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Design of CVT-Based Hybrid Passenger Cars

Theo Hofman, Maarten Steinbuch, Roëll van Druten, and Alex F. A. Serrarens

Abstract—In this paper, the hybridization of a small passenger car equipped with a continuously variable transmission (CVT) is investigated. Designing a hybrid drive train is a multiobjective design problem. The main design objectives are fuel consumption, emissions, and performance. However, it is difficult to find a global optimal integral design solution due to the interdependence of design choices (parameters) regarding the drive-train topology, component sizes, component technologies, and control strategy, as well as the unknown sensitivity of the design objectives to the design parameters. In this paper, a parametric optimization procedure is presented to solve the design problem, where the main design objective is fuel consumption. The effects of parameter variation on fuel consumption have been investigated. Furthermore, a reduced hybrid drive-train model is introduced, with which the effects of design parameter variation is very quickly studied with an average error of less than 1.6%.

Index Terms—Continuously variable transmission (CVT), energy source, hybrid electric vehicles (HEVs), vehicle simulation.

NOMENCLATURE

List of acronyms

Symbol	Description
ADVISOR	Advanced Vehicle Simulator.
AMT	Automated manual transmission.
AT	Automatic transmission.
BER	Brake-energy recovery.
CL	Wet-plate clutch.
CLC	Clutch closed.
CLO	Clutch open.
CVT	Continuously variable transmission.
CS	Control strategy.
EM	Electric machine.
EMA	Electromechanical actuation.
EMS	Energy management strategy.

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EON	Engine on.
EOFF	Engine off.
IMA	Integrated motor assist.
HDA	Hydraulic actuation.
JP10–15	Japanese 10–15 mode (drive cycle).
M1, M2	Simulation models 1 and 2.
NEDC	New European Drive Cycle.
NiMH	Nickel metal hydride.
OOL	Optimal operation line.
O	Observation.
P	Primary power source (engine).
PRI	Primary.
S	Secondary power source (battery/electric machine).
SI	Spark ignition.
SEC	Secondary.
T	Transmission technology.
TC	Torque converter.
WOT	Wide-open throttle.
V	Vehicle/drive cycle.

List of Symbols

List of Symbols	Description (Unit).
c_0, c_1	Fit coefficient. Characteristic parameter (-).
f_{fb}	Brake force ratio between the front and rear wheels (-).
g	Gravity (in meters per second squared).
g_i	Inequality constraints (in watts).
h_i	Equality constraints (in joules).
i	Iteration step (-).
$\dot{m}_f$	Fuel mass flow (in grams per second).
m_v	Total vehicle mass (in kilograms).
r_{CVT}	CVT speed ratio (-).
r_d	Final drive ratio (-).
r_t	Overall transmission speed ratio (-).
r_t^o	Optimal overall transmission speed ratio (-).
r_{ud}, r_{od}	Under- and overdrive speed ratios (-).
r_w	Wheel radius (in meters).
t	Time (in seconds).
t_f	Final time (in seconds).
v_M	Vehicle speed electric-only threshold value (in meters per second).
v_v	Vehicle speed (in meters per second).
$\dot{v}_v$	Time derivative of v_v (in meters per second squared).
x_i	Design parameter (-).
x_i^o	Optimal design parameter (-).
E_f	Energy content of used fuel (in joules).
E_s	Energy storage level of battery (in joules).

ΔE_s	Battery energy storage level difference (in joules).
E_v	Energy vehicle demand for driving over a drive cycle (in joules).
G	Matrix with inequality constraints (-).
J_w	Inertia of rotating parts, including wheels (in kilogram meter squared).
P_e	Engine power (in watts).
P_b	Battery output power (in watts).
$P_{b,\min}, P_{b,\max}$	Minimum and maximum battery output powers (in watts).
P_{em}	Electric machine power (in watts).
P_f	Fuel power (in watts).
P_s	Battery input power (in watts).
$P_{s,\min}, P_{s,\max}$	Minimum and maximum battery input powers (in watts).
P_{in}, P_{out}	Input and output powers (in watts).
P_{max}	Maximum output power. Characteristic parameter (in watts).
P_{BER}	Regenerative brake power (in watts).
P_v	Vehicle drive power demand (in watts).
$\mathbb{R}$	Set of rational numbers (-).
T_e	Engine crankshaft torque (in newton meter).
$T_{e,\min}, T_{e,\max}$	Minimum and maximum engine output torques (in newton meter).
T_e^o	Optimal engine crankshaft torque (in newton meter).
T_{em}	Electric machine output torque (in newton meter).
$T_{em,\min}, T_{em,\max}$	Minimum and maximum electric machine torques (in newton meter).
$T_{t,p}$	Transmission input torque (in newton meter).
T_{rl}	Road load torque (in newton meter).
ε	Error iteration step (in watts).
γ	Convergence ratio (-).
η_b	Battery efficiency (-).
η_{CVT}	CVT efficiency (-).
$\bar{\eta}_{CVT}$	Average CVT efficiency (-).
η_e	Engine efficiency (-).
η_{em}	Electric machine efficiency (-).
η_{fd}	Final drive set efficiency (-).
η_s	Secondary source efficiency (-).
η_{sys}	System efficiency (-).
η_t	Transmission efficiency (-).
ω_e	Angular engine speed (in radians per second).
ω_{em}	Angular electric machine speed (in radians per second).
$\omega_{em,\min}, \omega_{em,\max}$	Minimum and maximum angular electric machine speeds (in radians per second).
$\omega_{t,p}$	Angular transmission input speed (in radians per second).
ω_v	Angular vehicle wheel speed (in radians per second).
Φ_f	Objective function or total fuel consumption (in grams).

I. INTRODUCTION

SALES of hybrid vehicles are becoming more significant every year. The introduction of the Toyota Prius almost ten years ago significantly changed the spectrum of the automotive power train market. The Toyota Prius operates using an electric CVT principle [1], whereas the Honda Civic IMA [2] uses a mechanical belt-driven CVT in parallel with an electric motor. The CVT principle in the Prius excels in driving agility and fuel consumption. However, the CVT is relatively costly, in particular, due to the dual electric machines, power electronics, and battery pack. This is one of the reasons why this hybrid system is now introduced in higher class vehicles (e.g., large passenger cars and large off-roaders). The Honda IMA system uses mature and low-cost belt-driven CVT technology, in combination with a much smaller electric machine. This saves cost; however, overall performance is penalized due to the limited electrical assist, in particular if the relatively small electric machine is combined with a down-sized engine.

There are three main functional differences between a hybrid vehicle equipped with a CVT and a transmission with a discrete set of gear ratios (e.g., an AMT or an AT).

- 1) The engine or the electric machine (i.e., only if the electric machine is precoupled to the CVT) can be operated at the optimal operation points.
- 2) The electric machines are part of an electrical or electromechanical CVT (i.e., in case of a series or a series-parallel hybrid configuration, respectively).
- 3) A CVT with infinite ratio coverage (e.g., Toyota Prius) allows smooth and quick engine restart.

In this paper, the hybridization of a small passenger car equipped with a CVT is analyzed, and the component sizes (power specifications) of such a car are optimized. The baseline vehicle is a Toyota Yaris equipped with a four-cylinder 1.3-L gasoline engine, a TC, and a conventional push-belt CVT. A short overview of different CVT solutions is given in the next section.

A. Development of CVT Solutions

A pure electrical CVT consists of two electric machines [see Fig. 1 (top)]. Both electric machines can operate as motor or generator. If one electric machine is directly coupled to the engine and the other is directly coupled to the driven wheels, the size (maximum continuous power) of both machines has to be equal to the maximum power of the engine. The corresponding weight and cost would be too high for passenger car applications. Furthermore, the power conversion from the mechanical domain to the electrical domain and *vice versa* is not highly efficient [3]. To reduce the size of the electric machines and improve the efficiency of an electrical CVT, a power-split device (planetary gear set), which splits the engine power into a mechanical and an electrical path, can be used [see Fig. 1 (bottom)]. However, due to, e.g., the kinematics of the planetary gear or the constraints on the ratio coverage of the transmission, the reduction of the electric machine sizes is limited yet still significant, compared with a pure electrical CVT.

Fig. 1. Pure electrical, push-belt (discussed in this paper), and electrical power-split CVTs (Toyota Prius).

In summary, the advantages of a power-split CVT, compared with an electrical CVT, are given here.

- 1) The transmission efficiency is higher since most of the power is transmitted over the mechanical branch consisting of planetary gears.
- 2) A lower maximum power flow through the electrical branch is required, which results in smaller electric machines.

Developments to further reduce the size of the electric machines and further increase the transmission efficiency are done by Villeneuve [4], Schulz [5], and Kriegl *et al.* [6]. However, these developments mostly lead to a design that is close to that of existing or automated manuals, in which the clutches are replaced by electric machines to enable continuously variable shifts. This always leads to a transmission that is much more complex than a conventional transmission. Moreover, electronically controlled clutches for clutching shifts are so well designed nowadays that it is hard to feel that a shift took place.

The mechanical CVT, which is currently produced by Jatco, Aisin, ZF, Toyota, Subaru, Daimler Chrysler, Audi, etc., is considered to be a benchmark for all transmissions [7]. It outperforms the manual transmission regarding fuel consumption. Moreover, the mechanical CVT outperforms current automated manuals regarding cost and comfort [8]. In particular, due to the efforts of Japanese transmission manufacturers, the mechanical CVT will have high potential on a large scale. The potential for further improvement of the CVT efficiency and the reduction

of costs is high with new upcoming technologies, such as slip control [9], on-demand actuation systems [10], and involute chain technology [11].

II. PROBLEM DEFINITION

This paper presents a modeling and design approach in finding the optimal values for the following design parameters denoted as x_i : the drive-train topology, component technology, component size (in kilowatts), and control strategy. This is performed for a parallel hybrid configuration equipped with a push-belt CVT for a small passenger car (Toyota Yaris). The topology determines how the power sources (engine and electric machine/battery) are connected to the drive train.

The main design criterion, which is used to find the optimal parametric values, is the minimization of the overall fuel consumption, which is denoted as Φ_f , on a defined drive cycle. In addition, constraint functions $G(x_i)$ can be defined to keep the system properties and variables within certain bounds. The design optimization problem is formulated as [12]

$$\min_{x_i} \Phi_f(x_i) \text{ subject to } G(x_i) \leq 0, \quad x_i \in \mathcal{X} \subseteq \mathbb{R}^n \quad (1)$$

where the feasible design space is assumed to be embedded in set $\mathcal{X}$ with dimension n . From a hybrid vehicle propulsion design viewpoint, the control design is seen as a subproblem. Therefore, the control model uses a local design variable, such as the control power flow of the energy storage system (battery/electric machine).

III. CONTRIBUTION AND OUTLINE OF THIS PAPER

Finding the optimal parametric values is a complex task due to the strong interdependence of the design parameters. Furthermore, calculation of the optimal control strategy can be a computational burden, particularly if the control strategy is optimized with the control freedom of the component operation point on top of the power distribution problem between the main power sources. In the last part of this paper, to alleviate the complex design problem, the effects of employing a high-level modeling framework in determining the optimal design parameters are investigated. Initially, detailed component models are used for analyzing the sensitivity of the design parameters to the design objective. Moreover, the effects of control freedom in the operation point on the total fuel consumption are investigated. Within this context, one of the main underlying questions in this work is given here.

- Can a reduced hybrid drive train model with sufficient accuracy for design be employed?

This question is answered at the end of this paper.

The outline of this paper is given as follows: In Section IV, the baseline vehicle, engine, and push-belt CVT model used as a reference are discussed. Accordingly, the hybrid vehicle drive train and electrical storage model are discussed. The simulation methods and assumptions are discussed in Section V. In Section VI, the effects of parameter variation on the main design objective fuel consumption for a parallel hybrid vehicle are discussed. A method for the calculation

of the component efficiencies used with the simplified hybrid drive-train model is presented in Section VII. The component efficiencies are described using a few characteristic parameters. The fuel consumption using the reduced hybrid drive-train model is calculated and compared with that of a detailed hybrid drive train model. Finally, the conclusions are described in Section VIII.

IV. MODELING OF THE DRIVE TRAIN COMPONENTS

In this section, the used component models for the engine, transmission, and battery-storage system are discussed. Some of the used models, i.e., the electric machine and battery data, are obtained from ADVISOR [13]. The static-efficiency maps of the electric machines are (linearly) scaled up or down for their properties to match the desired values. Similar scaling approaches have been utilized by others [14]–[17]. Nevertheless, some of the used static-efficiency models, i.e., for the engine, push-belt CVT, and TC, have been derived based on the performed measurements. The control design method, as described in [18], is used for the determination of the optimal control strategy. The control method is based on the combination of rule-based and equivalent consumption minimization strategies having sufficient accuracy, with which fuel consumption can quickly be calculated. Next, the reference vehicle model and, accordingly, the hybrid vehicle drive trains, including the drive train components, are discussed.

A. Baseline Vehicle Model

The baseline vehicle is a Toyota Yaris (2007) equipped with a 1.3-L SI 64-kW engine, a TC, and a hydraulic actuated push-belt CVT. The fuel consumption of this vehicle was measured on a dynamo test bench for the NEDC (cold and hot engine starts), JP10–15 (only hot engine) cycles, additional different continuous driving speeds, and WOT measurement tests. Thereby, the injected fuel mass flow (in grams per second), engine crankshaft speed, drive shaft speed, and torque were measured. Since the engine crankshaft torque and the efficiency of the actual TC and CVT were not measured, available experimental validated models [19] for the TC and belt-driven CVT were used to calculate backward the required engine crankshaft torque. The collected measurement data and the reconstructed engine torque assumed to mimic the actual engine torque were used to reconstruct the (hot) engine fuel map. A forward-facing control model consisting of a driver model and a dynamic drive train model was used to validate the used component models with the measured data. The results are not discussed here and are beyond the scope of this paper. Instead, only the fuel consumption results of the baseline vehicle are given (see Table I). The simulation with a CL shows a relatively small fuel consumption improvement of 2%. The reconstructed fuel efficiency map as a function of engine torque T_e and speed ω_e is shown in Fig. 2. In the figure, the fitted or “smoothed” OOL is also shown. The OOL collects the set of engine operation points with maximum efficiency over the engine output power range.

TABLE I
REFERENCE VALUES: BASE LINE VEHICLE
(FORWARDS-FACING CONTROL MODEL)

Test	Fuel consumption on the JP10-15	
	l/100km	Relative values
Measured value (TC)	5.10	100.0%
Catalogue value (TC)	5.00	98.0%
Simulated value (TC)	5.00	98.0%
Simulated value (CL)	4.90	96.0%

TC = Torque Converter, CL = Wet-plate clutch

Fig. 2. Reconstructed 1.3-L VVTi 64-kW engine efficiency map and “smoothed” OOL.

B. Hybrid Drive Train Model

For the parallel hybrid configuration equipped with a push-belt CVT [Fig. 1 (middle)], the electric machine is coupled at the engine side of the transmission between the clutch and the transmission. This allows the engine to be turned off during the propulsion phases [20] since the vehicle can be propelled only by the electric machine. Smooth engine restart during standstill with this configuration cannot be done without an additional starter–alternator. This is however different for the Honda IMA, where the electric machine is directly connected to the engine crankshaft. The Honda IMA is able to restart the engine during standstill with the same electric machine that is used, e.g., for brake energy recuperation or motor assistance. However, the Honda IMA is not able to efficiently propel the vehicle solely by the electric machine due to the relatively large engine drag losses.

An electromechanical power-split CVT is used in the Toyota Prius [Fig. 1 (bottom)], which has only “one mode,” implying that the CVT has neither clutches nor brakes. Furthermore, engine restart can be done at vehicle standstill without using a starter–alternator. The depicted power-split CVT is characterized by the possible occurrence of recirculation power, which diminishes the overall transmission efficiency. The main power-flow cases are indicated by the arrow directions in the figure. In [4], [5], and [21], more details on this subject are given. In Fig. 3, the efficiency map at a transmission input speed of 209 rad/s (2000 r/min) as a function of the overall speed ratio

Fig. 3. Efficiency map for the belt-driven CVT at transmission input speed $\omega_{t,p} = 209$ rad/s (2000 rad/min).

TABLE II
BASE COMPONENT CHARACTERISTICS

Engine	Toyota Vitz, 4-cylinder, 64-kW peak power (at 6000 rpm), 1.3-l VVTi, 124-Nm peak torque (at 4400 rpm), 40% peak efficiency
Electric Machine/Controller	PM brushless AC motor, 58-kW peak power (at 1250 rpm), 403-Nm peak torque, 92% peak efficiency, mass 70 kg (MC-PM58*)
Battery Pack	Panasonic, Ni-MH type, current capacity 6 Ah, nominal voltage 1.2 V/cell, storage mass 0.166 kg/cell (ESS_NIMH6*)
Transmission	Push belt-driven Continuously Variable Transmission, Van Doorne type P920, underdrive $r_{ud} = 0.43$ and overdrive ratio $r_{od} = 2.39$, final drive ratio $r_d = 0.19$ [19]

*model data file from ADVISOR

and input torque is shown. The push-belt CVT efficiency is strongly influenced by the input torque for a fixed input speed. The efficiency increases with an increase of input torque and with an increase of speed ratio up to the “neutral” speed ratio $r_{CVT} = 1$. The maximum efficiency for the push-belt CVT is at neutral speed ratio $r_{CVT} = 1$ (which is indicated by the dashed line in the figure).

An overview of the used model data for the main power sources and the specific electric machine data is presented in Table II.

C. Battery Model

The maximum battery pack power is of the same size as the maximum possible output power specifications of the electric machine for the parallel hybrid drive train equipped with the push-belt CVT, i.e., 15 kW. Thereby, a minimum number of 40 battery modules for the NiMH with a nominal battery system voltage of 288 V are needed to meet the estimated minimum voltage requirement of 120 V and the maximum current of 125 A allowed by the motor controller/electric machine. These values are kept constant (see Table II).

Fig. 4. Power flow in the hybrid drive train (backward-facing control model).

TABLE III
VEHICLE MODEL PARAMETERS

Description	Symbol	Value	Unit
Total mass	m_v	1134	kg
Wheel radius	r_w	0.277	m
Inertia of two wheels	J_w	1.5	kg m ²

V. SIMULATION METHOD

In Fig. 4, the power flows in the hybrid drive train are schematically visualized, which yields a backward-facing model implying that the vehicle speed $v_v(t)$ (in meters per second) prescribed by the drive cycle is assumed to be exactly tracked. This model is used to compute the optimal control signals that minimize the fuel consumption.

The inputs are battery input power $P_s(t)$ (in watts), velocity $v_v(t)$ described by the given drive cycle, and vehicle drive power demand $P_v(t)$ (in watts). The outputs are engine speed $\omega_e(t)$ (in radians per second) and torque $T_e(t)$ (in newton meters), which are used to compute fuel mass flow $\dot{m}_f(t)$ (in grams per second). The vehicle parameters of the Toyota Yaris determine vehicle road load torque $T_{rl}(t)$ (in newton meter). The vehicle model parameters are listed in Table III. Note that the battery power can be affected by additional electrical (auxiliary) loads, which is not shown in the figure. The vehicle road load depends on vehicle speed $v_v(t)$ (assuming no wheel slip) and is the sum of the roll resistance and the air-drag torque, i.e.,

$$T_{rl}(t) = 36.8 + 0.416 v_v(t) + 0.113 v_v(t)^2 \quad (2)$$

where the vehicle power demand as a function of time $P_v(t)$ becomes

$$P_v(t) = T_{rl}(t)/r_w \cdot v_v(t) + (m_v + J_w/r_w^2) \cdot \dot{v}_v(t). \quad (3)$$

The vehicle mass (in kilograms), the inertia of the wheels (in kilogram meter squared), and wheel radius (in meters) are described by parameters m_v , J_w , and r_w , respectively. The energy levels (in joules) in the battery and in the fuel tank are represented by variables E_s and E_f , respectively.

The simulation procedure, which is described by sequential computation steps, to calculate the fuel mass-flow rate $\dot{m}_f(t)$ as

a function of battery input power $P_s(t)$, given $v_v(t)$ and $P_v(t)$, is discussed here.

Step 1: In the first step, transmission efficiencies η_{CVT} are computed for all admissible battery powers P_b , transmission input torques $T_{t,p}$, speeds $\omega_{t,p}$, and speed ratios $r_t = r_{\text{CVT}} \cdot r_d$ and are stored in multidimensional lookup tables. The transmission efficiency can be expressed as

$$\eta_{\text{CVT}} = \eta_{\text{CVT}}(\omega_{t,p}, T_{t,p}, r_t, P_b). \quad (4)$$

Step 2: In the second step, fuel mass flows $\dot{m}_f$ (in grams per second) are computed for all admissible battery output powers P_b , given drive power demand $P_v(t)$ and angular vehicle wheel speed $\omega_v(t) = v_v(t)/r_w$. The fuel mass flows are computed for two different control strategies focusing on maximizing engine η_e or system efficiency η_{sys} at each time instant. These two control strategies, which determine the engine operation point, are referred to as CS1 and CS2, respectively.

System efficiency (see Fig. 4) η_{sys} is defined as the product of engine η_e and transmission efficiency (without the battery) η_{CVT} and is written as

$$\eta_{\text{sys}} = \eta_e(\omega_e, T_e) \cdot \eta_{\text{CVT}}(\omega_{t,p}, T_{t,p}, r_t, P_b) \quad (5)$$

with the transmission input torque as the sum of the engine and the electric machine output torque $T_{t,p} = T_e + T_{\text{em}}$ and $\omega_{t,p} = \omega_e = \omega_{\text{em}}$, respectively, since the input shaft of the CVT is directly connected to the output shaft of the engine. Accordingly, the optimal speed ratio and engine torque are selected as

$$(r_t^o(P_b, t), T_e^o(P_b, t)) = \arg \max_{(r_t, T_e) \in \mathcal{R}} (\eta_* | P_b, P_v(t), \omega_v(t)) \quad (6)$$

for $* \in \{e, \text{sys}\}$ (i.e., CS1 or CS2, respectively), where set $\mathcal{R}$ covers the feasible solutions that satisfy the constraints

$$\begin{aligned} \mathcal{R} = \{ & (r_t, T_e) | \omega_{**,\min} \leq \omega_{**} \leq \omega_{**,\max} \\ & \wedge T_{**,\min}(\omega_{**}) \leq T_{**} \leq T_{**,\max}(\omega_{**}) \\ & \wedge r_{\text{ud}} \leq r_{\text{CVT}} \leq r_{\text{od}} \wedge P_{b,\min} \leq P_b \leq P_{b,\max} \} \end{aligned} \quad (7)$$

for $** \in \{e, \text{em}\}$, and r_{CVT} is limited by the under- and over-drive ratios r_{ud} and r_{od} , respectively, for the push-belt CVT (see Table II).

The optimal gear ratios and engine torques for all admissible battery powers for both control strategies (CS1 and CS2) are stored in multidimensional lookup tables and are used to compute the fuel mass flows using the engine fuel map. If the optimal gear ratios and engine torques at each time instant are known for different values of P_b , given $P_v(t)$ and $\omega_v(t)$, then the fuel mass flow can be expressed only as a function of the battery power, i.e.,

$$\dot{m}_f = \dot{m}_f(P_b, t | P_v(t), \omega_v(t)) \quad (8)$$

for CS1 and CS2.

Note that, in case of engine optimal operation (CS1), the transmission efficiency η_{CVT} is determined by speed ratio $r_t^o(P_b, t)$ and engine torque $T_e^o(P_b, t)$ (prescribed by the OOL),

given a certain vehicle speed $\omega_v(t)$. However, in a backward-facing control model, the required $r_t^o(P_b, t)$ and $T_e^o(P_b, t)$, given a certain speed $\omega_v(t)$, are determined by η_{CVT} and the given $P_v(t)$. Due to this causality conflict, it is impossible to determine $r_t^o(P_b, t)$ and $T_e^o(P_b, t)$, given a certain speed $\omega_v(t)$ exactly. In this paper, for CS1, the transmission losses are iteratively estimated and are compensated for engine power $P_e(t)$. The iteration error, which is denoted as ε , at time instant t in each iteration step i is defined as

$$\varepsilon(i, t) = |P_e(i+1, t) - P_e(i, t)|. \quad (9)$$

The iteration is repeated until the error $\varepsilon(i, t)$ between the iteration steps at a certain time instant becomes sufficiently small (i.e., $\varepsilon(i, t) < 10 \text{ W}$). A mathematical condition to prevent the iteration loop from instabilities, in terms of the estimation $P_e(i, t)$, is

$$|P_e(i+1, t) - P_e(i, t)| < \gamma |P_e(i, t) - P_e(i-1, t)| \quad 0 < \gamma < 1. \quad (10)$$

This condition implies that the error ε of each estimate decreases in each iteration step i . If the ratio of the error between the previous and subsequent steps becomes equal to 1, the estimates will not further improve. This corresponds to the transition between stable and unstable iteration loops. At later time instants, the required P_e is calculated using the known values for the efficiencies at the previous time instant. Thus, the requested $P_v(t)$ is divided by the computed η_{CVT} .

Step 3: In the third step, the optimal control strategy is determined using the control design model described in [18], where the engine efficiency (CS1), or the system efficiency (CS2) at each time instant t is maximized, and the total fuel consumption Φ_f is minimized. The optimization problem is finding control power flow $P_s(t)$, given a certain power demand at the wheels $P_v(t)$ and wheel speed $\omega_v(t)$, while Φ_f over a drive cycle with time length t_f is minimized, i.e.,

$$\Phi_f = \min_{P_s(t)} \int_0^{t_f} \dot{m}_f(P_s(t), t | P_v(t), \omega_v(t)) dt \quad (11)$$

subject to certain component constraints $\vec{h} = 0$, $\vec{g} \leq 0$, which are described here. The state is equal to the stored energy E_s in the battery in joules, and the control input is equal to the battery input power P_s in watts (see Fig. 4), i.e.,

$$P_s(t) = \min(P_b(t)/\eta_b(P_b(t)), P_b(t)\eta_b(P_b(t))). \quad (12)$$

Battery efficiency η_b is only a function of battery power P_b since the influence of the relatively small variation in the battery energy level on the battery efficiency is small and is, therefore, neglected. The energy level in the battery is a simple integration of the power and is calculated as follows:

$$E_s(t) = E_s(0) + \int_0^t P_s(\tau) d\tau. \quad (13)$$

The main constraints on the battery are the energy balance conservation of E_s over the drive cycle and constraints on power P_s , i.e.,

$$\begin{aligned} h_1 &:= E_s(t_f) - E_s(0) = 0 \\ g_{1,2} &:= P_{s,\min} \leq P_s(t) \leq P_{s,\max}. \end{aligned} \quad (14)$$

Computation steps 1–3 are performed for different electric machine sizes (1–15 kW), in combination with other different design parameters. In the next section, the modeling assumptions, descriptions, and symbols of the investigated design parameters are discussed.

Assumptions and Descriptions of the Design Parameters x_i : Initially, for the hybrid vehicle model, nine assumptions are made.

- A.1) The drive-train components (engine, transmission, electric machine, and battery) are modeled as quasi-static-efficiency models.
- A.2) The electric machine characteristics (maximum torque curve and static-efficiency map) are linearly scaled as needed. The effect of the vehicle mass increase by upscaling of the electric machine is neglected.
- A.3) For the whole drive cycle, the electrical auxiliary loads are assumed to be constant at 200 W. For the baseline vehicle, the electrical auxiliary loads increase the fuel consumption with 0.13 L/100 km for the JP10–15.
- A.4) The minimum engine speed during operation is 980 rad/min, which is equal to that of the reference vehicle with a conventional drive train.
- A.5) The braking forces are evenly distributed between the rear and front wheels. The brake force distribution ratio between the front and rear wheels is denoted as f_{fb} . To maximize the vehicle stability, an adaptive brake force distribution should be considered [22]. However, in this paper, ideal braking is not considered. Instead, the influence of different constant values for f_{fb} , which is also referred to as regenerative brake fraction, is investigated.
- A.6) The maximum electromechanical braking power is limited by the torque, speed, and power constraints of the electric machine and battery, respectively. Braking powers that are larger than the maximum electrical regenerative braking power are assumed to be dissipated in the front-wheel brake discs.
- A.7) The engine has no drag or idle losses at vehicle standstill. This start–stop function decreases fuel consumption with 0.66 L/100 km for the JP10–15. Fuel savings with the start–stop function during propulsion depends on the degree of hybridization.
- A.8) The fuel use for engine cranking is neglected since the engine can be started with the electric machine in a very short time period (typically 100–300 ms). The required electrical energy for engine start (usually, the engine is started when the vehicle is already driving) is therefore limited and very small. However, the power for engine start is relatively high.
- A.9) During braking, the clutch is open (CLO), and the engine is assumed to be off (EOFF) (i.e., inducing no additional drag torques and no idle fuel consump-

Fig. 5. EON/EOFF strategy.

tion). This assumption decreases fuel consumption with 0.45 L/100 km.

If, during braking, in a real vehicle, the engine is completely shut off and disengaged, then quick engine restart during driving can be uncomfortable and, therefore, unacceptable. Unless, for example, the electric machine is sufficiently large to deliver the relatively large engine start-up torques, which are superposed on the drive torque demand. Therefore, two other braking strategies are investigated where the engine remains on (EON) during braking, in combination with the electric-only vehicle speed threshold value, which is denoted as v_M . The engine is always on for vehicle speeds above this value, and vice versa. This will be discussed by the descriptions of the design parameters in more detail.

- A.10) The electric-only vehicle speed threshold value v_M is 20 km/h, which is equal to the measured value for the Toyota Prius on the JP10–15 cycle [23].
- A.11) During the electric-only mode (propulsion and braking), the CVT speed ratio is assumed to be controlled, such that the electric machine is operated at the highest efficiency points. In [24], it is shown that this braking strategy provides increased recuperated energy.

Next, the descriptions and symbols of the design (x_i for $i \in \{1, \dots, 7\}$) parameters are given.

- x_1 *Regenerative brake fraction:* This determines the distribution fraction f_{fb} between the front and rear wheels, where the regenerative braking power becomes $P_{BER}(t) = f_{fb} \cdot P_v(t|P_v(t) < 0)$, with $f_{fb} \in \{0\%, 50\%, 100\%\}$.
- x_2 *Drive cycle:* The vehicle load is influenced by using the NEDC or the Japanese drive cycle (JP10–15) [see (2)].
- x_3 *Component size:* The component efficiencies, power-flow control strategy, and potential fuel savings are affected by the size of the electric machine. The base electric machine size is linearly scaled as needed (see Table II).
- x_4 *Topology:* This determines how the electric machine is coupled to the transmission, i.e., at the primary side (engine side) (PRI) or the secondary side (wheel side) (SEC) of the transmission. When it is coupled at the secondary side, a reduction set of gearwheels with a fixed speed ratio is selected, such that the electric machine is able to reach the maximum drive cycle speed (70 km/h for the JP10–15). The efficiency of the reduction set is assumed to be constant at 98%. Note that the start–stop function is still possible with this configuration since the engine is, in principle, restarted with the starter–alternator. Without a starter–alternator, engine restart with an

Fig. 6. Fuel consumption as a function of electric machine size x_3 for different design parameters (x_1 , x_2 , and $x_4 - x_7$).

electric machine coupled at the wheel side requires a relatively large and more costly electric machine due to the transmission losses and relatively low torque multiplier factor [25].

x_5 *Actuation technology for the push-belt CVT*: One of the major power losses of the conventional push-belt CVT (which are on the order of 50% of the total power losses [26]) is caused by the hydraulic pump. Therefore, the influence of applying an EMA system, instead of an HDA system, on fuel consumption is investigated. The EMA power, which is only required during the operation of the push-belt CVT, is assumed to be constant at 90 W. The efficiency of the EMA system is assumed to be constant at 90%, which results in a battery output power demand of 100 W.

x_6 *Regenerative brake strategy (combined with an electric-only vehicle speed threshold value)*: The influence of the electric-only vehicle speed threshold value v_M on the fuel consumption is investigated, in combination with the clutch state (i.e., closed or open) during braking. Depend-

ing on the clutch state, the engine applies “fuel cutoff” (no fuel injection) and has engine drag losses or idle fuel mass flow during regenerative braking.

a) *CLO/EON*: The engine is on at an idle speed of 610 rad/min and disengaged. The engine has an idle fuel mass flow until $v_v(t) = v_M$. However, the engine has no drag losses. The electric machine is operated at the OOL during braking.

b) *CLC/EON*: The engine is on and engaged. The engine has no fuel mass flow (i.e., fuel cutoff) until $v_v(t) = v_M$. However, the engine has drag losses. The drag losses cause reduction in the recuperative brake energy. Notice that the push-belt CVT is controlled, such that the engine output shaft is rotating at the minimum speed (610 rad/min), while the engine drag losses are minimized.

For both cases, the engine is switched off and disengaged during braking only if $v_v(t) < v_M$. The strategies are schematically visualized in Fig. 5. At certain time instants, drive power demand P_v is still positive while the vehicle decelerates (due to relatively large road load losses at relatively high speeds). Brake

Fig. 7. Fuel consumption as a function of the technologies, control strategies, topologies, component sizes, and functions.

energy storage can be done at moments where the road load losses (air drag and roll resistance losses) are not sufficiently enough to decelerate the vehicle as desired.

x_7 *Speed ratio control strategy*: The CVT speed ratio is controlled based on an engine optimal operation (CS1) or system optimal operation (CS2).

In the next section, the effects of parameter variation ($x_1 - x_7$) on the total fuel consumption over a drive cycle Φ_f are investigated.

VI. RESULT

A. Effects of Parameter Variation ($x_1 - x_7$) on Design Objective Φ_f

The responses of design objective (fuel consumption) Φ_f as a function of electric machine size x_3 for the different design parameters x_1 , x_2 , and $x_4 - x_7$ are shown in Fig. 6. Although the response functions show small local oscillations (local minima), the global trend of the response functions shows a convex curvature. The optimal electric machine sizes are easily determined and are depicted by the white circular marks. The relatively fuel consumption values are shown in the figure at the right side. A bar diagram with the fuel consumption at optimal electric machine size x_3^* as a function of the different design parameters is shown in Fig. 7. Observations O, following the parameter variation study, are discussed here.

O.1) A 5-kW electric machine size is sufficient for the EMA push-belt CVT to minimize fuel consumption on the JP10-15. The optimal size is independent of topology choice x_4 , the regenerative braking strategy x_6 , and the CVT speed ratio control strategy x_7 .

Fig. 8. Influence of the electric machine size on the efficiency.

In Fig. 8, the static-efficiency maps of a small and a large electric machine is schematically visualized. The styled OOLs for both electric machines are also depicted. If the electric machine is operated during the electric-only modes at the OOL, then, for a larger electric machine, the desired optimal speed is lower than that for a smaller electric machine, given the same output power. However, the electric machine output torque is higher, which increases the push-belt CVT efficiency.

O.2) The optimal electric machine size is a tradeoff between the average electric machine and the CVT efficiency, which decrease and increase, respectively, with an increase in electric machine size.

The optimal electric machine size for the HDA push-belt CVT is approximately 10 kW. The optimal electric machine size is larger than that for the EMA push-belt CVT due to the higher static losses of the HDA push-belt CVT. For the NEDC, the optimal

Fig. 9. Overview of the relative effects of parameter variation on the fuel consumption.

electric machine size is also 10 kW due to the higher braking powers, compared with JP10–15. For $f_{fb} = 100\%$ (100% BER), the optimal electric machine size is 6 kW due to the increase in maximum braking power and regenerative brake energy on the JP10–15.

- O.3) The start–stop function at vehicle standstill relatively reduces the fuel consumption by approximately 13.8% or 14.3%, which depends on the type of actuation technology (compare test 4 with test 6 and test 5 with test 7 in Fig. 7).
- O.4) The fuel saving increases with increase in the brake distribution fraction. It is observed that, if f_{fb} is increased from 50% to 100% BER at the front wheels, the fuel consumption is relatively decreased by 12.8% (compare test 14 with test 19 in Fig. 7).
- O.5) The vehicle load prescribed by the NEDC increases the fuel consumption by 10%, compared with the JP10–15 (compare test 8 with test 14 of Fig. 7).
- O.6) Hybridization of the drive train with a 5-kW electric machine, which is connected at the engine side of the EMA push-belt CVT with 50% BER, reduces the fuel consumption by approximately 3.5% or 11.3%, depending on the regenerative brake strategy (excluding the start–stop function at vehicle standstill; compare test 7 with test 10 and test 7 with test 14 in Fig. 7).
- O.7) Connecting the electric machine at the wheel side of the push-belt CVT reduces the fuel consumption by 5.1% due to higher transmission efficiency during regenerative braking (compare test 14 with test 17 in Fig. 7).

However, due to the fixed gear ratio between the electric machine and wheels, the electric machine operation points are prescribed by the drive cycle and the vehicle power demand. The operation points are therefore not optimally selected. In addition, the average electric machine efficiency decreases with increase in electric machine size, which again causes a decrease in fuel savings.

- O.8) The EMA push-belt CVT, compared with the HDA push-belt CVT, reduces fuel consumption by 3.1% and 5.4% for the baseline and hybrid vehicles, respectively (compare test 4 with test 5 and test 11 with test 14 in Fig. 7).
- O.9) Fuel cutoff (CLC/EON) during regenerative braking is more beneficial than idling of the engine (CLO/EON). The relative fuel consumption with CLC/EON, compared with that with CLO/EON, is 4.7% lower (compare test 10 with test 13 in Fig. 7).
- O.10) The influence of system optimal operation CS2, compared to engine optimal operation CS1, on fuel saving for the hybrid vehicle is relatively small. Fuel consumption is reduced by 1.3% or 1.1%, which depends on the type of actuation technology (compare test 11 with test 12 and test 14 with test 15 in Fig. 7).

In Fig. 9, an overview of the relative effects of the parameter variation on fuel consumption change is shown. Some of the bars, which are indicated with an (lower)upper boundary value, represent the average values of the performed tests. The upper and lower boundary values represent the maximum and minimum values, respectively. Since the cross correlation

Fig. 10. Battery storage power and battery energy-level difference.

of parameter variation is not investigated, the relative effects that decrease or increase the fuel consumption cannot be summed up. Nonetheless, it can be concluded that potential fuel savings is strongly affected (5%–13%) by the following design parameters ($x_1 - x_4$): the regenerative brake fraction, drive cycle (vehicle load), component size (and start–stop function), and topology.

In Fig. 10, the power-flow control strategies $P_s(t)$, including the battery energy-level difference over time $\Delta E_s(t)$, are shown for the comparison of the HDA–PRI, EMA–PRI, and EMA–SEC push-belt CVTs. The other design parameters, which are kept constant, are shown in the figure. In the diagrams at the bottom of the figure, the strategies for the different regenerative brake fractions are shown. The transmission efficiencies during the electric-only modes are larger for the EMA–PRI and EMA–SEC CVTs, in comparison with the HDA–PRI CVT. Because of this, effectively more brake energy can be used for

electric driving. Furthermore, if brake fraction f_{fb} increases, then the electric machine is more intensively used during the electric-only modes.

In this section, the electric-only vehicle speed threshold value v_M was kept constant at 20 km/h for the different regenerative brake strategies. In the next section, the effects of changing v_M on the fuel consumption for the different brake strategies are investigated.

B. Effects of the Electric-Only Vehicle Speed Threshold Value v_M on Design Objective Φ_f for Different Brake Strategies x_7

In Fig. 11, the contour plots of the fuel consumption for the CS1–EMA–50% BER–PRI–JP1015 as a function of the electric machine size (in kilowatts) and electric-only vehicle speed threshold value v_M (in kilometers per hour) for the CLO/EOFF, CLO/EON, and CLC/EOFF braking strategies are

Fig. 11. Contour plots of the fuel consumption as a function of v_M and the electric machine size for different brake strategies.

shown, respectively. Two observations are made based on these results.

- O.11) The CLO/EOFF strategy has a negligible effect on the fuel consumption improvement, with increase in v_M of larger than 20 km/h. From the optimized control strategy, it followed that propulsion only by the electric machine during acceleration up to vehicle speeds of higher than 20 km/h does not occur.
- O.12) The CLC/EON strategy is preferable over the CLO/EON strategy for electric-only vehicle speed threshold values of lower than 40 km/h. The fuel savings by fuel cutoff and the recuperation of brake energy are effectively larger than the fuel costs of the engine idling during braking, although, with the CLC/EON strategy, the recuperative brake power is reduced.

C. Effects of Changing Topology x_4 and Actuation Technology x_5 on Design Objective Φ_f and Component Efficiencies

Table IV shows the fuel consumption results for the baseline vehicle and the hybrid vehicle equipped with the different CVT

TABLE IV
FUEL-CONSUMPTION RESULTS WITH CS1, 50% BER, AND $v_M = 20$ km/h

CVT technology	Fuel consumption (l/100km) on the JP10-15	
	Base/Hybrid	Relative change %
HDA-PRI	4.78/3.72	100%/79%
EMA-PRI	4.63/3.52	97%/74%
EMA-SEC	4.63/3.34	97%/70%

† 5 kW optimal electric machine size.

actuation technologies. For the baseline vehicle, the battery is not used, i.e., $P_s(t) = 0$. For the hybrid vehicle, the fuel-consumption results are similar to the results shown in Fig. 7. The lowest fuel consumption is achieved with EMA-SEC (see observation O.7).

For the hybrid vehicle, the average component efficiencies and the relative fuel savings as a function of the transmission technologies and topologies are shown in Fig. 12. The secondary power source efficiency, which is denoted as η_s , consists of the battery, electric machine, and transmission efficiency (e.g., the push-belt CVT or the reduction set, including the final drive set) from the battery to the wheels during the electric-only modes (i.e., regenerative braking and propulsion). The decrease in fuel consumption is in correspondence with increase in the

Fig. 12. Effects of the transmission technology and topology on the relative fuel saving, transmission, engine, and secondary source efficiency.

average component efficiencies. Fuel consumption is strongly correlated with secondary power source η_s and transmission efficiency η_t since engine efficiency η_e remains approximately constant.

VII. REDUCED HYBRID DRIVE-TRAIN MODEL

The computation steps (1–3), as discussed in Section V, are quite tedious to carry out. In this section, a reduced hybrid drive train model describing the main component models and the topology is discussed, which can be used for design analysis decoupled from the choice of specific components, hybrid drive train configurations, and control strategy. The idea of a high-level modeling and design framework, as presented in [3], is further investigated. First, the assumptions for simplified modeling of the components and topology are discussed.

- The component efficiencies are modeled as simplified parametric power-based fit functions, where, e.g., an affine relationship between the input and output powers is assumed: $P_{in} = c_1 \cdot \min(P_{max}, P_{out}) + c_0$.

Parameter P_{max} represents the output power limitation of the component. Parameters c_1 and c_0 represent the fit coefficients and correspond to the reciprocal of the internal efficiency and the static losses, respectively. The engine (Primary power source = P) and the electric machine/battery (Secondary power source = S) efficiency, which are similar to the efficiency of CVT (Transmission technology = T), are well described by an affine relationship. The operation points determine the values for the fit coefficients (c_1, c_0), which are also referred to as the characteristic component parameters. The

operations points, given a drive cycle and vehicle parameters, are prescribed by three assumptions.

- 1) The engine is operated over a whole drive cycle at the maximum efficiency points (speed ratio control strategy CS1).
- 2) The electric-only modes (regenerative braking and propulsion only by secondary power source S) over a whole drive cycle are used for S, whereas S is operated at the OOL (i.e., only if S is precoupled to T), and P is shut off (CLO/EOFF).
- 3) The engine-only mode over a whole drive cycle is used for transmission T, whereas engine P is operated at the OOL, and secondary power source S is shut off (assuming no drag or idle losses).

These assumptions are sufficiently accurate for estimating the component efficiencies since 1) the influence of system optimal operation, compared with engine optimal operation, on the fuel-consumption reduction for a hybrid vehicle, is relatively small (1%–2%), and 2) the influence of power exchange between S and P during driving (motor assisting and charging) on the efficiency of S and T is relatively small, as discussed here.

The battery power influences the CVT efficiency. For the hybrid vehicle, the CVT efficiency is decreased, compared with the baseline vehicle due to charging and motor assisting during driving with S. If the transmission input power is corrected with the battery power during the hybrid driving modes (i.e., in this case for charging and motor assisting during driving), then the CVT efficiency for the hybrid vehicle is slightly increased, compared with the CVT efficiency for the baseline vehicle. However, the difference is relatively small. This is observed by comparing the results for T, as listed in Table V.

TABLE V
TRANSMISSION EFFICIENCY η_{CVT} (CS1, JP10-15)

CVT technology	Vehicle	Efficiency model parameters		
		c_1 [-]	c_0 [W]	$\bar{\eta}_{CVT}$ [%]
EMA-PRI	Base	1.04	182	90.3
	Hybrid [†]	1.04	161	92.6
5-kW EM size	Hybrid	0.85	2040	79.4

[†] η_{CVT} corrected with battery power.

Fig. 13. Simplified hybrid drive train model. Component technology (c_0 , c_1), size (P_{max}), and topology models (location of S and P).

In Fig. 13, the reduced hybrid drive-train model for the different component technologies and the power-based fit function is shown schematically.

The effects of parameter variation on the fuel economy and optimal electric machine size x_3 using the reduced model are investigated next. This is performed to investigate if the assumptions, as posed in the beginning of this section, are sufficiently accurate for design optimization. The varied design parameters are regenerative brake fraction x_1 , drive cycle x_2 , and topology x_4 . For different sizes of S (x_3), the characteristic parameters (c_1 , c_0) during the electric-only modes and, for T, during the engine-only mode are calculated for the EMA CVT.

The fuel consumption results for both cycles of the detailed hybrid drive train model, which is denoted as M1, and the reduced model, which is denoted as M2, are both shown in Fig. 14.

The fuel consumption of the detailed model M1 is higher than that calculated with the reduced hybrid drive-train model M2. The differences tend to increase with increase in the electric machine size. The average relative errors and the maximum and minimum error values are shown in Fig. 15. It should be noted that the vehicle mass increases with increase in the electric machine size. This has a relatively small effect on the fuel consumption and is therefore neglected. Moreover, it was found that the function becomes more convex, yet there is no change in the observed optimal electric machine size.

For the detailed model M1, the engine is assumed to be operated at the OOL and therefore determines the operation points of the electric machine during the hybrid driving

Fig. 14. Fuel consumption as a function of the electric machine size. Fuel consumption is calculated using a detailed hybrid drive train model M1 and a reduced hybrid drive train model M2.

modes (i.e., charging and motor assisting during driving). This causes that the efficiency of S to decrease with increase in the electric machine size. This relatively small effect has not been taken into account using the reduced hybrid drive train model M2, which causes the objective function to sometimes not be convex; rather, a boundary optimum is determined at 15 kW. However, in this case, an additional design constraint, which is put on the sensitivity of the objective function to the electric machine size, can be used to find the optimal electric machine size.

A different transmission technology (see [3]) or actuation technology x_5 simply implies a different set of characteristic parameters for T. The same holds for power-split CVTs. For example, if the electric machine size at the wheel side of T is sufficiently large to maintain the transmission ratio coverage yet too small to fulfill the required functions of S, then the electric machine size at the wheel side of T needs to be increased.

For the electrical power-split CVT, the electric machines coupled at the wheel side of T, which perform the electric-only modes, are defined to be functionally part of S. For the power-split CVT, changing the size of S indirectly affects the transmission efficiency. This way, the characteristic parameters for T, which are determined by the engine-only mode, also become a function of the size of S. However, looking at the development of the transmission by Toyota, the increased size of S is mainly determined by performance and not by a reduction in fuel consumption or ratio coverage constraints.

Nevertheless, it can be concluded that the effects of component sizing on fuel consumption can be investigated using the reduced hybrid drive-train model with the assumptions posed at the beginning of this section very quickly and with sufficient accuracy. The maximum average relative error is 1.6%, which is shown in Fig. 15 for a configuration of 100% BER-JP1015-PRI.

Fig. 15. Average relative error and the maximum and minimum error values between the detailed model M1 and the reduced model M2 for the different design parameters.

VIII. CONCLUSION

The optimal electric machine size for a hybrid vehicle equipped with a CVT regarding the minimum fuel consumption was determined. Thereby, the effects of parameter variation, e.g., for the CVT technology and the topology, on fuel consumption were investigated. Other parameters that strongly influence the optimal electric machine size are related to the vehicle load (vehicle parameters and drive cycle) and the regenerative brake fraction. Finally, a reduced hybrid drive-train model with sufficient accuracy (maximum average error < 1.6%) is introduced, with which the influence of component technologies, sizes, and topology choice on the fuel consumption can be studied very quickly.

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