Studies on quasi-2D turbulence—the effect of boundaries

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Abstract
This paper addresses the effects of domain boundaries on the behaviour of quasi-two-dimensional flows, thereby distinguishing between lateral boundaries of horizontal flow domains and the horizontal boundaries confining shallow fluid layers. As already discussed in some recent papers, the lateral walls may play an essential role in acting as sources of filamentary high-amplitude vorticity, which usually affects the flow evolution in the interior of the domain. Besides, walls exert forces that may promote the self-organization of the flow, and hence contribute to a change in the net angular momentum of the flow. Both aspects will be reviewed briefly.

In contrast to what is commonly assumed, shallow-layer flows may develop an essentially three-dimensional structure, with vertical gradients in the principal horizontal flow field and with significant vertical velocity components. These features have been found recently in experiments on electromagnetically generated flows in a shallow fluid layer. In this paper, we will discuss some of these experimental observations as well as some numerical simulation results that are helpful in explaining the observed flow dynamics.

1. Introduction

Due to the absence of vortex stretching and tilting, two-dimensional (2D) turbulence behaves essentially differently from its 3D counterpart: it is characterized by a spectral transfer of kinetic energy to larger length scales, usually referred to as the ‘inverse energy cascade’. Phenomenologically, the action of this inverse cascade is observed in the emergence of coherent vortices, in particular well visible in slowly decaying 2D turbulent flows. The vortices thus emerging spontaneously from an initially random vorticity distribution may take the form of monopolar or dipolar structures, and even tripolar vortices have been observed to
emerge (see e.g., McWilliams 1984, Santangelo et al. 1989). This fascinating self-organization property of (mildly forced or decaying) 2D turbulence forms a remarkable contrast with the commonly observed behaviour of 3D turbulent flows. During the last few decades the dynamics and spectral characteristics of 2D turbulence—both forced and decaying—have received a lot of attention, and have been studied both theoretically and by numerical simulation. A recent review of work on this topic is given by Clercx and van Heijst (2009). In the majority of earlier studies on 2D turbulence any effects of lateral boundaries were ignored.

In addition to the theoretical and numerical studies, the (quasi-) 2D turbulence dynamics has also been studied experimentally in carefully designed laboratory experiments. In order to create conditions in which the flow component in one direction is suppressed, such experiments utilized density stratification (e.g., Maassen et al. 1999, 2002, 2003, Maassen 2000), background rotation (e.g., Hopfinger et al. 1982) or the geometrical confinement as a means of creating quasi-2D flows. In the latter category, experiments were conducted in soap films (e.g., Couder 1984, Kellay et al. 1995, Goldburg et al. 1997, Rivera et al. 1998, Shakeel and Vorobieff 2007) and in shallow fluid layers (e.g., Sommeria 1986, Tabeling et al. 1991, Dolzhanskii et al. 1992, Clercx et al. 2003, Wells and Afanasyev 2004, Boffetta et al. 2005, Rivera and Ecke 2005, Shats et al. 2005, 2007). Evidently, in laboratory experiments the flow is essentially bounded laterally by solid walls imposing a no-slip condition, and which may hence affect the flow evolution. For example, it has been observed that the final, organized state of slowly decaying quasi-2D flow depends on the shape of the fluid container (e.g., Clercx et al. 1999, Schneider and Farge 2008, Keetels et al. 2008, 2009). Apart from imposing an impermeability condition, solid no-slip walls also exert normal and tangential stresses that directly influence the flow: these forces may increase the net angular momentum of the flow and hence promote the self-organization process. Besides, they also imply high-amplitude vorticity present in thin boundary layers that may be peeled off from the walls, leading to injection of filamentary vorticity into the interior as discussed, e.g., by van Heijst et al. (2006).

In particular in the shallow-layer experiments (in either one-layer or two-layer configurations), the bottom of the fluid container plays a crucial role. In most studies it is tacitly assumed that the flow in this configuration behaves in a quasi-2D fashion, i.e. a principal horizontal motion with some weak vertical gradient. Recent experimental and numerical studies have shown, however, that shallow-layer flows may not at all be Poiseuille-like, see e.g. Sous et al. (2004, 2005) and Akkermans et al. (2008a, b). Such three-dimensionality may have a profound effect on the evolution of the principal horizontal flow component.

In this paper, we will review some aspects of the presence of solid no-slip walls, as encountered by definition in laboratory experiments. Section 2 provides an overview of the effects of lateral walls on evolving 2D flows. Section 3 presents some recent experimental and numerical studies on flows in a shallow-layer fluid, with a special focus on their 3D structure. Finally, some conclusions are drawn in section 4.

2. The effect of lateral boundaries

Decaying 2D turbulence is characterized by the emergence of larger coherent vortex structures, as a result of both the inverse energy cascade, according to which energy shows a spectral shift to larger scales, and the so-called selective decay mechanism, according to which smaller flow structures decay faster than those on larger scales (see Matthaeus and Montgomery 1980). This phenomenon was clearly demonstrated in various numerical simulations, see e.g. McWilliams (1984) and Santangelo et al. (1989). Most of the earlier flow simulations were performed on a double-periodic, square domain. In a numerical study of decaying 2D turbulence on a circular domain, it was shown by Li and Montgomery (1996)
and Li et al (1997) that the boundaries lead to a different decay scenario, somewhat depending on the imposed boundary condition (stress-free or no-slip). Additional support for their observations is found in recent high initial Reynolds number simulations, see Schneider and Farge (2005) and Keetels et al (2008, 2009). A similar effect of solid lateral boundaries was observed in the numerical study by Clercx et al (1999) of decaying 2D turbulence on a square domain, with either stress-free or no-slip conditions. Solid walls may exert a net torque (associated with normal and shear stresses) on the fluid, thus generating a nonzero angular momentum, which is often associated with a large domain-filling circulation cell. In some studies on forced 2D turbulence on bounded periodic domains this circulation cell is referred to as a Bose–Einstein condensate, an analogy put forward by Kraichnan (1967). Evidence of the emergence of this condensate regime has been provided by numerical simulations, see e.g. Smith and Yakhot (1993). Observation of this condensate regime in quasi-2D turbulence in laboratory experiments (in bounded domains) has been claimed by Paret and Tabeling (1998) and Xia et al (2008).

We will now review some observations on the behaviour of 2D turbulence on a square domain with no-slip walls, both for the decaying case (section 2.1) and for the case in which the flow is forced continuously (section 2.2). In addition, we will briefly address the effect of the domain shape (section 2.3), by comparing the flow evolution on a square domain with those observed in circular, elliptic and rectangular geometries.

2.1. Decaying 2D turbulence

The typical evolution of a decaying quasi-2D flow in a confined domain is nicely illustrated by laboratory experiments that were carried out in a square container with dimensions $100 \times 100 \times 30$ cm$^3$ (length $\times$ width $\times$ depth). In most cases, the tank was filled with a two-layer fluid, consisting of a layer of fresh water on top of a layer of salty fluid. The interfacial layer between these usually had a thickness of typically a few centimetres, as a result of some mixing and diffusion of salt.

The fluid was set in motion by horizontally traversing a linear grid of vertical rods at constant speed $V$ through the fluid. This grid was traversed from one side of the tank to the opposite side, and was then withdrawn vertically, thus leaving the fluid motion to evolve. In the homogeneous upper and lower layers, the turbulence introduced by passage of the grid was essentially 3D, and hence showed a relatively quick decay. In contrast, the motion in the interfacial region was suppressed in the vertical (through the action of density stratification) and was soon observed to become planar. Although not exactly 2D, the motion in this thin interfacial layer showed self-organization, in the sense that the irregular wake flow soon became organized in the form of layered vortical structures, which tend to fill the domain entirely. This behaviour of the flow, visualized by adding small white tracer particles illuminated in a proper way, is nicely observed in figure 1.

Although in the case of a square tank the ‘organized’ state consists of usually one large cell accompanied by a somewhat smaller one (see e.g. Clercx et al 1998, Maassen et al 2002), experiments in a rectangular tank have revealed that the flow in the later stages of the flow evolution takes on the form of a linear array of counter-rotating cells, their number being directly related to the length–width aspect ratio of the container (see Maassen et al 2003). Similar organization behaviour was observed when the experiment was carried out in a homogeneous fluid, but now in a rotating system. Stirring by the linear grid would result eventually in a number of columnar vortices (in agreement with the Taylor–Proudman theorem) that fill the horizontal cross-section of the container in a similar way as in the stratified non-rotating experiments.
Figure 1. Sequence of streak images of a laboratory experiment in a square tank filled with a stratified fluid. The fluid motion was initialized by towing a rake horizontally through the fluid from one side to the opposite side. The images are taken from 10 s until 55 min after towing the rake through the fluid. The initial state (at \( t = 10 \) s) is characterized by \( L_0 \approx 0 \). The tails of the streaks are generated after digital processing of the images (courtesy of Maassen 2000).

Numerical simulations of the evolution of 2D flow within a square (or rectangular) domain with no-slip walls have clearly demonstrated the role played by the boundaries during the flow evolution (see Clercx (1997) for details of the numerical simulation). Close inspection of the vorticity distribution in the flow field has revealed (in particular when comparing with stress-free and double-periodic runs) that a substantial number of intense vorticity filaments occur throughout the flow evolution, see figure 2.

The no-slip walls imply the presence of boundary layers with high concentrations of vorticity. When a vortical structure approaches, this boundary layer—containing opposite vorticity—may be peeled off from the wall, and advected away in the form of a filament, thus affecting the interior flow even at larger distances from the wall. This phenomenon demonstrates the role of the no-slip walls: they are sources of high-amplitude vorticity, usually seen in the form of intense filaments that are advected away, into the interior. It will be clear that this behaviour is very likely to destroy to some extent the picture that has been obtained for ‘unbounded’ 2D turbulence on a double-periodic domain, e.g. by the numerical simulations by McWilliams (1984) and Santangelo et al (1989). Since in many cases the vorticity filaments are wrapped around the vortices (which are usually the primary cause for their formation), the shielded character of these vortices will henceforth affect their interaction behaviour with neighbouring vortex structures to some extent. Hence, it can be expected that the spectral characteristics vortex statistics of the evolving 2D turbulent flow in a no-slip confinement will be different from that in the unbounded case. This has been confirmed by Clercx and van Heijst (2000) and Clercx and Nielsen (2000) for decaying
Figure 2. Sequence of vorticity plots showing the evolution of a randomly initialized 2D flow field on a square domain with no-slip walls. The numerical simulation was carried out with a spectral code (see Clercx 1997) for Re = 5000.

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The role played by the solid boundaries also becomes quite clear from the observed behaviour of the total angular momentum, \( L \), of the fluid measured with respect to the centre of the domain, as shown graphically in figure 3. The graph displays the scaled angular momentum \( L^* = |L/L_{sb}| \) measured in a number of experiments, where the scaling quantity \( L_{sb} \) represents the angular momentum of the same amount of fluid with the same total kinetic energy \( E(t) \) as the fluid in the experiment, but assumed to be in solid-body rotation. The observations were done for two sets of different experiments: one in which the initial angular momentum \( L_0 \) introduced by the rake traverse was zero (symmetric rake configuration), and one in which some angular momentum was introduced in the fluid (\( |L_0| \neq 0 \)) by applying an asymmetric grid configuration. For details, the reader is referred to Maassen et al (2002).

The graph shows the remarkable change in time of the normalized angular momentum to clearly nonzero (either positive or negative) values. This behaviour (termed ‘spontaneous spin-up’, see Clercx et al 1998) indicates another striking effect of the solid domain boundaries: the increase in the absolute value of \( |L(t)/L_{sb}| \) is brought about by the action of normal and shear stresses at the walls that may exert a net torque on the fluid. The laboratory observations (later supplemented by high-resolution numerical flow simulations) demonstrated that the increase of net angular momentum is significantly enhanced when some

turbulence simulations with Reynolds numbers up to 20 000 (based on the root-mean-square (rms) velocity of the initial flow field, the half-width of the container, and the kinematic viscosity). Obviously, this is a direct manifestation of the presence of no-slip walls.
initial amount $L_0 \neq 0$ is introduced. In these particular experiments, one observed that the self-organization process was speeded up in the latter case, resulting in a relatively quick formation of a central cell.

It should be stressed that the feature of spontaneous spin-up is typical for flows on a square domain with no-slip (or even stress-free) walls, again pointing out the crucial role played by such walls. Spontaneous spin-up is not restricted to 2D hydrodynamics as a similar phenomenon has recently been reported for decaying MHD turbulence in bounded domains, see Bos et al. (2008).

2.2. Continuously forced 2D turbulence

The behaviour of a confined 2D flow when stirred continuously was studied by numerical simulations. The stirring was modeled by adding a stochastic forcing term to the Navier–Stokes equation. In fact, the simulations were based on the vorticity equation, which is derived by taking the curl of this equation.

The simulations were performed for a square domain with no-slip walls, and some results are reported in Molenaar et al. (2004). For a certain forcing intensity the normalized angular momentum $L^* = L/L_{sb}$ was observed to show a remarkable behaviour, as displayed in figure 4: this integral quantity revealed sign changes at irregular intervals. While interpreting this signal $L^*(t)$, one should keep in mind that the magnitude of the forcing term in the governing vorticity equation was in these simulations found to be at least two orders of magnitude smaller than the other terms in this equation, implying that the wall effects (torques by normal and shear stresses) are dominant in the evolution of $L(t)$. Inspection of the vorticity distribution $\omega$ during the time evolution of the flow gives the key to this flipping between states of positive and negative $L^*$, see Molenaar et al. (2004). Earlier experiments in a shallow layer of mercury revealed a similar flipping between states of positive and negative large-scale circulation, see Sommeria (1986). However, the role of the lateral no-slip sidewalls as a source of strong small-scale vorticity that might destabilize the central cell was completely disregarded. Like in the unforced, decaying case the self-organization tendency of the 2D turbulent flow may result in an organized state, consisting of a large central cell that fills the square domain to a considerable extent, corresponding to a nonzero value of the net angular momentum $L$. At
Figure 4. Evolution of the normalized angular momentum $L^* = L/L_{sb}$ in a numerical simulation of randomly forced 2D flow in a square tank, revealing distinct features of repeated spin-up and breakdown of rotational motion (Molenaar et al 2004).

this stage, the filamentary vorticity activity, originating at the solid no-slip walls, may start to erode this larger flow cell at its edge. While the erosion continues, it may even lead to a complete destruction of the organized state, giving the flow an irregular appearance, with approximately zero net angular momentum. The self-organizing mechanism usually results in the relatively quick formation of a new domain-filling cell of either sign, as can be seen in the sudden increase of the net angular momentum, see figure 4.

It is clear that the no-slip walls play a crucial role in this continued process of build-up, erosion, breakdown and again build-up of the central cell, as the erosion is set in by the vorticity filaments originating at the walls. This is nicely illustrated by the direct relation between the total enstrophy $Z(t) = \frac{1}{2} \int \omega^2 \, dA$ in the flow domain and the (non-normalized) angular momentum $L(t)$ or the total kinetic energy $E(t)$ contained in the fluid, as shown graphically in figure 5. Apparently, once the cell is formed (large $|L|$-value), the enstrophy $|Z|$ soon increases as well, hinting at the increased enstrophy production at the no-slip walls at that stage. The flipping behaviour of $L^*(t)$ shown in figure 4 was observed for a particular forcing amplitude (for a given dissipation rate). It may be expected that the sequence of events will be different when the forcing amplitude $A$ is changed. For example, a weaker forcing usually leads to an organized state of a central cell, which is hardly affected by the weak filamentary activity at its edge and hence hardly shows any signs of erosion. On the other hand, stronger random forcing will prevent the formation of an organized state of a single cell. In later studies by Molenaar et al (2005, 2007), it was found how the organized flow state of a large domain-filling cell with little vortices in the corners changes when the forcing amplitude $A$ is gradually increased (stationary forcing was used). First, the reflection symmetry is broken when the cell becomes elliptical, accompanied by two larger corner cells and two smaller eddies in the opposite corners. Further increase of the forcing amplitude $A$ gives rise to unsteady motion, initially simply periodic, but with additional higher frequencies when $A$ is further increased. Power spectra show that an increasing number of higher harmonics (peaks) are excited when $A$ increases, until the flow has become chaotic. Further details of this transition to chaos are discussed by Molenaar et al (2005, 2007).

2.3. The effect of the domain shape

In the preceding sections we have considered decaying and forced (quasi-) 2D turbulence on a square domain, which seems to be the canonical geometry for this type of flow. This is largely due to the doubly periodic square domain used in the early numerical simulations,
which is commonly believed to mimic the unbounded flow domain. As discussed, the presence of lateral boundaries (either double-periodic, stress-free or no-slip) has a profound influence on the flow evolution: the lateral walls act as sources of filamentary vorticity structures and besides, they generally exert normal and shear stresses that may promote the self-organization process and lead to ‘spontaneous spin-up’. In view of this, the question emerges: how does the shape of the domain affect the flow evolution? As mentioned in the beginning of section 2, the numerical simulations by Li and Montgomery (1996) for the case of a circular domain have revealed the crucial role of the boundary conditions, by comparing stress-free and no-slip runs. With a nonzero initial angular momentum \(|L_0| > 0\) in the flow, the final state usually consists of a single domain-filling cell. This behaviour agrees with observations by Maassen et al. (1999, 2002) in experiments on decaying stratified turbulence in a circular container and confirmed in numerical studies by the same authors and by Keetels et al. (2009). The flow in these experiments was initialized by moving a rake of vertical bars through the stratified fluid along a straight diametrical path. A net amount of angular momentum could be introduced in the fluid by using an asymmetric grid configuration, as in the experiments described in section 2.1. Since normal (pressure and viscous) stresses at the circular wall do not produce any net torque relative to the tank centre, it can be expected that the wall effect in terms of change of \(L\) will be rather modest compared with the square configuration. In particular, spontaneous spin-up is virtually absent. Nevertheless, the laboratory experiments have revealed that the flow rapidly evolves towards a large, single cell structure (see

Figure 5. Time evolution plots of the total kinetic energy \(E(t)\) (solid line) and the absolute (non-normalized) angular momentum \(L(t)\) (broken line), in the upper graph, and the total enstrophy \(Z(t)\), in the lower graph, for the same run as shown in figure 4 (Molenaar et al 2004). For clarity of exposition, the curve representing the angular momentum has been shifted 2 units upwards.
Maassen et al. (2002). In terms of the vorticity: the final state in these experiments consists of a central cell surrounded by a band of oppositely signed vorticity.

In contrast, experiments with $L_0 \approx 0$ reveal the formation of a large dipolar cell, which slowly moves towards the wall. Vorticity residing in the boundary layers at the no-slip wall is subsequently advected away from the wall in the form of two filaments, which lead to the formation of two vorticity patches behind the dipole, giving the flow at this stage a quadrupolar appearance. Then, the ‘primary’ dipole becomes weaker, while the second becomes more pronounced. The newly formed dipole then starts to push against the wall like this original did, and the process described above is repeated. In fact, no clear quasi-steady ‘final’ state is reached in these experiments. The observed behaviour agrees with findings of Li and Montgomery (1996), Schneider and Farge (2005, 2008) and Keetels et al. (2009), obtained from numerical simulations of decaying 2D turbulence on a circular domain. Recently, Keetels et al. (2009) have analysed the quasi-stationary final states in the circular configuration in terms of a minimum-entrainment principle.

In order to better understand the strikingly different behaviour of the angular momentum during the flow evolution in the square versus the circular domain, Keetels et al. (2008) have studied the decaying flow in an elliptical geometry by numerical simulations. By continuously changing the eccentricity of the ellipse, the role of the normal (pressure and viscous) stresses in the flow evolution could be examined. In the circular case (eccentricity $= 1$) their contribution to a net torque on the fluid is by definition zero, and no spontaneous spin-up occurs. For eccentricity values larger than some critical value, strong and rapid spin-up was observed in all the ensemble runs. For smaller eccentricity values—between 1 and the critical value—spin-up events were encountered, but not in all runs. In this regime, small differences in the initial flow condition usually result in a completely different flow evolution and hence in a different final state.

In the case of a long rectangular domain, the evolution of the decaying flow is essentially different from that in the circular or square geometries: the final quasi-stationary state consists of a linear array of counter-rotating flow cells that fill the domain completely. Such organization patterns were observed in experiments in a rotating fluid (van Heijst et al. 1990) and in a stratified fluid in a rectangular container (Maassen et al. 2003). Although the no-slip walls exert forces that influence the flow evolution, in the rectangular geometry they do not lead to generation of net angular momentum. This is of course directly connected with the geometrical limitation of the rectangular tank, which naturally accommodates a linear array of cells rather than a single cell.

3. The effect of vertical confinement of shallow-layer flows

During the last two decades quite a few experimental studies on quasi-2D flows have been conducted in shallow-layer configurations, both in single- and two-layer fluids. It is commonly assumed that these shallow flows behave in a quasi-2D manner because the vertical length scale is much smaller than the horizontal length scale of the flow domain. The no-slip condition imposed by the solid bottom of the fluid container would then result in vertical gradients of the principal horizontal flow component. This effect has been parametrized by adding a linear friction term to the 2D Navier–Stokes equation, which is usually referred to as the ‘Rayleigh friction’ term, see e.g. Juettner et al. (1997), Danilov et al. (1996) and Clercx et al. (2003).

Flows in this shallow-layer configuration can be conveniently generated by applying electromagnetic forcing: by placing a magnet underneath a layer of electrolyte through which an electrical current is flowing, the Lorentz force induced by the combined magnetic and
Figure 6. Instantaneous velocity fields of a dipolar vortex in a horizontal plane at $z = 5 \text{ mm}$. Vectors represent horizontal velocity components and shades/colors indicate the magnitude of the vertical velocity. The circle denotes the position of the disc-shaped magnet. Experimental results obtained with SPIV at (a) $t = 0.50 \text{ s}$, (b) $t = 0.96 \text{ s}$ and (c) $t = 1.50 \text{ s}$. Numerical results at (d) $t = 0.50 \text{ s}$, (e) $t = 0.96 \text{ s}$ and (f) $t = 1.50 \text{ s}$. Experimental parameters: layer depth $H = 9.3 \text{ mm}$, electrical current $I = 4.4 \text{ A}$, sodium chloride solution 15% Brix, magnetic field strength near magnet $\approx 1 \text{T}$ (Akkermans et al 2008b).

electrical fields sets the fluid in motion locally. When using a flat, disc-shaped magnet underneath the thin tank bottom, one thus creates a dipolar vorticity structure that moves away from the forcing area. By applying a large array of magnets, with alternating polarity, one may thus generate a large collection of vortices that soon start to interact, giving the flow an irregular, turbulent appearance. This array configuration has been used by Tabeling et al (1991), Boffetta et al (2005) and Rivera and Ecke (2005) and more recently also by Xia et al (2008), who used an even larger array of $30 \times 30$ magnets.

In order to ‘shield’ the fluid motion somewhat from the no-slip bottom, in some studies a two-layer stratification was used in which the motion was generated in the upper fluid layer. Flow measurements in the upper layer have been used to derive spectral and other characteristics of the supposedly 2D turbulent flow (e.g. Paret et al 1999, Shats et al 2005, 2007, Tabeling et al 1991).

In recent studies by Akkermans et al (2008a, 2008b) on a single dipolar vortex, electromagnetically forced by applying a single disc-shaped magnet underneath the electrolytic fluid layer, it was found that this generic flow structure may contain and further develop significant 3D components. This is clearly illustrated by figure 6, which shows the evolution of the horizontal flow field as well as the vertical velocity in the horizontal cross-sectional plane at mid depth in the fluid layer. The upper row shows experimental results, obtained with stereo-PIV (for details about the experimental setup and the measurement technique, see Akkermans et al 2008a, 2008b). The flow was forced during a period of
$\Delta t = 1 \text{s}$, after which the electrical current was switched off at $t = 1 \text{s}$. The horizontal flow field is visualized by the black velocity vectors, while the colour coding indicates the vertical velocity component.

Apart from some noisy patches, the colour distribution in figure 6 clearly indicates that two regions of downward motion develop during the forcing stage (figures 6(a) and (b)). At the end of the forcing stage (figure 6(b)), upward motion is observed in the tail of the dipole structure. The same structures—but less noisy—are visible in the lower-row panels (d)–(f) in figure 6, representing results obtained by a numerical flow simulation carried out with a finite-element code (see Akkermans et al. 2008a, 2008b). The downward motion in the dipole cores is not driven by the vertical component of the Lorentz force, but due to a vertical pressure gradient in the horizontal swirling flow in each core. During the forcing in each swirl a cyclostrophic balance is established between the centripetal horizontal pressure gradient and the centrifugal force. Because the forcing is stronger closer to the magnet, the swirling motion is less intense higher up in the fluid layer, thus implying a vertical pressure gradient in each swirl region, with the lowest pressure near the bottom. As a result, a secondary, downward motion arises in each dipole core as long as the flow is forced.

As soon the forcing is stopped, this downward motion in the dipole cores changes into upward flow, see figures 6(c) and (f). Once the forcing has been switched off, no swirl is maintained in the lower part of the fluid layer. The continuing descending flow implies radial spreading near the bottom, and because of conservation of angular momentum ($\sim rv_\theta$) this implies a decrease in the swirl velocity $v_\theta$. This spin-down in the lower part of the fluid layer occurs on a time scale (here $\sim 1.5 \text{s}$) that is much smaller than the typical bottom friction timescale $\tau_E = 2H^2/\pi^2 \nu$ (with $H$ the layer depth and $\nu$ the kinematic fluid viscosity), which is approximately 18 s in this case. As a result of the spin-down, the pressure in the vortex core increases near the bottom, which starts to drive a vertical upward flow in the core of the swirling cell. It should be noted that this flow behaviour is independent of viscosity, but is entirely due to the vertical confinement of the flow.

As time progresses, the vertical flow in the dipole’s vortex cores shows an oscillatory behaviour, as clearly visible in the subsequent set of snapshots, see figure 7. These (inertial) oscillations result from an overshoot of the decelerated vertical motions in the cores, according to the mechanism just described. Under the conditions of the experiments reported here, two to three oscillations are observed, depending on the forcing strength.

Rather intense vertical motions are also seen to be present along the dipole axis and in an arc-shaped band at the front of the dipole. The double structure of upward and downward motion in this curved band suggests a ‘frontal circulation’ taking place in the form of a curved roll in front of the dipole, very similar to what has been observed by Sous et al. (2004).

It is to be noted that the numerical simulation results (figures 7(d)–(f)) are in good agreement with the experimental flow measurements, see figures 7(a)–(c). Since the numerical simulations were carried out with a flat stress-free upper surface, the good correspondence implies that free-surface deformations (dimples, waves) do not play a significant role in the flow evolution studied here.

Although initially the vertical vorticity $\omega_z$ of the dipole structure is organized in two coherent patches of opposite polarity, soon after the electromagnetic forcing is stopped, the vorticity distribution becomes fragmented, taking an irregular appearance.

Akkermans et al. (2008a, 2008b) have also carried out numerical simulations with stress-free conditions at both the bottom and the free surface. It was found that the flow evolution is very similar to that for the no-slip bottom. Evidently, the initial $z$-dependence of the flow as induced by the electromagnetic forcing is the cause of the further development of the 3D structure of the flow, and not the no-slip bottom per se. In the experiments and numerical
Figure 7. Continuation of figure 6 for the flow during the post-forcing phase, but with a slightly different viewing area. Experimental results at (a) $t = 1.50 \text{ s}$, (b) $t = 2.00 \text{ s}$ and (c) $t = 2.60 \text{ s}$. Snapshots of numerical simulation at (d) $t = 1.50 \text{ s}$, (e) $t = 2.00 \text{ s}$ and (f) $t = 2.60 \text{ s}$ (Akkermans et al. 2008b).

Simulations reported by Akkermans et al. (2008a, 2008b) the ‘degree of three-dimensionality’ of the dipolar flow structure has been quantified in a number of different ways, e.g. by calculating the ratio of the kinetic energies associated with the vertical and horizontal velocity components, and by calculating the horizontal divergence. All these quantities reveal a substantial three-dimensionality of the dipole flows, both during and after the forcing. Since the vortex dipole can be considered as a prototype flow structure, it is to be expected that more complicated flows in the shallow fluid layers will also show 3D features to some extent.

Recent laboratory experiments on shallow two-layer flows, with a non-conducting bottom layer, have revealed a flow evolution similar to that shown in figures 6 and 7, i.e. with substantial vertical velocities both during the forcing and the post-forcing stages. This behaviour has also been observed in numerical simulations of such two-layer fluid configurations, as will be reported by Akkermans et al. (2009).

In a related study, Cieślik et al. (2009a) have investigated the collision of a shallow-layer vortex dipole against a solid vertical lateral boundary, both experimentally and numerically. This work has revealed that the 3D structure of the dipole vortex becomes even more complex during the collision, thus providing evidence that the lateral domain boundaries further contribute to deviations from 2D behaviour.

Shallow-layer configurations with electromagnetic forcing by a regular $n \times n$ array of magnets have been used by, e.g., Tabeling et al. (1991), Clercx et al. (2003), Rivera and Ecke (2005), Shats et al. (2005, 2007) and Xia et al. (2008), aiming at experimental verification of (spectral) characteristics and vortex statistics of 2D turbulence. Under the assumption of two-dimensionality, the vortices induced by the magnets would interact and gradually give rise to larger coherent vortex structures, as illustrated for example by the numerical simulations.
by McWilliams (1984). Recent experiments by Cieślik et al (2009b, 2009c) on shallow flows driven electromagnetically by a regular array of $10 \times 10$ magnets have revealed a different flow evolution; however, in the post-forcing stage the flow shows large-scale meandering structures rather than vortices. This is clearly observed in the streak photographs presented in figure 8: during the forcing (figure 8(a)) the flow is organized in a regular array of $10 \times 10$ counter-rotating cells, but some time after the forcing has stopped large meandering currents are visible throughout the flow domain (figure 8(b)). Stereo-PIV measurements in horizontal cross-sectional planes in the fluid layer have revealed the rather complex 3D structure of the flow, both during and after the forcing. Figure 9 shows the evolution of the vertical vorticity $\omega_z$ as well as the instantaneous ‘flow line’ pattern of the horizontal flow component measured in a plane at $h = 5$ mm above the bottom. These ‘flow lines’ are defined as curves of which the tangent directions indicate the direction of the horizontal component of the 3D flow velocity vector in that particular horizontal plane. Since this horizontal part of the flow field does not constitute an incompressible flow, these lines are generally not closed and may extend over the whole area. The plots only cover a limited part of the domain, somewhat less than $3 \times 3$ magnets. At the end of the forcing stage ($t = 0$ s, see figure 9(a)), one clearly observes a number of dipolar vortex structures. Owing to their self-propulsion these dipoles translate, which quickly leads to head-on collisions all over the domain. During the frontal collisions the dipoles exchange partners, resulting in two new couples that propagate away from the collision area in directions perpendicular to the original dipole axes (see figure 9(b)). At this stage one observes that the vorticity patches become fragmented, with even oppositely signed vorticity formed within the dipole cores (see figures 9(b) and (c)).

The newly formed dipoles propagate over a short distance until they collide frontally with neighbouring dipoles (see figure 9(c)). The dipolar structures emerging after this second set of collisions are usually somewhat asymmetric, leading to the formation of larger-scale flow structures. This is clearly seen in figure 9(c), in particular in the shape of the instantaneous flow lines. The symmetry of the flow is progressively broken as time goes on, see figures 9(c) and (d). At this stage the flow line pattern reveals meandering current structures extending over the whole measurement domain, while the vorticity is observed to become organized in elongated filaments rather than in vortices. Moreover, the vorticity filaments tend to be aligned with the flow lines.

A set of corresponding plots of the vertical velocity $w$ measured at the same level $h = 5$ mm is displayed in figure 10. Throughout the evolution significant vertical motion is observed. Just before the first set of dipole collisions (figure 10(a)), regions of pronounced upward flow are present at the front of each dipole, as well as at its rear; regions of downward motion are seen in the dipole cores. This is in agreement with the vertical flow structures found in single dipole vortices in a shallow fluid layer (Akkermans et al 2008a, 2008b, Sous et al 2004). The maximum value of the vertical velocities between two approaching dipoles is typically $20 \text{ mm s}^{-1}$, while the rms value of the horizontal flow components is $28 \text{ mm s}^{-1}$, indicating that locally the flow is fully 3D.

Regions of upward and downward flow remain visible in the next stages (see figures 10(b) and (c)), i.e. when the vortex structures are interacting, and even in the stage when any symmetry is lost and the flow has taken the appearance of larger meandering structures (figure 10(d)). The measurements reveal that the strongest vertical motion is associated with downward rather than upward flow. The regions of downward motion tend to be correlated with the meandering flow structures, while the weaker upward motion is mainly observed inside vortical structures.

Numerical simulations have been carried out with a finite-element code, based on the Navier–Stokes equation including a realistic representation of the electromagnetic forcing.
A no-slip condition was prescribed at the bottom, while a stress-free condition was applied at the free surface, which was taken to be flat. These simulations have revealed that the flow throughout the post-forcing stage is essentially 3D, containing locally substantial vertical velocities as well as a vertical structure that is not Poiseuille-like, see Cieślak et al (2009b, 2009c). The numerical simulation results show good agreement with the experimentally observed flow evolution.

Apparently, shallow flows generated under the conditions of the experiments reported by Akkermans et al (2008a, 2008b, 2009) and Cieślak et al (2009b, 2009c) do not behave in a quasi-2D fashion, as is commonly assumed in experimental shallow-flow studies related to 2D turbulence, see e.g. Tabeling et al (1991), Danilov et al (2002) and Shats et al (2005, 2007). Of course, by applying a two-layer fluid configuration (as the latter authors do) the upper layer is somewhat shielded from the no-slip bottom, by which the vertical gradients are
Figure 9. Vertical vorticity component $\omega_z$ (colour, in s$^{-1}$) and instantaneous flow lines representing the horizontal flow components in a horizontal plane 5 mm above the bottom, measured at $t = 0$ (a), 1 s (b), 2 s (c) and 5 s (d) in the experiment of figure 8, for a limited field of view (Cieślak et al 2009b).

Figure 10. Vertical velocity $w$ (colour, in mm s$^{-1}$) and instantaneous flow lines representing the horizontal flow components in a horizontal plane 5 mm above the bottom, measured at $t = 0$ (a), 1 s (b), 2 s (c) and 5 s (d) in the experiment of figure 8, for a limited field of view (Cieślak 2009).

Recent high-resolution stereo-PIV measurements in such two-layer configurations by Akkermans et al (2009), however, have revealed that significant vertical motions do still occur in the upper layer, so that the shielding by the lower layer thus has a limited effect.

4. Discussion

Obviously, the presence of solid domain boundaries has a profound effect on the flow evolution. In the case of even purely 2D confined flow, solid lateral boundaries imply forces that may promote the self-organization process. The normal and tangential stresses exerted by the solid walls may result in a net torque on the fluid, which may lead to a rapid, spontaneous spin-up of the contained fluid. In the case of compact domains (circular, square) this is observed in the rapid emergence of a single domain-filling flow cell. On the other hand, no-slip walls act as sources of filamentary vorticity structures, which are advected into the flow interior, thus influencing the flow evolution and its spectral characteristics. As was demonstrated for the case of forced 2D turbulence on a square domain with no-slip walls, these filamentary vorticity structures may even cause erosion of the central flow cell in the organized state, eventually leading to its complete breakdown. After the collapse of this organized flow state, a new domain-filling cell may emerge due to the self-organizing tendency of the 2D flow, with the same or with opposite sense of rotation. This process of build-up and breakdown of the cell may be repeated, depending on the forcing strength. All these cases illustrate the subtle effects introduced by the solid domain boundaries.

During the last two decades quite a few experiments have been carried out in shallow-layer flows, with the purpose of studying the characteristics of quasi-2D turbulence. It was found recently, however, that such flows do not behave in a quasi-2D fashion, i.e. as planar flows with a Poiseuille-like structure in the vertical, in order to satisfy the no-slip bottom condition. High-resolution stereo-PIV measurements on single dipole vortices generated by electromagnetic forcing have revealed rather complicated 3D structures and substantial vertical motions in the shallow fluid layer. These experimental observations have been confirmed by detailed numerical flow simulations.

In the case of single dipoles these 3D effects are seen in oscillating upwelling/downwelling flows in the vortex cores, upwelling in the tail of the dipole, and a frontal circulation roll ahead of the translating dipole. Forcing with a large array of magnets—often applied when attempting to generate a turbulent flow field—is under certain conditions found to lead to large meandering flow structures rather than to the emergence of larger coherent vortices, as predicted by the theory of 2D turbulence. Again, this behaviour is due to the 3D structures present in the shallow-layer flow, whose origin lies in the presence of the horizontal boundaries.

Although substantial progress has been made in understanding the effects of solid boundaries on the evolution of confined 2D flows and on the development of the 3D structure of shallow flows, the properties of tracer transport in such flow configurations are still not well understood. A careful study of tracer dispersion in (quasi-)2D flows in the presence of solid boundaries forms an important topic of future investigations.

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