I. INTRODUCTION

Determination of the turbulent flame velocity in premixed combustion is one of the key problems in combustion science. Still, it is not so easy to define this value unambiguously, which leads to serious problems how to measure it in a particular burning geometry in experiments and numerical simulations, how to interpret the measurements, and how to compare to other results obtained in different configurations. As a discussion of these problems, we may refer to numerous recent reviews on the subject. In a relatively simple case of statistically stationary burning and turbulence, the concept of turbulent flame velocity denotes the mean propagation velocity $U_w$ of the flame front with respect to the fuel mixture. Turbulence is typically characterized by the root-mean-square (rms) velocity $U_{rms}$ of the flow fluctuations, which should be separated from the average flow. In the pioneering papers by Shelkin and Damköhler, the turbulent flame velocity was assumed in a functional form

$$U_w/U_f = f(U_{rms}/U_f) ,$$

where $U_f$ is the planar flame speed (the unstretched laminar burning rate). Numerous papers in turbulent combustion tried to obtain a formula like Eq. (1), see the reviews, Refs. 2–6. A simple functional form of Eq. (1) was especially popular within the artificial model of turbulent “burning” with zero thermal expansion. However, realistic flames involve considerable thermal expansion, with density ratio of the fuel mixture and the burnt matter about $\Theta = \rho_f/\rho_b \approx 5$–8. In that case, the propagating flame strongly modifies the initial turbulent flow, and the dependence like Eq. (1) should involve many other parameters such as the Reynolds number, the Markstein number, etc. If the turbulent flow is not uniform in space and not statistically stationary, then defining the turbulent flame velocity becomes even more difficult. Sometimes, researchers try to apply the concept of turbulent flame velocity like Eq. (1) locally in space and time. In that case, values $U_w$ and $U_{rms}$ imply local averaging on small length and time scales. This is one of the main ideas behind different numerical methods of large-scale combustion modeling such as the method of thickened flames in large eddy simulations. Interpretation of experimental results is usually different from this local approach. The turbulent flame velocity measured experimentally is not a local, but an integral value characterizing the whole flame front. Therefore, any empirical formula for a turbulent flame velocity, like those proposed in Refs. 21, 23, and 26, reflects not only properties of turbulent burning but also particular features of the experimental setup. Recent renormalization theory of turbulent flame velocity with realistic thermal expansion demonstrated the importance of the large-scale effects in the experimental measurements.

Thus, it is incorrect to look for a universal integral formula for a turbulent flame velocity like Eq. (1), which could be applied to any combustion experiment and simulation irrespective of the flow geometry. Still, we may ask a much less restrictive question: do we have at least some universal
qualitative properties of turbulent burning, which can be put in a form of a scaling law? By assuming a positive answer to this question, one should consider flame interaction with a vortex (or a vortex couple) as a basic step in understanding of turbulent burning. The problem of flame-vortex interaction has been studied quite intensively, see the reviews\textsuperscript{2–6} and the papers.\textsuperscript{19,20,31–33} These studies have been summarized in Refs. 19 and 20 in a formula suggested for renormalization of turbulent flame velocity similar to Eq. (1). In order to avoid a misunderstanding, we reproduce the formula in detail:

\[
U_w/U_f = 1 + 0.75\alpha \exp \left[ \frac{-1.2}{(U_{rms}/U_f)^{0.3}} \right] \left( \frac{1}{\alpha L_f} \right)^{2/3} U_{rms}^{2/3} U_f. \tag{2}
\]

Formula (2) was quite scattered in Ref. 19 all over several pages; here, we collect all coefficients together. The value \( \lambda \) in Eq. (2) is the scale length of the flame (presumably, the vortex size in the geometry of a single vortex), \( L_f \) is the flame thickness, \( \alpha \) and \( \alpha \) are correction coefficients. The flame thickness is defined with the help of the thermal diffusivity in the fuel mixture \( \chi \) as \( L_f = \chi / U_f \). Colin \textit{et al.}\textsuperscript{19} varied the ratio \( \lambda / L_f \) by making the flame front thicker; this dependence may be also interpreted as a dependence of the turbulent flame velocity versus the stretch rate. In the case of \( \text{Pr} =0.68 \), the correction factor \( \alpha \) was chosen by Colin \textit{et al.}\textsuperscript{19} as \( \alpha=4 \text{ Pr}=2.72 \). The other correction factor was specified in a more complicated way

\[
\alpha = \beta \frac{2 \ln 2}{3c_{mx}(\text{Re}^{1/2} - 1)}, \tag{3}
\]

where \( \beta \) and \( c_{mx} \) are some empirical constants and \( \text{Re} \) is the Reynolds number characterizing the turbulent flow (the vortex). Colin \textit{et al.}\textsuperscript{19} suggested the value \( c_{mx}=0.28 \). The parameter \( \beta \) was specified not so accurately; according to Ref. 19, \( \beta \) is of an order of unity, so that one has to take \( \beta=1 \) in the absence of a more detailed definition. We stress that Eq. (2) is an empirical formula, which is supposed to describe flame interaction with a single vortex at least for \( U_{rms}/U_f \) and \( \lambda / L_f \) studied in the simulations (though the parameter investigated in Ref. 19 and previous papers is rather wide). Selle \textit{et al.}\textsuperscript{20} employed Eq. (2) as a basis for large eddy simulations of combustion in a complicated flow of gas turbines; formula (2) was used to renormalize the flame parameters and to get rid of small-scale features of the flow. The usage of formula (2) for renormalization means that it reproduces correctly universal properties of turbulent burning (at least locally) independent of the large-scale flow. Particularly, it should work in the case of statistically stationary burning. In order to check that, recent direct numerical simulations\textsuperscript{34} investigated flame propagation in a vortex array in open tubes, which may be interpreted as statistically stationary. An attempt to describe the turbulent flame velocity obtained in Ref. 34 with the help of Eq. (2) was not encouraging; quantitative predictions of Eq. (2) did not come even close to the numerical results of Ref. 34. This finding demonstrated one more time that a universal formula for the turbulent flame velocity, probably, does not exist even for a local flow; and the validity domain of Eq. (2) is quite limited.

Still, in spite of quantitative disagreement, the numerical simulations\textsuperscript{34} reproduced the main qualitative tendency of Eq. (2) for sufficiently strong turbulence

\[
U_w/U_f - 1 \propto U_{rms}^{2/3}. \tag{4}
\]

We stress that tendency (4) has not been predicted by any theory of turbulent flame velocity so far (see Refs. 9–12 and 28–30 for comparison), and the validity domain of Eq. (4) is not clear at present. Colin \textit{et al.}\textsuperscript{19} tried to explain the tendency (4) by appealing to the Kolmogorov spectrum, but Kolmogorov spectrum has nothing to do either with a single vortex of Ref. 19 or with a single-mode vortex array of Ref. 34. Still, in spite of no theoretical explanation, the tendency Eq. (4) holds at least in two geometries of turbulent burning: in the case of flame interaction with a single vortex and for a turbulent flame propagating in open tubes for a rather wide parameter domain of \( U_{rms}/U_f \) and \( \lambda / L_f \). In that case, we may reformulate the question asked above: are the tendencies of Eq. (4) universal for turbulent burning? Do we obtain the same tendency for combustion in a closed chamber, like a combustion bomb\textsuperscript{21} or a spark-ignition engine? For example, if we have to achieve faster combustion in a spark-ignition engine for a fixed turbulent intensity, then, according to Eqs. (2) and (4), we should create vortices of largest possible size. Is that true? In the present work, we demonstrate that this is not true; the qualitative scaling (4) does not hold for the case of burning in a closed chamber. We also stress that both Eqs. (2) and (4) have been obtained for two-dimensional (2D) flows. It is well known that 2D turbulence may be quite different from realistic three-dimensional (3D) one. By this reason, it is questionable if one can use Eq. (2) as a basis for 3D simulations.\textsuperscript{20} Still, in the present work, we are interested in checking Eqs. (2) and (4), so that our work is also inevitably limited to 2D flows.

In the present paper, we perform direct numerical simulations of combustion in a closed burning chamber, which resembles geometrically clearance of a spark-ignition engine. In this study, we are mainly interested in the fundamental combustion properties. Though keeping engines in mind as an application, we do not try to imitate any particular engine. The chamber is initially filled with vortex arrays similar to Ref. 34. Our purpose is to compare basic features of combustion in a closed chamber and in unconfined situations previously studied. We consider a 2D flow in a closed box with nonslip at the walls. We investigate combustion for a wide range of vortex rms velocities, from initially quiescent gas to quite strong vortices, \( U_{rms}/U_f =0–20 \). We also take different vortex sizes: the largest possible size corresponds to the chamber height \( H \); the smallest size is \( H/32 \). We investigate the burning rate in the chamber and compare it to predictions of Eq. (4). In agreement with Eq. (4), in the case of sufficiently strong turbulence, the burning rate depends almost linearly on the initial turbulent rms velocity of the vortex array. However, the dependence on the vortex size is much more complicated than predictions of Eq. (4). As we decrease the vortex size from \( H \) to \( H/6 \), we obtain the results opposite to the tendency of Eq. (4): the burning rate increases noticeably with decreasing vortex size. Still, the dependence is nonmonotonic. Decreasing the vortex size fur-
ther from $H/6$ to $H/32$, we find decrease in the burning rate. As a result, there is an optimal vortex size, which provides the maximal burning rate and the minimal time of burning. This feature is qualitatively different from properties of turbulent combustion in unconfined situations, Eq. (4). In the present calculations, the optimal vortex size is about six times smaller than the height of the burning chamber.

The present paper is organized as follows: In Sec. II, we describe the details of the simulations; the initial conditions for the flame and the flow are discussed in Sec. III; in Sec. IV, we present and discuss the results obtained; the paper is concluded with a short summary.

II. BASIC EQUATIONS AND THE DETAILS OF THE DIRECT NUMERICAL SIMULATIONS

We perform direct numerical simulations of the 2D hydrodynamic and combustion equations including transport processes (thermal conduction, diffusion, viscosity) and chemical kinetics with an Arrhenius reaction. The equations read

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}(\rho u_i) = 0, \quad (5)$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j + \delta_{ij} P) - \xi_{ij} = 0, \quad (6)$$

$$\frac{\partial}{\partial t}\left(\rho e + \frac{1}{2} \rho u_i u_i\right) + \frac{\partial}{\partial x_i}\left(\rho u_i h + \frac{1}{2} \rho u_i u_i + q_i - u_j \xi_{ij}\right) = 0, \quad (7)$$

$$\frac{\partial}{\partial t}(\rho Y) + \frac{\partial}{\partial x_i}(\rho u_i Y - \frac{\mu}{Sc} \frac{\partial Y}{\partial x_i}) = -\frac{\rho Y}{\tau_r} \exp(-E_a R_p T), \quad (8)$$

where $Y$ is the mass fraction of the fuel, $e=QY+C_vT$ is the internal energy, $h=QY+C_pT$ is the enthalpy, $Q$ is the energy release in the reaction, $C_v$ and $C_p$ are the heat capacities at constant volume and pressure, respectively. It is assumed that the heat capacities do not depend on the chemical composition. The stress tensor $\xi_{ij}$ and the energy diffusion vector $q_i$ take the form

$$\xi_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij}\right), \quad (9)$$

$$q_i = -\mu \left(\frac{C_p}{Pr} \frac{\partial T}{\partial x_i} + \frac{Q}{Sc} \frac{\partial Y}{\partial x_i}\right), \quad (10)$$

where $\mu$ is the dynamic viscosity, $Pr$ and $Sc$ are the Prandtl and Schmidt numbers, respectively. To avoid the Zeldovich (thermal-diffusion) instability, we took a unit Lewis number $Le=Pr/Sc=1$, with $Pr=Sc=0.7$. Unlike in our previous studies, in the present work, we have taken into account temperature dependence of the transport coefficients. The dynamical viscosity has been specified as

$$\mu = \mu_f(T/T_f)^{\frac{1}{2}}, \quad (11)$$

to be a perfect gas with a constant molar mass $m = 2.9 \times 10^{-2}$ kg/mol, with $C_v=5R_p/2m$, $C_p=7R_p/2m$, and the equation of state

$$P = \rho R_f T/m, \quad (12)$$

where $R_p=8.31$ J/mol K is the universal gas constant. The flame thickness is conventionally defined as

$$L_f = \frac{\mu_f}{Pr\rho_f U_f}, \quad (13)$$

where $\rho_f=1.16$ kg/m$^3$ is the initial mixture density. However, we would like to stress that the value (13) is just a thermal-chemical parameter of length dimension in the problem; the characteristic size of the burning zone may be an order of magnitude larger. Equation (8) describes a single irreversible reaction of first order, where the temperature dependence of the reaction rate obeys an Arrhenius law with an activation energy $E_a$ and a reaction time $\tau_r$. In the case of a second-order reaction, Eq. (8) should be replaced by

$$\frac{\partial}{\partial t}(\rho Y) + \frac{\partial}{\partial x_i}(\rho u_i Y - \frac{\mu}{Sc} \frac{\partial Y}{\partial x_i}) = -\frac{\rho^2 Y}{\rho_h \tau_r} \exp(-E_a R_p T). \quad (14)$$

We performed a test simulation run demonstrating that the solutions obtained by using first and second-order reaction are quite close, see below.

Typical experimental geometries of closed combustion chambers are that of a spherical combustion bomb and a box resembling qualitatively a spark-ignition engine, e.g., see. Refs. 21 and 41. We consider the later geometry; following the resemblance, we take aspect ratio (height:width, $H:D$) equal 1:8, with central flame ignition at one of the walls. In order to choose reasonable chamber size and flame velocity, one has to take care of the numerical resolution. Good resolution means a sufficient number of numerical grid points inside the flame front, with the characteristic small length scale specified by $L_f$ (the smallest length scale of the flow structure is typically much larger than the flame thickness). The length scale $L_f$ is inversely proportional to the planar flame speed $U_f$, see Eq. (13). The larger $U_f$, the smaller $L_f$, and the finer mesh size should be used to resolve the internal flame structure. On the other hand, the computational time is inversely proportional to the Mach number Ma= $U_f/c_s$, where $c_s=347$ m/s is the sound speed. Taking $U_f = 34.7$ cm/s typical for hydrocarbon flames with $Ma=10^{-5}$ we find $L_f=8.42 \times 10^{-3}$ cm from Eq. (13). By choosing chamber parameters $H=1.05$ cm, $D=8.2$ cm similar to spark-ignition engines, we obtain $H=125 L_f$, $D=8H=10^3 L_f$, which is attainable in direct numerical simulations. With the numerical cell size $L_f/3$, this gives a reasonable total number of numerical points; these parameters were used for a test simulation run. Still, the choice $Ma=10^{-5}$ required too much computational time. In order to reduce the computational time, we increased the Mach number by an order of magnitude larger.
to an order of magnitude smaller flame thickness, \( L_f = 8.42 \times 10^{-3} \) cm. In order to reproduce the main features of hydrocarbon burning correctly, we have to keep the Reynolds number of the flow \( \text{Re} = \rho U_f / \mu \) unchanged. For that purpose, increasing the flame velocity ten times, we have to decrease the height and width of the burning chamber ten times, and we come to the same ratio \( H / L_f = 125, \, D / L_f = 8H / L_f = 10^3 \) as before. We would like to stress that such a model increase in the Mach number is typical for direct numerical simulations of compressible Navier–Stokes equations.\(^{34–37}\) Again, we are interested in the scaled value \( U_{\text{rms}} / U_f \) instead of the dimensional \( U_f \). Reducing the chamber size, we also reduce the decay time of vortices. However, increasing flame velocity decreases also the burning time. By keeping the Reynolds number fixed, we obtain the same ratio of burning time versus the decay time. In addition, a test simulation run performed for the chamber height and width \( H = 1.05 \text{ cm}, \, D = 8.2 \text{ cm} \) and the Mach number \( \text{Ma} = 10^{-3} \) demonstrates that the rescaling works properly. Flame behavior for \( \text{Ma} = 10^{-3} \) reproduces the main feature of burning observed for \( \text{Ma} = 10^{-2} \) quite well.

In the simulations, the planar flame speed \( U_f \) is determined by the choice of the fuel temperature \( T_f \), pressure \( P \), and the chemical parameters of burning: \( Q, \, E_a, \, R_p \). The energy release in the reaction \( Q \) specifies the thermal expansion in the burning process \( \Theta = T_f / T_0 = 1 + Q / C_p T_f \). Similar to Ref. \(^{34}\), we took initially \( P_0 = 10^5 \text{ Pa}, \, T_0 = 300 \text{ K}, \) and the initial expansion factor \( \Theta_0 = 8 \) (i.e., \( T_0 = 2400 \text{ K} \)). Due to adiabatic compression in burning, these values vary with time. At the end of burning, the burnt and unburned gas temperatures reached \( T_{b1} \approx 4000 \text{ K} \) and \( T_{f1} \approx 640 \text{ K} \), respectively, with \( \Theta_1 = 0.25 \). As a result, the final pressure was \( P_f = P_0 T_{b1} / T_{f1} = 1.33 \times 10^6 \text{ Pa} \). The increase in pressure and the fuel temperature may influence the planar flame speed. The dynamics of \( U_f \) may be estimated as \( U_f (P, T_f) \approx P^{-\kappa} T_f^{-\theta} \), where the factors \( \kappa \) and \( \theta \) depend upon the equivalence ratio and other flame parameters.\(^{42–45}\) According to the measurements,\(^{45} \kappa = 0.3, \, \theta = 1 \) for stoichiometric hydrocarbon flames. In that case, the pressure and temperature variations influence the planar flame speed in the present simulations very slightly. Indeed, by the end of burning \( U_{f1} / U_{f0} = (P_f / P_0)^{0.3} (T_{f1} / T_{f0})^{0.98} \). To justify this result, we also performed few test simulation runs with different \( P \) and \( T_f \).

In general, the test simulations were in line with the power law \( U_f \sim P^{0.3} T_f^{0.7} \). In particular, we have found \( U_f = 0.96 U_{f0} \) for \( P = 1.33 \times 10^6 \text{ Pa}, \, T_f = 640 \text{ K} \). Thus, the planar flame speed is almost independent of the pressure and temperature variations in the present work. Hereafter, \( U_f = U_{f0} \) denotes an initial value related to \( P_0, \, T_{f0} \).

In most of the simulations, the activation energy was taken as large as \( E_a = 7R_f T_{f0} \), which allows smoothing the reaction zone over few computational cells. As long as the Lewis number is equal to unity, numerical results typically do not depend on a particular choice of the activation energy \( E_a \). This tendency may be violated at extremely large turbulent intensity, when turbulent flow distorts the reaction zone beyond the limits of the flamelet regime of burning. Kagan et al.\(^{17}\) investigated the artificial model of zero thermal expansion \( \Theta = 1 \) and found how the turbulent flame velocity depends on \( E_a \) for the flow rms velocities up to \( U_{rms} / U_f = 120 \) and activation energies within the domain \( E_a / R_f T_{f0} = 0.5 – 20 \). In particular, the simulations demonstrated almost no dependence on the activation energy for the vortex intensities below \( U_{rms} / U_f = 20 \). To avoid dependence on the activation energy, in the present studies, we keep turbulent intensity within the same limits, \( U_{rms} / U_f = 20 \). In addition, we performed a test simulation run with \( E_a / R_f T_{f0} = 4 \) and \( U_{rms} / U_f = 20 \). The difference between the solutions obtained using \( E_a / R_f T_{f0} = 4 \) and \( E_a / R_f T_{f0} = 7 \) is negligible, see below. To save more computational time, we reduced the simulation domain twice taking the axis \( z = 0 \) as a mirror. Thus, we considered a box \( H \times (D / 2) = 125 L_f \times 500 L_f \) with one slip/symmetry wall and with nonslip adiabatic boundary conditions at other three walls.

\[
\mathbf{u} = 0, \quad \mathbf{n} \cdot \nabla T = 0,
\]

where \( \mathbf{n} \) is the normal vector at the wall. We used a uniform square grid with the grid walls parallel to the coordinate axes. Unlike many of our previous simulations,\(^{35,38,39}\) the grid was uniform all over the chamber since we have to resolve not only the flame front but also the flow structures. Of course, such a grid requires much more computational facilities than nonuniform rectangular grids used before. We chose the mesh size \( \Delta = L_f / 3 \) in both directions. Such a grid is sufficiently fine to resolve the inner flame structure (indeed, we obtain about three to five grid points inside the active reaction zone and 15–20 grid points inside the flame; note that \( L_f \) is much smaller than the effective flame thickness). To be sure, we have performed an additional simulation run for a grid with \( \Delta = L_f / 6 \). The results obtained for both grids are almost identical.

Similar to Ref. \(^{34}\), we used a 2D Cartesian Navier–Stokes solver developed at Volvo Aero. Basic elements of the code are presented in Refs.\(^{46,47}\) Since that time, the code was continuously modified, extended and updated, see Refs.\(^{48–50}\). Particularly, we work with the version of the code adapted for parallel computations. The numerical scheme of the code is of second-order accuracy in time and fourth order in space for convective terms, and second order in space for diffusive terms. The code is both robust and accurate, and it was utilized quite successfully in studies of laminar and turbulent burning, hydrodynamic flame instabilities, flame acceleration, flame-sound interaction, and similar phenomena.\(^{34–40,51}\) The code imitates quite well even aeroacoustic applications, which are extremely critical in terms of accuracy, since both the turbulent flow and the resulting acoustic waves have to be captured. For example, the jet noise predictions\(^{48,49}\) demonstrated very good results comparable to the best in the field. The code is available in 2D (Cartesian and cylindrical axisymmetric) and 3D Cartesian versions. In the present paper, we performed only 2D simulations to save the computational time and to be able to perform a large number of simulation runs required for thorough investigation of the problem. 3D simulations are very time consuming; at present, studies devoted to 3D simulations are usually limited to one or few simulation runs.
III. INITIAL CONDITIONS FOR THE FLAME AND THE FLOW

We studied flame propagation from the ignition point near the wall. Similar to Ref. 37, the ignition point was approximated by a hemispherical flame front of radius \( r_0 \approx H \ll R=D/2 \) (see Fig. 1). The smaller \( r_0 \) we use, the better we reproduce “point” ignition of a flame. Still, taking a too small radius of the hemisphere, we would not be able resolving the initial structure of the burning zone. In the present simulations, we chose \( r_0=10L_f=0.08H=0.02R \). The initial temperature distribution was chosen in the form

\[
T/T_f \sim \begin{cases} 1+(\Theta-1)\exp\left(-\left(r_1 - r_0\right)/L_f\right), & \text{if } r_1 \geq r_0, \\ \Theta, & \text{if } r_1 < r_0, \end{cases} \tag{16}
\]

\[
Y = \Theta - T/T_f \tag{17}
\]

where \( r_1 = \sqrt{x^2 + z^2} \). Equations (16) and (17) imitate the hemispherical counterpart of the Zeldovich–Frank–Kamenetskii solution for a planar flame. 44 A much more difficult question is how to imitate a turbulent flow, since turbulence in itself is one of the most unresolved and complicated problems in classical physics. Simulations performed in the approach of constant density 13–18 typically imitate the velocity field by a Fourier decomposition like

\[
u_z = \sum_{i=1}^{N} U_i \cos(k_i z + \varphi_{z,i}) \cos(k_i x + \varphi_{x,i}), \tag{18}\]

where \( k_i \) are the wave numbers, \( U_i \) are the mode amplitudes and \( \varphi_{z,i}, \varphi_{x,i} \) are the random phases. The rms velocity of the flow (18) and (19) is calculated as

\[
U_{rms}^2 = \langle u_z^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} U_i^2, \tag{19}\]

In the present work, we used only arrays of coherent vortices

\[
u_z = -U_i \sin(k_i z) \cos(k_i x), \tag{20}\]

\[
u_z = U_i \cos(k_i z) \sin(k_i x), \tag{21}\]

with

\[
U_{rms} = U_i/2, \tag{22}\]

and the wave numbers \( k_i = i\pi/H \) controlled by the tube width. The vortices fill initially the whole chamber, as shown in Fig. 1, for \( i=1,2,4 \).

In the model of constant gas density, the turbulent flow is an external vector function in the equations for energy and species, which may be easily specified. In the case of realistic thermal expansion, the situation is not so simple. This problem has been discussed in detail in Refs. 29 and 34. To first order in \( L_f/H \), the intensity of a coherent structure (21) and (22) decays in time as

\[
U_{rms} = \frac{U_{rms,0}}{\exp\left(-2k_i^2\mu/\rho\right)} \tag{23}\]

In our previous paper on turbulent burning, 34 we tried to imitate statistically stationary combustion. In that case, to avoid fast decay of turbulent intensity, we used an artificially small Pr. Still, turbulence in combustion experiments decays because of viscous friction, which allows using a realistic value of the Prandtl number in the present work, Pr=0.7. To check the formula Eq. (24), we simulated the decay of vortices of size \( H \times H \) with initially large rms velocity \( U_{rms,0}/U_f=10 \), shown in Fig. 2 by the solid lines. The dashed lines in Fig. 2 present the theoretical predictions (24) for \( i =1,2,4,8,16 \). Large vortices demonstrate quite moderate
decrease in the turbulent intensity (say, 5%–20%) during the characteristic time of burning \( \tau_b = (0.1–0.5)H/U_f \). These vortices can live long enough to provide flame-vortex interaction. On the contrary, small vortices with large wave numbers \( i \gg 8 \) decay in a very short time. Figure 2 demonstrates good agreement of the theoretical predictions and the simulations. As long as there is no burning, an initially isotropic vortex array remains isotropic all the time. Flame propagation typically violates isotropy. In that case, evolution of the rms velocities in \( x \) and \( z \) directions, \( U_{rms,x} \) and \( U_{rms,z} \), and the total rms velocity \( U_{rms} \) were calculated as

\[
U_{rms,x}^2 = \langle u_x^2 \rangle - \langle u_z \rangle^2, \quad U_{rms,z}^2 = \langle u_z^2 \rangle - \langle u_x \rangle^2, \quad U_{rms}^2 = \frac{1}{2}(U_{rms,x}^2 + U_{rms,z}^2),
\]

where the averaging is taken over the whole simulation domain.

IV. RESULTS AND DISCUSSION

In the present work, we simulated flame propagation in a closed 2D chamber of height \( H = 125L_f \) and length \( R = 500L_f \). We investigated flame interaction with a flow specified initially by Eqs. (21) and (22). The main parameters of the flow are the scaled flow rms velocity \( U_{rms}/U_f \) and the vortex size of the flow determined by the scaled wave number \( i \). To investigate the burning rate, we determined the mass of the burnt matter \( M_b \) as a fraction of the total mass \( M \) of the gas in the chamber. First, we kept the wave number of the coherent structure fixed \( (i = 1, k = \pi/H) \) and performed the simulations for a wide range of the flow velocities \( 0 \leq U_{rms,0}/U_f \leq 20 \). Then, the simulations for different harmonics \( 1 \leq i \leq 32 \) were performed at fixed \( U_{rms,0}/U_f = 10 \).

A. Flame propagation in an initially quiescent gas

We started with the simulations of laminar flame propagation in an initially quiescent gas. Figure 3 presents evolution of the flame shape and position in that case. The snapshots in Figs. 3(a)–3(d) are related to the time instants \( U_f/H = 0.098; 0.184; 0.361; 0.583 \), which correspond to the mass fractions of the burnt matter \( M_b/M = 0.05; 0.2; 0.5; 0.95 \), respectively. The colors show the temperature distribution from \( T = 300 \) K (blue) in the fuel mixture at the ignition till \( T = 4000 \) K (red) in the burnt gas at the end of burning. In black-and-white version the green color is shown by light, the red is half-dark, and the blue is dark. We stress one more time that such high temperature is achieved in the mixture because of adiabatic compression. The flame front corresponds to light blue separating the fuel mixture (dark blue) from the burnt gas (green-yellow-red). It is interesting that laminar burning in a closed chamber goes much faster than one should expect. Instead of a natural evaluation of the burning time as \( D/2U_f = 4H/U_f \), combustion is completed in an order of magnitude faster, in a time interval about \( 0.6H/U_f \). We suggest that characteristic laminar burning rate in the process should be evaluated as \( \Theta U_f \) rather than \( U_f \) due to the strongly curved flame shape. Still, one should be careful when interpreting the physical reason for such an evaluation. For example, it is well known that a planar flame propagates from a closed tube end to an open one with the speed \( \Theta U_f \).

In that case, the increased velocity with respect to the walls, \( \Theta U_f \), does not mean any increase in the mass burning rate, which remains equal \( U_f \) because of the planar flame shape. In the present case, the evaluation \( \Theta U_f \) has another physical meaning. It characterizes deformation of the flame shape like that observed experimentally and explained theoretically in Refs. 37 and 52, see below for more details. Thus, in the present case, the evaluation \( \Theta U_f \) does mean increase in the mass burning rate.

Another curious point of Fig. 3 is that the snapshot (c) corresponds to only \( \frac{1}{2} \) of the fuel mixture burnt (measured by mass). Visually, this snapshot looks like almost the end of burning; the deceitful impression is caused by adiabatic pre-compression of the fuel mixture. We can identify two distinctive intervals of flame propagation. In the first half in the burning process (measured by mass), the flow is dominated by thermal expansion of the burning matter. This stage of burning resembles flame acceleration from the closed tube end observed and described within the problem of “tulip flames.” An analytical theory of this process has been developed recently in Ref. 37 and validated by numerical simulations. At the beginning, a quasispherical flame front expands from the ignition point at the corner pushing a flow in the fuel mixture. As one of the flame skirts approaches the opposite wall, the wall stops the flow in the \( z \) direction, which makes it stronger in the \( x \) direction and modifies the flame front to a “finger-shape.” In a semi-infinite tube, this effect leads to exponential acceleration of the flame front. The burning rate increases because of the curved flame shape; the maximal burning rate is about \( 2\Theta U_f \) for a cylindrical geometry and \( \Theta U_f \) for a 2D geometry. This theoretical result explains the evaluation \( \Theta U_f \) for the characteristic burning rate in a closed chamber suggested above. In the present case of a closed burning chamber, the effect of finger-shape is weaker; still it may be easily recognized in Figs. 3(a) and 3(b). The second half in the burning process (measured by mass) involves well-developed instabilities of the flame front and in the burnt matter. Presumably, these are related to the Darrieus–Landau (DL) instability and/or the parametric instability in an acoustic wave. The DL instability develops if the perturbation wavelength exceeds the cutoff wavelength \( \lambda_c \). By using parameters of the present simulations, we obtained the characteristic DL cutoff \( \lambda_c \approx 35.6L_f \), which is noticeably smaller than the chamber height and allows a relatively strong DL instability. Besides, we have to remember that the flame velocity increases in the burning process in a closed chamber because of adiabatic pre-compression of the fuel mixture. Velocity increase leads to decrease in the flame thickness, Eq. (16), and in the DL cutoff, which implies a stronger DL instability. Recent numerical studies of the DL instability have been reviewed in Refs. 6 and 51. Figures 3(c) and 3(d) demonstrate multiple cusps at the flame front expected for the DL instability at sufficiently large length scales. The parametric instability happens because of flame interaction with acoustic waves generated by the flame front, see Refs. 53–55. Recent numerical results on flame interaction with acoustic waves may be found in Refs.
36 and 40, which include violent flame folding because of the acoustic resonance. There is one more hydrodynamic instability, which has not been discussed yet within the problem of combustion in a closed chamber. Looking at temperature distribution in the burnt matter in Figs. 3(a)–(d), we can clearly observe a “tongue” of heavier (colder) gas penetrating the lighter (warmer) one, and a similar “bubble” of lighter gas floating into the colder one. This flow pattern is related to the Richtmyer–Meshkov/Rayleigh–Taylor instabilities developing in a gas with density gradients under the action of acoustic waves or weak shocks.56–59 The relation of this process to acoustic waves is especially obvious in a movie of the combustion process, which shows noticeable growth of the cold tongue every time as the compression wave sweeps through the chamber from right to left.

Thermal expansion of burning matter generates a flow in the burning chamber. Figure 4 demonstrates vorticity snapshots in the flow for the same time instants as in Fig. 3. Positive and negative vorticities are shown by colors: from $-0.3U_f/H$ (blue) to $0.3U_f/H$ (red); the green color corresponds to zero vorticity. We can see that the flow remains mostly irrotational in Figs. 4(a) and 4(b) except for the boundary layers at the chamber walls. This result agrees quite well with the theory,37 which describes the flow of burnt gas in the process of flame acceleration as mostly potential one with properties resembling a stagnation flow.
Mark that respective two snapshots Figs. 3(a) and 3(b) do not demonstrate any instability at the flame front. At this stage of burning, vorticity is produced at the boundary layers because of nonslip at the walls. Figures 4(c) and 4(d) show vorticity generated at the curved flame front due to the DL instability and other instabilities. It is interesting to observe vorticity of different sign produced at the adjacent sides of every cell at the flame front. The vortices produced at the flame front are drifted by the flow and eventually dissipate because of viscosity. To understand the intensity of the flow, we investigate the rms velocities. First, we average over the whole simulation domain according to Eq. (25). In that case, however, we forget about the difference between the flow in the fresh and burnt gases, which may be considerable. To take into account the difference, we have also calculated the average rms velocities only in the fresh fuel mixture as

\[ U_{\text{rms},j,f}^2 = \frac{\langle u_j^2 Y \rangle}{\langle Y \rangle} - \left( \frac{\langle u_j Y \rangle}{\langle Y \rangle} \right)^2, \quad j = x \text{ or } z, \]

and only in the burnt gas as

\[ U_{\text{rms},j,b}^2 = \frac{1}{2} \left( U_{\text{rms},x,b}^2 + U_{\text{rms},z,b}^2 \right). \]
Finally, we can specify the “weighted” rms velocities as

\[
U_{\text{rms,}b} = \frac{1}{2}(U_{\text{rms,x,b}} + U_{\text{rms,z,b}}),
\]

and investigated the flame dynamics for a wide range of the intensities

\[
U_{\text{rms,0}}/U_f = 1; 2; 5; 10; 20.
\]

In the case of moderate initial rms velocity \( U_{\text{rms,0}}/U_f = 1, 2 \), we observed little difference from the laminar case; the flame dynamics resembles the respective snapshots of Fig. 3, with the flame shape a little more corrugated at snapshot \( c \) corresponding to \( M_f/M = 0.5 \). The flame shape becomes noticeably different from that of Fig. 3.
only at considerable vortex intensity $U_{rms,0}/U_f=5$. The snapshots related to $M_b/M=0.05; 0.2; 0.5; 0.95$ are shown in Figs. 6–9 for $U_{rms,0}/U_f=5$. Figures 6 and 8 present the temperature distribution, while Figs. 7 and 9 demonstrate vorticity. In the case of $U_{rms,0}/U_f=5$ (Figs. 6 and 7), combustion is completed approximately twice faster, until $0.3H/U_f$, in comparison with the case without flow (Fig. 3). The flame shape in Fig. 6 differs considerably from that of Fig. 3 too. The front becomes much more corrugated, now we can see deep caves at the flame front and pockets of fuel mixture trapped in the burnt gas both close to the outer wall and to the symmetry axis of the chamber. Still, much resemblance remains between snapshots (a) and (b) of Figs. 3 and 6, which indicates an important role of thermal expansion on burning even at relatively large vortex intensity. In addition, snapshots of Fig. 7 demonstrate noticeable vorticity in the case of $U_{rms,0}/U_f=5$ in comparison with Fig. 4. In Figs. 7(a) and 7(b), vorticity is mainly located in the fresh gas, while the flow behind the flame is mostly irrotational similar to Figs. 4(a) and 4(b). It looks as in that case the original vortices have been compressed together with the fuel mixture, while the relatively smooth flame front has not produced any additional vorticity. The picture is especially obvious in Fig. 7(a), where we can see three out of four original vortices slightly compressed in the $x$ direction, while the first vortex is pushed to the upper wall and strongly squeezed. We observe the other three vortices squeezed in a similar way in Fig. 7(b): they remain as large as they were originally in the
z direction with strong compression in the x direction. We remind that flame has burnt only 0.05 and 0.2 of the fuel mixture measured by mass in Figs. 7a and 7b, though burnt matter occupies a good deal of the chamber volume (more than half in figure (b)). The situation changes in Figs. 7c and 7d: the flame front is strongly corrugated in that case already, and it produces vorticity, which fills the whole chamber until the end of burning. The characteristic vortex size at the end of burning is much smaller than the original vortices. This is quite different from the model studies of turbulent “flames” with zero thermal expansion\(^{13-18}\) where flame passes the vortices without changing them. Finally, the effect of pockets becomes even stronger at \(U_{\text{rms},0}/U_f=20\), see Figs. 8 and 9. In that case, combustion is completed in a very short time, about 0.12\(H/U_f\). The flow distortion in Fig. 9 is very strong, starting with “quasicoherent” structures in Fig. 9a, and ending with “scrambled” vorticity of different scales in Figs. 9c and 9d. Still, the absolute value of vorticity remains approximately the same during the whole combustion process. Mark that there is no regions of zero vorticity even at the initial stage of burning in Figs. 9a and 9b, which makes the case different from Figs. 3a, 3b, 7a, and 7b.

The same tendencies are demonstrated quantitatively in Figs. 10 and 11. Figure 10 shows the mass fraction of the burnt matter \(M_b/M\) versus time for different initial flow intensities \(U_{\text{rms},0}/U_f=0; 1; 2; 5; 10; 20\) (solid lines). The markers in Fig. 10 are related to the flame snapshots/time instants.
presented in Figs. 3, 6, and 8 and (and Figs. 4, 7, and 9) for $U_{\text{rms},0}/U_f=0, 5, 20$, respectively. Other lines in Fig. 10 show the results of the test simulation runs. One test run was performed for $U_{\text{rms},0}/U_f=20$ and $U_f=0.347$ m/s, which corresponds to realistically small Mach number $Ma=10^{-3}$ (the dashed line). The difference between the plots with $Ma=10^{-2}$ and $Ma=10^{-3}$ is extremely small, which justifies our rescaling of the planar flame speed, the flame thickness and the chamber size. The dotted line in Fig. 10 is related to the test simulation run for $U_{\text{rms}}/U_f=10$ with second-order reaction, Eq. (14), used instead of the first order one, Eq. (8). We can see that the numerical results for these two cases are very close too. Finally, we performed a test simulation run with another activation energy $E_a/R_sT_{b0}=4$ at $U_{\text{rms}}/U_f=20$. The result is shown in Fig. 10 by the dot-dashed line. A hardly seen difference between the curves describing the case of $U_{\text{rms}}/U_f=20$ demonstrates that simulation results are practically independent of the choice of the activation energy. Figure 11 shows the inverse total burning time (measured for $M_b/M=0.99$) versus the initial vortex intensity. The inverse burning time characterizes average burning rate in the process. In Fig. 11, we can see tendencies, both similar to and different from turbulent combustion in the open channel, Eq. (4). The average burning rate increases almost linearly with turbulent intensity for sufficiently large $U_{\text{rms}}/U_f$; this is similar to turbulent combustion in the open channel. However, quantitative comparison of two combustion geometries demonstrate a difference. For comparison,
we take the geometry of a flame propagating in a vortex array,\textsuperscript{34} which looks closer to the present case than flame interaction with a single vortex. Taking into account empirical coefficients of Ref.\textsuperscript{34}, the tendency $U_w/U_f$ should be rewritten as

$$U_w/U_f \approx 1 + 0.5 \frac{U_{\text{rms}}}{U_f} \left( \frac{2H}{\lambda_c} \right)^{2/3}. \tag{29}$$

As explained in Ref.\textsuperscript{34}, the DL cutoff $\lambda_c$ plays the role of effective flame thickness, which is often much more useful than $L_f$. Taking into account parameters of the present simulations $H=125L_f$ and the DL cutoff $\lambda_c \approx 35.6L_f$, the empirical estimate Eq. (29) suggests burning rate strongly increased in comparison to the laminar case already for $U_{\text{rms}}/U_f = 1$, approximately by a factor of three, $U_w \approx 2.8U_f$. On the contrary, in the present geometry of a closed burning chamber initial vortex intensity $U_{\text{rms}}/U_f = 1$ increases the burning rate (decreases the burning time) only slightly, from $H/t_b U_f = 1.68$ in the laminar case of $U_{\text{rms}}/U_f = 0$ to $H/t_b U_f = 1.85$ for $U_{\text{rms}}/U_f = 1$. Roughly speaking, turbulence becomes equally important as intrinsic laminar flame properties when burning time is about half the laminar value. In the present case, this happens at a noticeable vortex intensity, about $U_{\text{rms}}/U_f = 5$. By an order of magnitude, this critical vortex intensity may be evaluated as $U_{\text{rms}} = (\Theta - 1)U_f$, which agrees with the discussion presented in Sec. IV A.

It is also interesting to study the flow in the case of strong initial vortices. Figure 12 presents evolution of the
rims-flow velocities calculated over the whole chamber according to Eq. (25) in the case of $U_{rms,0}/U_f=20$ (solid lines). Flow intensity varies a little during the simulation run and remains more or less the same in both directions; the curves for $U_{rms,1}$ and $U_{rms,2}$ are quite close. The dashed line in Fig. 12 present $U_{rms}$ for the test simulation run with $U_f=34.7$ cm/s. Again, the difference between the plots for $Ma=10^{-2}$ and $Ma=10^{-3}$ is minor. It concerns mostly the period and amplitude of small acoustic pulsations imposed on the average rms flow. The acoustic time for $Ma=10^{-3}$ is ten times smaller than that for $Ma=10^{-2}$, and the dashed curve in Fig. 12 consists of numerous short, but weak pulsations. Plots for rms velocity in the fuel mixture and in the burnt gas show the same tendency, and therefore, we do not present the plots here. In Fig. 13, we compare the rms velocities $U_{rms}$ averaged over the whole simulation domain, Eq. (25), for different initial flow intensities $U_{rms,0}/U_f=0\ldots20$. The dashed lines in the plot show the expected decay of the vortices in case of no burning (see also Fig. 2). According to Fig. 13, there is almost no difference between the plots for $U_{rms,0}/U_f=0; 1; 2$, except for a short transition time at the beginning. These plots demonstrate a noticeable flame generated flow, with the characteristic rms velocity up to $U_{rms}/U_f=3\ldots4$, which dominates over the initial vortices. Only initial vortices of really high intensity $U_{rms,0}/U_f\geq 5$ become of principal importance for burning.

C. How does the burning rate depend on the vortex size?

We have also investigated the influence of vortex size on the burning rate. The purpose was to check if the tendency (4) holds for the present geometry. Immediately, we have to say some words of caution. First, we have demonstrated in the previous subsection that original vortices are strongly deformed in the burning process. By this reason, for a fixed time instant, vortex size becomes an anisotropic value, which makes quantitative comparison of the present case to Eq. (4) practically impossible. As we can see in Figs. 7(b) and 7(c), the original vortices in the fuel mixture typically retain their initial size in the $z$ direction, but they may be strongly compressed in the $x$ direction. Below, we demonstrate the same effect in the case of small initial vortices. Taking vortex deformation into account, we can check the tendency (4) mostly qualitatively with the initial vortex size (not the cur-

FIG. 10. The mass fraction of the burnt matter $M_b/M$ vs time for different initial flow intensities $U_{rms,0}/U_f=0; 1; 2; 5; 10; 20$ (solid lines). The markers are related to flame snapshots/time instants presented in Figs. 3, 6, and 8 (and Figs. 4, 7, and 9) for $U_{rms,0}/U_f=0, 5, 20$, respectively. The other lines in the plot show the results of the test simulation runs: for $U_{rms,0}/U_f=20$ with $U_f=0.347$ m/s (dashed); for $U_{rms,0}/U_f=10$ with second-order reaction, Eq. (14), used instead of the first order one, Eq. (8) (dotted); and for $U_{rms,0}/U_f=20$ with $E_p/R_T=4$ (dot-dashed).

FIG. 11. The scaled inverse burning time $H/t_bU_f$ vs the initial flow intensity $U_{rms,0}/U_f$.

FIG. 12. Evolution of the scaled flow rms velocities $U_{rms}/U_f$ averaged over the whole chamber according to Eq. (25) in the case of $U_{rms,0}/U_f=20$ (solid lines). The dashed line presents $U_{rms}/U_f$ for the test simulation run with $U_f=34.7$ cm/s.

FIG. 13. Evolution of the scaled flow rms velocity $U_{rms}/U_f$ averaged over the whole simulation domain, Eq. (25), for different initial flow intensities $U_{rms,0}/U_f=0, 2, 5, 10, 20$. The dashed lines show expected decay of the vortices in the case of no burning.
rent one!) as the main parameter for comparison. According to Eq. (4), one should expect a noticeably larger burning time (smaller burning rate) in the case of small initial vortices. For example, taking initial vortices of the size $H/6$, we should expect total burning time increased by a factor of about 3.3. Such a considerable difference cannot be missed even if the factor is not exactly the same as predicted by Eq. (4). The qualitative tendency may be easily checked, we do it below and demonstrate that it does not work in the present burning geometry.

The second word of caution concerns the scaling law Eq. (4) itself. Before validating or refuting the scaling law, we have to discuss its origin, the physical meaning, and the validity domain in more detail. As far as we understand, for the first time, this formula was suggested in Ref. 19, see Eq. (2), as a summary of numerous works on flame interaction with a single vortex (or a vortex couple taking into account symmetry). In that sense, the value $\lambda$ should stand for the vortex size or about. Colin et al. proposed Eq. (2) as a formula for local renormalization of turbulent flow velocity, where $\lambda$ played the role of a characteristic local length scale of the turbulent flow. Of course, it is quite a bold step to use the data for flame interaction with a single vortex in order to describe all possible situations of turbulent burning. Particularly, Eq. (2) demonstrated strong quantitative disagreement with numerical simulations of burning in a vortex array with a fixed wave number. Besides, the simulations have shown considerable quantitative difference between the cases of a single-mode vortex array and the pseudo-Kolmogorov spectrum. The numerical simulations demonstrated only qualitative agreement with Eq. (2); the numerical data of Ref. 34 may be approximated by Eq. (29).

However, even this empirical formula described only a part of data. namely, the results obtained for a sufficiently large vortex size. In the case of small vortices, the burning rate decreased with vortex size much faster than $\lambda^{2/3}$. The boundary between “large” and “small” vortices in Ref. 34 correlated strongly with the cutoff wavelength $\lambda_c$ of the DL instability. This correlation is not a coincidence; it follows from the theory, see also, Refs. 29, 30, and 61 and it was supported by experiments. The same effect was also found in numerical simulations of a flame propagating along the vortex axis. This correlation does not necessarily mean any important role of the DL instability in a turbulent flow (though sometimes the instability may be of importance). Rather say, the DL cutoff $\lambda_c$, plays the role of the characteristic length scale (effective flame thickness). Below $\lambda_c$, the effects of flame stretch and thermal conduction dump strongly hydrodynamic wrinkling of the flame front. The damping works nor matter if wrinkling happens because of the DL instability or external turbulence. From that point of view, study of the DL instability is just a convenient way to determine the cutoff wavelength. In the case of $\Theta = 8$, unit Lewis number and thermal conduction depending on temperature as Eq. (11), we obtain the DL cutoff $\lambda_c = 35.6l_f$. Thus, the scaling law $\lambda^{2/3}$ is expected only for vortex sizes sufficiently larger than $\lambda_c$. The physical meaning of the scaling $\lambda^{2/3}$ is not clear at present, since there is no theoretical explanation of this empirical law. In Ref. 19, this tendency has been interpreted as influence of the flame stretch on the burning rate employing also the Kolmogorov spectrum. We do not argue with this explanation because of the lack of a better one. Still, to our mind, it is much more appropriate to talk about influence of flame stretch for small vortex sizes, below and comparable to $\lambda_c$. Besides, we point out that neither flame interaction with a single vortex, nor flame propagation in a single-mode vortex array involve the Kolmogorov spectrum.

What does it mean in the present combustion geometry? Taking into account the wavelength of the vortex array $2H/i$, see Eqs. (21) and (22), we evaluate the scaled “cutoff” wave number $i_c$ from $2H/i_c = \lambda_c$, which corresponds approximately to $i_c = 7$ in our case (we remember that $i$ is an integer value because of the flow confinement). From this evaluation, one should expect that the role of turbulent vortices in the burning rate goes down quite fast for $i \geq i_c = 7$. On the contrary, the tendency like Eq. (4) should be expected for vortex wave number sufficiently smaller than $i_c$. However, in the closed chamber, we observed the tendency opposite (!) to Eq. (4) for $i = 6$, see below.

We performed the simulations for the initial flows determined by Eqs. (21) and (22) with $i = 1–32$. In this set of simulation runs, the initial flow intensity was taken as large as $U_{rms,0}/U_f = 10$. The temperature and vorticity snapshots for $M_f/M = 0.05; 0.2; 0.5; 0.95$ are shown in Figs. 14–17 for $i = 4; 12$ corresponding to cases of relatively large and small vortices. In the case of $i = 4$, Fig. 14, the flame front is much stronger fractalized in comparison with largest possible vortices in the chamber, $i = 1$; the system of caves and pockets is more developed. In Figs. 15(a) and 15(b), we observe an organized array of small vortices ahead of the front, with numerous chaotic vortices behind the flame. Similar to Fig. 7(b) vortices in the fuel mixture keep their original size in the $z$ direction, but they are strongly squeezed in the $x$ direction. Vortex size in the burn matter has no visible correlation with the original vortex size. Figures 16 and 17 are the counterparts of Figs. 14 and 15 for the case of very small vortices, $i = 12$. In Fig. 16, we also observe a fine fractal structure at the flame front. However, this time, the caves are quite small, which reduces the total surface area of the flame front, and the burning rate. Besides, small vortices decay faster than the large ones (see Fig. 2). If we keep decreasing the vortex size, the flame front becomes less and less fractal, and the influence of initial vortices becomes weaker. In the case of $i = 32$, flame evolution practically coincides with the laminar case of Fig. 3 in spite of strong vortex intensity, $U_{rms,0}/U_f = 10$. Figure 18 shows the mass fraction of the burn matter $M_f/M$ for $U_{rms,0}/U_f = 10$ and different initial vortex sizes $i = 1–32$. Figure 19 presents the average burning rate (inverse burning time) for different vortex sizes. The maximal burning rate corresponds to $i = 6$; it exceeds the burning rate of large vortices with $i = 1$ approximately twice. In agreement with the theoretical reasoning above, we observe strong decrease in the burning rate for sufficiently small vortices $i > 7$. In spite of the quite strong turbulent flow $U_{rms,0}/U_f = 10$, in the case of $i = 32$ the burning rate/time practically coincides with that in case of an initially quiescent gas, $H/\sqrt{i_f}U_f = 1.68$, shown by the straight dashed line in Fig. 19.
The same tendency may be observed in the snapshots of Fig. 16, where turbulent wrinkling is effectively smoothed by flame stretch and thermal conduction.

The case of relatively large vortices ($i < 7$) is much more interesting, since most of the turbulent energy is typically stored in the large-scale structures. This is also the domain where the tendency of Eq. (4) is expected to work. As we pointed out above, following Eq. (4), we should expect that decreasing the vortex size from $H$ to $H/6$ (increasing $i$ from 1 to 6) leads to decrease in the burning rate approximately by the factor of 3.3. The second dashed line in Fig. 19 shows the tendency Eq. (4) with $i = 1$ taken as the reference point. We observe no agreement between the present simulations and Eq. (4). Figure 19 shows the opposite tendency: vortices with $i = 6$ provide a considerably larger burning rate. Strongly fractal flame shape for $i = 4$ shown in Fig. 14 illustrates the same tendency qualitatively. Explanation of this effect looks deceitfully simple, even trivial: smaller vortices produce a more fractal flame front, which leads to the larger burning rate. The second part of this statement is indeed trivial in the combustion science, at least, as long as we work in the flamelet regime. However, the first part of the explanation is questionable, and, in general, incorrect. Do smaller vortices lead to a more fractal flame, that is, to a flame with a larger surface area? As a counter example, let us imagine the same burning chamber, but with the flame ignited as a planar front at $x = 0$ and with slip at the walls. Because of the symmetry, flame propagation in that case may be split into burning in narrow closed “tubes” of width $H/i$ and length $D/2$. In that case, smaller vortex size does not mean larger flame surface...
On the contrary, we come to the situation resembling the study\textsuperscript{34} quite close with the scaling Eq. (4). Thus, the question of a burning rate is not only a question of vortex size. It is not even a question of closed or open chamber. It is the question of combustion geometry as a whole, including vortex size, type of ignition, open or closed chamber, etc. The studies\textsuperscript{19,34} demonstrated increase in the burning rate with vortex size in the open channel, and came to the same scaling law Eq. (4). The present study has shown the opposite tendency of burning rate decreasing with vortex size in a closed chamber. Igniting flame in a different way, we may obtain other possibilities.

Finally, flow intensity in the chamber depends also on the size of initial vortices. Figure 20 shows rms-flow velocity for initial vortex intensity $U_{\text{rms},0}/U_f=10$ and $i=1, 6, 12, 32$. The minimal burning time was observed for $i=6$, and the largest flow rms velocity was obtained for the wave numbers close to $i=6$. The flows with the initial vortex size in the domain $i=4–8$ are approximately twice stronger than the initial vortices.

V. SUMMARY

We started this paper with the question: do we have some universal qualitative properties of turbulent burning, which can be put in a form of a scaling law like Eq. (1)? As an example, we have taken the scaling law (2) suggested in Ref. 19 on the basis of numerical simulations of flame inter-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig15.png}
\caption{(Color online) Evolution of the flow vorticity from $-0.3U_f/H$ (blue) to $0.3U_f/H$ (red) for $U_{\text{rms},0}/U_f=10$ and $i=4$. In black-and-white version the green color is shown by light, the red is half-dark, and the blue is dark. The snapshots (a)–(d) correspond to the time instants $U_f/H =0.046; 0.068; 0.099; 0.13$ (related to $M_b/M=0.05; 0.2; 0.5; 0.95$, respectively).}
\end{figure}
action with a single vortex in the open channel. We did not look for quantitative agreement, since recent studies of flame propagation in a vortex array in open tubes have already refuted that possibility. What we asked was only qualitative tendency like Eq. (4). To test the scaling law, in the present work, we studied burning in a closed chamber in a form of a box with aspect ratio 1:8 filled with vortex arrays. Such a chamber resembles geometrically clearance of a spark-ignition engine. Still, keeping engines in mind as a possible application, we did not try to imitate any particular engine. We have performed direct numerical simulations of the complete set of hydrodynamic/combustion equations including transport processes (thermal conduction, diffusion, and viscosity) and chemical kinetics with an Arrhenius reaction. We have investigated how the burning rate and the flow intensity depend on the initial vortex intensity and size.

We have obtained some common features of combustion with and without confinement. Similar to all other studies of turbulent combustion, in the present work, the burning rate increases with vortex intensity. The increase is approximately linear for a sufficiently strong turbulent intensity. However, even at that point, we obtain considerable quantitative difference between combustion in a closed chamber and in an open tube. For example, in open tubes, taking vortices as large, as shown in Fig. 1(a), with typical intensity $U_{\text{rms}}/U_f = 1$, one finds the burning rate exceeding the laminar value by a considerable factor about 3, see Eq. (29) and Ref. 34. In contrast, in the present geometry of a closed burning
chamber, vortices of the same strength and size provide only slight increase in the burning rate over the laminar value, approximately by 10%. In order to obtain the burning rate twice larger than the laminar one, in the closed chamber, we have to create vortices of considerable intensity, about \( \frac{U_{rms}}{U_f} = 5 \). This difference is explained by a much stronger flow produced by a laminar flame in a closed chamber.

Still, this difference is only quantitative. The situation becomes much worse with the dependence of the burning rate versus the vortex size (the turbulent length scale). Unlike the previous studies of burning without confinement, the present work demonstrates a nonmonotonic dependence of the burning rate on the vortex size for combustion in a closed chamber. The dependence may be divided roughly into two

![FIG. 17](image1).

![FIG. 18](image2).
domains of large and small vortices with the border between them correlated with the DL cutoff $\lambda_c$. In the case of small vortex size, approximately below $\lambda_c$, burning is strongly influenced by thermal conduction and finite flame thickness (by stretch), which try to smooth all possible flame wrinkles. In agreement with the previous theory, experiments and simulations, the role of small vortices decreases strongly with decreasing vortex size. Even in the case of strong vortex intensity, burning rate approaches the laminar value for small vortices. In the case of relatively large vortex size, above the cutoff $\lambda_c$, we observe the opposite tendency. The burning rate increases as we decrease the vortex size. This tendency is also opposite to the scaling law Eq. (4), obtained for flames in the open channel. As a result, we find an optimal vortex size, which provides maximal burning rate (minimal burning time). In the present study, the optimal vortex size is about 1/6 of the chamber height. Still, the optimal size may depend on vortex intensity, on the flame thickness, on the chamber geometry and other parameters. More detailed investigation of the optimal vortex size is left for future studies.

Thus, the present study confirms only two general qualitative tendencies of turbulent burning: (1) the burning rate increases approximately linear at sufficiently strong turbulent intensity; (2) the burning rate goes down to the laminar value as the vortex size becomes considerably smaller than the cutoff. Still, even these two tendencies can hardly be formulated as quantitative laws, since the border between strong and weak turbulence may vary by an order of magnitude depending on the burning geometry. Other tendencies [like those formulated in Eq. (2)] are not universal. For different burning geometries, we obtain qualitatively different (opposite!) dependence of burning rate versus the vortex size.

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FIG. 19. The scaled inverse burning time $H/\tau U_f$ vs the scaled initial vortex size $i$. The markers show the present simulations. The tendency Eq. (4) with $i=1$ taken as the reference point is shown by the dashed line. One more dashed line (straight) is related to an initially quiescent gas.

FIG. 20. The scaled flow rms velocity $U_{rms}/U_f$ averaged over the whole simulation domain, Eq. (25), vs time for different initial vortex size $i =1, 6, 8, 32$. 

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