation rate, quantified here for the first time in a laboratory setting. This result is consistent with qualitative observations by Hopfinger *et al.*³

It is also consistent with the numerical results reported by Godeferd and Lollini.²⁹ Another remarkable result of this study is the linear (anomalous) scaling of the structure function exponents in the presence of rotation. This type of anomaly has not been reported until recently by Baroud *et al.*¹³ For as far as the vertical dependence of the rotating turbulence is concerned, it should be noted that the vertical decay observed in the nonrotating case is almost entirely inhibited by rotation. Hopfinger *et al.*³ reported a similar observation, viz., that the turbulent flow field becomes independent of depth for local Rossby numbers smaller than 0.2.

In all of the experiments discussed so far, the turbulence was generated with an electrical current of $I=4$ A. Most of the experiments have been repeated with $I=8$ A to study the effect of the forcing parameter on the turbulence. Basically, increasing the electrical current implies increasing the Reynolds number. None of the *normalized* quantities presented in Secs. III and IV were notably affected by the change in electrical current. This goes for both the reference and the rotating experiments. In both cases the Reynolds number decays linearly with height, at least for the investigated range of current, i.e., $I \in [4, 8]$ A. The corresponding slope is approximately proportional to $\sqrt{I}$. Moreover, no turbulence exists beyond a certain critical height, which is found to be somewhat larger than $H=100$ mm for both values of I in the present experiments. The vertical dependence of Re_λ , and in addition that of $Ro_\lambda \equiv \langle u' \rangle_{TA} / (2\Omega \langle \lambda \rangle_{TA})$, has also been studied for the rotating experiments. Both dimensionless numbers weakly depend on H , a dependency that decreases with increasing Ω . Finally, although increasing the forcing current I only marginally affects most of the statistics considered here, it does notably affect the longitudinal structure functions. When $I=8$ A, self-similar scaling of the longitudinal spatial structure function exponents is no longer observed for the lowest rotation rate ($\Omega=1$ rad/s), see Fig. 17(c). Moreover, for higher rotation rates ($\Omega=5, 10$ rad/s) the slope is slightly steeper than before, viz., $C_\zeta \approx 0.75$. The results for higher Re_λ indicate saturation of the scaling exponents according to $\zeta_p \approx 0.75p$ (for $p \leq 4$). This would imply with $\zeta_2 \approx 1.5$ that $E(k) \sim k^{-2.5}$.

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