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Citation for published version (APA):

Dijk, van, W., Geer, van der, S. B., Wiel, van der, M. J., & Brussaard, G. J. H. (2008). Parameter study of acceleration of externally injected electrons in the linear laser wakefield regime. *Physics of Plasmas*, 15(9), 093102-1/9. <https://doi.org/10.1063/1.2977765>

DOI:

[10.1063/1.2977765](https://doi.org/10.1063/1.2977765)

Document status and date:

Published: 01/01/2008

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Parameter study of acceleration of externally injected electrons in the linear laser wakefield regime

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(Received 28 April 2008; accepted 12 August 2008; published online 9 September 2008)

A parameter study for laser wakefield acceleration is presented, in which externally injected electrons are accelerated in low amplitude plasma waves, represented by an analytical two-dimensional description. Results have been obtained for plasma densities up to $2.6 \times 10^{24} \text{ m}^{-3}$, plasma lengths up to 300 mm, laser intensities up to $3.5 \times 10^{21} \text{ W/m}^2$, and injection of Gaussian model bunches at energies up to 12 MeV. For the range of parameters studied, effects of laser depletion and the influence of the electron bunch on the plasma can be ignored. In the parameter space, a region is identified where final energies of over 100 MeV are reached, at an energy spread of less than 5% and a rms emittance of a few micrometers. © 2008 American Institute of Physics. [DOI: 10.1063/1.2977765]

I. INTRODUCTION

Laser wakefield acceleration¹ allows the generation and use of accelerating electric fields orders of magnitude larger than those achieved in conventional accelerators, allowing the reduction of the size of the accelerating structure from meters or kilometers to the scale of millimeters or meters. Recent demonstrations have shown the ability to create electron bunches with energies of 100 MeV (Refs. 2–4) and even up to 1 GeV (Ref. 5) with low energy spread.

One of the interesting features that sets electron bunches created using laser wakefield acceleration apart from those created with conventional radio-frequency accelerators is the length of the bunches created. Simulations predict that this length is on the order of a few to ten femtoseconds,^{3,5–7} below the current electron bunch length measurement lower limit of 50 fs.⁸ The generation of electron bunches with such short duration opens up new applications in fields such as coherent THz generation⁹ and compact, femtosecond free-electron lasers.¹⁰

In addition to having severe requirements on energy, energy spread, bunch duration, and emittance, these applications also require a stable and reliable source of electrons, something current laser wakefield acceleration experiments have not been able to achieve. The main reason for the unstable behavior is the way the electrons are injected into the wakefield. Electrons are trapped from the plasma itself in a process known as wave-breaking. The onset of wave-breaking requires the generation of very nonlinear fields; these fields will then trap electrons from the plasma. The highly nonlinear behavior and necessary instability make control over the injection process very difficult. A possible way^{6,11,12} to prevent this problem is to use externally injected electrons instead of electrons trapped from the plasma itself. The use of an external electron source would allow more control over the injection process and avoid the instabilities

inherent to the highly nonlinear regime needed for wave breaking. In addition, the approach of external injection enables the use of low amplitude plasma waves which require less laser power for their generation, reducing the complexity and costs of the laser system.

A previous article⁷ has already focused on the specific design and optimization of an injector beamline for injecting electrons into a low amplitude wakefield. This article first describes the framework and simulation methods used to obtain the results. This model will then be used to explore the effects of several important experimental parameters on the properties of the final electron beam. The parameters are the density and length of the plasma, the energy of the injected electrons, and the intensity/power of the laser used to generate the wakefield. When using external injection, these parameters can be separately varied and more or less individually tuned to obtain the desired electron beam properties. The final electron beam will be characterized by energy, energy spread, charge, emittance, and duration. The resulting trends of the parameter study will be discussed.

In the parameter space under consideration, the plasma wave will function not only as a compact accelerator, but will also act as a temporal slicer, transforming the picosecond injected bunch into a train of femtosecond sub-bunches (see Fig. 1). This will allow diagnostics using electrons or radiation created by electrons on the femtosecond scale using picosecond electron bunches.

II. METHOD

The tool used to evaluate the effects of the different laser wakefield and electron parameters on the initial electron bunch is the general particle tracer code (GPT).¹³ The GPT code solves the relativistic equations of motion of charged (macro-)particles in three dimensions in the presence of both external electromagnetic fields and internal space-charge forces. To include the effects of a plasma wave, an analytical approximation of the fields inside the plasma wave (described in Sec. I) has been used.

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FIG. 1. Typical example of the effect of the plasma wave on a bunch with a length greater than the plasma wavelength. The initial Gaussian bunch (a) is transformed into a train of sub-bunches (b) corresponding to similar phases in the plasma wave while retaining the overall Gaussian profile.

One advantage of this approach compared to the use of particle in cell (PIC) codes^{14–16} is the possibility to include all the components used for the generation of the injected electrons and their transport to the plasma. This allows the use of nonidealized electron bunches and evaluation of the effect of the injection beamline on the final electron bunch, as was shown in Ref. 7.

More relevant for the present work is the fact that the current implementation of the laser wakefield module allows the execution of many thousands of simulations with different parameters on a regular personal computer in one or a few days instead of requiring the same time for a single run on a cluster, as is the case with the more elaborate PIC codes. This allows a much more complete sampling of the parameter space given constraints on time and computing power.

A. Laser

Low amplitude wakefields can be described by simple harmonics with a single frequency (the plasma electron frequency). As a contrast to the regime of high amplitude non-linear plasma waves, this regime will be referred to as the linear laser wakefield regime. Because this article focuses on the linear laser wakefield regime, the default laser parameters considered are those of a relatively modest laser system. The specifications of the laser system used for the simulations (except where stated otherwise), are those given in Table I.

TABLE I. The default parameters used when exploring the parameter space.

Parameter	Default value
Bunch energy	6.68 MeV
Bunch charge	10 pC
Bunch length (FWHM)	500 fs
Bunch FWHM radial size	30 μm
Laser central wavelength	800 nm
Energy per laser pulse	100 mJ
Laser pulse length	50 fs
Laser spot size ($1/e^2$ radius)	30 μm
Laser intensity	$0.7 \times 10^{21} \text{ W/m}^2$

1. Laser guiding

To maintain a high laser intensity over a long distance, it will be assumed that the laser is guided^{17–21} in the plasma. For a parabolic plasma channel, the matched spot size²¹ is then given by

$$W = \sqrt[4]{\frac{e^2}{8\epsilon_0 m_e c^2} \frac{\partial^2 n_e}{\partial r^2}}. \quad (1)$$

Here, n_e is the electron density, r the distance from the center, e is the elementary charge, ϵ_0 is the vacuum permittivity, m_e is the electron mass, and c is the speed of light in vacuum.

B. Electron bunches

External injection of electron bunches with a duration of less than one plasma wavelength has not been demonstrated and may be very difficult to achieve.^{22,23} It used to be thought that such bunches would be necessary for the generation of (quasi-)monoenergetic electrons bunches. However, recently, simulations and theory have been published^{7,11,24} showing that bunches significantly longer than the plasma wavelength can still lead to the generation of electrons with low energy spread and a bunch length on the order of 10 fs. In this scenario, the plasma wave acts as a slicer and as a compressor in addition to an accelerator, as can be seen in Fig. 1.

An additional advantage of using electron bunches which span several plasma periods lies in the relaxed demands for synchronization between the laser and the electrons. For bunches significantly shorter than the plasma wavelength, synchronization on time scales below the plasma period is needed to select the proper injection phase. With bunches longer than the plasma wavelength, injection needs to take place before significant wave damping takes effect. This relaxes the synchronization demands by more than an order of magnitude from a few to ten femtoseconds up to hundreds of femtoseconds. Such synchronization has already been demonstrated and can be achieved routinely.²⁵

In the simulations, an initial electron bunch with a length of 500 fs rms will be used. This bunch spans several plasma periods, but is short enough to limit the number of accelerated (sub-)bunches to a few up to 20 per injected bunch, as can be seen in Fig. 1. The temporal profile will be assumed to be a Gaussian distribution. It is possible to use arbitrary distributions, but the intention is to give a general parameter scan. Effects of a non-Gaussian distribution will be discussed in Sec. IV.

The transverse size of the electron bunch should be as small as possible to ensure acceleration of as many electrons as possible in the region with the highest laser intensity. In reality the emittance of the bunch and the space-charge forces inside the bunch will limit the minimum size to which the electrons can be focused. Based on previous investigations of realistic beamlines,^{7,24} it can be shown that the electrons can be focused down to at least 30 μm . That is the value will be assumed for the full width at half maximum (FWHM) of the Gaussian radial distribution so that both laser and electron bunch have the same spot size. The results

are representative for other electron beam radii as long as the laser spot size and laser power are adjusted to maintain the same laser intensity and relative spot sizes compared to the ones presented here.

The effects of the bunch charge on the plasma are not included in the model. This means that the bunch charge should be low enough not to significantly influence the plasma it is traveling through. Limiting the total charge to 10 pC will result in space-charge fields that are less than 1% of the fields generated by the wakefield. In experiments, the total injected charge will depend on the electron source used. Therefore, the final charge in the simulation results is expressed as a percentage of the injected charge for more convenient evaluation.

C. Plasma wave description

A general three-dimensional (3-D) fully analytical treatment of the laser wakefield acceleration process is extremely difficult if not impossible. However, under the conditions explored in this article, an analytical approximation of the wakefield can be made.^{11,26–28} The main restriction is that the laser intensity is relatively low. Under this condition the plasma wave can be described by simple harmonics. The second assumption is that the problems is quasi-static. This requires both the depletion of the laser and the effect of the accelerated bunch on the plasma wave to be negligible. In the parameter space explored, both these conditions are fulfilled. The plasma is assumed to be a hydrogen plasma and fully ionized in the region of interest.

With the assumptions mentioned, the electrical fields of the plasma wave can be described (analogous to Ref. 29) by

$$E_z(r, z) = k_p U_0 \sin(k_p(z - v_g t) + \phi_0) e^{-(r^2/W^2)}, \quad (2)$$

$$E_r(r, z) = \frac{2U_0 r_0}{r_l} \cos(k_p(z - v_g t) + \phi_0) e^{-(r^2/W^2)}, \quad (3)$$

with k_p the plasma wave number, U_0 the potential amplitude of the wave, v_g the group velocity of the wave, ϕ_0 the initial phase of the wave, W the matched spot [Eq. (1)] of the plasma channel, and r_l the $1/e^2$ radius of the laser beam.

The plasma wave number k_p is related to the electron density n_e , electron mass m_e , and vacuum permittivity ϵ_0 by

$$k_p^2 = \frac{e^2 n_e}{\epsilon_0 m_e c^2}. \quad (4)$$

The group velocity v_g of the plasma wave is described by

$$v_g = c \sqrt{1 - \frac{n_p}{n_{cr}}}, \quad (5)$$

with n_{cr} the critical plasma density [$n_{cr} = (\epsilon_0 m_0 / e^2) \omega_l^2$] for angular frequency ω_l of the laser that is used to generate the wakefield.

The amplitude of the wave U_0 can be expressed as a function of the parameters of laser pulse and plasma channel as described in Ref. 30,

$$U_0 = \frac{eP\sqrt{\mu_0}}{m_e c^2 \sqrt{\pi \epsilon_0}} \left(\frac{\lambda_l}{\lambda_p} \right) \left(\frac{\sigma_z}{2Z_R} \right) e^{-(k_p^2 \sigma_z^2 / 4)}. \quad (6)$$

Here, μ_0 is the permeability of vacuum, P is the total laser power, λ_l the laser wavelength, λ_p the plasma wavelength ($\lambda_p = 2\pi/k_p$), σ_z the temporal half width of the laser pulse and Z_R the Rayleigh length in vacuum, $Z_R = \pi r_l^2 / \lambda_l$.

The electric fields described by Eqs. (2), (3), and (6) have been incorporated into the GPT code allowing the simulation of both the injector beamline and the plasma wakefield within the same code. The results of the two-dimensional (2-D) low amplitude approximation have been compared to those of the more complete 3-D particle-in-cell/fluid model developed by Reitsma³¹ and within the stated limits of validity, the results agree.

During acceleration, part of the electrons are expelled in the radial direction by the radial component of the wakefields. These electrons are removed from the final beam in the simulation as soon as their radial distance from the optical axis and center of the wakefield exceeds $150 \mu\text{m}$ within the plasma, well separated from the trapped part of the beam.

III. PARAMETER EVALUATION

When evaluating the effect of the various system parameters on the accelerated electrons, it is important to first identify which ones are interesting to perform a full parameter scan and which are the ones that can be accounted for by scaling.

The plasma serves a dual role in the system: It serves both as a medium to accelerate the electrons, driven by the laser, and as a laser guide to maintain the same laser intensity over the entire plasma length. The efficiency as a laser guide can be adjusted by changing the way the plasma is generated, but since these are specific to the kind of plasma used, it will be assumed that these adjustments have been made and the plasma acts as a laser guide. Two important remaining parameters are the density and the length of the plasma, which can both significantly influence the characteristics of the final electron bunch, as is shown in subsection C.

For the injected electrons, the most important characteristics are energy, energy spread, charge, bunch length, and radial size. Within the limitations stated in Sec. II B, the effects of charge, bunch length, and radial size can be obtained by scaling the given results. Since most injection sources can provide beams with low energy spread, its effect can also be neglected. That leaves the effect of the initial energy of the injected electrons, which is explored in subsection D.

The laser pulse is characterized by wavelength, energy, duration, and spot size. While the wavelength and minimum duration are usually fixed for the laser system chosen, the energy, and spot size can be varied. As can be seen by examining Eq. (6), the electric fields within the plasma wave are proportional to the intensity of the laser pulse. Since both the laser spot and electron bunch have been set at the same size, the laser spot size remains constant while the energy in the laser pulse is varied to study the effect of the laser intensity. These results are presented in subsection E.

Default parameters

When evaluating the effect of one particular parameter, the other parameters of the injected electrons and laser are kept at the default values which have been summarized in Table I.

IV. SIMULATION RESULTS

A. Parameters explored

In this section, there follows an evaluation of the effects of the plasma length (subsection C), electron energy (subsection D), and laser intensity (subsection E) on the electrons accelerated in a laser wakefield. The plasma density has a strong effect on the final beam characteristics, even in combination with other parameter changes. Therefore, all results will also be presented as a function of the plasma density in a range of $(0.2\text{--}2.6) \times 10^{24} \text{ m}^{-3}$. The lower limit represents the practical lower limit of the current guiding plasmas. The upper limit denotes the current lower limit for electron injection by wavebreaking.^{2–5}

B. Beam transformation

When a bunch with a total length exceeding the plasma period is injected into a plasma wave, parts of the initial electron bunch will be radially expelled. This leads to a breakup of the original bunch into a train of sub-bunches, as is shown in Fig. 1.

When considering the properties of the final bunch, the integral over all the sub-bunches of the entire bunch train will be taken.

1. Beam criteria

Since the initial electron bunches span more than one period of the plasma wave, the fields and energy gain experienced by the different electrons vary greatly. In addition, there are electrons that neither remain trapped nor are expelled far enough from the plasma to be removed in the simulation scheme. In order to reduce the effect of these few outliers on the macroscopic properties of the final train of electron bunches, only the central 95% of the distribution of the final electron bunches will be considered when determining the average energy, relative energy spread, charge, and emittance.

2. Emittance definition

There are many definitions that can be used to quantify the emittance of a beam. The generally used rms emittance is not suited for describing the emittance of the bunches created since it assumes a Gaussian distribution in phase space which, as mentioned before, is often not the case for the bunches exiting the wakefield. Moreover, the rms emittance is very sensitive to a few outliers. Therefore, the area of an ellipse in the transverse phase-space containing 95% of the electrons, divided by π and normalized by the Lorentz factor will be used.

In order to be able to compare this emittance with the more conventional rms emittance, it can be noted that for a

uniform phase-space distribution, the area of the ellipse containing 95% of the electrons is a factor of 4 larger than the rms emittance.

C. Effect of plasma length

The effect of the length of the plasma on energy, energy spread, charge, and emittance can be seen in Fig. 2. Normally, one would expect the highest energy to be achieved at the dephasing length (L_d) (Refs. 32–34) given by

$$L_d = \frac{\pi c \omega^2}{\omega_{pe}^3}. \quad (7)$$

As can be seen in Fig. 2(a), the length for the plasma which yields the highest energy, differs slightly from the dephasing length [Eq. (7)]. The general picture of dephasing is preserved, however: The electrons are accelerated until they outrun the accelerating phase and enter the decelerating phase. After deceleration, they again enter the accelerating phase and gain energy, and so on.

The energy spread [Fig. 2(b)] reaches its minimum for a given density at around the same plasma lengths for which the energy reaches its maximum. At that length, the gradient in potential energy is minimal leading to the least energy difference between those electrons in the forward part of the bunch and those in the back. The total energy spread is reduced from almost 100% during the initial phase of the acceleration to only a few percent at this plasma length.

The total charge [Fig. 2(c)] varies very little: After about 3 mm, over 75% of the electrons are expelled in the radial direction and those trapped by the plasma wave generally stay trapped. The total accelerated charge drops both with high and low plasma densities, showing a peak around 15% at intermediate values within the range evaluated.

The emittance [Fig. 2(d)] at the lengths that yield the highest energy and lowest energy spread is $1\text{--}8 \mu\text{m}[\text{rad}]$ ($0.25\text{--}2 \mu\text{m}$ rms). The values of the emittance can vary over two orders of magnitude over the entire parameter space explored.

While studying the effect of the energy of the injected electrons (subsection D) and the laser intensity (subsection E), Eq. (7) will be used to set the length of the plasma. This is close to the length providing maximum energy and minimum energy spread, as can be seen in Fig. 2.

D. Effect of the energy of the injected electrons

The effect of the injection energy on energy, energy spread, charge, and emittance is shown in Fig. 3. Because the initial energy of the electrons determines the range of the phase of the plasma wave where electrons can be trapped, it is expected to influence energy, energy spread, and especially accelerated charge.

There are regions in Fig. 3 where all of the electrons are expelled and none reach the end of the plasma. These regions are denoted by a white color. The data from Fig. 3 show two different regions, depending on the initial energy of the electrons separated around an injection energy of 4.75 MeV for

FIG. 2. (Color online) Effect of the plasma length and density on the final energy (a), energy spread (b), total charge (c), and emittance (d) of the train of bunches. The solid line represents the dephasing length [Eq. (7)]. The laser and injected electron parameters are described in Table I.

the default laser parameters. Electrons with an energy below 4.75 MeV are mostly expelled beyond the 150 μm radial limit, resulting in no accelerated electrons. There are however also cases, mainly at higher plasma densities, where the lower energy electrons seem to gain (almost) no energy, exhibit low to moderate energy spread (0%–15%), and the remaining charge varies up to almost 100%. Closer inspection of the evolution of these electron bunches reveals the following: The electrons are not trapped and accelerated by the plasma wave during their journey through the plasma, but are radially expelled from the region of the plasma wave. However, these expelling fields are not strong enough to remove them beyond the 150 μm radial limit from the optical axis where the particles are removed within the length of the plasma. They therefore continue inside the plasma without further interaction with the plasma wave on a trajectory roughly parallel to the optical axis on which they entered the plasma with a small radial velocity due to the radially defocusing fields. The fact that these electrons still remain in the simulation is therefore a result of the criteria chosen to determine which electrons are trapped and which are expelled and should be disregarded in the results.

Electrons with injection energy higher than the 4.75 MeV trapping threshold are trapped by the plasma wave and their final energy is almost independent of the initial

energy, as can be seen in Fig. 3(a); there is even a slight decrease in average energy when the injection energy is increased due to electrons also being trapped in phases of the plasma wave that lead to smaller final energies.

For a given plasma density, the energy spread [Fig. 3(b)] starts out high as the injection energy is increased beyond 4.75 MeV, as part of the electrons are trapped and accelerated to higher energies while another part is not trapped or does not remain trapped over the entire length of the plasma. Just above the threshold, all the electrons that are trapped are being trapped at nearly the same phase of the plasma wave. The resulting energy spread is small, on the order of a few percent. When the initial energy is increased, a larger part of the initial electron bunch is trapped over a larger phase of the plasma wave and thus the electrons experience different accelerating fields, resulting in a larger final energy spread.

When considering the charge of the accelerated electrons [Fig. 3(c)], the effect of more electrons being trapped when injected with higher initial energy is clearly visible in the final total charge. Higher initial energy leads to more trapping, which leads to more charge. In the region of plasma densities between 0.2×10^{24} and $0.8 \times 10^{24} \text{ m}^{-3}$ and injection energies between 7 and 12 MeV where the energy spread is below 20%, the total fraction of the initial charge accelerated can be as high as 40%.

FIG. 3. (Color online) Effect of the plasma density and injection energy of the electrons on the final energy (a), energy spread (b), total charge (c), and emittance (d) of the train of bunches. The fixed laser and injected electron parameters are described in Table I and the plasma length is equal to the dephasing length [Eq. (7)]. The dotted line denotes the approximate trapping limit of 4.75 MeV.

The emittance [Fig. 3(d)] is higher near the trapping limit due to the presence of both trapped and untrapped but unexpelled electrons. At electron energies just above the trapping threshold, the emittance drops as electrons stay either trapped or are radially expelled. At injection energies further above the trapping threshold, the emittance starts to increase as electrons are trapped in a larger range phases of the plasma wave experiencing different radial fields. In the region of plasma densities between 0.2×10^{24} and $0.8 \times 10^{24} \text{ m}^{-3}$ and injection energies between 7 and 12 MeV where the energy spread is below 20%, the emittance varies between 3 and 20 μm (0.75–5 μm rms).

E. Effect of the laser intensity

The effect of the laser intensity on energy, energy spread, charge, and emittance is shown in Fig. 4. According to the equation for the wakefield amplitude [Eq. (6)], the electric fields scale linearly with the laser intensity.

As expected, the energy of the final electron bunch [Fig. 4(a)] typically scales linearly with the intensity above a certain threshold where trapping occurs. The average final energy varies between the initial injection energy of 6.7 MeV to energies well above 250 MeV.

The energy spread [Fig. 4(b)] increases with increased laser intensity. At higher laser intensities more electrons are trapped over a larger phase of the plasma wave. Similar to the process described for increased injection energy, this leads to a higher energy spread. Around $0.5 \times 10^{21} \text{ W/m}^2$, electrons start getting trapped and there is similar trapping behavior as for the injection energy with an increased energy spread due to electrons remaining trapped and electrons being trapped, but then escaping again. The total energy spread typically varies from 50% at high laser intensities to below 5% just above the trapping threshold.

The charge in the final bunch [Fig. 4(c)] increases as a function of the laser intensity as noted above. The charge also increases with higher plasma densities due to the larger electric fields [linear with the plasma density; see Eq. (6)], resulting in a higher trapping efficiency. However, higher plasma densities lead to lower final energies due to a shorter dephasing length. The total trapped fraction can get as high as 45% for high plasma densities and 30% at lower plasma densities.

The emittance [Fig. 4(d)] is again higher near the trapping limit due to the presence of both trapped and untrapped but unexpelled electrons. Similarly, at laser intensities just above the threshold, it drops as electrons stay either trapped

FIG. 4. (Color online) Effect of the plasma density and laser power on the final energy (a), energy spread (b), total charge (c), and emittance (d) of the train of bunches. The fixed laser and injected electron parameters are described in Table I and the plasma length is equal to the dephasing length [Eq. (7)]. The dotted line denotes the approximate trapping limit of $0.5 \times 10^{21} \text{ W/m}^2$.

or are radially expelled. At intensities further above the trapping threshold, the emittance starts to increase as electrons are trapped in a larger range phases of the plasma wave experiencing different radial fields. The emittance is between 5 and $20 \mu\text{m}$ ($1.25\text{--}5 \mu\text{m}$ rms) for plasma densities between 0.2×10^{24} and $0.8 \times 10^{24} \text{ m}^{-3}$ and laser intensities between 0.75×10^{21} and $2.0 \times 10^{21} \text{ W/m}^2$.

V. DISCUSSION AND CONCLUSION

From the results presented in Sec. IV, it is possible to draw several conclusions regarding the effect of various experimental parameters on the process of laser wakefield acceleration in the linear regime. The study helps select the experimental parameter ranges appropriate for obtaining desired beam characteristics for applications.

For most applications, the following characteristics of the accelerated beam are desirable: High energy, low energy spread, high charge, and low emittance. From Figs. 2–4 the following can be concluded regarding those characteristics:

The final energy of the accelerated bunch is mostly determined by the plasma density (combined with the appropriate plasma length) and laser power. Lower plasma densi-

ties and higher laser power result in a high electron energy, which is to be expected.

The energy spread of the accelerated electrons varies greatly from a few percent to almost 100% depending on the parameters chosen. With regard to the plasma length, the energy spread generally has a minimum at the same length where the energy is at its maximum. Low energy spread is possible at all plasma densities evaluated. With regard to laser power and the energy of the injected electrons, the energy spread reaches a minimum slightly above the trapping limit for the electrons.

The total fraction of accelerated charge is fairly uniform over most of the parameter space with 5%–10% of the initial charge being accelerated, though this fraction drops when either the laser power or the initial electron energy is too low for trapping. More charge is preserved with a higher laser power or initial electron energy, but this will also lead to an increased energy spread. There are many cases which show significantly more than 10% charge in the final bunch below the trapping limit, but these all have a large fraction of electrons that are radially expelled from the plasma wave, but with insufficient force to remove them completely leading to a large energy spread and emittance.

The emittance of the accelerated electrons shows minima around the areas where the energy spread is also at a minimum, usually just above the trapping limits.

The use of nonidealized bunches will have some effects, as will be discussed in Sec. VI, but all the final characteristics will still remain roughly the same with the exception of the total charge.

It was noted before that there is a difference between the dephasing length [Eq. (7)] and the length offering the lowest energy spread and emittance as can be seen in Fig. 2. The effect of this difference is most evident at higher plasma densities, leading to a suboptimal result at higher plasma densities in Figs. 3 and 4. In addition, at small plasma lengths (generally below 3 mm) there is still a fraction of untrapped electrons present that have not been radially expelled far enough to be removed. These electrons still influence the macroscopic bunch properties. This leads to a lower average energy and a higher energy spread, charge, and emittance. The general picture is preserved, however, and can help narrow down the region of interest for further detailed evaluation.

The length and number of the (sub-) bunches has thus far not been discussed. With respect to the total bunch train length, that length remains the same unless a very large fraction of the electrons is expelled (leading to shorter trains/no electrons) or when there is an untrapped fraction traveling along the accelerated electrons. Since the untrapped electrons usually have a lower energy, the bunch train will become longer (with less distinct sub-bunches). The length of the sub-bunches varies little, typical values are a few to ten femtoseconds for each sub-bunch. The reason for the almost constant bunch length lies in the fact that the electrons, in the regions of interest, are almost all trapped at nearly the same phase in the plasma wave. The energy spread will however lead to (sub-)bunch lengthening after the plasma. The number of bunches is almost exclusively determined by the plasma density (provided enough electrons are trapped by the plasma wave). Since trapping occurs at specific phases of the plasma wave, the various sub-bunches are separated by the wavelength of the plasma wave. Higher densities lead to shorter wavelengths and more sub-bunches while lower densities lead to less and more separated sub-bunches. The total number of sub-bunches is roughly equal to the initial bunch length divided by the plasma wave wavelength.

The model does not include the phase front curvature induced by the radial gradient in the plasma density (leading to a radial gradient in the group velocity [Eq. (5)]. This phase front curvature can lead to an increased overlap between the longitudinally accelerating [Eq. (2)] and radially focusing [Eq. (3)] parts of the plasma wave, as was discussed in Ref. 11. However, the exact amount of increased (or decreased) overlap depends on the plasma density, matched spot size, and the exact position the electron bunch is injected into the plasma wave with respect to the laser pulse generating the wave. This would therefore reduce the general aim and scope of the current paper.

Concluding, there is an area within the parameter space that offers most of the desired characteristics for the accelerated electrons: Moderately high energy, low energy spread,

TABLE II. The bunch properties of the injected electron bunch of the model study in this paper and the results obtained for the setup described in Ref. 7.

Bunch parameter	Model	Design
Energy, MeV	6.68	6.68
Energy spread, %	0	0.6
Charge, pC	10	10
Rms emittance, μm	0	1.96
Length (FWHM), fs	696	333
Radial size (FWHM), μm	30	30

reasonable charge, low emittance, and a sub-bunch duration <10 fs. This area can be found at plasma densities between 0.2×10^{24} and $1.1 \times 10^{24} \text{ m}^{-3}$, initial electron energies between 4.75 and 9 MeV, and a laser power between 0.5×10^{21} and $2.5 \times 10^{21} \text{ W/m}^2$ (1.75 and 9 TW). The optimal plasma length depends on the plasma density and within the region mentioned above, is between 25 mm for a density of $1.1 \times 10^{24} \text{ m}^{-3}$ and just over 300 mm for a density of $0.2 \times 10^{24} \text{ m}^{-3}$. For plasma densities between 0.7×10^{24} and $1.1 \times 10^{24} \text{ m}^{-3}$, guiding plasmas are available. At the densities below $0.7 \times 10^{24} \text{ m}^{-3}$ and lengths exceeding 70 mm, guiding plasmas with a small matched spot size currently do not exist yet, this seems a good incentive for the further development of long, low density guiding plasmas.

VI. COMPARISON WITH A DETAILED DESIGN STUDY OF A REALISTIC EXPERIMENT

So far, the initial bunches considered had model Gaussian spatial and temporal profiles for the injected beam. In this section, the properties of the accelerated bunches will be compared to those of a realistic design for an injection beamline, as described in Ref. 7, in order to determine the effect of nonidealized features found in realistic bunches. The main parameters of the two injected beams are shown in Table II.

As can be seen from Table II, the model bunch that most closely resembles the optimized bunch has been chosen for comparison. The two bunches differ in energy spread, emittance, and length with the design bunch having a nonzero energy spread and emittance and a shorter bunch length. In addition, the design bunch has a non-Gaussian spatial and temporal profile. The beamline described in Ref. 7 has been specifically designed to keep the injected electron bunch as short as possible. This beamline includes the components needed to focus both laser and electrons into the plasma, leading to a bunch length increase due to space charge, energy spread, and path length differences.

The parameters of the resulting bunches when both the model and design bunch are accelerated in the same plasma wave as used in Ref. 7 can be found in Table III.

The energy (97 MeV versus 97 MeV) and energy spread (4.6% versus 4.1%) are about equal since they are mainly determined by the plasma wave (plasma density and laser power) and the injection energy which have been chosen the same. The emittance ($11 \mu\text{m}$ versus $10 \mu\text{m}$ ($2.75 \mu\text{m}$ versus $2.5 \mu\text{m}$ rms) of the accelerated bunches is also about the

TABLE III. Comparison of the bunch properties (see Sec. I) of the accelerated electron bunch of the generalized study in this paper and the results obtained for the setup described in Ref. 7 using the same plasma and laser parameters ($N_e = 7 \times 10^{23} \text{ m}^{-3}$, $L_{\text{plasma}} = 70 \text{ mm}$, $P = 0.7 \times 10^{21} \text{ W/m}^2$).

Accelerated bunch parameter	Model	Design
Energy, MeV	97	97
Energy spread, %	4.6	4.1
Charge, pC	1.1	0.5
95% Emittance, μm	11	10
Equivalent rms emittance, μm	2.75	2.5
Sub-bunch length (FWHM), fs	<10	<10
Max. charge/sub-bunch, pC	0.19	0.27

same in both situations with the final emittance of the design bunch being comparable to the initial emittance of the design injected bunch.

The total accelerated charge in the design case is lower than in the general case (0.5 pC versus 1.1 pC). This is due mainly to the initial spread in energy and divergence of the beam in the design case. Because the charge is spread over less sub-bunches (3 versus 13), the maximum charge per sub-bunch is still 50% higher (0.27 pC versus 0.18 pC).

Concluding, the generalized Gaussian bunches used in this study give a prediction good of most the final parameters of a comparable nonidealized bunch from an actual beam-line. The charge is more sensitive to the actual beam properties and warrants attention when evaluating the effect of a designed beamline on the final electron bunch.

ACKNOWLEDGMENTS

This work is part of the research programme of the “Stichting voor Fundamenteel Onderzoek der Materie (FOM),” which is financially supported by the “Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).”

¹T. Tajima and J. M. Dawson, *Phys. Rev. Lett.* **43**, 267 (1979).

²S. P. D. Mangles, C. Murphy, Z. Najmudin, A. G. R. Thomas, J. L. Collier, A. E. Dangor, E. J. Divall, P. S. Foster, J. G. Gallacher, C. J. Hooker, D. A. Jaroszynski, A. J. Langley, W. B. Mori, P. A. Norreys, F. S. Tsung, R. Viskup, B. R. Walton, and K. Krushelnick, *Nature (London)* **43**, 535 (2004).

³C. G. R. Geddes, C. Toth, J. van Tilborg, E. Esarey, C. B. Schroeder, D. Bruhwiler, C. Nieter, J. Cary, and W. P. Leemans, *Nature (London)* **43**, 538 (2004).

⁴J. Faure, Y. Glinec, A. Pukhov, S. Kiselev, S. Gordienko, E. Lefebvre, J.-P. Rousseau, F. Burgy, and V. Malka, *Nature (London)* **43**, 541 (2004).

⁵W. P. Leemans, B. Nagler, A. J. Gonsalves, C. Toth, K. Nakamura, C. G. R. Geddes, E. Esarey, C. B. Schroeder, and C. J. Hooker, *Nat. Phys.* **2**, 696 (2006).

⁶A. G. Khachatryan, F. A. van Goor, K.-J. Boller, A. J. W. Reitsma, and D. A. Jaroszynski, *Phys. Rev. ST Accel. Beams* **7**, 121301 (2004).

⁷W. H. Urbanus, W. van Dijk, S. B. van der Geer, G. J. H. Brussaard, and M. J. van der Wiel, *J. Appl. Phys.* **99**, 114501 (2006).

⁸J. van Tilborg, C. B. Schroeder, C. V. Filip, C. Tóth, C. G. R. Geddes, G.

Fubiani, R. Huber, R. A. Kaundl, E. Esarey, and W. P. Leemans, *Phys. Rev. Lett.* **96**, 014801 (2006).

⁹W. P. Leemans, C. G. R. Geddes, J. Faure, C. Tóth, J. van Tilborg, C. B. Schroeder, E. Esarey, G. Fubiani, D. Auerbach, B. Marcellis, M. A. Carnahan, R. A. Kaundl, J. Byrd, and M. C. Martin, *Phys. Rev. Lett.* **91**, 074802 (2003).

¹⁰F. Grüner, S. Becker, U. Schramm, T. Eichner, M. Fuchs, R. Weingartner, D. Habs, J. M. ter Vehn, M. Geissler, M. Ferrario, L. Serafini, B. van der Geer, H. Backe, W. Lauth, and S. Reiche, *Appl. Phys. B: Lasers Opt.* **86**, 431 (2007).

¹¹D. F. Gordon, R. F. Hubbard, J. H. Cooley, B. Hafizi, A. Ting, and P. Sprangle, *Phys. Rev. E* **71**, 026404 (2005).

¹²A. F. Lifschitz, J. Faure, V. Malka, and P. Mora, *Phys. Plasmas* **12**, 093104 (2005).

¹³B. van der Geer and M. de Loos, Ph.D. thesis, Eindhoven University of Technology, 2001.

¹⁴A. Pukhov, *J. Plasma Phys.* **61**, 425 (1999).

¹⁵R. G. Hemker, F. S. Tsung, V. K. Decyk, W. B. Mori, S. Lee, and T. Katsouleas, in *Proceedings of the 1999 Particle Accelerator Conference*, edited by A. Luccio and W. MacKay (IEEE, Piscataway, 1999) Vol. 5, p. 3672.

¹⁶C. Nieter and J. R. Cary, *J. Comput. Phys.* **196**, 448473 (2004).

¹⁷P. Volfbeyn, E. Esarey, and W. P. Leemans, *Phys. Plasmas* **6**, 2269 (1999).

¹⁸P. Sprangle, E. Esarey, A. Ting, and G. Joyce, *AIP Conf. Proc.* **193**, 376 (1989).

¹⁹F. Dorchies, J. R. Marquès, B. Cros, G. Matthieussent, C. Courtois, T. Vélakorousov, P. Audebert, J. P. Geindre, S. Rebibo, G. Hamoniaux, and F. Amiranoff, *Phys. Rev. Lett.* **82**, 4655 (1999).

²⁰V. V. Ivanov, P. S. Antsiferov, K. N. Koshelev, M. R. Akdim, and F. Bijkerk, *Phys. Rev. Lett.* **97**, 205007 (2006).

²¹N. A. Bobrova, A. A. Esaulov, J. I. Sakai, P. V. Sasorov, D. J. Spence, A. Butler, S. M. Hooker, and S. V. Bulanov, *Phys. Rev. E* **65**, 016407 (2002).

²²M. J. de Loos, S. B. van der Geer, Y. M. Saveliev, V. M. Pavlov, A. J. W. Reitsma, S. M. Wiggins, J. Rodier, T. Garvey, and D. A. Jaroszynski, *Phys. Rev. ST Accel. Beams* **9**, 084201 (2006).

²³S. B. van der Geer, M. J. de Loos, T. van Oudheusden, W. P. E. M. op 't Root, M. J. van der Wiel, and O. J. Luiten, *Phys. Rev. ST Accel. Beams* **9**, 044203 (2006).

²⁴A. Khachatryan, M. Luttikhof, A. Irman, F. van Goor, J. Verschuur, H. Bastiaens, and K.-J. Boller, *Nucl. Instrum. Methods Phys. Res. A* **566**, 244 (2006).

²⁵F. B. Kiewiet, A. H. Kemper, O. J. Luiten, G. J. H. Brussaard, and M. J. van der Wiel, *Nucl. Instrum. Methods Phys. Res. A* **484**, 619 (2002).

²⁶R. Hubbard, D. Gordon, J. Cooley, B. Hafizi, T. Jones, D. Kaganovich, P. Sprangle, A. Ting, A. Zigler, and J. Dexter, *IEEE Trans. Plasma Sci.* **33**, 712 (2005).

²⁷W. Kimura, in *IEEE Conference Record—Abstracts. The 31st IEEE International Conference on Plasma Science ICOPS 2004* (IEEE, New York, 2004), p. 158.

²⁸J. R. Marquès, J. P. Geindre, F. Amiranoff, P. Audebert, J. C. Gauthier, A. Antonetti, and G. Grillon, *Phys. Rev. Lett.* **76**, 3566 (1996).

²⁹K. Nakajima, *Phys. Plasmas* **3**, 2169 (1996).

³⁰H. Dewa, H. Ahn, H. Harano, M. Kando, K. Kinoshita, S. Kondoh, H. Kotaki, K. Nakajima, H. Nakanishi, A. Ogata, H. Sakai, M. Uesaka, T. Ueda, T. Watanabe, and K. Yoshii, *Nucl. Instrum. Methods Phys. Res. A* **410**, 357 (1998).

³¹The 3-D model is not yet published. It is an extension of the 2-D code described in Ref. 35.

³²R. Hubbard, P. Sprangle, and B. Hafizi, *IEEE Trans. Plasma Sci.* **28**, 1159 (2000).

³³T. Esirkepov, S. Bulanov, M. Yamagiwa, and T. Tajima, *Phys. Rev. Lett.* **96**, 014803 (2006).

³⁴R. F. Hubbard, D. Kaganovich, B. Hafizi, C. I. Moore, P. Sprangle, A. Ting, and A. Zigler, *Phys. Rev. E* **63**, 036502 (2001).

³⁵A. J. W. Reitsma, V. V. Goloviznin, L. P. J. Kamp, and T. J. Schep, *Phys. Rev. E* **63**, 046502 (2001).