Turbulent oscillating channel flow subjected to a free-surface stress.

Citation for published version (APA):

Kramer, W., Clercx, H. J. H., & Armenio, V. (2010). Turbulent oscillating channel flow subjected to a free-surface stress. *Physics of Fluids*, 22(9), 095101-1/12. [095101]. <https://doi.org/10.1063/1.3481149>

DOI:

[10.1063/1.3481149](https://doi.org/10.1063/1.3481149)

Document status and date:

Published: 01/01/2010

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Turbulent oscillating channel flow subjected to a free-surface stress

W. Kramer,¹ H. J. H. Clercx,¹ and V. Armenio²

¹*Department of Physics, Fluid Dynamics Laboratory, Eindhoven University of Technology,
P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

²*Dipartimento di Ingegneria Civile e Ambientale, Università di Trieste,
Piazzale Europa 1, 34127 Trieste, Italy*

(Received 27 May 2010; accepted 14 July 2010; published online 1 September 2010)

The channel flow subjected to a wind stress at the free surface and an oscillating pressure gradient is investigated using large-eddy simulations. The orientation of the surface stress is parallel with the oscillating pressure gradient and a purely pulsating mean flow develops. The Reynolds number is typically $Re_\omega = 10^6$ and the Keulegan–Carpenter number—the ratio between the oscillation period and advection time scale—is $KC = 80$. Results compare favorably to the data from direct numerical simulations obtained over a single period. A slowly pulsating mean flow occurs with the turbulent flow essentially being statistically steady. Logarithmic boundary layers are present at both the bottom wall and the free surface. Turbulent streaks are observed in the bottom and free-surface layer. The viscous sublayer below the free surface is, however, much thinner. This observation is verified by simulations we performed for a purely wind-driven channel flow. For the oscillating flow, additional low-speed splats (localized regions of upwelling) occur at the free surface when the mean velocity and stress are in the same direction. © 2010 American Institute of Physics.

[doi:10.1063/1.3481149]

I. INTRODUCTION

In this paper, we present the results from large-eddy simulations (LES) of a periodic channel flow subjected to an oscillating pressure gradient and stress at the free surface. No-slip boundary conditions are applied at the bottom of the channel. This flow is a simple model for a water column in a tidal channel or estuary. The parameters for the simulation are loosely based on the data for the Westerschelde estuary. This estuary of the Schelde river is situated in the south western part of the Netherlands. As the depth is of the order $h = 10$ m, the estuary can be considered to be shallow. We are here particularly interested in the vertical variation of the flow in the fluid column. The flow in the estuary is mainly driven by the tide with the relating volume flow ranging from 12 000 m³/s at the point the Schelde river enters to 80 000 m³/s at the outlet in the North Sea. With a volume flow of 110 m³/s, the inflow of the Schelde river is only a fraction of the tidal flow. Maximum fluid velocities are in the range $U_0 = 0.2$ – 1.0 m/s. The lunar semidiurnal tide with an angular frequency of $\omega = 1.40 \times 10^{-4}$ rad/s is the main component of the total tide. Typical wind speeds in this region are $U_{10\text{ m}} = 5$ – 10 m/s.

The Reynolds number based on the height of the domain, $Re_h = U_0 h / \nu$ with ν the kinematic viscosity, is of order of $\mathcal{O}(10^6)$ – $\mathcal{O}(10^7)$, indicating that the flow is turbulent. Turbulent time scales are much smaller than the tidal period as indicated by the large Keulegan–Carpenter number, $KC = U_0 / \omega h = \mathcal{O}(100)$, which expresses the ratio between the tidal period and the advection time scale. For oscillating flows, it is common to define a Reynolds number based on the frequency, $Re_\omega = Re_h KC = U_0^2 / \omega \nu$, which is of the order of $\mathcal{O}(10^8)$ – $\mathcal{O}(10^9)$ in the Westerschelde. Essentially, the length scale used in the Reynolds number Re_ω is equal to

U_0 / ω , which is related to the distance a drifter would travel between high and low tide. From the wind speed at a reference altitude of 10 m, the stress exerted at the free surface can be calculated using $\tau_{fs} = \rho_{air} C_D U_{10\text{ m}}^2 = 0.16$ N/m² with ρ_{air} , the density of air and $C_D = 1.3 \times 10^{-3}$.¹ The bottom stress due to the tidal flow can be estimated using $\tau_w = \frac{1}{2} \rho f_w U_0^2$ with ρ , the density of water and f_w , the friction coefficient. For large Reynolds numbers, the friction coefficient is $f_w = 2 \times 10^{-3}$.² Depending on the fluid velocity, the bottom stress ranges at $\tau_w = 0.04$ – 1 N/m². Comparing the wall stress and the wind stress, one can conclude that the flow behavior can be either oscillatory ($\tau_w \gg \tau_{fs}$) or current dominated ($\tau_w \ll \tau_{fs}$). In this paper, we will only consider the case with $\tau_{fs} = \frac{1}{2} \tau_w$ and the wind stress parallel with the oscillating pressure gradient. To inquire the effect of all parameters, we provided a brief discussion of key observations from the literature first.

The purely oscillating channel flow is a much studied configuration for geophysical and industrial applications (for an overview see Refs. 3 and 4). The governing dimensionless number is the Reynolds number $Re_S = U_0 \delta_S / \nu$, based on the thickness of the Stokes boundary layer $\delta_S = \sqrt{2\nu/\omega}$. This Reynolds number also follows from $Re_\omega = Re_S^2 / 2$. Four different regimes can be distinguished: the laminar, the disturbed-laminar, the intermittent turbulent, and the turbulent regime.⁵ At $Re_S \approx 100$, small two-dimensional disturbances appear on the base laminar Stokes flow. For $Re_S > 500$, the flow enters the intermittent turbulent regime. Here, explosive turbulent near-wall bursts appear during the deceleration phase, while the flow relaminarizes during acceleration.⁶ For higher Re_S , the flow becomes turbulent in earlier phases and remains turbulent for longer periods. A study by Jensen *et al.*² showed that fully developed turbu-

lence is already present during most of the oscillation cycle at $Re_S \sim 1800$, while the flow remains turbulent during the complete cycle for $Re_S > 3500$.

For an oscillating flow, it is important how deep the turbulence created in the shear layers near the wall can penetrate into the bulk of the flow. Scotti and Piomelli⁷ performed turbulence simulations in a periodic channel domain driven by a pulsating pressure gradient. They introduced the turbulent Stokes length $\delta_t = \sqrt{2(\nu + \nu_t)/\omega}$ as a measure for the turbulent penetration depth, where ν_t is an eddy viscosity. For $\delta_t \gg h$, the turbulence created in the shear layers extends quickly over the full height. For every phase of the cycle, the production of turbulence is balanced by the dissipation. The flow is essentially not different from a turbulent boundary-layer flow driven by a constant pressure gradient. For $\delta_t = \mathcal{O}(h)$, the time needed for the turbulence generated in the boundary layer to extend over the channel height is of the same order as the oscillation period. Production and dissipation of turbulent fluctuations are no longer in phase. For $\delta_t < h$, the interior is not affected by the turbulence created in the shear layers. This typically leads to a plug flow in the interior where turbulence is essentially frozen. Only in thin wall layers, turbulent production and dissipation are out of phase. These high frequency pulsating flows are typical for aeroacoustics. In estuaries, the tidal period is large compared to the turbulent time scales and one would expect turbulence to be in the statistically steady state.

Lodahl *et al.*⁸ performed an experimental study of pulsating flow in a cylindrical pipe. The character of the flow changes depending on the ratio V/U_0 with V , the time mean velocity and U_0 , the amplitude of the oscillatory part. If the current-only case is already turbulent, the added oscillatory part can either suppress or enhance turbulence. Essentially, when the flow is current dominated with $V/U_0 \gg 1$, there is basically no change in the wall stress. For $V/U_0 \lesssim 1$, a laminarization of the flow could occur when the oscillatory component in itself would be in the laminar regime. This leads to a decrease in the mean wall stress. If the oscillatory part is in the turbulent regime, the total flow will be turbulent and increase of the wall stress is observed.

The effect of the fixed wind stress is not limited to driving a constant current. Depending on the conditions, many processes are related to the production of turbulence near the free surface. In our studies, we will restrict to a nondeformable free surface and a uniform wind stress. This excludes processes such as wave breaking and Langmuir circulation.⁹ The assumption of using a nondeformable free surface has been motivated in several studies.^{10–13} Of particular interest is the paper by Tsai *et al.*¹³ on a numerical study of a stress-driven turbulent shear flow with a nondeformable free surface. They concluded that despite the idealization, the flow exhibits many features observed in laboratory and field experiments. Two of these features are elongated high-speed streaks and localized low-speed spots at the free surface. Just below the free surface, a thin viscous layer is present followed by a logarithmic layer.

The present investigation concerns an oscillating channel flow subjected to a wind stress at the free surface, where the novel aspect is the introduction of a constant surface stress to

the oscillating channel flow problem. Interesting aspects are the influence of the turbulence created in the free-surface layer on the interior turbulent flow, and, vice versa, how the free-surface layer is affected by the interior flow. The paper is organized in the following sections. The problem is described in Sec. II. The flow is studied by performing large eddy simulations. The numerical approach is described in Sec. III. In Sec. IV, we briefly summarize our results of a simulation of the turbulent channel flow driven by a constant wind stress. This allows us to distinguish the phenomena induced by the constant surface stress only and those by the oscillatory component. Then, in Sec. V, the main results for the oscillating channel problem with a constant wind stress are presented. Here we discuss the LES at different resolutions and compare them to results from direct numerical simulations (DNS). Furthermore, we investigate a number of mean and turbulent properties of the flow in the interior and boundary layer regions. Finally, conclusions are drawn in Sec. VI.

II. PROBLEM DESCRIPTION

We aim to model the tidal flow in an estuary by numerically solving the incompressible Navier–Stokes equations on a periodic channel domain. In the coordinate system, x and y are chosen in the horizontal directions and z in the vertical. The cubic domain measures $l_x \times l_y \times h$ with the bottom wall located at $z=0$ and the free surface at $z=h$. As the fluid density ρ is considered to be constant, the velocity field $\mathbf{u}=(u,v,w)$ is divergence free, i.e., $\nabla \cdot \mathbf{u}=0$. The Navier–Stokes equation reads

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\rho^{-1} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}, \quad (1)$$

where p is the pressure, ν is the kinematic viscosity, and $\mathbf{f}$, any external forces. At the bottom wall, no-slip boundary conditions are applied,

$$\mathbf{u} = 0 \quad \text{for } z = 0. \quad (2)$$

A constant wind stress τ_{fs} is imposed at the free surface and is orientated along the x axis,

$$\rho \nu \frac{\partial u}{\partial z} = \tau_{fs}, \quad \frac{\partial v}{\partial z} = 0, \quad w = 0 \quad \text{for } z = h. \quad (3)$$

For both horizontal directions, periodic boundary conditions are applied. To mimic the tidal oscillation in the estuary, an external oscillating pressure gradient is applied over the periodic x direction. It can be written as a volume force,

$$\mathbf{f}_p = U_0 \omega \cos \omega t \mathbf{e}_x, \quad (4)$$

with ω , the frequency. In the case of an unbounded domain, this oscillating pressure gradient would drive a free stream velocity $(U_0 \sin \omega t, 0, 0)$ with amplitude U_0 and period $T_p = 2\pi/\omega$. In this case, the flow is accelerated until it reaches maximum velocity at $\varphi = 90^\circ$. Subsequently, the flow decelerates until the flow direction is reversed at $\varphi = 180^\circ$. Then, the flow is accelerated in the negative x direction until a maximum negative velocity is reached at $\varphi = 270^\circ$ followed by a deceleration of the flow. The model can be considered to

FIG. 1. The domain describes a water column of size $l_x \times l_y \times h$ bounded by a no-slip bottom and a free surface at the top. In the simulations periodic boundary conditions are assumed for both the streamwise and spanwise directions. An oscillating pressure gradient $f_p = U_0 \omega \cos \omega t$ is applied over the x direction, while a wind stress τ_{fs} is acting on the free surface.

describe a rectangular fluid column in an estuary. The effects of the lateral boundaries or a deformable free surface are, hence, not included in the simulation. A sketch of the domain is given in Fig. 1.

Three independent dimensionless numbers can be derived for this problem. The first is the Reynolds number, $Re_\omega = U_0^2 / \omega \nu$. Second, the Keulegan–Carpenter number gives the ratio between the tidal period and the advection time scale, $KC = U_0 / \omega h$. The third dimensionless number relates the two forces driving the flow, i.e., the oscillating pressure gradient and the free-surface stress. Here, the ratio $\tau_{fs} / \tau_{w,max}$ between the applied surface stress τ_{fs} and the maximum stress observed at the bottom, $\tau_{w,max}$, can be used. For the simulations, all quantities are made dimensionless by using h and U_0 as the typical length and velocity scales, respectively. The advection time scale is then $T_a = h / U_0$.

III. NUMERICAL SETUP

For the numerical investigations, we resort to LES. Several studies revealed that LES can be used for oscillating wall flows.^{7,14} Nonetheless, we will also perform DNS for our specific flow problem, as in none of those studies a free-surface stress was applied in combination with an oscillatory forcing. In LES only the dynamics of the large scales in the flow are calculated directly. By applying a top-hat filter to the incompressible Navier–Stokes equations¹⁵ we obtain

$$\frac{\partial \hat{u}_i}{\partial x_i} = 0, \quad (5)$$

and

$$\frac{\partial \hat{u}_i}{\partial t} + \frac{\partial \hat{u}_i \hat{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial x_i} + \nu \frac{\partial^2 \hat{u}_i}{\partial x_j \partial x_j} + \hat{f}_i - \frac{\partial \tau_{ij}}{\partial x_j}. \quad (6)$$

Equation (6) for filtered variables closely resembles the Navier–Stokes equations for the unfiltered variables. The additional term in the right-hand side of Eq. (6) describes the effect of the subgrid-scale stresses on the filtered velocity $\hat{u}_i$. Note that the subgrid-scale stress term, given by $\tau_{ij} = u_i u_j - \hat{u}_i \hat{u}_j$, depends on the unfiltered velocity field u_i . A closure model for the subgrid-scale stresses is required for solving Eqs. (5) and (6) to provide an approximation for $\hat{u}_i$. For our simulations, we use a dynamic-mixed model, a combination of an eddy viscosity model and scale-similarity model¹⁶ with a dynamic determination of the eddy viscosity coefficient.¹⁷

More details on the applied subgrid-scale model are given in Ref. 18.

We aim to investigate the case with the surface stress aligned with the pressure gradient. For this purpose, Eqs. (5) and (6) are solved for a rectangular cuboid domain, bounded by a no-slip bottom at $z=0$ and a free surface at $z=1$, using a finite difference method based on the method of Zang *et al.*¹⁹ The horizontal x and y direction are periodic. Recall that the flow quantities are made dimensionless using the channel height, the velocity amplitude of the tidal flow and the density of water. Dimensionless time is then expressed in advection time units. An oscillating pressure gradient (4) is applied over the x direction with $U_0=1$ and $\omega=1/80$. This leads to a Keulegan–Carpenter number of $KC=80$. The density in the simulations is scaled by the density of water and hence is set to $\rho=1$. The viscosity is set to $\nu=8 \times 10^{-5}$ to obtain the desired Reynolds number of $Re_\omega=10^6$. The horizontal dimensions of the domain must be large enough to contain the large energy containing eddies. The domain dimensions are set to $l_x \times l_y \times h = 4 \times 2 \times 1$, which is comparable to the one used by Scotti and Piomelli.⁷ The largest correlation scale in the velocity field we observe in the simulations is about 0.3, i.e., much smaller than the horizontal domain dimensions.

The number of grid points required for accurate LES has been determined by performing a number of simulations at different resolutions. These results have been compared to a DNS at an even higher resolution solving all the flow scales directly. The setting for the simulations is specified in Table I. For the bottom boundary layer to be resolved, the first grid cell is set equal to one wall unit $\Delta z^+ = \Delta z / z^* = 1$. Here, $z^* = \nu / u_{\tau,max}$ with the friction velocity following from the maximum wall stress, $u_{\tau,max} = \sqrt{\tau_{w,max} / \rho}$. Resolving the free-surface layer proved to require a finer resolution. Here, the first grid cell height is $\Delta z^+ = \frac{1}{2}$. At this resolution there are always at least 8 grid points inside the viscous sublayer at the wall and below the free surface. The Kolmogorov scale, $\eta = (\nu^3 / \epsilon)^{1/4}$, can be considered as the smallest turbulent length scale in the flow. For a DNS the grid resolution needs to be of the same order as the Kolmogorov scale. The maximum value of the energy dissipation rate ϵ was obtained from the LES runs (and later validated by the DNS) for the boundary layers and the interior. This leads to a Kolmogorov scale of $\eta_w = 1.8 \times 10^{-3}$ in the wall boundary layer, $\eta_{fs} = 3 \times 10^{-3}$ in the free-surface layer and $\eta_{int} = 4 \times 10^{-3}$ in the interior. In the wall boundary layer the resolution expressed in the Kolmogorov scale is $\Delta x = 15.4 \eta_w$, $\Delta y = 4.3 \eta_w$, and $\Delta z = 0.32 \eta_w$. This resolution is in line with the resolution specified by Moin and Mahesh²⁰ to obtain accurate first and second order statistics for the boundary layer. In the interior, the resolution is $\Delta x = 7 \eta_{int}$, $\Delta y = 2 \eta_{int}$, and $\Delta z = 2 \eta_{int}$, which is close to a required grid spacing of at highest 4.5η . The resolution that is according to Moin and Mahesh yields accurate results for isotropic turbulence.

The following initialization procedure is adopted for the different simulations. The LES2 run is started from a simulation with only an oscillating pressure gradient. Within two periods, the flow is adjusted to the free-surface stress. These two initial periods are then discarded from the data set used

TABLE I. Specification of the resolution, domain size and control parameters used for the different simulations. The flow is driven by a free-surface stress ($\tau_{fs}=10^{-3}$) and an oscillating pressure gradient ($U_0=1$ and $\omega=1/80$). The domain size is $l_x \times l_y \times h=4 \times 2 \times 1$ for all simulations. Consequently, the Reynolds number is $Re_\omega=U_0^2/\omega\nu=10^6$ or using the Stokes boundary layer thickness $Re_s=\sqrt{2}U_0/\sqrt{\omega\nu}=1.4 \times 10^3$. The Keulegan–Carpenter number is for all cases $KC=U_0/\omega h=80$. The grid spacing is nonuniform in the z direction. Hence, for the grid size the value at the bottom wall, the maximum value and the value at the free surface are specified, respectively. Each simulation is run for a set net duration, which is expressed is the number of advection times T_a or the number of oscillation periods T_p .

	$n_x \times n_y \times n_z$	ν	Δx^+	Δy^+	Δz^+	Duration
Constant surface stress						
WO	$64 \times 96 \times 96$	8×10^{-5}	25	8.3	0.46, 10, 0.23	$600T_a$
Oscillating pressure gradient and constant surface stress						
LES1	$48 \times 64 \times 64$	8×10^{-5}	72	27	1.0, 35, 0.50	$12T_p$
LES2	$64 \times 96 \times 96$	8×10^{-5}	54	18	1.0, 21, 0.50	$10T_p$
LES3	$96 \times 128 \times 128$	8×10^{-5}	36	13	1.0, 14, 0.50	$5T_p$
DNS	$144 \times 256 \times 256$	8×10^{-5}	24	6.7	0.50, 7.0, 0.25	T_p

for analysis and phase averaging. The initial fields for the LES1 and LES3 runs are interpolated from the adjusted LES2 field. In these cases, the first period is discarded from the results. The initial field for the DNS is obtained by interpolating the LES3 field at $\varphi=345^\circ$, i.e., when turbulence intensities in the bottom boundary layer are weakest. The first 15° are discarded from the data set and a full period from $\varphi=0^\circ$ on is available for comparison with our LES. The transient time is a multiple large eddy turnover times, i.e., long enough for the spectrum to develop at the smaller scales. (Note that in Table I, we always report the net duration of the LES and DNS runs, thus without the initial transient).

To draw a comparison, we have also performed an LES for the case the flow is only driven by the constant wind stress at the same resolution as the LES2 run. The initial field is taken from the (adjusted) LES2 run at $\phi=0^\circ$ and run for $1100T_a$. Results for the initial $500T_a$ are discarded. Averages are determined over 30 subsequent snapshots with a time lap of $20T_a$ between the snapshots. Note that due to the much smaller wall stress, this LES is at nearly DNS resolution and can be considered to be accurate.

In Fig. 2, we have plotted the longitudinal spectra along the x axis at four different phases for the three LES and the DNS. We have to make sure that the cut-off scale is in the inertial range. The universal Kolmogorov spectrum for the inertial range is given by

$$E_{11} = \frac{18}{55} C_s \epsilon^{2/3} k^{-5/3}, \quad (7)$$

where ϵ is the dissipation rate and $C_s=1.5$. Due to the oscillating pressure gradient, the turbulence intensity is varying with time. At the phase $\varphi=90^\circ$, turbulent production is the strongest and hence has the largest Kolmogorov wave number. The longitudinal energy spectra then reveal a clear inertial range. The cut-off scale for all the LES resolutions is in the inertial range. For the small wave numbers all the spectra approximately collapse onto a single curve, thus clearly indicating that the intermediate and large-scale flow phenomena are well resolved in our LES. The strong fall-off of the spectrum near the cut-off scale is related to the use of a

top-hat filter. For certain phases, the turbulence intensities close to the bottom are low, e.g., for $\varphi=0^\circ$, and a dissipative spectrum is observed. Due to the low turbulence intensity in these cases all the spectra for the LES simulations then collapse to the DNS result. A small pile up of energy is observed in the tail of the spectrum, which does not affect the spectra for smaller wave numbers. The pile up is a numerical artifact, which is most likely caused by the use of the central difference scheme for the dissipation and subgrid-scale terms.²¹ It is possible that the pile up is enhanced due to the anisotropy of the grid. At the free surface the turbulence intensity is constant. A clear inertial range is present for all phases shown here which nicely coincides with the Kolmogorov spectrum (7).

IV. TURBULENCE DRIVEN BY A FREE-SURFACE STRESS

The wind-driven channel flow is taken as a baseline for the simulations with the additional oscillating pressure gradient. Here, we will briefly discuss the most important results of the former case. From now on, we use $(U, V, W) = (U_1, U_2, U_3)$ to denote the mean velocity components and $(u, v, w) = (u_1, u_2, u_3)$ to denote the velocity fluctuations. The mean and fluctuating pressure are denoted by P and p , respectively.

The constant surface stress ($\tau_{fs}=10^{-3}$) drives a flow, which reaches a statistical steady state at $t \approx 5 \times 10^2$ (time is expressed in advection time units h/U_0). In the steady state, the mean velocity is obtained by averaging over the horizontal plane and over 30 realizations each $20T_a$ apart. The mean velocity in the interior ranges from $U=0.5-0.7$ [Fig. 3(a)]. Using the mean velocity at half height $U_{h/2}=0.6$, the Reynolds number is $Re_h=Uh/\nu=7500$. The Reynolds numbers based on the friction velocity—which is identical at the free surface and the bottom wall—is $Re_\tau=u_\tau h/\nu=395$. A boundary layer is present at the no-slip bottom in which the velocity is reduced to zero. In the free-surface layer, the velocity is increasing to the value at the surface, i.e., $U_{fs}=1.0$.

FIG. 2. The longitudinal energy spectrum in (a) the wall layer (at $z=\delta_s$) and (b) the free-surface layer (at $z=h-\delta_s$). Spectra of LES1 (dotted), LES2 (dash-dotted) and LES3 (drawn) are phase averaged, while the DNS spectrum (drawn/gray) is instantaneous. The dashed lines correspond with the universal Kolmogorov spectrum $E_{11}=(18/55)C_s\epsilon^{2/3}k^{-5/3}$ with $C_s=1.5$.

Averaging of the Navier–Stokes equation over the horizontal plane yields for the streamwise component

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial z} \left[-\overline{uw} + \nu \frac{\partial U}{\partial z} \right] - \frac{1}{\rho} \frac{\partial P}{\partial x}. \quad (8)$$

In the absence of a pressure gradient the total stress, $-\overline{uw} + \nu(\partial U/\partial z)$, is constant for the steady mean flow. Hence, the total stress in the fluid has to be equal to the applied surface stress. In the interior, the Reynolds shear stress $-\overline{uw}$ is the main component of the total stress [Fig. 3(b)]. Viscous stresses take over from the Reynolds stress in the boundary layers. A similar observation can be made in a turbulent Couette flow.²² The Reynolds shear stress vanishes in the immediate region above the no-slip bottom,²³ whereas in the free-surface layer it is only zero exactly at the free surface.

The mean shear is the only cause behind the generation of turbulence. The shear production of turbulent kinetic energy is given by $-\overline{u_i u_j}(\partial U_i/\partial x_j)$. As the only nonzero mean shear component is $\partial U/\partial z$, only streamwise fluctuations are the direct effect of shear production. The streamwise fluctuations are most intense in the bottom and free-surface boundary layers, reaching peak values of $u_{\text{rms}}=10^{-1}$ [Fig. 3(c)]. The intensity is reduced to $u_{\text{rms}}=7 \times 10^{-2}$ in the interior. Turbulent kinetic energy is transferred to the other components due to the pressure-strain correlation. In the interior, the

spanwise root-mean-square (rms) velocity, v_{rms} , is just above the values for the vertical rms velocity w_{rms} . Both components are about 60% of the streamwise rms velocity. Note that vertical velocity fluctuations vanish at the free surface as a rigid lid approach has been used.

FIG. 3. The mean velocity (a), the Reynolds shear stress (b), and the rms velocities (c) for a turbulent channel flow driven only by a constant wind stress. In (c) the three components of the rms velocity, $\overline{u_i u_i}^{1/2}$ (drawn), $\overline{u_i}^{1/2}$ (dashed) and $\overline{w_i}^{1/2}$ (gray) are plotted. In the interior the Reynolds shear stress $-\overline{uw}$ is equal to the wind stress given by the dotted line.

FIG. 4. The mean velocity in wall units for the wind-driven channel flow at (a) the bottom wall and (b) the free surface. The gray lines in the left plot corresponds with z^+ for $z^+ < 10$ and $\kappa^{-1} \log z^+ + 5$ for $z^+ > 10$. For the right plot the dashed gray line relates to $\kappa^{-1} \log z^+ + 1.5$ (for comparison with the wind-driven oscillating channel flow) and the drawn gray line to $U^+ = \kappa^{-1} \log z^+$ with $\kappa = 0.41$.

The structure of the steady turbulent wall layer is well known.²⁴ In Fig. 4, the velocity profile above the wall and below the free surface is plotted in wall units. Wall units are based on the friction velocity at the wall and free surface, respectively. Immediate to the wall up to $z^+ < 5$, the viscous sublayer that is present is described by the linear profile $u^+ = z^+$ with $u^+ = u/u_\tau$. For $z^+ > 30$, a log-layer can be found where $u^+ = \kappa^{-1} \log z^+ + C$ with $\kappa = 0.41$ the von Kármán constant and $C = 5$. The transient area between the viscous sublayer and log layer is called the buffer layer. This composition of the boundary layer describes the velocity profile near the no-slip bottom wall in our simulations [Fig. 4(a)]. At the free surface, a viscous sublayer is also observed but only up to $z^+ < 3$. A log layer develops, $U^+(z=h) - U^+ = \kappa^{-1} \log z^+$ for $z^+ > 10$, i.e., the constant C vanishes. Tsai *et al.*¹³ also found a thinner viscous sublayer beneath the free surface. They observed a reduced value for C ranging between 1.1 and 1.9 (thus much smaller than the value $C \approx 5$ for the no-slip boundary). In their simulations, stress-free boundary conditions are applied to the bottom wall and the flow is driven purely by a wind stress. A thinner viscous sublayer beneath the free surface was also observed by Refs. 25 and 26. For a no-slip wall, a decrease in C relates to additional surface roughness, thus inducing substantial horizontal velocity fluctuations near the no-slip wall. The boundary conditions for the turbulent fluctuations at the free surface, i.e., $\partial u / \partial z = 0$ and $\partial v / \partial z = 0$ (thus allowing horizontal velocity fluctuations at the free surface), are less restricting than the no-slip boundary condition, $\mathbf{u} = 0$. Tsai *et al.* therefore conjectured that at the free surface the presence of horizontal turbulence leads to an effect similar to surface roughness.

FIG. 5. The phase-averaged wall stress for the three LES runs. For comparison data obtained with a DNS for one period is also plotted.

V. OSCILLATING CHANNEL FLOW SUBJECTED TO A SURFACE STRESS

In this section, we present the results of the oscillating channel flow simulations with wind stress acting on the free surface. LES are available for a number of resolutions and time spans (see Table I). All the mean quantities from the LES are obtained by applying both plane averaging and phase averaging. Only one period of DNS data is available, hence only plane averaging is applied here. As more periods are available for the lower resolutions, these simulations (LES1, LES2) provide better converged statistics. Instantaneous fields are, however, better resolved by the higher resolution data from LES3 and DNS. Throughout this section, we will present the data, which is most suitable. All the reported phase angles are relative to the applied pressure gradient.

A. Mean flow

The wall stress in a turbulent channel flow is strongly dependent on the turbulent fluctuations in the boundary layer, and hence an excellent measure to assess the accuracy of the LES model. In Fig. 5, we plot the bottom wall stress for LES runs at different resolutions and for the DNS. The observed wall stress profile is strongly dependent on the turbulent fluctuations in the boundary layer. An important difference between the LES and DNS is the phase at which the sudden increase in wall stress is observed. This transition occurs at earlier phases in the LES runs compared to the DNS results. The wall stress amplitude follows correctly from the LES runs at the medium and finest resolutions, while in the lowest resolution simulations (LES1), the wall stress amplitude is clearly underestimated. Conditions in the second half cycle are less demanding as the wall stress is much smaller. The wall stress is still underestimated in the LES1 run.

The streamwise velocity is given in Fig. 6. The surface stress drives a time-independent mean flow in the interior with shear layers at the bottom and the free-surface layer. In the first half cycle, this time-independent mean flow is in the same direction as the oscillating part driven by the applied pressure gradient. As a consequence, the mean velocity at half the channel width is $|U_{\max}| = 1.4$ and the shear in the bottom layer is strong. When the oscillatory part is opposite,

FIG. 6. The mean streamwise velocity for oscillating channel flow subjected to a wind stress for different phases. Results are plotted for the LES2 run (dash-dotted) and the DNS run (drawn/gray). For each following phase the plot is shifted by one unit.

the wall stress reduces and, subsequently, changes sign close to $\varphi=180^\circ$ as the streamwise velocity ($|U_{\max}|=0.7$) is reversed. The velocity at the free surface is only opposite to the wind stress for $\varphi=240^\circ-300^\circ$. At $\varphi=330^\circ$, the mean streamwise velocity is again positive in the complete vertical column. In the bottom, the boundary layer flow acceleration occurs from $\varphi=330^\circ-70^\circ$ and from $\varphi=180^\circ-270^\circ$ in the positive and negative x direction, respectively. The difference between the LES2 run and the DNS run is limited to the acceleration phase in the first half cycle. At 30° , the transition to turbulence already has occurred for the LES runs. Hence, the bottom boundary layer is here thinner than in the DNS run. After the transition a slightly larger mean shear is observed for the DNS run in the interior.

The mean velocity is not a simple superposition of an oscillating turbulent channel flow (driven by the pressure gradient) and a constant part (driven by the wind stress). The pressure gradient alone would result in a maximum wall stress of $\tau_w \approx 2 \times 10^{-3}$ as can be determined using the results from Jensen *et al.*² Adding the wind stress (which yields $\tau_w \approx 3 \times 10^{-3}$) is not enough to explain the observed maximum wall stress of 4.8×10^{-3} (see Fig. 5). This increase of wall stress, when an oscillatory component is added to a constant mean flow, was also observed by Lodahl *et al.* for pulsating flows in a cylindrical pipe.⁸ The time-averaged value of the wall stress is (as it should be) equal to the wind stress. An above-average wall stress is observed during a shorter period ($20^\circ \leq \varphi \leq 160^\circ$) than a below-average value. The maximum Reynolds number $\text{Re}_{\omega, \max} = |U_{\max}|^2 / \omega \nu = 2 \times 10^6$ in the first half cycle and 5×10^5 in the second half cycle. The Reynolds number based on the friction velocity are $\text{Re}_\tau = u_\tau h / \nu = 866$ and 515, respectively. The wall-stress

profiles in the first and second half cycle are in agreement with the profiles Jensen *et al.*² [see their Fig. 9(e) for $\text{Re}_\omega = 6.5 \times 10^5$ and Fig. 9(f) for $\text{Re}_\omega = 1.6 \times 10^6$]. Now following their observations, the Reynolds numbers in our simulations indicate that both the first half cycle and second half cycle are in the intermittent turbulent regime. The increase of the wall stress amplitude around $\varphi=270^\circ$ is due to a burstlike production of turbulence, as was observed by Hino *et al.*⁶ in the deceleration phase. When the Reynolds number increases, the transition to turbulence shifts to earlier phases, $\varphi=260^\circ$ in the second half cycle and $\varphi=30^\circ$ in the first half cycle.

Averaging of the Navier–Stokes equation (1) over the horizontal plane yields an expression for the mean x momentum, see Eq. (8). As the flow is homogeneous in the horizontal plane, all horizontal derivatives vanish except for the mean pressure gradient. The change of the mean velocity in time is due to the oscillating pressure gradient, while viscosity and the Reynolds shear stresses $\overline{uw}$ cause a vertical redistribution of mean momentum. The Reynolds shear stress is plotted in Fig. 7. There is a constant uniform contribution to the Reynolds stress by the free-surface stress, see Fig. 3(b) and the discussion in Sec. IV. At $\varphi=30^\circ$ $\overline{uw}$ equals the steady wind stress in the upper part of the domain. Additional negative $\overline{uw}$ stresses appear in the wall region during the early acceleration phase, and subsequently increase in amplitude and extend over the entire fluid column until $\varphi=90^\circ$. Then, in the deceleration phase $\overline{uw}$ relaxes toward a nearly uniform profile at $\varphi=180^\circ$. In the second half cycle the oscillatory component results in a Reynolds stress of opposite sign. In the following deceleration phase the $\overline{uw}$ relaxes toward zero in the lower half of the domain.

FIG. 7. The Reynolds shear stress $\overline{uw}$ for the channel flow driven by an oscillating pressure gradient and free surface stress during different phases. For each subsequent phase the profiles are shifted 3×10^{-3} units. Data is given for LES2 (dash-dotted) and LES3 (drawn).

FIG. 8. The mean velocity in wall units of the oscillating turbulent channel flow with a wind stress at (a) the bottom wall and (b) the free surface. The plots are given with 30° phase intervals with each subsequent plot shifted upwards by 20 and 10 units in the left and right figure, respectively. Results are given for the LES2 runs, marked by a plus (even points are omitted), and for the DNS run (drawn). The gray line in the left plot corresponds with z^+ for $z^+ < 10$ and $\kappa^{-1} \log z^+ + 5$ for $z^+ > 10$. For the right plot the dashed gray line relates to $\kappa^{-1} \log z^+ + 1.5$ and the drawn gray line to $U^+ = \kappa^{-1} \log z^+$. For all cases $\kappa = 0.41$.

B. Scaling of the boundary layers

The mean velocity profile above the wall and below the free surface is plotted in wall units in Fig. 8. The wall units used in the figure are based on the friction velocity at the wall or at the free surface. The structure of the steady turbulent wall layer is well known²⁴ and also briefly discussed in Sec. IV. It consists of the viscous sublayer, the buffer layer, and the log layer and this decomposition of the boundary layer describes the velocity profile near the no-slip bottom wall in our simulations for the phases with high turbulent activity very well. Note, that the agreement between the LES2 (and LES3) simulation and DNS is good. Differences occur for 30° and 180° due to a phase difference between the LES runs and DNS run for the transition to turbulence and flow reversal, respectively. Before the transition to turbulence occurs a thicker viscous sublayer is observed, notably for $\varphi = 0^\circ$ and 240° .

At the free surface, a viscous sublayer is also observed but here only up to $z^+ < 3$. Note that this is the reason that a higher normal resolution is required for the free surface than for a no-slip wall. For most phases, a log-law scaling region can be observed for $|U^+(z=h) - U^+| = \kappa^{-1} \log z^+ + C$ with $\kappa = 0.41$ for $z^+ > 10$. Note that the thickness of the viscous sublayer increases around $\varphi = 180^\circ$ and decreases again after $\varphi \approx 330^\circ$. This also relates to a different scaling of in the log layer. For $0^\circ \leq \varphi \leq 150^\circ$ a value of $C \approx 0$ is obtained, while for $210^\circ \leq \varphi \leq 330^\circ$ this value increases to $C \approx 1.5$. This obviously points to different dynamics of the turbulent flow below the free surface for these different phases. We will investigate the structure of the turbulent fluctuations in Sec. VI.

As already mentioned in Sec. IV, Tsai *et al.*¹³ also found a thinner viscous sublayer beneath the free surface with C typically between 1.1 and 1.9. Tsai *et al.* reasoned that at the

free surface the presence of horizontal turbulence leads to an effect similar to surface roughness. In their simulations, no shear production of turbulence near the bottom wall is observed as stress-free conditions are used. However, the turbulence produced in the bottom boundary layer cannot totally explain why the viscous boundary layer below the free surface is thinner in the first half cycle than in the second half cycle. Especially at 0° and 30° , when the transition to strong turbulent fluctuations did not yet occur, we still observe a thinner viscous sublayer.

C. Turbulent velocity fluctuations

To get more insight in the structure and creation of the turbulent structures we investigate the behavior of the Reynolds stress components. The balance of the Reynolds stress is given by

$$\begin{aligned} \frac{D}{Dt} \overline{u_i u_j} = & \underbrace{- \left(\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} + \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} \right)}_{\text{Shear Production}} - \underbrace{\frac{1}{\rho} \left(\overline{u_i \frac{\partial p}{\partial x_j}} + \overline{u_j \frac{\partial p}{\partial x_i}} \right)}_{\text{Pressure Diffusion}} \\ & - \underbrace{\frac{\partial}{\partial x_k} \overline{u_i u_j u_k}}_{\text{Turb. Transp}} + \underbrace{\nu \frac{\partial^2}{\partial x_k^2} \overline{u_i u_j}}_{\text{Viscous Diff.}} - \underbrace{2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}}}_{\text{Dissipation}}. \end{aligned} \quad (9)$$

Let us focus on the kinetic energy of the turbulent fluctuations, i.e., the diagonal components of the Reynolds stress tensor. The rms velocities are given in Fig. 9. There is only shear production $[-\overline{uw}(\partial U / \partial z)]$ of the streamwise velocity fluctuations. In the free-surface layer, the streamwise rms velocity is about 10% of U_0 . The rms velocities in the wall

FIG. 9. The rms velocities in the channel flow driven by an oscillating pressure gradient and free surface stress from LES2. For each subsequent phase the profiles are shifted 3×10^{-3} units. Streamwise (drawn), spanwise (gray) and vertical (dashed) rms velocities are plotted.

layer reach 20% of U_0 when the wall stress is maximum and decrease strongly when the wall stress vanishes. In the interior, the rms velocity is about 5% of U_0 .

Turbulent kinetic energy in the streamwise velocity component is redistributed to the spanwise and vertical directions or transported to other fluid layers. As shown in Fig. 10, shear production, denoted by $|\overline{uw}|\partial U/\partial z = |\overline{uw}|S$, is balanced locally by viscous dissipation ϵ in a substantial part of the fluid column where $|\overline{uw}|S/\epsilon \approx 1$. Only in the boundary layer production is significantly stronger than dissipation. This indicates that the turbulence in the interior is in a quasisteady regime. The time scale of the tide (the period of the oscillating pressure gradient) is much larger than the turbulent time scale. Turbulence is mainly produced in the shear layers, but subsequently fills the interior domain. The related time scale $\tau_\tau = h/u_\tau$ is much shorter than the oscillation period $T_p = 2\pi/\omega$. Hence, the flow is in the quasisteady regime as observed by Scotti and Piomelli.⁷ A way to verify the quasisteady behavior of the turbulence is calculating the structure parameter $a_1 = \overline{uw}/2K$ with $K = \frac{1}{2}\overline{u_i u_i}$ the turbulent kinetic energy (Fig. 11). A value of $|a_1| \approx 0.15$ is typical for steady turbulence in the logarithmic layer.⁷ Lower values are observed in strongly time-dependent flows and three-dimensional boundary layers. In the interior we observe values close to $|a_1| = 0.15$ during most phases. For $270^\circ \leq \varphi \leq 330^\circ$ the $\overline{uw}$, and hence a_1 , goes through zero around $z = 0.45$. Deviations appear mainly in the bottom layer for $\varphi = 0^\circ$ and $180^\circ - 240^\circ$, when turbulence intensities are weak.

The pressure diffusion term in the balance of the Reynolds stresses (9) can be rewritten as

$$\frac{p}{\rho} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{\partial}{\partial x_k} \left(\frac{p}{\rho} (u_i \delta_{jk} + u_j \delta_{ik}) \right). \quad (10)$$

The first term is responsible for the exchange of turbulent kinetic energy among the three velocity components. It is found in several numerical studies that this exchange plays an important role in the boundary layer below the free surface (see, e.g., Handler *et al.*¹⁰ and Nagaosa¹¹ who pointed out the importance of exchange of turbulent kinetic energy between the surface-normal direction and the spanwise directions). The second term relates to vertical and horizontal transport of turbulent kinetic energy. In Fig. 12, we investigate how the first term, the pressure-strain correlation, $\Pi_{ij} = p(\partial u_i/\partial x_j + \partial u_j/\partial x_i)/\rho$, changes directly below the free surface. Streamwise velocity fluctuations are enhanced by the mean shear. Turbulent kinetic energy is then transferred from the streamwise fluctuations, i.e., a negative Π_{11} to the spanwise and vertical fluctuations. This is true for the entire fluid column as indicated by negative values for Π_{11} and positive values for Π_{22} and Π_{33} . The only exception is the viscous sublayer below the free surface for $\varphi = 0^\circ - 180^\circ$. Here, kinetic energy is mainly transferred from the vertical fluctuations to the spanwise velocity fluctuations. The transfer from the streamwise turbulent energy is strongly reduced in this region and is typically zero at the free surface. This specific energy redistribution is related to the appearance of the low-speed splats at the free surface.¹¹

FIG. 10. The ratio of shear production $S|\overline{uw}|$ and dissipation ϵ for LES2 for different phases. For each following phase the plot is shifted by 2 units.

FIG. 11. The structure parameter $a_1 = \overline{u'w'}/2K$ with $K = \frac{1}{2}\overline{u_i u_i}$ the turbulent kinetic energy for different phases of the LES2 phase averaged data. For each subsequent phase the profile is offset by 0.2 units. The dotted lines give $|a_1| = 0.15$ that is typical for the log layer in the case of the steady turbulent boundary layer.

D. Structure of the turbulent flow

The coherent structure of the turbulent vortices is visualized by using the Q -criterion.²⁷ Here, Q is the second invariant of the velocity gradient tensor $\nabla \mathbf{u}$ and is defined as

$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}), \quad (11)$$

with $\Omega_{ij} = \frac{1}{2}(\partial u_i / \partial x_j - \partial u_j / \partial x_i)$ and $S_{ij} = \frac{1}{2}(\partial u_i / \partial x_j + \partial u_j / \partial x_i)$, the antisymmetric and symmetric components of the velocity gradient tensor, respectively. The varying wall stress has an impact on the intensity and structure of the turbulence. In the first half cycle the strong shear causes the formation of turbulent low-speed streaks in the bottom layer (see Fig. 13). The streaks are typically found in wall layers. These streaks are caused by turbulent bursts of slower moving fluid into the layer above.²⁸ Clear streaks first appear in regions of the bottom boundary layer in the acceleration phases just after $\varphi = 30^\circ$ and at 210° . For the first half cycle high turbulent activity is then observed in the entire fluid column up to the late deceleration phase. In the second half cycle wall streaks are much weaker, and turbulence levels in general are lower. The appearance of wall streaks and relaminarization of the flow was also observed for the purely oscillating turbulent boundary layer with a similar Reynolds and Keulegan-Carpenter number as in the present study, see Ref. 14. Lam and Banerjee²⁹ investigated the condition for streak formation in bounded turbulent flow. For streaks to appear in wall layers or free-surface layers, they found that $S|\overline{u'w'}|/\epsilon > 1$. Clearly this condition is satisfied in the wall layer when streaks are present (see Fig. 10). However, in the early acceleration phase no clear streaks are present, while the $S|\overline{u'w'}|/\epsilon$ is substantially larger than one.

At the free surface $S|\overline{u'w'}|/\epsilon > 1$ for all phases. As the free surface is moving faster than the flow underneath the surface, streaks are observed throughout the period. There is a difference between the flow structures for the phases $\varphi = 210^\circ - 300^\circ$ compared to $330^\circ - 180^\circ$. In the latter period splats are present. This is more clear when we investigate the turbulent velocity field just below the free surface. In Fig. 14, the streamwise and vertical velocity fluctuations for $\varphi = 90^\circ$ and 270° are plotted. Note that essentially, streaks in the buffer layer have a higher absolute velocity than the mean for $\varphi = 90^\circ$, while for 270° these streaks are low-speed. For $\varphi = 90^\circ$, the streaks are disturbed by low-speed spots or splats. A splat is a confined oval region of upwelling, as can be seen in Fig. 14(c). As the upwelling is halted at the free surface, the horizontal velocity field is divergent at this point, leading to spanwise velocity fluctuations. As downwelling occurs over larger regions it is less intense. Due to this asymmetry, the splats contribute more to the pressure-strain relation, and the vertical turbulent energy is transferred to the spanwise fluctuations (Fig. 12). In Fig. 13, the splats can be noticed by the particular dark structures at the free surface. Splats first occur for $\varphi = 330^\circ$ and are present until 180° . Thereafter, no splats are present and streaks are much more similar to the low speed streaks observed in the wall layer. The free-surface flow pattern obtained with LES3 reveals good agreement with the DNS data. The appearance of the splats seems to be related to the alignment of the free-surface stress and the mean velocity. When splats are present, the mean velocity and free-surface stress are both in the positive x direction. This is also the case in the study of Nagaosa,¹¹ where the flow is solely driven by the free-surface stress.

FIG. 12. The pressure-strain correlation below the free surface for the oscillating channel flow subjected to a wind stress at a 0° angle for different phases. Only the diagonal terms, Π_{11} (drawn), Π_{22} (dash-dotted) and Π_{33} (gray) are plotted from LES2 data. For each of the following phase, the plot is shifted by 5×10^{-3} .

FIG. 13. Isosurface of the second invariant of the velocity gradient tensor, where the data is taken from the DNS run. In each plot the upper slice corresponds with the free-surface layer, the lower slice with the bottom layer. The contour level in each plot is given by Q_0 .

When the mean velocity is opposite to the free-surface velocity (and close to zero), splats are absent and regular, undisturbed streaks are present. Also note that when turbulent splats are present the viscous sublayer is noticeably thinner (see Fig. 8) than when the streak pattern is dominant.

To study the anisotropy of the turbulence in the entire domain we have constructed the turbulence triangle or Lumley triangle³⁰ in Fig. 15 (for more information on the Lumley triangle the reader can consult the textbook by Pope³¹). The anisotropic component of the Reynolds stress tensor is given by $b_{ij} = \overline{u_i u_j} / \overline{u_k u_k} - \delta_{ij}/3$. In each horizontal plane, we compute the second and third invariants of b_{ij} , Π_b and III_b , respectively. The combination of the two invariants relate to an anisotropic state of the Reynolds stresses independent from the orientation of the coordinate frame. Alternative coordinates (ξ, η) are used for plotting the Lumley triangle with $6\eta^2 = -2\Pi_b$ and $6\xi^3 = 3\text{III}_b$. As illustration of the use of the Lumley triangle we consider turbulence driven by wind stress only. The line (with plusses) in Fig. 15 indicates the horizontally averaged Reynolds stresses as function of distance from the wall in a developed steady boundary layer (see, e.g., Ref. 23). In the viscous sublayer ($z^+ < 5$), the Reynolds stress is essentially two-component with $\overline{u^2}$ larger than $\overline{v^2}$ and $\overline{w^2}$ is close to zero. In the buffer layer and the log-law region the turbulence is nearly axisymmetric with $\overline{u^2}$ larger than the other two components.

FIG. 14. Turbulent streaks and splats at $z^+ = 3$ below the free surface. Left column: $\varphi = 90^\circ$; right column: $\varphi = 270^\circ$. In the upper and middle panels, the streamwise-velocity fluctuations are plotted, for the LES3 and DNS runs, respectively. In the lower panels the vertical velocity field is plotted. Positive and negative velocities are colored white and black, respectively. The white arrow in panel (e) points to a localized region of upwelling, which correlates with the low-speed spot in panel (c).

In our simulations for the pulsating turbulent channel flow, the same signature is observed for the Reynolds stresses in the wall boundary layer for almost all phases (Fig. 16). The only exceptions are found when the flow reverses in the viscous sublayer at $\varphi = 180^\circ$ and 330° . Here, the Reynolds stresses in the viscous sublayer are still two-component but with $\overline{v^2}$ closer to $\overline{u^2}$. The same behavior was observed by Salon *et al.*¹⁴ for the purely oscillating channel flow. For $240^\circ \leq \varphi \leq 270^\circ$ we do not perceive a different structure in the free-surface layer. This is in agreement with the structure of the streaks having the same appearance as wall layer streaks. For $30^\circ \leq \varphi \leq 150^\circ$, the Reynolds stresses are further away from the axisymmetric limit. The stream-

FIG. 15. The Lumley triangle in invariant coordinates η and ξ . In this triangle three-dimensional (3D) isotropic structures map to (0,0), two-dimensional (2D) isotropic to $(-1/6, 1/6)$ and one-dimensional (1D) structures to $(1/3, 1/3)$. All realizable values for the Reynolds stress invariant fall within the triangle. Characteristic states have been indicated. The line (with plusses) corresponds to a steady turbulent boundary layer at the no-slip wall as was obtained for the purely wind-driven case, see Sec. IV.

FIG. 16. The turbulence triangle measuring the isotropy of the Reynolds stresses, where 3D isotropic structures map to (0,0), 2D isotropic to $(-1/6, 1/6)$, and 1D structures to $(1/3, 1/3)$. The line (drawn) quantifies the structure of the turbulence at different depths within the fluid column for the LES2 run. In the Stokes boundary layer, points are marked with a plus sign and in the free-surface layer (down to $z=0.75$) points are marked with a circle. For comparison the LES3 results are given by the dash-dotted line.

wise fluctuations are still dominant, but the spanwise fluctuations become stronger. This is related to the splats that occur at the free surface.

VI. CONCLUSION

The two driving forces, i.e., an oscillating pressure gradient and an aligned fixed wind stress at the free surface, result in a pulsating mean flow. The shear layer below the free surface is a second source of turbulent fluctuations (together with the one near the no-slip bottom). Turbulence produced in the bottom and free-surface layer extends over the entire fluid column. Turbulent time scales are much shorter than the oscillation period. A number of measures substantiates that the turbulence is essentially in a quasisteady state for most phases of the cycle. This results in the presence of clear logarithmic mean-velocity profiles in the wall boundary layer. Turbulent streaks are also present in the free-surface buffer layer. This is in agreement with the condition $S|\overline{uw}|/\epsilon > 1$ as was proposed by Lam and Banerjee.²⁹ When the mean velocity and stress are aligned at the free surface, splats occur, i.e., low-speed oval spots caused by upwelling fluid. The presence of the splats results in a much thinner viscous sublayer below the free surface. When stress and mean velocity are opposite no splats occur. The turbulent

streaks then resemble the turbulent wall streaks, although the spacing between streaks is larger for the former.

Turbulence fluctuations are present within the entire fluid column for most phases. The logarithmic scaling of the mean-velocity profile for the wall boundary layer does not defer from the classical steady boundary layer. This implies that it is not affected by the additional production of turbulent fluctuations in the free-surface layer. The free-surface layer consists of very thin viscous sublayer. This results in an almost zero value for C in the logarithmic scaling law $\frac{1}{\kappa} \log z^+ + C$ for the phases where turbulent splats are present. This is even smaller than $C=1.1-1.9$ Tsai *et al.* observed in their simulations¹³ (where a stress-free bottom wall was used). The decrease in C can be contributed to horizontal turbulent fluctuations present at the free surface, opposed to the zero velocity fluctuations at a no-slip wall. A large value $C=1.5$ is typically observed when the low-speed free-surface spots are absent. Note that for the latter case the mean velocity is opposite to the mean shear in the free-surface layer. Then, like in a wall boundary layer, the shear is constraining the flow instead of driving the flow.

An interesting next step is to change the direction of the wind stress. A setup similar to the one used by Lam and Banerjee²⁹ could be useful for this purpose. Then, the flow is driven by a constant pressure gradient and the wind stress can be altered in both direction and amplitude. Such a study with the surface stress and pressure gradient still aligned can give more insight in the presence of the turbulent splats. In the study of Lam and Banerjee, the surface stress is always constraining the flow and not used to drive the flow. Another configuration that has to be addressed is the one with the stress and the mean velocity not aligned at the free surface. This will have an impact at the formation and structure of the turbulent fluctuations in the free-surface boundary layer.

ACKNOWLEDGMENTS

This program is funded by the Netherlands Organisation for Scientific Research (NWO) and Technology Foundation (STW) under the Innovational Research Incentives Scheme Grant No. ESF.6239.

- ¹M. Yelland and P. K. Taylor, "Wind stress measurements from the open ocean," *J. Phys. Oceanogr.* **26**, 541 (1996).
- ²B. L. Jensen, B. M. Sumer, and J. Fredsøe, "Turbulent oscillatory boundary layers at high Reynolds numbers," *J. Fluid Mech.* **206**, 265 (1989).
- ³M. Y. Gündoğdu and M. Ö. Çarpınlioğlu, "Present state of art on pulsatile flow theory. 1. Laminar and transitional flow regimes," *JSME Int. J., Ser. B* **42**, 384 (1999).
- ⁴M. Y. Gündoğdu and M. Ö. Çarpınlioğlu, "Present state of art on pulsatile flow theory. 2. Turbulent flow regime," *JSME Int. J., Ser. B* **42**, 398 (1999).
- ⁵R. Akhavan, R. D. Kamm, and A. H. Shapiro, "An investigation of transition to turbulence in bounded oscillatory stokes flows. Part 1. Experiments," *J. Fluid Mech.* **225**, 395 (1991).
- ⁶M. Hino, M. Kashiwayanagi, A. Nakayamai, and T. Hara, "Experiments on the turbulence statistics and the structure of a reciprocating oscillatory flow," *J. Fluid Mech.* **131**, 363 (1983).
- ⁷A. Scotti and U. Piomelli, "Numerical simulation of pulsating turbulent channel flow," *Phys. Fluids* **13**, 1367 (2001).
- ⁸C. R. Lodahl, B. M. Sumer, and J. Fredsøe, "Turbulent combined oscillatory flow and current in a pipe," *J. Fluid Mech.* **373**, 313 (1998).
- ⁹I. Langmuir, "Surface motion of water induced by wind," *Science* **87**, 119 (1938).

- ¹⁰R. A. Handler, T. F. Swann, Jr., R. I. Leighton, and J. D. Swearingen, "Length scales and the energy balance for turbulence near a free surface," *AIAA J.* **31**, 1998 (1993).
- ¹¹R. Nagaosa, "Direct numerical simulation of vortex structures and turbulent scalar transfer across a free surface in a fully developed turbulence," *Phys. Fluids* **11**, 1581 (1999).
- ¹²R. Nagaosa and T. Saito, "Turbulence structure and scalar transfer in stably stratified free-surface flow," *AIChE J.* **43**, 2393 (1997).
- ¹³W.-T. Tsai, S.-M. Chen, and C.-H. Moeng, "A numerical study on the evolution and structure of a stress-driven free-surface turbulent shear flow," *J. Fluid Mech.* **545**, 163 (2005).
- ¹⁴S. Salon, V. Armenio, and A. Crise, "A numerical investigation of the stokes boundary layer in the turbulent regime," *J. Fluid Mech.* **570**, 253 (2007).
- ¹⁵Here, we introduce an alternative notation $\mathbf{x}=(x_1, x_2, x_3)$, $\mathbf{u}=(u_1, u_2, u_3)$, etc. Furthermore, Einstein's summation convention is used.
- ¹⁶J. Bardina, J. H. Ferziger, and W. C. Reynolds, "Improved subgrid-scale models for large-eddy simulation," AIAA Paper No. 80-1357, 1980.
- ¹⁷M. Germano, U. Piomelli, P. Moin, and W. H. Cabot, "A dynamic subgrid-scale eddy viscosity model," *Phys. Fluids A* **3**, 1760 (1991).
- ¹⁸V. Armenio and U. Piomelli, "A Lagrangian mixed subgrid-scale model in generalized coordinates," *Flow Turbul. Combust.* **65**, 51 (2000).
- ¹⁹Y. Zang, R. L. Street, and J. R. Koseff, "A non-staggered grid, fractional time step method for time-dependent incompressible Navier-Stokes equations in curvilinear coordinates," *J. Comput. Phys.* **114**, 18 (1994).
- ²⁰P. Moin and K. Mahesh, "Direct numerical simulation: A tool in turbulence research," *Annu. Rev. Fluid Mech.* **30**, 539 (1998).
- ²¹N. Park and K. Mahesh, "Analysis of numerical errors in large eddy simulation using statistical closure theory," *J. Comput. Phys.* **222**, 194 (2007).
- ²²J. Komminaho, A. Lundblad, and A. V. Johansson, "Very large structures in plane turbulent Couette flow," *J. Fluid Mech.* **320**, 259 (1996).
- ²³J. Kim, P. Moin, and P. Moser, "Turbulence statistics in fully developed channel flow in a turbulent boundary layer," *J. Fluid Mech.* **177**, 133 (1987).
- ²⁴S. J. Kline, W. C. Reynolds, F. A. Schraub, and P. W. Runstadler, "The structure of turbulent boundary layers," *J. Fluid Mech.* **30**, 741 (1967).
- ²⁵G. T. Csanady, "The free surface turbulent shear layer," *J. Phys. Oceanogr.* **14**, 402 (1984).
- ²⁶J. Wu, "Viscous sublayer below a wind-distributed water surface," *J. Phys. Oceanogr.* **14**, 138 (1984).
- ²⁷J. C. R. Hunt, A. Wray, and P. Moin, "Eddies, stream, and convergence zones in turbulent flows," Center for Turbulent Research Report No. CTR-88, 1988.
- ²⁸P. K. Kundu and I. M. Cohen, *Fluid Mechanics* (Academic, New York, 2004).
- ²⁹K. Lam and S. Banerjee, "On the condition of streak formation in a bounded turbulent flow," *Phys. Fluids A* **4**, 306 (1992).
- ³⁰J. L. Lumley, *Lumley* (Academic, New York, 1970).
- ³¹S. B. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2000).