On the broadcast capacity of wireless multihop interference networks

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Abstract—This paper is concerned with wireless broadcasting in multihop networks where a selected number of relay nodes may aid the source node in the broadcast under a given total energy and hop constraint. We study an ad-hoc network with infinitely many nodes and analytically find the number and positions of rebroadcasting relay nodes to achieve the optimal broadcast capacity. The interference due to multiple transmissions in the geographical area is taken into account. Based on the theoretical findings, we then propose one distributed and one centralized heuristic for relay selection in wireless broadcasting. We discuss the broadcast capacity performances and CSI (channel state information) requirements of these algorithms. The results illustrate that the benefits of peer-assisted broadcasting are more pronounced in the centralized relay selection algorithm when compared to the fully randomized and distributed selection under a realistic system model.

I. INTRODUCTION

As wireless ad-hoc networks proliferate, the problem of broadcasting a common information from a source node to all the other nodes in the network has gained particular importance. In a wireless ad-hoc network, one of the important objectives of broadcasting is to deliver all the information to all nodes successfully with minimum pre-roll delay. In such a system, some of the nodes can be selected as rebroadcasting relays to substantially improve the overall system capacity. However, using additional nodes as cooperating relays increases the total energy consumption and pre-roll delay for some of the receiving nodes. In addition, allowing retransmissions from multiple cooperating relay nodes in addition to the source node creates interference for the receiving nodes since wireless transmission from each node using an omnidirectional antenna is broadcast in nature. Furthermore, given a network topology, CSI between each user and a total energy constraint, selecting the set of re-broadcasting relays that maximizes the broadcast capacity is already a difficult problem without taking the interference into account [1]. Therefore, it is both necessary and challenging to analyze and subsequently improve the broadcast capacity of a wireless network under total energy, and hop constraints when interference is taken into consideration.

In recent years, much of the study on wireless ad-hoc broadcast networks concentrated on deriving capacity limits [2], [3] and scaling laws [4], [5] of source-destination pairs. Although the capacity limits of unicast transmission in wireless multihop networks have been extensively studied, the studies on broadcast capacity limits are scarce. On this front, there are two main papers studying the theoretical capacity of wireless multihop networks for broadcasting. In [6] Zheng provides a definition for the broadcast capacity and proves that multihop broadcasting is more beneficial than single-hop transmission in extended networks while the reverse is true for dense networks. In [7] Keshavarz-Haddad and Riedi develop bounds for the broadcast capacity of arbitrarily connected networks under different channel models and power regimes. Although both of these papers focus on broadcast capacity, their results are based on theoretical analysis and asymptotic bounds where interference from retransmissions as well as the impact of CSI is ignored. In this paper, we derive the optimal broadcast capacity, i.e., the maximum achievable broadcast capacity for a given hop and energy constraint, for a dense network with infinitely many nodes where interference, total energy consumption, total transmission delay, required CSI as well as the receiver capabilities are all taken into account.

The underlying problem in our study is the selection of the set of re-broadcasting nodes that achieves the optimal broadcast capacity. When the wireless channel is modeled using a path-loss model and interference due to retransmissions is neglected, this problem is shown to be NP-complete for a finite population of nodes [1]. The relay selection problem becomes even more difficult as the interference observed by each node is tightly coupled to the identity of rebroadcasting nodes. In this paper, we first show that for a wireless network with infinitely many nodes, this problem is analytically tractable. In such a scenario, we find the optimal positions of rebroadcasting nodes that maximize the broadcast capacity. Based on our optimal analysis of the infinitely populated system, we propose two relay selection methods for networks with finite node population. We discuss the CSI requirements and broadcast performances of the proposed methods.

The rest of the paper is organized as follows. In Section II, we define the system model, assumptions and necessary definitions with a discussion on the complexity of our problem. Then, a method for computing the capacity for the system with infinitely many nodes is presented in Section III. Then, for finite population networks, we propose two distinct relay selection methods in Section IV. Simulation results to assess
the performances of the proposed heuristics are provided in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

The problem at hand considers a source node that broadcasts data to all $n$ relay nodes and $m$ receiver nodes continuously. Meanwhile, $n$ relay nodes are assisting to the broadcast by decoding-and-forwarding the received signal from the source to the receiver nodes. In order to decode a symbol, all relays process received signals for a certain duration because relays are assumed to be using symbol-by-symbol decoding as in [8]. Therefore, both propagation time and processing time are considered as done in [9] for system modeling. To achieve diversity from signals transmitted by different nodes, each node is assumed to have a Rake receiver with $R$ fingers.

We assumed a network topology of a 2-D circular region, $\Gamma$. Let $r$ denote the radius of the circular region. $S, N$ and $M$ denote the source node, the set of relay nodes, and the set of nodes that are not transmitting, respectively, where $S = \{0\}$, $N = \{1, \ldots, n\}$, $M = \{n+1, \ldots, n+m\}$. Assume the source node is placed at the center of the region $\Gamma$. Since the broadcast capacity is defined by the node having the minimum capacity, the optimal locations of the relays form a set of concentric circles $\Phi = \{\varphi_i \mid i = 1, \ldots, L\}$ centered around the source node with associated set of radii $D = \{r_i \mid i = 1, \ldots, L\}$ as shown in Fig. 1 due to the circular symmetry.

In this work, only large-scale path loss characteristics are considered in the channel model, which is commonly used in papers on wireless capacity analysis. Furthermore, generalized physical model is used for broadcast capacity analysis since this model includes interference. On the other hand, the source node has transmission power $P$ while there is a total power constraint, $TP$, on all cooperating relay nodes in the system. However, no power adaptation is used, i.e., each cooperating relay node transmits equally at full power, which is equal to $TP/n$. The broadcast capacity analysis is done for one-ring scenario, i.e., $|\Phi| = 1$ with the optimal broadcast capacity achieving radius $r_1$.

In theoretical analysis, nodes selected as relays are assumed to be equally spaced around a ring. For a given total energy constraint, radius of this ring has to be optimized for the maximization of the broadcast capacity. This leads to the optimal broadcast capacity for the given total energy and hop constraint due to the point symmetry of the circular network. However, incorporating a second ring to the capacity analysis increases the complexity of the solution exponentially since at each feasible radius of the inner ring, an optimal outer ring radius has to be found. Therefore, we stick to the one-ring scenario as it improves the broadcast capacity substantially with a reasonable optimization complexity. Also, to provide full connectivity of the network, all relay nodes lock on to source’s signal.

A. Broadcast Capacity

The broadcast capacity of a wireless multihop ad-hoc network can be formulated as the minimum of the channel capacity over all transmitter-receiver pairs. For a network with region $\Gamma$, the broadcast capacity is:

$$C_B(\Gamma) = C_B(S, R_0) = \min_{i \in N \cap M} [B \log_2(1 + \Psi_i)]$$

(1)

$$C_B(S, R_k) = \min[C_B(S, N), C_B(S, M_S), C_B(N, M)],$$

(2)

where $S_i$ and $R_k$ denote the transmitter-receiver pair with the bottleneck channel capacity while $M_S$ and $M_N$, where $M = M_S \cup M_N$, denote the non-transmitting nodes receiving service from the source node and the relay nodes, respectively. Also, $\Psi_i$ denotes the signal-to-interference-plus-noise-ratio (SINR) of user $i$ and $B$ is the channel bandwidth.

B. Rake Receiver

In a wireless multihop broadcasting scenario, a receiving node is subject to a number of signals carrying the same information with different time delays. This situation can be viewed as a unicast transmission with multipath fading. To mitigate multipath fading in a unicast transmission, multipath diversity can be employed using direct sequence code division multiple access (DS-CDMA) with Rake receiver. Using maximal-ratio-combining (MRC), a Rake receiver with $R$ fingers ($R \leq n+1$) can achieve an SINR, which is the sum of the SINRs on each finger. At each finger, unresolvable paths contribute to both signal and interference power based on the delay difference while remaining resolvable paths contribute to the interference power. Then, the total SINR achieved at a receiver $i$, $\Psi_i$, can be calculated as follows:

$$\Psi_i = \sum_{k=1}^{R} \frac{P d_{ij}^{-\alpha}}{N_0 B} + \sum_{w \in H_k} P d_{wi}^{-\alpha} \left(1 - \frac{\tau_{ik} - \tau_{ij}}{T_c}\right)$$

(3)

where $d_{ij}$ is the distance between node $i$ and $j$, $\alpha$ is the pathloss exponent, $N_0/2$ is the power spectral density of the additive white Gaussian noise (AWGN), $\tau_i$ denote the time delay of path $i$, $k$ is the finger index, $t_k$ is the index of the transmitter that is locked by $k$th finger, $T_c$ is the chip time ($T_c \approx 1/B$), and $H_k$ and $I_k$ denote the set of irresolvable and resolvable paths with the $k$th finger, respectively.
C. Complexity Issues

Cagalj et al. [1] proved that the minimum energy broadcast problem, where wireless channel is described by a simple pathloss model, in two-dimensional Euclidean metric space is NP-complete while interference is not taken into account. The problem is to find a node power assignment vector such that all the nodes are covered and connected while satisfying a total power constraint. The link cost between two nodes only depends on the pathloss exponent and the distance between them. However, for a complete realization of a wireless ad-hoc network, interference, transmission time of cooperating nodes and the diversity at the receiver side has to be included in the equation of SINR. Therefore, in this paper, we model the wireless channel using (3).

When wireless channel is modeled by (3), an achievable rate between two nodes can only be computed by knowing the node power assignment vector, \( A = [p^1, p^2, \ldots, p^n] \). But, the problem is whether there exists a node power assignment vector such that all the nodes in the network are covered and connected while the sum of assigned powers is less than or equal to a total power constraint. Although the question is similar to one in [1], the solution of our problem requires apriori knowledge of the node power assignment vector in order to check whether the given network is covered and connected, and total power constraint is satisfied. As a result, when interference is taken into account, finding a node power assignment vector that makes the network connected while satisfying a given data rate and total power constraint can not be solved practically.

III. OPTIMAL BROADCAST CAPACITY

Since the broadcast capacity is defined to be the minimum channel capacity among each transmitter-receiver pair, we can find the points, which have the minimum capacity, in a network with infinitely many nodes. According to point symmetry, the optimal broadcast capacity of a circular region is equal to the optimal broadcast capacity of \( \pi/n \)-degree pie slice when relay nodes form a ring. Therefore, we can simply analyze the broadcast capacity of a pie slice between 0 and \( \pi/n \)-degree to find the optimal broadcast capacity. Although this property simplifies the analysis, we need to consider the chip time and the number of fingers at the receiver.

At a given geographical point, the Rake receiver utilizes some part of the signal delayed by less than \( T_c \) other than the intended transmission according to its phase difference given by (3). Let us call these geographical points as the region of intended transmission according to its phase difference given by (3).

\[ A_{\Pi,ij} = \{ p \in \Pi \mid |d_{i,p} - d_{j,p}| < c.T_c \} \]  (4)

where \( c \) is the speed of light. A simple demonstration of this utilization region for nodes 1 and 2, depicted with grey color, is shown in Fig. 2.

In a practical scenario, each user is equipped with a Rake receiver which has more than one fingers. However, as MRC is used to combine signals from each finger of the Rake receiver, the capacity analysis for more than one-finger case becomes complex. However, SINR is a smooth, continuous, and well-behaving, i.e., includes no rapid fluctuations and abrupt changes, function except at points that are very close to any transmitter. So, the critical points of a region can be found using a method, which we call as feasible direction method (FDM), that converges to a local critical point (LCP), which is the nearest critical point to the initial guess, of the region.

1) Feasible Direction Method: For a given initial point, we can compute SINR over a ball, defined by \( \epsilon \) and \( \rho \) set of direction vectors \( d_i \) with \( \|d_i\| = 1 \) for \( i = 1, \ldots, k \), where \( k \) denotes the number of intervals on the ball, around an interior point. The selected interior point is only an initial guess, and it is updated if there is a point on the ball with a worse SINR. If there is none, \( \epsilon \) can be decreased (e.g. halved) for a finer-grained search. Ultimately, a critical point is found when \( \epsilon < \epsilon_i \), where \( \epsilon_i \) is a lower bound on \( \epsilon \).

Algorithm 1 FDM

Require: \( p_{init} = (x_{init}, y_{init}) \): Initial guess point
Ensure: \( cap : \text{Broadcast capacity of region } \Phi \)

1: \( x \leftarrow C_B(S_{p_{init}}, p_{init}) \)
2: \( \epsilon \geq \epsilon_i \)
3: for \( \theta = 0; \theta \leq 2\pi; \theta = \theta + \theta_{step} \) do
4: \( dx \leftarrow \cos(\theta) \); \( dy \leftarrow \sin(\theta) \)
5: \( p_{new} \leftarrow (x_{init} + \epsilon dx, y_{init} + \epsilon dy) \)
6: if \( p_{new} \in \Phi \) then
7: \( \text{compute } C_B(S_{p_{new}}, p_{new}) \)
8: if \( cap > C_B(S_{p_{new}}, p_{new}) \) then
9: \( cap \leftarrow C_B(S_{p_{new}}, p_{new}) \)
10: \( p_{crc} \leftarrow p_{new} \)
11: end if
12: end if
13: end for
14: if \( p_{crc} = p_{init} \) then
15: \( \epsilon \leftarrow \epsilon/2 \)
16: else
17: \( p_{init} \leftarrow p_{crc} \)
18: end if
19: end while

The feasible direction method finds the local critical point for a given initial guess. In order to select an initial guess, the region at hand has to be divided into subregions. We used both Voronoi tessellations [10] and Delaunay triangulation, which corresponds to the dual graph for the Voronoi diagram, for determining these subregions. Then, we can find the local critical point of a subregion for a given system setting as shown in Fig. 2. The convergence of FDM from an initial guess (IG) to an LCP for different guesses is also shown in the same figure. Yellow lines denote the Voronoi tessellations, red lines denote the Delaunay triangulation while grey area is the utilization region. As we can see, different initial guesses in the same subregion converges to the same local critical point.

As a result, we can use FDM to locate the local critical
points of a given region. Then, the maximum achievable broadcast capacity is the minimum over the capacity of those local critical points and the capacity between source and relays, according to (2).

**IV. PROPOSED HEURISTICS**

As mentioned previously, for a given total energy constraint, selecting optimal broadcast capacity achieving subset of nodes as relays is a complicated problem in a practical broadcasting scenario. To find a good suboptimal solution to this problem, we performed an analysis in the previous section, considering the case where there are infinitely many nodes in the network. This analysis constitutes a basis for our proposed methods that can be used in practical scenarios, because the optimized ring radius is used in our methods heuristically.

The proposed relay selection procedures can be applied to the existing ad-hoc networks in order to increase the broadcast capacity since our proposition does not require any change in the hardware. Here, we present two simple algorithms, based on the parameters optimized under the continuum model, as suboptimal but practical solutions. First one, random selection (RS), is a distributed algorithm and requires node-to-source CSI, while second one, selection by triangularization (ST), is a centralized algorithm based on not only node-to-source CSI but also node-to-node CSI.

**A. Random Selection (RS)**

The source node computes two threshold SINR values per rings using the optimal radii found by the continuum model. These threshold SINR values determine the number of relay candidates as users having an SINR in between these thresholds become candidates for relay selection. Depending on the targeted number of relay nodes, source computes a number in between 0 and 1 for selection and broadcasts this number. Then, each node generates a random number between 0 and 1 independent of each other. If the number generated at a node is less than the number sent by the source, then the node starts to cooperate. As seen, RS is a randomized and distributed relay selection method and it requires only the source-to-node CSI. Therefore, in this method, \((m+n)\), which is \(O(m+n)\), channel information is necessary to initiate the broadcast session.

**B. Selection by Triangularization (ST)**

In ST, the goal is to imitate the ring structure created under the continuum model. First, source node selects a node, which has an SINR value closest to the SINR value of the optimal ring found in the continuum model, as the first relay.

![Convergence of FDM for different initial guesses.](image)

Then a second node is searched by using triangularization of the source node and the first relay. The second selected node is approximately \(2\pi/n\) degree apart from the first selected node while it has the closest SINR to the SINR of the first selected node. Following relays are selected by the source node and two neighboring relay nodes using the same strategy. As source node plays an active role in deciding each relay node, this method is centralized. On the other hand, as triangularization is used, not only node-to-source CSI but also node-to-node CSI is required for this algorithm. As a result, in ST, \([\frac{3}{2}(n(n-1) + 3m(n-1) + 1)]\), which is \(O(n(m+n))\), number of channel information is required prior to broadcasting.

**Algorithm 2 RS**

**Require:** \(r_{\text{star}}\) : optimal ring radius computed by the continuum model

**Ensure:** \(\text{relays}\) : Indexes of relay nodes

1. \((\text{thres}_1, \text{thres}_2) \leftarrow \text{comp\_thres\_RS}(r_{\text{star}})\)
2. \(k = 1; k \leq |R| ; k = k + 1\) do
3. \(\text{if} \text{thres}_1 \leq \text{sinr}(S, k) \leq \text{thres}_2 \text{then}\)
4. \(\text{addElement}(\text{cands}, k)\)
5. \(\text{end if}\)
6. \(\text{end for}\)
7. \(\text{sno} \leftarrow S \text{ generates a number between }[0,1]\)
8. \(k = 1; k \leq \text{length}(\text{cands}); k = k + 1\) do
9. \(\text{nrand} \leftarrow \text{Each candidate generates a random number}\)
10. \(\text{if} \text{nrand} \leq \text{sno} \text{then}\)
11. \(\text{addElement}(\text{relays}, \text{candidates}(k))\)
12. \(\text{end if}\)
13. \(\text{end for}\)

**Algorithm 3 ST**

**Require:** \(r_{\text{star}}\) : optimal ring radius computed by the continuum model

**Ensure:** \(\text{relays}\) : Indexes of relay nodes

1. \(\text{sr\_sinr} : \text{source-to-relay SINR of the theoretical model}\)
2. \(\text{rr\_sinr} : \text{nearest relay-to-relay SINR}\)
3. \(\text{rr\_sinr2} : \text{second nearest relay-to-relay SINR}\)
4. \((\text{sr\_sinr}, \text{rr\_sinr}, \text{rr\_sinr2}) \leftarrow \text{comp\_thres\_ST}(r_{\text{star}})\)
5. \(\text{relays}(1) \leftarrow \text{index}\{\text{min}(\text{sinr}(S, R) - \text{sr\_sinr})\}\)
6. \(\text{St} \leftarrow \text{index}\{\text{sinr}(S, R) - \text{sr\_sinr}\}\)
7. \(\text{Rset} \leftarrow \text{index}\{\text{sinr}(\text{relays}(1), R) - \text{rr\_sinr}\}\)
8. \(\text{relays}(2) \leftarrow \text{intersect}(\text{St}, \text{Rset})\)
9. \(\text{for} k = 3; k \leq \text{no\_relays}; k = k + 1\) do
10. \(\text{Rset} \leftarrow \text{index}\{\text{sinr}(\text{relays}(k-1), R) - \text{rr\_sinr}\}\)
11. \(\text{Rset2} \leftarrow \text{index}\{\text{sinr}(\text{relays}(k-2), R) - \text{rr\_sinr2}\}\)
12. \(\text{relays}(k) \leftarrow \text{triangularize}(\text{St}, \text{Rset}, \text{Rset2})\)
13. \(\text{end for}\)

**V. SIMULATIONS**

**A. Simulation Parameters**

In this section, we conduct simulations to observe the behavior of the broadcast capacity under different system configura-
where \( r_\min \leq r_1 \leq r_\max \) and \( C_B(\mathcal{Y}, N, r_1) \) denotes the broadcast capacity of the network where the system parameters are defined by \( \mathcal{Y} \) and \( N \) relays are positioned on the ring with a radius of \( r_1 \). The computation of the broadcast capacity is done via FDM. At last, the optimal ring radius is used for computing the broadcast capacity of the proposed heuristics.

To observe the gains of the multihop strategy, we consider the capacity ratio between multihop and direct transmission as a performance measure. The broadcast capacity for direct transmission is:

\[
C_B(\mathcal{Y}) = B \log_2 \left( 1 + \frac{P_r^{\max} - \alpha}{N_0 B} \right)
\]

where \( r^{\max} = \max(r) \), \( i = 1, ..., n + m \), i.e., \( r^{\max} \) is the maximum distance between the source and all the nodes in the network.

As this work aims to consider practical aspects of wireless broadcasting, system parameters are selected realistically. In simulations, the ratio of symbol duration to chip duration, i.e., processing gain \( N = T_s/T_c \), is taken to be 100, where \( N >> 1 \) is common [11]. When computing the optimal ring radius, a lower bound, \( r_\min = 50 \) m, is set in order to avoid small values of \( r_1 \) as the channel model is not accurate for small distances. Threshold SINRs for RS are selected such that the number of candidate nodes is at least two times the desired number of relay nodes, where this value is set heuristically. We measure broadcast capacity under two different network densities, scarcely populated (250 users), and highly crowded (2500 users). We compare the performance of the proposed methods under three different system configurations given in Table I. This comparison can be observed in subplots of Fig.s 3-6 where \( TP = k \) denotes that the total power constraint on all the cooperating nodes is \( k \) times the source power and each node is transmitting at \( kP_j/n \).

### B. Results

In the infinitely many user scenario, the broadcast capacity ratio (BCR) decreases slowly as the targeted number of cooperating peers (TNCP) increases since the total power consumed by the network is kept the same as shown in Fig. 3. According to (2), the broadcast capacity is defined by two elements, the relays that is being served by the source and the geographical point having the minimum capacity over the region. When the number of cooperating peers is increased, the distance between two neighboring peers and the transmission power per peer decreases. This leads to a degradation in the capacity of listening peers, which are served by the relays, and an improvement in the capacity of relays as the total interference power from neighboring relays decreases. For a given system configuration and the number of cooperating peers, the ring radius is optimized such that the minimum of the capacities of the relays and the user with the minimum capacity is maximized. Thus, the broadcast capacity values given in Fig. 3 constitutes an upper bound for the heuristics applied to a highly crowded network.

The performance results for the proposed heuristics are given in Fig.s 4-7. We can observe the fast diminishing behavior of BCR for ST when the network population is scarce. This is expected since ST tries to imitate the strict ring structure and the probability of finding a peer set that forms this desired ring decreases when there are few users in the network. However, the gap between the BCR values for the optimal capacity and ST closes when the network becomes crowded as shown in Fig.s 3 and 5. On the other hand, RS performs better at wider operating bandwidths. An increase in bandwidth decreases the chip time and the average number of resolvable paths for a specific user in the network. As RS randomly selects peers closer to the optimal ring radius, it is probable that both two users that are very close to each other are selected as cooperating peers. Then, the capacity between one of these relays and the source is highly degraded due to a powerful interferer. In a highly crowded network, the probability of selecting two peers in such a proximity increases as users are more uniformly distributed over the geography.
VI. CONCLUSION

The goal of this work is to close the existing gap between scarce theoretical and practical studies on wireless multihop broadcast capacity in the literature where real life impairments are not ignored. We derive the wireless multihop broadcast capacity for an infinite population network where total energy consumption, interference from multiple transmitters and the utilization of signals by multiple fingers at a receiver via diversity combining, all part of a realistic wireless system model, are taken into account. We then propose a pair of heuristics, one distributed and one centralized, for finite population as suboptimal but practical solutions. The results illustrate that although the proposed centralized relay selection method outperforms the fully randomized and distributed one, significant capacity gains can also be achieved using a random peer selection algorithm at wider bandwidths without the need for CSI availability at intermediate nodes.

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