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Citation for published version (APA):

Gajdusek, M., Damen, A. A. H., & Bosch, van den, P. P. J. (2010). L-inf norm and clipped L2-norm based commutation for ironless over-actuated electromagnetic actuators. In *Proceedings of the 19th International Conference on Electrical Machines (ICEM), 6-8 September 2010, Rome, Italy* (pp. 1-6). Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/ICELMACH.2010.5608136>

DOI:

[10.1109/ICELMACH.2010.5608136](https://doi.org/10.1109/ICELMACH.2010.5608136)

Document status and date:

Published: 01/01/2010

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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l^∞ -norm and clipped l^2 -norm based commutation for ironless over-actuated electromagnetic actuators

M. Gajdušek, A. A. H. Damen, and P. P. J. van den Bosch

Abstract -- Norm-based commutation methods are discussed for the class of ironless, over-actuated actuators with a linear relation between applied current and produced force/torque. Three norm-based commutation methods are compared. The l^∞ -norm and clipped l^2 -norm based commutation methods are novel alternatives to the known l^2 -norm based commutation. The main benefit of the proposed new commutations is the limitation of maximum current. All three methods are compared on a model and experimental setup of a magnetically levitated planar actuator with moving magnets.

Index Terms -- Commutation, magnetic levitation, ironless motor, over-actuated motor, planar actuator.

I. INTRODUCTION

IN this paper, norm-based commutation is discussed for the class of ironless over-actuated electromagnetic actuators (IOEA) that can be described by a linear relation between the current $\underline{i}$ applied to the individual coils and the final wrench vector $\underline{w}$ (composition of forces and/or torques) acting on the actuator (e.g. [1], [2]):

$$\underline{w} = \mathbf{K}(\underline{q}) \underline{i} \quad (1)$$

The $m \times n$ coupling matrix $\mathbf{K}$ is usually a nonlinear function of the position and orientation $\underline{q}$ of the translator/rotor of the actuator. Ironless actuators do not suffer from cogging force and reluctance effects due to the permanent magnets can be neglected [2]. Therefore, the relation (1) is effectively linear as it is dictated only by the Lorentz force acting on a piece of wire carrying an electrical current in a magnetic field. The decoupling can be described by the quasi-static model (1) also during transient operation assuming that [2]: 1) magnetic fields are quasi-static 2) the force caused by eddy currents is negligible 3) the wire diameter of the coil is smaller than skin-depth.

Over-actuated actuator means that the number of active coils is always greater than the number of degrees of freedom (DOF). In mathematical sense, the set of equations (1) is under-determined or $\dim(\underline{i}) > \dim(\underline{w})$. The rank of the matrix $\mathbf{K}$ must be equal to the amount of DOFs for all positions $\underline{q}$ for the system of equations to be consistent.

To linearize and decouple an IOEA, an inverse mapping

of (1) is necessary:

$$\underline{i} = \underline{\mathbf{K}}^-(\underline{q}, \underline{w}_{\text{des}}), \quad (2)$$

where $\underline{w}_{\text{des}}$ is the desired wrench vector and $\underline{\mathbf{K}}^-$ is a mapping from the desired wrench vector to the current vector $\underline{i}$ at the actual position $\underline{q}$. The transformation provided by this position-dependent inverse mapping is called *commutation*. The inverse mapping $\underline{\mathbf{K}}^-$ is not necessarily linear in $\underline{w}$, because over-actuation brings freedom in the choice of the current vector. If $\underline{\mathbf{K}}^-$ is linear in $\underline{w}$, the inverse mapping, being a vector function of $\underline{w}$, can be written as a position-dependent matrix $\mathbf{K}^-(\underline{q})$ independent of $\underline{w}$: $\underline{\mathbf{K}}^-(\underline{q}, \underline{w}) \approx \mathbf{K}^-(\underline{q})\underline{w}$.

Classically, for most rotational (and linear) actuators this inverse mapping is achieved by using dq0- or Park's transformation [3]-[7]. For linear and planar actuators, this transformation can be used directly to derive a commutation that decouples only the force components. In moving-coil planar actuator, such as [8], [9], it is possible to use design symmetries, which reduce the complexity of the torque equations. Then an additional transformation can be derived, which allows for decoupling of the torques [10]. Moving-magnet planar actuators with integrated magnetic bearing [11]-[14] have complex torque equations [15]. As a result, the dq0-transformation is not convenient for them. In literature attempts can be found to decouple torque using additional transformation after applying dq0-transformation [16], [17]. Nevertheless, the resultant disturbance torque was still significant. The algorithm was further improved by Binnard et al. [18], [19] resulting in commutation for 6-DOF planar actuator, but the full torque equations are still not included.

In the next section of this paper, three norm-based commutation methods will be described of which two methods are presented for the first time as alternative commutation methods for IOEA. All three methods are compared on an example of a magnetically levitated planar actuator in the third and fourth section.

II. NORM-BASED COMMUTATION

Three norm-based approaches are discussed in this section, l^2 , l^∞ , and clipped l^2 -norm based commutations.

A. l^2 -norm based Commutation (L2C)

To overcome the problem of torque decoupling, a method for direct wrench-current decoupling has been developed independently and in parallel by [20] and [21]. The under-determined set of equations (1) offers the possibility to impose extra constraints while creating an inverse mapping. An interesting additional constraint is to minimize the sum of the Ohmic losses in the coils (or dissipated power). This

This IOP-EMVT project is funded by SenterNovem. SenterNovem is an agency of the Dutch Ministry of Economical Affairs.

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can be done by minimizing the l^2 -norm of the current vector:

$$\min_{\mathbf{K}(\underline{q})\underline{i}=\underline{w}_{\text{des}}} \|\underline{i}\|_2 = \|\mathbf{K}_2^-(\underline{q})\underline{w}_{\text{des}}\|_2 \quad (3)$$

The matrix $\mathbf{K}_2^-$, which minimizes the l^2 -norm, is a *reflexive generalized inverse* of $\mathbf{K}$, also known as a *pseudo-inverse* of $\mathbf{K}$ [22], and can be calculated explicitly:

$$\mathbf{K}_2^-(\underline{q}) = \mathbf{K}^T(\underline{q})(\mathbf{K}(\underline{q})\mathbf{K}^T(\underline{q}))^{-1}. \quad (4)$$

This method minimizes the necessary power to operate the actuator. The second benefit is that the commutation is obtained in a single step (no iterations are necessary). The third benefit is the short calculation time due to the single step solution and the simplicity of calculation of the pseudo-inverse. The inverse in (4) is calculated only from a $m \times m$ matrix, where $m = \dim(\underline{w})$ and therefore for any over-actuated actuator $m \leq 6$. The last, but very important property is the continuous current variation with the position $\underline{q}$ or the wrench $\underline{w}_{\text{des}}$ variation. For application of the commutation on a real over-actuated actuator, continuous current change is important, because too high current variation di/dt causes peaks in the terminal voltage u of the coil [23]:

$$u = iR + L \frac{di}{dt} + \frac{\partial \Lambda}{\partial x} \frac{dx}{dt}, \quad (5)$$

where R and L are resistance and inductance, respectively, of the coil, $\partial \Lambda / \partial x$ is a change of the flux linkage of the permanent magnets with the coil.

The drawback is that the l^2 -norm based commutation cannot imply any constraints on the current maximum. The pseudo-inversion (4) can easily generate currents that are over the physical limits of the amplifiers or coils. In such a situation amplifiers clip the current or, even worse, they turn themselves off due to overload/overheat. In any case, forces and torques of the actuator are no more linearized and decoupled introducing considerable control errors. For that reason it might be better to use different criteria for minimization.

B. l^∞ -norm based commutation (LiC)

The infinity-norm is the only norm that directly puts constraints on the maximum current. The criterion for minimization is given as:

$$\min_{\mathbf{K}(\underline{q})\underline{i}=\underline{w}_{\text{des}}} \|\underline{i}\|_\infty = \|\mathbf{K}_\infty^-(\underline{q}, \underline{w}_{\text{des}})\|_\infty. \quad (6)$$

In mathematical literature, this problem is known under the term: *Chebyshev solution of an underdetermined system of linear equations* [24], [25]:

$$\min_{\underline{x}} \{\|\underline{x}\|_\infty \mid \mathbf{A}\underline{x} = \underline{y}\}, \quad (7)$$

where $\|\underline{x}\|_\infty = \sup(|x_1|, |x_2|, \dots, |x_n|)$ is called l^∞ - or Chebyshev norm. From the nature of the problem, the inversion mapping $\mathbf{K}_\infty^-$ is not affine in $\underline{w}$ anymore, and, consequently, $\mathbf{K}_\infty^-$ cannot be written as a matrix as in the l^2 -norm situation. The main difficulty of solving this mathematical problem is that it is not possible to obtain the solution explicitly. All the known methods reach the solution of (7) iteratively. The methods differ in memory requirements and in the necessary calculation time, which is mostly dependent on the number of iterations. An extensive

summary of the methods for calculation of underdetermined system of linear equations can be found in [24]. However, one additional, fast and effective algorithm is discussed in [25]. The algorithm in [25] employs a linear programming algorithm for the solution of a set of over-determined linear equations in the l^1 -norm to obtain a minimum l^∞ -norm solution to the set of consistent linear equations. To minimize the number of extraneous variables, the vector $\underline{x}$ is normalized first. Then the solution is iteratively obtained via the dual l^1 problem. The main benefit is a lower number of necessary iterations, which depends on the number of equations and less on the number of variables. Thus the algorithm is supposed to lead to a shorter calculation time. This is in contrast to a similar principle for the calculation that can be found in [24], where linear programming is used to obtain only the initial solution. After this step, the iteration process employs a slightly modified simplex method.

A few examples have been used to compare the effectiveness of both algorithms. It was found that the number of iterations in the algorithm in [25] depends more on the number of equations (m) whereas the algorithm form [24] is dependent on the number of variables (n). Both algorithms obtain the same solution whereas the one form [25] is always about twice as fast regardless the dimensions of the matrix $\mathbf{A}$.

The advantage of the l^∞ -norm minimization is direct minimization of the maximum currents. This goes so far that $n_a - m + 1$ variables have the same minimized absolute value [25], where n_a is number of variables that have nonzero coefficient in at least one of the equations (number of the active coils). From a physical point of view, it means that as many coils ($n_a - m + 1$) will be energized to the same level even if their contribution to the final wrench is very small. In consequence, although the maximum current is reduced, the actuator needs much more power than the l^2 -norm solution. In addition, the energized coils will produce more force, which will cancel each other (the final wrench must be the same); hence, the sensitivity to coupling matrix errors will be higher. Another drawback, from an application point of view, is a discontinuous current-variation with a change of position $\underline{q}$.

C. Clipped l^2 -norm based commutation (CL2C)

So far, two solutions for commutation of an IOEA have been discussed in this paper. The first solution minimizes the l^2 -norm of the current vector and, therefore, the dissipated power with no constraints on the maximum current. The second one minimizes the l^∞ -norm of the current vector or the maximum currents whereas the power usage increases rapidly. Obviously, neither of the solutions is perfect. The compromise is to minimize the used power whereas the maximum current can be limited if necessary:

$$\min_{\mathbf{K}(\underline{q})\underline{i}=\underline{w}_{\text{des}}, \|\underline{i}\|_\infty \leq i_{\text{clip}}} \|\underline{i}\|_2, \quad (8)$$

where i_{clip} is the limit on the current. Minimization of l^2 -norm of constrained system of linear equations can be solved by *quadratic programming* (e.g. [26]) where a cost function $f(\underline{x})$ is minimized subject to inequality and equality constraints:

$$f(\underline{x}) = \frac{1}{2} \underline{x}^T \mathbf{Q} \underline{x} + \underline{c}^T \underline{x}, \quad (9)$$

$$\mathbf{A} \underline{x} < \underline{b}, \quad \mathbf{E} \underline{x} = \underline{d}.$$

By using the notation of (8), the quadratic programming problem (9) can be defined as:

$$f(\underline{i}) = \frac{1}{2} \underline{i}^T \underline{i}, \quad (10)$$

$$-\underline{i}_{\text{clip}} < \underline{i} < \underline{i}_{\text{clip}}, \quad \mathbf{K} \underline{i} = \underline{w}_{\text{des}},$$

where $\underline{i}_{\text{clip}}$ is the $n \times 1$ vector of the current limit values.

The large calculation time, for solving this quadratic programming problem, prohibits real-time commutation. For that reason a novel, fast commutation method has been developed that, as well, puts constraints on the maximum current whereas the current vector is still minimized in l^2 -norm. The calculation process itself is also iterative, but the number of iteration is low and each iteration step is calculated fast.

The algorithm is based on the fact that if the currents are calculated with L2C and exceed the limit ($|i_j| > i_{\text{clip}}$), they will the most probably have the limit value $i_j := i_{\text{clip}} \text{sgn}(i_j)$ when solved according to (8). The algorithm has the following steps:

- 1) Calculate the pseudo-inversion $\mathbf{K}_2^-$ (4) of the coupling matrix $\mathbf{K}$, which minimizes current vector in l^2 -norm (3).
- 2) If any of the values exceeds the clipping limit $|i_j| > i_{\text{clip}}, j = 1, \dots, n$, continue, otherwise go to end.
- 3) All exceeding currents are saturated - a vector of clipped currents ${}^c \underline{i}$ is generated:

$$[{}^c i]_j := \begin{cases} i_{\text{clip}} \text{sgn}(i_j) & \text{if } |i_j| > i_{\text{clip}} \\ 0 & \text{if } |i_j| \leq i_{\text{clip}} \end{cases}, j = 1, \dots, n. \quad (11)$$

- 4) A new matrix ${}^{l2} \mathbf{K}$ is created from $\mathbf{K}$, where columns corresponding to nonzero elements of vector ${}^c \underline{i}$ are set to zero. A new desired wrench vector ${}^{l2} \underline{w}_{\text{des}}$ is calculated:

$${}^{l2} \underline{w}_{\text{des}} = \underline{w}_{\text{des}} - \mathbf{K} {}^c \underline{i}. \quad (12)$$

- 5) By using the pseudo-inversion (4), a new current vector ${}^{l2} \underline{i}$ is calculated from the reduced system:

$${}^{l2} \mathbf{K} {}^{l2} \underline{i} = {}^{l2} \underline{w}_{\text{des}}. \quad (13)$$

- 6) The final current vector ${}^{cl2} \underline{i}$ is the summation of the clipped values ${}^c \underline{i}$ and the vector of l^2 -norm minimized values ${}^{l2} \underline{i}$:

$${}^{cl2} \underline{i} = {}^{l2} \underline{i} + {}^c \underline{i}. \quad (14)$$

- 7) The process loops back to the step 2 (with $\underline{i} := {}^{cl2} \underline{i}$) while any of the current values exceeds the clipping limit.

It is important to mention that the solution of this heuristic algorithm is not necessarily optimal. It might happen that by saturating one of the currents some other, which was also saturated, could be lowered to obtain the optimal solution, but it is not lowered. The optimal solution can be obtained for example by quadratic programming. The

main advantage of this approach in comparison to the quadratic programming is in the calculation time, which is just k -times the calculation time of l^2 -norm solution, where k is number of iterations needed.

The number of iterations mostly depends on the clipping value i_{clip} for the current limitation. If the clipping value is higher than the actual maximum current obtained with l^2 -norm, only one iteration step is needed. If the value is lower than the maximum current, at least one additional iteration step is required. The number of iterations increases as the clipping value decreases down to its lowest limit, which is equal to the value of the minimized l^2 -norm of the current vector $\min \|\underline{i}\|_{\infty}$. The maximum theoretical number of iterations is thus $n_a - m + 2$. This situation occurs only when in each iteration step one (extra) current is saturated.

Although the algorithm is heuristic, the obtained solution shows an exact match with the solution obtained with quadratic programming in the majority of tests. From simulations, the same solution as from the quadratic programming was obtained with higher probability if the clipping value was further from the minimum l^2 -norm solution (fewer currents had to be saturated). Even if the obtained solution is suboptimal, it still satisfies given constraints and the non-saturated currents are minimized in l^2 -norm. Therefore, the dissipated power will always be lower than those calculated by LiC, so:

$${}^{l2} \|\underline{i}\|_2 \leq {}^{cl2} \|\underline{i}\|_2 \leq {}^{l\infty} \|\underline{i}\|_2. \quad (15)$$

Moreover, the current steps caused by the CL2C are not as severe as in the LiC, because only the currents close to i_{clip} value can make step to/from the saturation limit. Other currents (further from the boundary) will also make small step in consequence, to satisfy the desired wrench production. With known parameters of an IOEA and by using (5), one can calculate how big current step the current amplifiers can handle. For example, with a coil inductance of 10 mH, sampling frequency of 1 kHz, and current step of 0.1 A, the peak in the voltage will have amplitude of 1 V. Current amplifiers can usually handle much higher voltage peaks.

Continuous current-change is guaranteed if the consecutive solutions are the same as those obtained by quadratic programming and hence optimal. Continuity of solution obtained via quadratic programming is shown e.g. in [27]-[29].

In a real application, current amplifiers should not be the limiting factor during the whole trajectory of the actuator. On the contrary, current limitation via commutation should be considered as a safety layer for unexpected circumstances (increased load, disturbing forces etc.) or just for several spots in the trajectory of the actuator. If a significant number of coils need to be limited for most of the time of the motion, the actuator will have increased dissipated power. Obviously, this is not a good design. If only several currents are limited, the obtained solution is with high probability optimal and hence continuous. Therefore, applicability of CL2C on a real actuator should be possible.

For a predefined trajectory of the actuator, the current continuity and the peak voltages in the case of the discontinuity can be tested in simulation in advance.

In Table I, all three norm-based commutation methods

are compared. The calculation time is based on a model of an actuator with $n = 100$ coils and $m = 6$ degrees of freedom. From the comparison, clipped l^2 -norm based commutation comes out as the best trade-off between the algorithm performance and the constraint satisfaction.

Commutation methods based on minimization of other l^n -norms with $2 < n < \infty$ have no practical sense, since they cannot limit the peak current, nor they minimize the dissipated power.

TABLE I
COMPARISON OF THE NORM-BASED COMMUTATION METHODS

Commutation method	l^2 -norm	l^n -norm	clipped l^2 -norm
Current limitation	–	++	+
Current continuity	++	–	+/-
Power losses	++	–	+
Calculation time (μ s)	70	450	$iter \times 70^a$
Overall Performance	+	0	++

^a Calculation time is approximately equal to the calculation time of the L2C multiplied by the number of iterations.

III. EXAMPLE ON MODEL OF PLANAR ACTUATOR

The presented norm-based commutation algorithms have been first tested on a model of a magnetically levitated and propelled planar actuator (PA) illustrated in Fig. 1 [14]. Since the number of active coils is always greater than the number of DOFs, the system satisfies the definition of over-actuated actuator. Because the commutation is a static mapping between the desired wrench vector $\underline{w}$ and the calculated current vector $\underline{i}$, simulation of the commutation gives the same results as we would obtain from the real setup. The dimensions of the coupling matrix $\mathbf{K}$ from (1) are $n = 99$ and $m = 6$. The dimensions and properties of PA are indicated in the drawing in Fig. 2 and listed in Table II.

Fig. 1. Drawing of moving-magnet planar actuator.

Fig. 2. Definition of dimensions of the planar actuator.

TABLE II
DIMENSIONS OF THE PLANAR ACTUATOR

Parameter	Symbol	Dimension
Halbach magnet array		10×10 magnet poles
Magnet-pole pitch	τ	40 mm
Magnet width	m_w	25.6 mm
Magnet height	m_h	10.3 mm
Coil array		9×11 coils
Coil spacing	c_s	$4/3\tau$ (53.33 mm)
Coil diameter	c_w	51 mm
Coil height	c_h	11.4 mm
Coil bundle width	c_{bw}	21 mm
Coil inductance	L	8.1 mH
Coil resistance	R	4.4Ω
Levitated mass	m	20.5 kg
Maximum velocity	v	0.5 m/s
Sampling frequency	f_s	3 kHz
Designed levitation height	h	0–2 mm

Fig. 3 shows the energized coils example for one position of the PA when the commutation is calculated with the L2C and LiC for $\underline{w}_{des} = [F_x, F_y, F_z, T_x, T_y, T_z]^T = [200 \text{ N}, 200 \text{ N}, 200 \text{ N}, 0, 0, 0]^T$. The difference is visible immediately. With the L2C, only a few coils are energized to the significant level (those that have the highest effect on the commutation). On the other hand, with the LiC all the coils with nonzero effect on commutation are energized. Moreover, most of them are energized to the same absolute value, which is less than the maximum value reached by the L2C.

Fig. 3. Distribution of currents in the coils calculated with L2C (a) and LiC (b). The solid square represents actual position of the PA. The dashed rectangle is working area of the PA.

Fig. 4. Distribution of currents in the coils calculated with L2C, CL2C and LiC for the actual position of the PA. Only first 50 coil currents shown.

The current values are also plotted in Fig. 4 where all three types of commutation are used: L2C, CL2C and LiC with $^{l2}\underline{i}$, $^{cl2}\underline{i}$, and $^{li}\underline{i}$ current vectors, respectively. The clipping of the maximum current shifts the CL2C towards the LiC.

The dissipated power, calculated as $P = \underline{i}^T \mathbf{R} \underline{i}$, where $\mathbf{R}$ is a diagonal matrix of the coil resistances, for different clipping currents of the CL2C is presented in Fig. 5. From the figure, one can conclude that reducing the maximum current moderately will not cause significant power increase, however trying to get close to the minimum l^∞ -norm current limit, where most of the coils are limited, will increase the power demands substantially. In this example, it is more than 50%.

Fig. 5. Dissipated power in dependence on clipping current i_{clip} . For $i_{clip} > ^{l2}i_{max}$ the dissipated power is the same.

IV. EXAMPLE ON THE PROTOTYPE

The L2C and CL2C have also been tested on the real experimental setup (see Fig. 6). The figure shows different variables during point-to-point movement of the PA at $v_{max} = 0.6$ m/s, $a_{max} = 14$ m/s², and $j_{max} = 1400$ m/s³. The clipping (saturation) current i_{clip} in the CL2C was set to 1.8 A. The plots compare the CL2C with the original L2C and with L2C with artificially saturated currents (also 1.8 A). The L2C with the saturated currents demonstrates the system behavior in case of saturated current amplifiers.

In the case of the L2C with saturated amplifiers, the plots with the position errors show several times increased tracking errors. The large tracking errors are caused by the total applied wrench, which does not correspond to the

Fig. 6. Comparison of commutation methods; L2C, L2C with saturated amplifiers and CL2C (solid black, dashed black, and gray, respectively, if not defined otherwise).

desired wrench when the currents are saturated. On the other hand, the tracking errors for the CL2C and L2C are almost identical (see the inset). This proves that the total wrench vectors produced by the both commutation algorithms are equivalent. The plots with the maximum current show how much was the peak current reduced (up to 45 %). The current profiles in all the coils are also shown. The plots with the maximum absolute current slew rate ($\max |\Delta \underline{i} / \Delta t| = \max |(\underline{i}_k - \underline{i}_{k-1}) / \Delta t|$) show no visible discontinuity in the solution. The increased current change is only caused by the necessity of faster variation of the non-saturated currents.

With the known value of the inductance $L = 8.1$ mH, the terminal voltage of the coil due to slew rate di/dt is at most 3.6 V. The clipping current limit of 1.8 A was chosen to obtain the solution in the maximum of five iterations, while up to 20 coil currents were saturated. The plots also show that for the CL2C the dissipative power increased only by about 8 % in comparison to the L2C.

V. CONCLUSIONS

In this paper, three norm-based commutation methods have been compared. The l^2 -norm based commutation (L2C), has main benefit in the minimization of dissipated power. The drawback is its inability to constraint the maximum current, which can cause saturation of the amplifiers and consequently the difference between the desired and the real forces and torques. This problem can be solved by two novel norm-based commutation techniques. The benefit of l^∞ -norm based commutation (LiC) is that the maximum current is always minimized. The negative aspect is increased dissipated power. Therefore, we proposed clipped l^2 -norm based commutation (CL2C), which puts constraints on the maximum current whereas the non-saturated coils are minimized in l^2 -norm. By varying the clipping (saturation) value the obtained solution is closer to either the L2C or LiC.

The heuristic algorithm for the CL2C does not necessarily lead to the optimal solution, but from the tests performed the discontinuity is not visible in the real system. Benefit of this algorithm is in calculation time, which is reduced considerably in comparison to optimal CL2C obtained via quadratic programming.

With the presented novel commutation techniques, a higher acceleration of the translator can be achieved and/or less powerful (cheaper) current amplifiers can be utilized and/or fewer commutation errors arise.

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