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Efficient Training Sequence for Joint Carrier Frequency Offset and Channel Estimation for MIMO-OFDM Systems

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Abstract—Combining Orthogonal Frequency Division Multiplexing (OFDM) and Multiple Input and Multiple Output (MIMO) technologies enables high spectral efficiency data transmission over frequency selective fading channels. However, carrier frequency offset (CFO) and channel estimation become more complex for MIMO-OFDM systems. In this paper, we propose a class of efficient shift-orthogonal training sequences to assist joint CFO and channel estimation. We show that with the proposed training sequence, the CFO estimation touches Cramer-Rao bound (CRB) at practical SNR values. Such sequences are efficient as the overhead used for channel estimation can be significantly reduced compared to using conventional training sequence designs. We also derive a tight lower bound on the MSE of the channel estimation in the presence of the residual CFO.

I. INTRODUCTION

The capacity of rich scattering wireless fading channels can be increased enormously by using multiple antennas at both the transmitter and the receiver [1]. For wide-band systems where the channel becomes frequency selective, combining MIMO with OFDM is one effective solution. MIMO-OFDM transforms the frequency selective system to a flat fading MIMO system on each subcarrier. MIMO-OFDM, therefore, has been chosen as the transmission technology for IEEE 802.11n high-throughput wireless local area network (WLAN) standard [2]. It is also included in some other standards of wireless communication systems such as IEEE 802.16 WiMAX and 3GPP long term evolution (3GPP-LTE).

Similar to the single input single output (SISO) OFDM systems, carrier frequency offset (CFO) is still a major impairment for MIMO-OFDM systems. The CFO is present due to the difference between the reference frequencies of the local oscillators (LO) at the transmitter and receiver, as well as the doppler effect of the channel. Such CFO destroys the orthogonality between different subcarriers and causes inter-carrier interference (ICI). The ICI could cause severe degradation to the system performance if not properly compensated. Therefore, accurate estimation and compensation of CFO is essential for both SISO and MIMO OFDM systems. CFO estimation for SISO-OFDM systems have been extensively studied in literature such as [3] [4]. For collocated MIMO-OFDM systems, all the transmit antennas are driven by a single local oscillator and so are all the receive antennas. Therefore, the CFO between the transmitter and the receiver is a single scalar parameter. In such systems, the CFO estimation is very

similar to its SISO counterpart. The problem is mainly how to optimally combine the received signals from different receive antennas. Some CFO estimation algorithms for MIMO-OFDM systems have been proposed in [5] [6].

The channel estimation for MIMO-OFDM systems is more complicated than for SISO systems. For a MIMO-OFDM system with n_t transmit and n_r receive antennas, $n_t \times n_r$ channel coefficients must be estimated per subcarrier. To be able to differentiate the transmitted signal from different transmit antennas, some orthogonality among them is required. In [7], a frequency orthogonal training sequence design was proposed. The training sequences from different transmit antennas occupy different subcarriers in the same OFDM symbol interval. The length of the training sequence must be at least n_t OFDM symbols to get the channel estimate on all the subcarriers. Another approach, adopted in the IEEE 802.11n draft standard [2], uses different Walsh Hardward codes to spread one training sequence to different OFDM symbols such that the training sequences from different transmit antennas are orthogonal in the code domain. This approach again requires a training sequence of at least n_t OFDM symbols. Both these frequency domain approaches require significantly larger overhead for channel estimation compared to SISO-OFDM system. In [8], a time domain training sequence design was proposed for MIMO-OFDM systems. This time domain approach requires only training sequences of length $n_t \times L$ where L is the length of the channel impulse response, which is much smaller than the OFDM symbol interval. However, such training sequences are impulses in time and hence have very high peak to average power ratio (PAPR).

In this paper, we propose an efficient training sequence design based on a shift-orthogonal polyphase sequence, first proposed in [9] in CDMA system, for joint CFO and channel estimation for MIMO-OFDM systems. We first formulate the Maximum Likelihood (ML) CFO and channel estimator. Similar to [5] [6], we show that ML CFO estimation can be efficiently implemented by transmitting two identical copies of the same training sequence. After compensating the estimated CFO, the channel estimation can be performed in time domain. The advantages of the proposed training sequences are:

- One training sequence for both CFO and channel estimation.
- No calculation of matrix inversion is required to obtain channel estimates.
- Significantly reduced overhead. The minimum length

required for channel estimation is only $n_t \times L$.

- The PAPR of the training sequence in time domain is 1.

The residual CFO due to the error in the CFO estimation will degrade the performance of the channel estimation. In this paper, we derive a lower bound on the mean square error (MSE) of channel estimation in the presence of such residual CFO. We show from computer simulations that this lower bound is tight. From the lower bound, we can see that the degradation caused by the residual CFO in the channel estimation is negligible.

This paper is organized as follows. In Section II, we develop the system model for the MIMO-OFDM system and the maximal likelihood CFO and channel estimator. We then propose the shift-orthogonal polyphase training sequence and the corresponding CFO and channel estimator in Section III. We also show that using such training sequences, the overhead of training can be significantly reduced compared to conventional frequency domain training. In Section IV we analyze the MSE of the channel estimation in the presence of the residual CFO. A tight lower bound on the MSE is derived. Simulation results are presented in Section V and conclusions are drawn in Section VI.

II. SYSTEM MODEL

In this paper, we deal with a collocated MIMO-OFDM system, where the CFO is quantified by a single parameter ϕ . We assume perfect timing synchronization has been achieved. In this case, the received signal at the i th receive antenna can be written as

$$r_i(k) = e^{j\phi k} \sum_{m=1}^{n_t} \sum_{d=0}^{L-1} h_{i,m}(d) s_m(k-d) + n_i(k). \quad (1)$$

where k is the time index, and L is the length of the multipath channel. After removing the cyclic prefix, we can write the received signal in the equivalent matrix form

$$\mathbf{r}_i = \sum_{m=1}^{n_t} \mathbf{E}(\phi) \mathbf{S}_m \mathbf{h}_{i,m} + \mathbf{n}_i \quad (2)$$

where $\mathbf{E}(\phi)$ is the CFO matrix, which is a diagonal matrix with diagonal elements equal to $[1, \exp(j\phi), \dots, \exp(j(L-1)\phi)]$. We use $\mathbf{S}_i$ to denote the transmitted signal matrix from the i th transmit antenna, which is a circulant matrix with the first column defined by $[s_i(0), s_i(1), \dots, s_i(L-1)]^T$. The channel impulse response vector between the m th transmit antenna and the i th received antenna is denoted by $\mathbf{h}_{i,m}$.

For training sequences of length N , we can gather the received signal from all the receive antennas into one matrix as follows

$$\tilde{\mathbf{R}} = \mathbf{E}(\phi) \mathcal{S} \tilde{\mathbf{H}} + \mathcal{N}, \quad (3)$$

where

$$\tilde{\mathbf{R}} = [\mathbf{r}_1 \cdots, \mathbf{r}_{n_r}]_{\{N \times n_r\}},$$

$$\mathcal{S} = \frac{1}{\sqrt{n_t}} [\mathbf{S}_1, \cdots, \mathbf{S}_{n_t}]_{\{N \times (N \times n_t)\}}$$

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{\mathcal{H}}_1 \\ \vdots \\ \tilde{\mathcal{H}}_{n_t} \end{bmatrix}_{\{(N \times n_t) \times n_r\}},$$

with $\tilde{\mathcal{H}}_i = [\tilde{\mathbf{h}}_{1,i}, \dots, \tilde{\mathbf{h}}_{n_r,i}]_{\{N \times n_r\}}$. For ease of understanding, we use a subscript inside the curved bracket to denote the dimensions of the matrices. Here we assume the length of the training sequence $N \geq L$ and that $\tilde{\mathbf{h}}_{i,m}$ is an $N \times 1$ vector obtained by appending the original $L \times 1$ vector $\mathbf{h}_{i,m}$ with $N-L$ zeros. The noise matrix is given by $\mathcal{N} = [\mathbf{n}_1, \dots, \mathbf{n}_{n_r}]$.

Because the noise is Gaussian and uncorrelated, the likelihood function for the channel $\tilde{\mathbf{H}}$ and CFO values ϕ can be written as

$$\Lambda(\tilde{\mathbf{H}}, \phi) = \frac{1}{(\pi \sigma_n^2)^{N \times n_r}} \exp \left\{ -\frac{1}{\sigma_n^2} \left\| \tilde{\mathbf{R}} - \mathbf{E}(\phi) \mathcal{S} \tilde{\mathbf{H}} \right\|^2 \right\}. \quad (4)$$

III. SHIFT ORTHOGONAL POLYPHASE SEQUENCES FOR JOINT CFO AND CHANNEL ESTIMATION

In this paper, we propose to use a class of constant modulus polyphase training sequences, initially proposed by Suehiro and Hatori [9] for CDMA systems, for joint CFO and channel estimation¹. The polyphase sequence has a length equal to N where $N = K^2$ with K being any positive integer. Each element of the sequence has a amplitude equal to one. The polyphase sequences also have the following properties:

- 1) The autocorrelation function of s_i satisfies

$$\phi(k) = \sum_{n=0}^{N-1} s_i(n) s_i(n \oplus k) = \begin{cases} N & k=0; \\ 0 & k \neq 0. \end{cases} \quad (5)$$

where $\oplus$ denote cyclic addition. This means this sequence is orthogonal to all the non-zero cyclic shifts.

- 2) The cross correlation function of s_i and s_m where $i \neq m$ satisfies

$$\varphi(k) = \sum_{n=0}^{N-1} s_i(n) s_m(n \oplus k) = N/K, \quad (6)$$

for all values of k .

Let $\mathbf{S}_1$ to be a circulant matrix with the first column equal to $[s_1(0), s_1(1), \dots, s_1(N-1)]^T$. The auto-correlation property of such polyphase sequences can be written in equivalent matrix form as

$$\mathbf{S}_1 \mathbf{S}_1^H = N \mathbf{I}. \quad (7)$$

This means that $\mathbf{S}_1$ is both a unitary and a circulant matrix.

We let the training sequence from the first transmit antenna be $\mathbf{s}_1$. The training sequence from the m th transmit antenna is the cyclic shifted version of $\mathbf{s}_1$, i.e. $s_m(n) = [s_1(n \ominus \tau_m)]^T$, where $\ominus$ denotes cyclic subtraction and τ_m denotes the shift value. As a result, the matrix $\mathbf{S}_m$ can be obtained by circularly shifting the rows of $\mathbf{S}_1$ τ_m rows downwards. Hence, we have

$$\mathbf{S}_m \tilde{\mathbf{h}}_{i,m} = \mathbf{S}_1 \tilde{\mathbf{h}}_{i,m}^{\tau_m}, \quad (8)$$

¹For details on the construction of such sequences, please refer to [9].

where $\tilde{\mathbf{h}}_{i,m}^{\tau_m} = [\tilde{h}_{i,m}(N - \tau_m), \dots, \tilde{h}_{i,m}(0), \dots, \tilde{h}_{i,m}(N - \tau_m - 1)]^T$ is obtained by cyclically shifting $\tilde{\mathbf{h}}_{i,m}$ τ_m rows downwards. Making use of this property, we can rewrite the received signal at receive antenna i as

$$\mathbf{r}_i = \frac{1}{\sqrt{n_t}} \mathbf{E}(\phi) \sum_{m=1}^{n_t} \mathbf{S}_m \tilde{\mathbf{h}}_{i,m} = \frac{1}{\sqrt{n_t}} \mathbf{E}(\phi) \mathbf{S}_1 \sum_{m=1}^{n_t} \tilde{\mathbf{h}}_{i,m}^{\tau_m}. \quad (9)$$

Collecting the received signals from all receive antennas, we get

$$\tilde{\mathcal{R}} = \frac{1}{\sqrt{n_t}} \mathbf{E}(\phi) \mathbf{S}_1 \mathcal{H} + \mathcal{N}, \quad (10)$$

where

$$\mathcal{H} = \left[\sum_{m=1}^{n_t} \tilde{\mathbf{h}}_{1,m}^{\tau_m}, \sum_{m=1}^{n_t} \tilde{\mathbf{h}}_{2,m}^{\tau_m}, \dots, \sum_{m=1}^{n_t} \tilde{\mathbf{h}}_{n_r,m}^{\tau_m} \right]_{\{N \times n_r\}}.$$

Without loss of generality, let us examine the first column of the $\mathcal{H}$ matrix, $\mathcal{H}_1 = \sum_{m=1}^{n_t} \tilde{\mathbf{h}}_{1,m}^{\tau_m}$. We know that only the first L elements in the $N \times 1$ vector $\tilde{\mathbf{h}}_{1,m}$ are nonzero, i.e. $\tilde{\mathbf{h}}_{1,m} = [\mathbf{h}_{1,m}^T, \mathbf{0}_{1 \times (N-L)}^T]^T$. We choose the length of the polyphase sequence N such that $N \geq n_t \times L$. In this case, we can make $\tau_m - \tau_{m-1} \geq L$ for $m = 2, \dots, n_t$, in $\mathcal{H}_1$. In this case, there will be no overlap between the non-zero impulse responses of the channels from different transmit antennas. As a result, obtaining $\mathcal{H}_1$, we can obtain the channel impulse responses from different transmit antennas by taking appropriate samples of $\mathcal{H}_1$. The same principle applies to the other columns of $\mathcal{H}$. In this case, the estimation of the channel impulse response of $\mathbf{h}_{i,m}$ for $i = 1, 2, \dots, n_r$ and $m = 1, 2, \dots, n_t$ is converted equivalently to the estimation of the matrix $\mathcal{H}$.

In [3] and [4], CFO estimation methods using periodic training sequences were proposed. Here, we adopt a similar approach. Now, we transmit two identical training sequences in time, i.e. the training sequence from the i th transmit antenna, $\mathbf{x}_i = [\mathbf{s}_i^T, \mathbf{s}_i^T]^T$, is of length $2N$. Assuming the channel does not change within the duration of $2N$, the received signal can be written as:

$$\mathcal{R}_{\{2N \times n_r\}} = \mathbb{E}_{\{2N \times 2N\}}(\phi) \mathbf{X}_{\{2N \times N\}} \mathcal{H}_{\{N \times n_r\}} + \mathcal{N}_{\{2N \times n_r\}}, \quad (11)$$

where

$$\mathbf{X} = \frac{1}{\sqrt{n_t}} \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_1 \end{bmatrix}.$$

Using the received signal model given in (11), the likelihood function of the channel response $\mathcal{H}$ and the CFO ϕ can be written as

$$\Lambda(\mathcal{H}, \phi) = \frac{1}{(\pi \sigma_n^2)^{2N \times n_r}} \exp \left\{ -\frac{1}{\sigma_n^2} \text{tr} \left\{ [\mathcal{R} - \mathbb{E}(\phi) \mathbf{X} \mathcal{H}]^H [\mathcal{R} - \mathbb{E}(\phi) \mathbf{X} \mathcal{H}] \right\} \right\} \quad (12)$$

where tr denotes the trace of a matrix.

Following a similar approach as in [10], we find that by keeping ϕ fixed, the ML estimate of the channel $\mathcal{H}$ is given by

$$\hat{\mathcal{H}}(\phi) = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbb{E}^H(\phi) \mathcal{R}. \quad (13)$$

Substituting (13) into (12), and after some algebraic manipulation we get the ML estimator of the CFO ϕ is given by

$$\hat{\phi} = \arg \max_{\phi} \left\{ \text{tr} \left\{ \mathcal{R}^H \mathbb{E}(\phi) \mathbf{B} \mathbb{E}^H(\phi) \mathcal{R} \right\} \right\}, \quad (14)$$

where

$$\begin{aligned} \mathbf{B} &= \mathbf{X} (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H = \frac{1}{2N} \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_1 \end{bmatrix} \mathbf{I}_{\{N \times N\}} [\mathbf{S}_1^H, \mathbf{S}_1^H] \\ &= \frac{1}{2} \begin{bmatrix} \mathbf{I}_{\{N \times N\}} & \mathbf{I}_{\{N \times N\}} \\ \mathbf{I}_{\{N \times N\}} & \mathbf{I}_{\{N \times N\}} \end{bmatrix}. \end{aligned} \quad (15)$$

We can also rewrite $\mathbb{E}(\phi)$ and $\mathcal{R}$ in the following form

$$\mathbb{E}(\phi) = \begin{bmatrix} \mathbf{E}(\phi) & \mathbf{0} \\ \mathbf{0} & e^{jN\phi} \mathbf{E}(\phi) \end{bmatrix},$$

where $\mathbf{E}(\phi) = \text{diag}[1, e^{j\phi}, \dots, e^{j(N-1)\phi}]$, and

$$\mathcal{R} = \begin{bmatrix} \mathcal{R}_1 \\ \mathcal{R}_2 \end{bmatrix},$$

where $\mathcal{R}_1$ denotes the first N rows of $\mathcal{R}$ and $\mathcal{R}_2$ denotes the $N + 1$ th to the $2N$ th rows of $\mathcal{R}$. Substituting these new expressions into the cost function in (14), we can further simplify the cost function to

$$\mathcal{J}(\phi) = \Re \left\{ e^{jN\phi} \text{tr}(\mathcal{R}_2^H \mathcal{R}_1) \right\}, \quad (16)$$

where $\Re(\bullet)$ denotes the real part of a complex number. The maximal likelihood estimate of ϕ is therefore reduced to

$$\hat{\phi} = \arg \max_{\phi} \Re \left\{ e^{jN\phi} \text{tr}(\mathcal{R}_2^H \mathcal{R}_1) \right\}. \quad (17)$$

Using scalar notations, the ML CFO estimator in (17) can be written equivalently as

$$\hat{\phi} = \frac{1}{N} \angle \left\{ \sum_{k=1}^N \sum_{m=1}^{n_r} r_m^*(k) r_m(k + N) \right\}, \quad (18)$$

where $\angle(\bullet)$ denotes the angle of a complex number and $r_m(k)$ denotes the received signal from the m th receive antenna at time k . This ML estimate is very similar in form to its SISO counterpart as shown in [4] [10] and it can be easily implemented using simple correlators. It is of the same form as the other CFO estimators for MIMO-OFDM system using periodic sequences [11].

The performance of the ML CFO estimator for SISO-OFDM systems has been extensively studied in literature [3], [4], [10]. Following a similar approach as the SISO-OFDM systems, we can show that the CRB of the CFO estimation is given by [6]

$$E[(\hat{\phi} - \phi)^2] \geq \text{CRB} = \frac{1}{n_r N^3 \gamma}, \quad (19)$$

where γ is the SNR per receive antenna.

Having estimated the CFO value of $\hat{\phi}$, we can simply plug it into (13) to get the channel estimate as follows

$$\begin{aligned} \hat{\mathcal{H}} &= (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbb{E}^H(\hat{\phi}) \mathcal{R} = \frac{n_t}{2N} \mathbf{X}^H \mathbb{E}^H(\hat{\phi}) \mathcal{R} \\ &= \frac{\sqrt{n_t}}{2N} [\mathbf{S}_1^H, \mathbf{S}_1^H] \begin{bmatrix} \mathbf{E}^H(\hat{\phi}) & \mathbf{0} \\ \mathbf{0} & e^{-jN\hat{\phi}} \mathbf{E}^H(\hat{\phi}) \end{bmatrix} \begin{bmatrix} \mathcal{R}_1 \\ \mathcal{R}_2 \end{bmatrix} \\ &= \frac{\sqrt{n_t}}{2N} \left(\mathbf{S}_1^H \mathbf{E}(-\hat{\phi}) \mathcal{R}_1 + e^{-jN\hat{\phi}} \mathbf{S}_1^H \mathbf{E}(-\hat{\phi}) \mathcal{R}_2 \right). \end{aligned} \quad (20)$$

From (20), we can see that the channel estimate can be simply obtained using simple multiplications. Due to the orthogonal property of the $\mathbf{S}_1$ matrix, the complicated matrix inversion can be avoided. This provides a large saving in the computation complexity of channel estimation. Another advantage of this training sequence is the low overhead. For channel estimation alone, we only need training sequences of length N where $N \geq n_t \times L$. By comparison, in frequency based channel estimation in MIMO-OFDM systems, the channel estimation requires transmitting of minimum $n_t \times K$ training symbols, where K is the length of the OFDM symbol. In a practical OFDM system, the length of the channel impulse response L is similar to the length of the cyclic prefix which is much less compared to the length of the OFDM symbol K . Therefore, by using the proposed training sequence for channel estimation, we could provide overhead saving of $K/L - 1$ times. Moreover, the training signal is constant amplitude in time and therefore has PAPR equal to 1.

IV. MSE ANALYSIS OF THE CHANNEL ESTIMATION IN THE PRESENCE OF RESIDUAL CFO

In practical systems, there exists some residual CFO after CFO compensation due to the inaccuracy of the estimation. In this section, we examine how such inaccuracy affects the performance of the channel estimation.

Substituting the expression for $\mathcal{R}_1$ and $\mathcal{R}_2$ into (20) and let the residual CFO $\phi_d = \phi - \hat{\phi}$ we get

$$\begin{aligned} \hat{\mathcal{H}} &= \frac{1}{2N} \left(\mathbf{S}_1^H \mathbf{E}(\phi_d) \mathbf{S}_1 \mathcal{H} + \sqrt{n_t} \mathbf{S}_1^H \mathbf{E}(-\hat{\phi}) \mathcal{N}_1 \right. \\ &\quad \left. + e^{jN(\phi_d)} \mathbf{S}_1^H \mathbf{E}(\phi_d) \mathbf{S}_1 \mathcal{H} + e^{-jN(\hat{\phi})} \sqrt{n_t} \mathbf{S}_1^H \mathbf{E}(-\hat{\phi}) \mathcal{N}_2 \right). \end{aligned} \quad (21)$$

As $\mathbf{E}(-\hat{\phi})$ is a unitary matrix, $\mathbf{E}(-\hat{\phi}) \mathcal{N}_1$ is statistically the same as $\mathcal{N}_1$. Same applies to $\mathcal{N}_2$. Therefore, for ease of notation, we still use $\mathcal{N}_1$ and $\mathcal{N}_2$ to denote the new noise. With this simplification of notation, the channel estimate can be rewritten as

$$\hat{\mathcal{H}} = \frac{1}{2N} (1 + e^{jN\phi_d}) \mathbf{S}_1^H \mathbf{E}(\phi_d) \mathbf{S}_1 \mathcal{H} + \frac{\sqrt{n_t}}{2N} (\mathbf{S}_1^H \mathcal{N}_1 + \mathbf{S}_1^H \mathcal{N}_2). \quad (22)$$

The MSE of the channel estimation is given by

$$\text{MSE} = \frac{1}{N n_r} \text{tr} \left\{ E \left[(\mathcal{H} - \hat{\mathcal{H}})^H (\mathcal{H} - \hat{\mathcal{H}}) \right] \right\} \quad (23)$$

When the CFO estimation is perfect, i.e. $\phi_d = 0$, it can be easily shown that

$$\begin{aligned} \text{MSE} &= \frac{n_t}{4N^2} \frac{1}{N n_r} \text{tr} \left\{ E \left[\mathcal{N}_1^H \mathbf{S}_1 \mathbf{S}_1^H \mathcal{N}_1 \right] + E \left[\mathcal{N}_2^H \mathbf{S}_1 \mathbf{S}_1^H \mathcal{N}_2 \right] \right\} \\ &= \frac{n_t \sigma_n^2}{2N}. \end{aligned} \quad (24)$$

Making use of the Taylor's series expansion, we can write the exponential function as $e^{\phi} = 1 + \sum_{k=1}^{\infty} \frac{\phi^k}{k!}$. Substituting this into (22), we get

$$\hat{\mathcal{H}} = \frac{1+\delta}{N} \mathbf{S}_1^H (\mathbf{I} + \Delta(\phi_d)) \mathbf{S}_1 \mathcal{H} + \frac{\mathcal{N}'}{N}, \quad (25)$$

where $\delta = 1/2 \sum_{k=1}^{\infty} \frac{(jN\phi_d)^k}{k!}$ and $\Delta(\phi_d)$ is a diagonal matrix with the n th element equal to $\sum_{k=1}^{\infty} \frac{(j(n-1)\phi_d)^k}{k!}$. Here we use one AWGN noise matrix $\mathcal{N}'$ to denote $\frac{\sqrt{n_t}}{2} (\mathbf{S}_1^H \mathcal{N}_1 + \mathbf{S}_1^H \mathcal{N}_2)$. Note that the variance of each element of $\mathcal{N}'$ is $\frac{N n_t}{2} \sigma_n^2$. Using this new expression, we can write the difference between the channel estimate and the actual channel as

$$\hat{\mathcal{H}} - \mathcal{H} = \mathcal{H} \delta + \frac{1+\delta}{N} \mathbf{S}_1^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} + \frac{\mathcal{N}'}{N}. \quad (26)$$

Here the first two terms in the summation describe the extra estimation error caused by the residual CFO error ϕ_d while $\frac{\mathcal{N}'}{N}$ is caused by the AWGN noise.

Using (26), we can write

$$\begin{aligned} &(\hat{\mathcal{H}} - \mathcal{H})^H (\hat{\mathcal{H}} - \mathcal{H}) \\ &= \frac{1}{N^2} \left\{ N^2 |\delta|^2 \mathcal{H}^H \mathcal{H} + N(\delta^* + |\delta|^2) \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} \right. \\ &\quad + N\delta^* \mathcal{H}^H \mathcal{N}' + N(\delta + |\delta|^2) \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d)^H \mathbf{S}_1 \mathcal{H} \\ &\quad + |1 + \delta|^2 \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d)^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} + (1 + \delta^*) \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d)^H \mathbf{S}_1 \mathcal{N}' \\ &\quad \left. + N\delta \mathcal{N}'^H \mathcal{H} + (1 + \delta) \mathcal{N}'^H \mathbf{S}_1^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} + \mathcal{N}'^H \mathcal{N}' \right\}. \end{aligned}$$

Taking expectation over the AWGN noise $\mathcal{N}'$, we get

$$\begin{aligned} E_n \left[(\hat{\mathcal{H}} - \mathcal{H})^H (\hat{\mathcal{H}} - \mathcal{H}) \right] \\ &= \frac{1}{N^2} \left\{ N^2 |\delta|^2 \mathcal{H}^H \mathcal{H} + N(\delta^* + |\delta|^2) \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} \right. \\ &\quad + N(\delta + |\delta|^2) \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d)^H \mathbf{S}_1 \mathcal{H} \\ &\quad \left. + |1 + \delta|^2 \mathcal{H}^H \mathbf{S}_1^H \Delta(\phi_d)^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} + \frac{N^2 n_t}{2} \sigma_n^2 \mathbf{I}_{n_r} \right\}. \end{aligned}$$

Here E_n denotes expectation over noise n and $\mathbf{I}_{n_r}$ is an identity matrix of size $n_r \times n_r$. After the trace operation, we get the MSE of channel estimation conditioned on a particular channel realization $\mathcal{H}$, i.e.

$$\begin{aligned} \text{MSE}_{|\mathcal{H}} &= \frac{1}{N n_r} \text{tr} \left\{ E_n \left[(\hat{\mathcal{H}} - \mathcal{H})^H (\hat{\mathcal{H}} - \mathcal{H}) \right] \right\} \\ &= \frac{1}{N^2} \frac{1}{N n_r} \left\{ N^2 |\delta|^2 \text{tr}(\mathcal{H}^H \mathcal{H}) \right. \\ &\quad + 2N \Re \left[(\delta^* + |\delta|^2) \text{tr} \left[\Delta(\phi_d) \mathbf{S}_1 \mathcal{H} \mathcal{H}^H \mathbf{S}_1^H \right] \right] \\ &\quad \left. + |1 + \delta|^2 \text{tr} \left[\Delta(\phi_d)^H \Delta(\phi_d) \mathbf{S}_1 \mathcal{H} \mathcal{H}^H \mathbf{S}_1^H \right] \right\} + \frac{n_t \sigma_n^2}{2N}. \end{aligned} \quad (27)$$

We assume that the channels between different transmit and receive antennas are uncorrelated in space and the different paths in the multi-path channel are also uncorrelated. We define $\mathbf{p}_{i,m} = [p_{i,m}(1), \dots, p_{i,m}(N)]^T$ as the power delay profile of the channel between the m th transmit antenna and the i th receive antenna. Since the length of the channel impulse response is L , $p_{i,m}(N) = 0$ for $N > L - 1$. Using this, we can write

$$\Psi_{\mathcal{H}} = E(\mathcal{H} \mathcal{H}^H) = \text{diag} \left[\sum_{i=1}^{n_t} \left(\sum_{k=1}^{n_r} \mathbf{p}_{k,i}(n \ominus \tau_i) \right) \right]. \quad (28)$$

We normalize the channel such that the total transmit signal power equals the total receive signal power. As a result, the

power of the channel taps between all the transmit and receive antenna pairs adds up to $n_t \times n_r$. Therefore, we have

$$\text{tr}[\Psi_{\mathcal{H}}] = n_t n_r.$$

Now we average the MSE of the channel estimation over all the channel realizations.

$$\begin{aligned} \text{MSE} &= E_{\mathcal{H}} \{ \text{MSE}_{|\mathcal{H}} \} = \frac{1}{N^3 n_r} \left\{ N^2 |\delta|^2 \text{tr} \{ \Psi_{\mathcal{H}} \} \right. \\ &+ 2N \Re \left[(\delta^* + |\delta|^2) \text{tr} \left[\Delta(\phi_d) \mathbf{S}_1 \Psi_{\mathcal{H}} \mathbf{S}_1^H \right] \right] \\ &+ \left. |1 + \delta|^2 \text{tr} \left[\Delta(\phi_d)^H \Delta(\phi_d) \mathbf{S}_1 \Psi_{\mathcal{H}} \mathbf{S}_1^H \right] \right\} + \frac{n_t \sigma_n^2}{2N}. \end{aligned} \quad (29)$$

Let us define $\mathbf{F} = \mathbf{S}_1 \Psi_{\mathcal{H}} \mathbf{S}_1^H$. It can be shown that the i th diagonal element of $\mathbf{F}$, $F(i, i)$ is given by

$$F(i, i) = \sum_{m=1}^N \Psi(j, j) = \text{tr}[\Psi_{\mathcal{H}}] = n_t n_r. \quad (30)$$

As $\Delta(\phi_d)$ is a diagonal matrix, we have

$$\text{tr} [\Delta(\phi_d) \mathbf{S}_1 \Psi_{\mathcal{H}} \mathbf{S}_1^H] = n_t n_r \text{tr} [\Delta(\phi_d)],$$

and similarly

$$\text{tr} [\Delta(\phi_d)^H \Delta(\phi_d) \mathbf{S}_1 \Psi_{\mathcal{H}} \mathbf{S}_1^H] = n_t n_r \text{tr} [\Delta(\phi_d)^H \Delta(\phi_d)].$$

As we are going to show later from the simulations, the CFO estimation closely approaches the CRB at practical SNR regions. Therefore, the residual CFO value ϕ_d is usually very small. We can use first order Taylor series expansion to get a good approximation of the exponential function, i.e. $\delta = 1/2 \sum_{k=1}^{\infty} \frac{(jN\phi_d)^k}{k!} \approx jN\phi_d/2$ and $\Delta(\phi_d) \approx \text{diag}[0, j\phi_d, j2\phi_d, \dots, j(N-1)\phi_d]$. Substituting these into (29), we get

$$\begin{aligned} \text{MSE} &= \frac{n_t N \phi_d^2}{4} + 2\Re \left\{ \frac{(N-1)\phi_d^2 n_t}{4} + \frac{jN(N-1)\phi_d^3 n_t}{8} \right\} \\ &+ \left(1 + \frac{N^2 \phi_d^2}{4} \right) \frac{n_t (N-1)(2N-1)\phi_d^2}{6N^2} + \frac{n_t \sigma_n^2}{2N} \\ &= \frac{n_t (9N^3 - 2N^2 - 6N + 2)}{12N^2} \phi_d^2 \\ &+ \frac{n_t (N-1)(2N-1)}{24} \phi_d^4 + \frac{n_t \sigma_n^2}{2N} \\ &\approx \frac{n_t (9N^3 - 2N^2 - 6N + 2)}{12N^2} \phi_d^2 + \frac{n_t \sigma_n^2}{2N}. \end{aligned} \quad (31)$$

Here we drop the ϕ_d^4 term in the MSE expression because the estimation error ϕ_d is very small in practical SNR regions and higher order terms have only negligible contribution to the overall MSE. From the previous section, we know that the variance of ϕ_d touches the CRB at moderate to high SNR regions. Therefore, we can approximate ϕ_d^2 in (31) with the CRB expression, i.e.

$$\phi_d^2 \approx \frac{1}{n_r N^3 \gamma}.$$

TABLE I
EXTRA MSE CAUSED BY RESIDUAL CFO FOR DIFFERENT TRAINING SEQUENCE LENGTH AND NUMBER OF RECEIVE ANTENNAS .

Training Sequence Length	2 Rx Ant	3 Rx Ant
25	0.127 dB	0.085 dB
36	0.089 dB	0.060 dB
49	0.066 dB	0.044 dB

Substituting this into (31), we get the MSE of channel estimation in the presence of residual CFO ϕ_d given by

$$\begin{aligned} \text{MSE} &\approx \frac{n_t (9N^3 - 2N^2 - 6N + 2)}{12N^5 \gamma n_r} + \frac{n_t \sigma_n^2}{2N} \\ &= \sigma_n^2 \left(\frac{n_t (9N^3 - 2N^2 - 6N + 2)}{12N^5 n_r} + \frac{n_t}{2N} \right) \end{aligned} \quad (32)$$

The first term in (32) is the extra MSE caused by the residual CFO ϕ_d while the second term is the MSE due to the AWGN noise. The ratio between these two MSE terms amounts to

$$\frac{\text{MSE}_{RCFO}}{\text{MSE}_{AWGN}} = \frac{(9N^3 - 2N^2 - 6N + 2)}{6N^4 n_r}. \quad (33)$$

As N is larger than 16 in practical systems and the denominator is one order larger than the numerator, the MSE caused by the interference is usually negligible compared to the MSE caused by the AWGN noise. Computer simulations, which will be presented later, show that the bound given by (32) is very close to actual MSE obtained from simulations.

Table I shows the extra MSE in channel estimation caused by the residual CFO ϕ_d obtained through the lower bound in (32) for different sequence length and number of receive antennas. From the table, we can see that the MSE degradation is indeed very marginal.

V. SIMULATION RESULTS

We performed computer simulations to study the performance of the CFO and channel estimation using the proposed shift orthogonal polyphase sequences. We simulated a MIMO OFDM system with 2 transmit and 2 receive antennas. The number of subcarriers is equal to 64 with length 16 cyclic prefix. The CFO is fixed at 0.5 of the subcarrier spacing, i.e. $\phi = 0.5 \times \frac{2\pi}{64}$. We use polyphase sequences of length 36 and the cyclic shift between transmit antenna 1 and 2 is 18 taps. The total length of the training sequence is 72 as two periods of the same polyphase sequence are transmitted for CFO estimation. The channel is a 16-tap multipath fading channel with uniform power delay profile.

Figure 1 shows the performance of the CFO estimation using the proposed training sequence. Here, the MSE of the CFO estimation is normalized with respect to the subcarrier spacing, i.e. $2\pi/64$. In comparison, we also plotted the CRB of the CFO estimation given by (19). We can see that the performance of the CFO estimator touches the CRB when SNR is larger than 5 dB.

Fig. 1. MSE of CFO estimation using the proposed training sequence.

Fig. 2. Performance of channel estimation using the proposed polyphase sequence in the presence of CFO estimation errors.

Figure 2 shows the performance of channel estimation using the proposed shift orthogonal training sequences in the presence of residual CFO ϕ_d . Comparing the channel estimation with perfect CFO compensation, we can see that the performance degradation due to the residual CFO ϕ_d is really negligible, which is consistent with the theoretical prediction. In the lower left corner of the figure, we plotted the zoomed in MSE performance of the channel estimation. We can see that the theoretical MSE lower bound by (32) is very tight compared to the MSE obtained through simulations.

VI. CONCLUSIONS

In this paper, we proposed an efficient polyphase training sequence to perform CFO and channel estimation for MIMO-OFDM systems. We derived the CFO and channel estimator when such sequence is used. We showed that using this sequence, the training overhead of the system can be significantly reduced compared to conventional frequency domain training. Moreover, no matrix inversion needs to be performed to obtain the channel estimate. We derived a closed-form tight lower bound on the MSE performance of channel estimation

in the presence of the residual CFO. We showed that the extra degradation due to the residual CFO is negligible.

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