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Citation for published version (APA):

Moreira-Tamayo, O., & Pineda de Gyvez, J. (1995). Time-domain analog wavelet transform in real-time. In *Proceedings of the 1995 IEEE International Symposium on Circuits and Systems, 1995, ISCAS '95, 30 April - 3 May 1995, Seattle, Washington* (Vol. 3, pp. 1640-1643). Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/ISCAS.1995.523724>

DOI:

[10.1109/ISCAS.1995.523724](https://doi.org/10.1109/ISCAS.1995.523724)

Document status and date:

Published: 01/01/1995

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Time-Domain Analog Wavelet Transform In Real-Time

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ABSTRACT

This paper presents a time-domain approach for the implementation and continuous generation of wavelet transform coefficients. The wavelet generation relies on amplitude modulation techniques. This approach offers two extra degrees of freedom through the appropriate use of the modulation index and the selected envelope signal. The added flexibility opens the possibility to create new families of wavelets which are specially suitable for analog implementations. The theoretical fundamentals for the generation of wavelets using the amplitude modulation scheme are presented. Simulated and experimental results for our hardware prototype are presented.

I. INTRODUCTION

Limitations in the Short Time Fourier Transform has motivated the development of the Wavelet Transform (W_ψ), where W_ψ generalizes the kernel of the Fourier Transform. This kernel allow us to have flexible time windows and other oscillatory signals in order to have different resolutions in the time and frequency axis. Different types of basic wavelets (also called mother wavelet) have been proposed [1].

Hardware implementations of wavelets have been scarcely explored; they are mostly based on frequency-domain approaches [2,3,4]. The time domain implementation of the wavelet transform involves the design of wavelet generators, multipliers, and integrators in order to perform the convolution of the signal with the wavelet. Some wavelets are especially suitable for time domain analog implementations, among them are the Meyer wavelets, Spline wavelets, and Gabor wavelets [5]. Other types of wavelets are those based on the window functions for discrete Fourier transform [6] (see Table 1). It will be shown in this paper that these wavelets have the advantage of requiring small hardware for their implementation.

Table 1. Window functions for STFT.

Barlett (triangular)	$\begin{cases} 2t/M & 0 \leq t \leq M/2, \\ 2 - 2t/M & M/2 \leq t \leq M, \\ 0, & \text{otherwise} \end{cases}$
Hanning	$\begin{cases} 0.5 - 0.5 \cos(2\pi t/M), & 0 \leq t \leq M, \\ 0, & \text{otherwise} \end{cases}$
Hamming	$\begin{cases} 0.54 - 0.46 \cos(2\pi t/M), & 0 \leq t \leq M, \\ 0, & \text{otherwise} \end{cases}$
Blackman	$\begin{cases} 0.42 - 0.5 \cos(2\pi t/M) \\ + 0.08 \cos(4\pi t/M), & 0 \leq t \leq M, \\ 0, & \text{otherwise} \end{cases}$

II. WAVELET TRANSFORM COMPUTATION USING AMPLITUDE MODULATED SIGNALS

The wavelet transform is defined as [4]:

$$(W_\psi f)(b, a) := \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-b}{a} \right) dt \quad (1)$$

where ψ is called a "basic wavelet" (also called mother wavelet), ψ^* is its complex conjugate and a is the scaling constant.

Many practical applications involving spectral analysis, require the analysis of the signal in a time interval. Wavelet transforms of the signal can be computed repeatedly using some scaled and shifted version of the basic wavelet. In order to perform these operations a *wavelet chain* is proposed. A wavelet chain consists of a sequence of wavelets generated consecutively one after another. Fig. 1 shows a chain of Spline [5] wavelets. Each wavelet in the chain can be maintained equal or be adjusted to a specific size depending on the application. For instance, in Fig. 1 the wavelets are being successively scaled.

Chains of sinusoidal wavelets can easily be generated by modulating a sinusoidal signal with a lower frequency sinusoidal signal, i.e.,

$$\psi_s(t) := \begin{cases} A_c e^{-j\omega_c t} [1 + m \cos(\omega_p t)], & \text{for } -\pi < \omega_p t < \pi, \\ 0, & \text{elsewhere} \end{cases} \quad (2)$$

where A_c is a reference magnitude of the wavelet, $2\pi\omega_p$ is the time window size, ω_c is the frequency being analyzed and m is the modulation index. It can be seen from the limits imposed by eq. 2, that the wavelet has a finite support. This wavelet is suitable for time domain analog implementations since it is formed by sinusoidal signals which can be generated with simple circuits. Moreover, the envelope signal can be changed to a triangular signal (Barlett window [6]) which also can be easily implemented by integrating square signals.

The modulation index adds an extra degree of freedom as different time windows can be computed using the same expression by varying only m . For example, if the modulating index m is 1 or 23/27 we have wavelets based on the Hanning window or Hamming window respectively (see Table 1).

Figure 1. Chain of wavelets.

III. IMPLEMENTATION PROCEDURE

A wavelet function is generated by multiplying two periodic functions, that give origin to a modulated "analyzing" signal. The wavelet is then multiplied by the signal to be analyzed, $f(t)$, and integrated. The final result is a convolution of the signals. The integration to be performed in hardware is:

$$(W_s f)(a, b) = \int_{l_i}^{l_f} f(t) \cdot \left[g\left(\frac{t-b}{a}\right) \cdot v\left(\frac{t-b}{a}\right) \right] dt \quad (3)$$

where $g(t)$ is the analyzing signal and $v(t)$ is a periodic window signal used as envelope in the modulation process. The limits l_i and l_f are selected to have the same limits that the time window establishes.

For the sinusoidal wavelet (see eq. 3), we have that the analyzing function $g(\cdot)$ specifies a complex sinusoidal consisting of two orthogonal functions expressed as follows

$$g(t) = e^{-j\omega_c t} = \cos(\omega_c t) + j \sin(\omega_c t) \quad (4)$$

Hence, the wavelet transform can be separated in a real part and a imaginary part as follows

$$(W_s f)(a, b) = \int_{l_i}^{l_f} f(t) \cos(\omega_c t) \frac{1}{2} [1 + m \cos(\omega_p t)] dt + j \int_{l_i}^{l_f} f(t) \sin(\omega_c t) \frac{1}{2} [1 + m \cos(\omega_p t)] dt \quad (5)$$

The limits of integration, l_i and l_f , are taken from the envelope function for any two zero consecutive points. This case occurs when the cosine function reaches a minimum, e.g., $v(t) = 0$ if $m = 0$. The integration process is carried out independently for each wavelet in the chain of wavelets, and the wavelet coefficient is obtained just before the resetting occurs.

IV. HARDWARE IMPLEMENTATION

Fig. 2 shows the block diagram for the system implementation. In order to generate orthogonal signals (90° phase shift) with integer multiple frequencies, the signals are generated using a pulse signal (master clock) as the time reference. The master clock, is divided using two different flip-flops, one of them triggered by the positive edge and the other by the negative edge. These two signal are filtered to pass only the fundamental signal, producing two orthogonal sinusoids. The envelope can also be generated by dividing the master clock and filtering it afterwards, specially when short wavelets are required. The next step is to add a DC component according to the modulation index desired to compose the windowing signal. The modulation index is equal to the magnitude of the sinusoidal divided by the DC component. Then the two sinusoids are multiplied by the envelope. For our prototype we used the analog multiplier MC1494. At this point the chain of wavelets similar to those in Fig. 1 has been generated. Notice that by changing the frequency of the master clock the shape of the wavelet is scaled proportionally, but if this happens the amplitude of the wavelet A_c and the filters' cut off frequency need to be readjusted in order to have a scaled version of the same wavelet.

The wavelet signals are then applied to a convolution section. The wavelets are multiplied by the signal to be analyzed using analog multipliers and then integrated. In this case two MC1494 were used for this purpose.

For the integration section the circuit shown in Fig. 2 was used. The integrating elements are the OTA and the capacitor C_1 . The OTA was chosen because it has the advantage of being electronically programmable, i.e., its transconductance is a function of the current I_g . If the wavelet is scaled, a different integration constant may

Figure 2. Implementation scheme.

be wanted. The integrator is resetted right after the end of each wavelet. Using a pulse ϕ_1 the transistor M_1 resets the capacitor C_1 . Many applications require interfacing the circuit with a digital system, therefore, a sample and hold circuit is needed. The buffer, transistor M_2 and C_2 perform this function. The pulses that reset the capacitors (ϕ_1 and ϕ_2) are two pulses, their duration must be sufficient to fully discharge the capacitors. The pulse ϕ_2 is used to trigger ϕ_1 , i.e., the output is sampled first, and then the integrator is resetted. The pulse ϕ_2 is triggered by a timed signal produced by a comparator circuit. A delayed version of the envelope signal is compared with a threshold voltage producing a square wave. The threshold for the comparator is adjusted to produce a positive flank just before the wavelet ends. This flank triggers an astable multivibrator that produces ϕ_2 , i.e., the sampling pulse.

V. SIMULATION RESULTS

The first part of the simulation consists of illustrating the response of the system to different input signals. Fig. 3 shows the response of the system to three sinusoidal signals. Fig. 3.a shows the sinusoidal wavelet used, it has a duration of 5 ms and an analyzing frequency of 2 KHz. Fig. 3.b shows the input signals of frequencies 2.0, 2.05 and 2.1 KHz. Fig. 3.c shows the output voltage of the circuit at capacitor C_1 of the real part. That is, the result of the integration. The final value, i.e., before it is resetted, corresponds to the real part of the wavelet coefficient. Notice that the integration curve 1 reaches 1 V before it is resetted. That is because the input sine wave is in phase with the real part of the analyzing signal. As the input signal frequency departs from 2.0 KHz the output becomes smaller reaching zero at 2.1 KHz. In this last case the output is imaginary and its magnitude is 0.92 V. (not shown).

Fig. 3. Time response of a sinusoidal wavelet. (a) Wavelet 2 KHz. (b) Input signals. (c) Real part of WT.

For the time delay of the system one has to consider that a discrete wavelet transform is being computed (not discrete-time wavelet transform) since a coefficient is obtained only at the end of the integration period. Therefore, to compute the time response characteristic, a set of delayed wavelets were generated. Using the same wavelet length of 5 ms, a coefficient is obtained at the end of this period. Therefore the maximum delay is also 5 ms. To have the time-response plot with 100 points we generated 100 wavelets, each one was delayed 0.5 μ s with respect to the previous one. The wavelet coefficients were obtained for a sinusoidal signal with a starting point t_0 , i.e., this signal is zero before the starting point, the purpose is to find the experimental delay of the system. The result of this computation will indicate how long after this point the system is able to capture the sine signal. Fig. 4 shows the results. Notice that the signal has reached its maximum after 4ms. This can be considered the practical delay of the circuit.

Fig. 4. Time response of the sinusoidal transform.

VI. EXPERIMENTAL RESULTS

The test consisted in applying different signals to find their wavelet coefficients. The wavelet used for this case-study has an analyzing frequency $\omega_c = 3700$ Hz, and the envelope has a frequency is $1/16$ of the inner frequency, i.e., $\omega_p = 3700\text{Hz} / 16 = 231.25$ Hz (The analyzing frequency is a integer multiple of the envelope). Fig. 5 shows the results of analyzing a pure sinusoidal signal at the exact same frequency (3700 Hz) of the wavelet but with $+90^\circ$ phase shift with respect to the real part. Figures 5.a and 6.a show the product of the wavelet and the input signal. Figures 5.b and 6.b show the results of the integration. As expected, the result of integrating the signal in Fig. 5.a is zero, and the result of integrating Fig. 6.a is -0.72V which corresponds to the imaginary part of the wavelet coefficient. Since the real part is zero, the magnitude of this wavelet coefficient is 0.72V and the phase is $\pi/2$.

Fig. 5. Real part of WT. (a) Multiplication of the wavelet and the input signal. (b) WT Coefficient.

VII. CONCLUSION

A systematic approach to generate wavelets in time domain was developed. Starting from this approach, a new family of wavelets, namely Amplitude Modulated Wavelets, was developed. A wavelet transform circuit was implemented in hardware. The results were suitable for those applications that require fast computation of wavelet coefficients. By using a bank of these circuits a spectral decomposition can be achieved with small computation time.

Figure 6. Imaginary part of WT (a) Multiplication of the wavelet and the input signal. (b) WT Coefficient.

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