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An analytical model for the illuminance distribution of a power LED

Hongming Yang,[1,2,*] **Jan W. M. Bergmans,**[1,2] **Tim C. W. Schenk,**[2]
Jean-Paul M. G. Linnartz[1,2] **and Ronald Rietman**[2]

[1]*Department of Electrical Engineering, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands.*
[2]*Philips Research Eindhoven, High Tech Campus 37, 5656 AE Eindhoven, the Netherlands.*

h.m.yang@tue.nl

Abstract: Light-emitting diodes (LEDs) will play a major role in future indoor illumination systems. In general, the generalized Lambertian pattern is widely used as the radiation pattern of a single LED. In this letter, we show that the illuminance distribution due to this Lambertian pattern, when projected onto a horizontal surface such as a floor, can be well approximated by a Gaussian function.

OCIS codes: (150.2950) Illumination; (230.3670) Light-emitting diodes.

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1. Introduction

Due to the rapid development of solid state lighting technologies, light-emitting diodes (LEDs) will largely replace incandescent and fluorescent lamps in future indoor illumination systems. An LED based illumination system may consist of a large number, e.g., hundreds or even thousands, of spatially distributed LEDs with narrow beams. The reason for this large number mainly lies in the fact that a single state-of-the-art LED [1], which can produce a luminous flux

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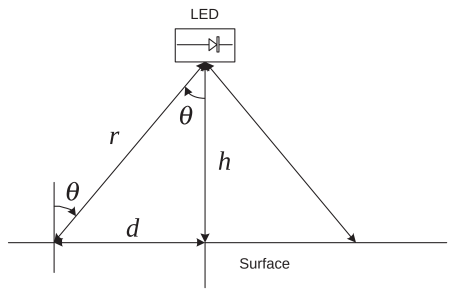

Fig. 1. LOS path geometry between an LED and a flat surface.

of 200 lumen, still cannot provide sufficient illumination for an indoor environment, where an illuminance of about 400 – 1000 lux (lumen per m^2) is normally needed. An appealing feature of such a system with narrow beam LEDs is that it can provide localized, colorful, and dynamic lighting effects, especially because the intensity level of each LED can be easily changed.

For such an illumination system, in order to optimize the intensity levels of all the LEDs to achieve certain desired lighting effects, it is essential to have an accurate model for the illuminance distribution of a single LED. In particular, in this letter, we consider the lighting effect rendered on a flat surface, e.g., the floor, by a single LED, assuming the symmetry axis of the LED's radiation pattern to be perpendicular to the floor. More specifically, a two-dimensional (2D) model for the illuminance distribution is proposed.

In the literature, e.g [2, 3, 4, 5], as well as in the datasheets of actual LED products, e.g., [6], various radiation patterns of the LEDs are provided as functions of the observation angle with respect to the LEDs. One of the most widely used patterns is the generalized Lambertian pattern [2]. In this letter, by contrast, we provide a 2D analytical model, as a function of the location on the floor, for the lighting pattern due to a single LED. More specifically, based on the generalized Lambertian pattern, we provide a simple yet accurate analytical model of the lighting effect on the floor due to a single LED.

The emitted light from an LED propagates through free space, illuminating a target location, e.g., the floor. The free space optical channel in principle consists of the line of sight (LOS) path and diffuse reflections. In this paper, we focus on the modeling of the illuminance distribution due to the LOS path. The optical power from diffuse reflections is known, by a good approximation, to be uniformly distributed and to be much smaller than that from the LOS path [7], and therefore is neglected in this paper.

The proposed model for the illuminance distribution is presented in Section 2. Section 3 concludes this letter.

2. Illuminance distribution

Figure 1 depicts the geometry of an LED and an illuminated location with a flat surface, where r is the distance between the LED and the illuminated location, the projection of r onto the flat surface has length d, and h denotes the vertical distance between the LED and the flat surface. The polar angle of the location with respect to the LED is denoted by θ, and the angle of light incidence on the location is clearly equal to θ.

Thus, from the generalized Lambertian pattern, the illuminance, i.e., the optical power per unit area, at the location is a function of d or equivalently θ. For convenience in describing the illuminance distribution on a flat surface at a distance h, we write it as a function of d, denoted

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by $f_{\mathrm{L}}(d)$,

$$f_{\mathrm{L}}(d) = \frac{m+1}{2\pi} f_0 \cos^m(\theta) \frac{\cos(\theta)}{r^2} = \frac{(m+1) f_0}{2\pi h^2} \left(1 + \frac{d^2}{h^2}\right)^{-\frac{m+3}{2}}, \tag{1}$$

where f_0 is the total illuminance, m is the Lambertian mode number and $m > 0$. The mode number is a measure of the directivity of the light beam and is related to the semiangle of the light beam at half power, denoted by $\Phi_{1/2}$, by $m = -\ln(2)/\ln(\cos(\Phi_{1/2}))$ [2]. Therefore, a larger m corresponds to a narrower beam. Commercially available LED lenses can shape the beam of the Lambertian-type LEDs into $\Phi_{1/2} = 10^{\mathrm{o}}$ to $\Phi_{1/2} = 5^{\mathrm{o}}$ [8, 9, 10], which correspond to $m = 45$ and $m = 181$, respectively. Hence, for the sake of convenience in this paper, we focus on the range from $m = 25$ to $m = 200$.

2.1. Gaussian approximation

For the sake of analytical conveniences and tractability when discussing the illumination effects of multiple LEDs, we would like to use an approximate model for the actual $f_{\mathrm{L}}(d)$. For instance, in [11], the two-dimensional (2D) Fourier transform is used as $F_{\mathrm{L}}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\mathrm{L}}(x,y)\exp(-j2\pi(ux+vy))\mathrm{d}x\mathrm{d}y$, where $f_{\mathrm{L}}(x,y)$ is obtained by writing $f_{\mathrm{L}}(d)$ into the 2D Cartesian coordinate system (x,y) through the relation $d^2 = x^2 + y^2$. The analytical form of $F_{\mathrm{L}}(u,v)$ for an integer m can be obtained as

$$F_{\mathrm{L}}(u,v) = \begin{cases} \frac{(-2)^{m/2} f_0 h^{m+1}}{(m-1)!!} \left(\frac{\partial}{\partial\xi}\right)^{m/2} \left[\frac{1}{\sqrt{\xi}} \exp(-2\pi\sqrt{\xi}\sqrt{u^2+v^2})\right]\Bigg|_{\xi=h^2}, & \text{if } m \text{ is even} \\ \frac{(-2)^{(m+1)/2} f_0 h^{m+1}}{(m-1)!!} \left(\frac{\partial}{\partial\xi}\right)^{(m+1)/2} \left[K_0(2\pi\sqrt{\xi}\sqrt{u^2+v^2})\right]\Bigg|_{\xi=h^2}, & \text{if } m \text{ is odd} \end{cases} \tag{2}$$

where $m!!$ denotes the double factorial and $K_0(\cdot)$ is the modified Bessel function of the second kind. We can see that it is cumbersome to evaluate the values of $F_{\mathrm{L}}(u,v)$ for a large integer m. Moreover, to our best knowledge, there is in general no analytical expression of $F_{\mathrm{L}}(u,v)$ for a non-integer m. Therefore, we are particularly interested in the approximation models that can potentially bring convenience in the analysis of illumination effects by multiple LEDs. More particularly, in this paper, we propose a Gaussian approximation of Eq. (1).

It can be observed from Eq. (1) that the value $f_{\mathrm{L}}(d)$ at $d = 0$ is the largest, and decreases as d increases. Moreover, for an illumination effect, the human visual system tends to focus on the bright region rather than the background. Hence, we start from $d = 0$ and approximate the rate of decrease in $f_{\mathrm{L}}(d)$.

We take the derivative of Eq. (1) with respect to d and get

$$f_{\mathrm{L}}'(d) = \frac{(m+1) f_0}{2\pi h^2}\left(-\frac{m+3}{2}\right)\left(1 + \frac{d^2}{h^2}\right)^{-\frac{m+5}{2}} \frac{2d}{h^2} = -\frac{(m+3)d}{d^2+h^2} f_{\mathrm{L}}(d). \tag{3}$$

When d is small compared to h, i.e., $d^2 << h^2$, $d^2 + h^2 \approx h^2$. Hence

$$f_{\mathrm{L}}'(d) \approx -\frac{(m+3)d}{h^2} f_{\mathrm{L}}(d) \propto -d \cdot f_{\mathrm{L}}(d), \tag{4}$$

which is a property that defines the Gaussian function. This motivates us to approximate $f_{\mathrm{L}}(d)$ as a Gaussian function. The approximation error in $f_{\mathrm{L}}'(d)$ can be obtained as

$$\Delta f_{\mathrm{L}}'(d) = -\frac{m+3}{h^2} \frac{d^3}{d^2+h^2} f_{\mathrm{L}}(d). \tag{5}$$

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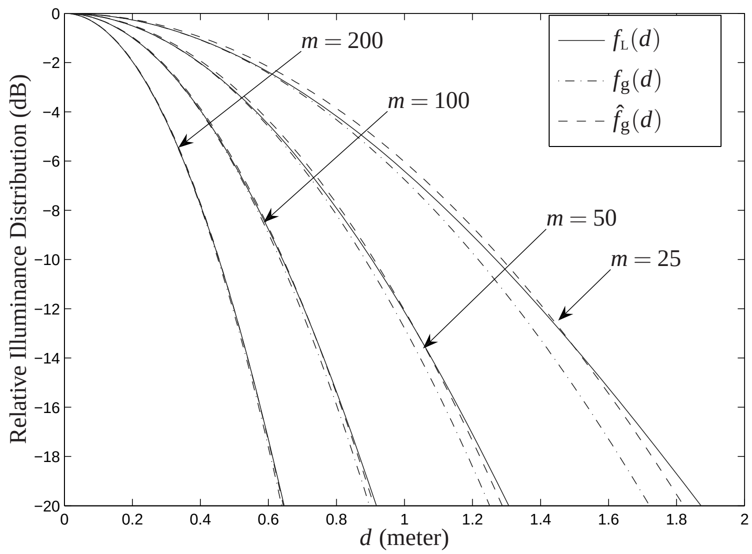

Fig. 2. The illuminance distribution at $h = 3$ meter due to a single LED.

From Eq. (5), the approximation error remains small even when d gets larger, since $f_L(d)$ decreases quickly, especially when m is large, with the increase of d (see Fig. 2).

Next, we derive the key parameters in the Gaussian approximation of $f_L(d)$, denoted by $f_g(d)$. Let $f_g(d) = \frac{c}{\pi\sigma^2}\exp\left\{-\frac{d^2}{\sigma^2}\right\}$, where σ^2 is the variance and c is a normalization factor. Thus, the derivative of $f_g(d)$ with respect to d is

$$f_g'(d) = -\frac{2d}{\sigma^2} f_g(d). \tag{6}$$

Comparing Eq. (4) and Eq. (6), we get $\sigma^2 = \frac{2h^2}{m+3}$. Further, letting $f_L(0) = f_g(0)$, we get $c = f_0\frac{m+1}{m+3}$. Thus we have

$$f_g(d) = \frac{(m+1)f_0}{2\pi h^2}\exp\left\{-\frac{m+3}{2}\cdot\frac{d^2}{h^2}\right\}. \tag{7}$$

As an example, the comparison between $f_L(d)$ and $f_g(d)$ is illustrated in Fig. 2 for the case $h = 3$ meter and for different m. The illuminance at every d is normalized by the value at $d = 0$, i.e. the curves shown in Fig. 2 are actually $10\log_{10}\frac{f_L(d)}{f_L(0)}$ and $10\log_{10}\frac{f_g(d)}{f_g(0)}$. Here, we look at the numerical data on a logarithmic scale, since human eyes perceive brightness logarithmically, which property is known as Weber's law. Further, the range of relative illuminance is considered to be between 0 and -20 dB. This range is taken because illuminance levels below -20 dB are no longer visible to human eyes [12] when one is focused on the center part of the light pattern. It can be seen that the Gaussian approximation is very accurate when m is large, i.e., when the light from the LED is quite focused. The difference between $f_L(d)$ and $f_g(d)$ is slightly larger for a smaller m, e.g., there is a 1 dB difference for $m = 50$ at $d = 1.2$m.

The difference between $f_L(d)$ and $f_g(d)$ can be explained as follows. Comparing Eq. (1) and Eq. (7), we observe that the approximation we make is actually $(1+\frac{d^2}{h^2})^{-\frac{m+3}{2}} \approx \exp(-\frac{m+3}{2}\frac{d^2}{h^2})$,

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or, $-\frac{m+3}{2}\ln(1+\frac{d^2}{h^2}) \approx -\frac{m+3}{2}\frac{d^2}{h^2}$ on the logarithmatic scale. Through the approach of Taylor expansion, we know

$$-\frac{m+3}{2}\ln\left(1+\frac{d^2}{h^2}\right) = -\frac{m+3}{2}\frac{d^2}{h^2} + \frac{m+3}{2}\left(\frac{1}{2}\frac{d^4}{h^4} - \frac{1}{3}\frac{d^6}{h^6} + \mathscr{O}\left(\frac{d^8}{h^8}\right)\right). \tag{8}$$

Hence in the above Gaussian approximation, we take only the first term in Eq. (8). Moreover, from Fig. 2, the range of d of interest is $0 \leq d < h$. In this range, $-\frac{m+3}{2}\ln(1+\frac{d^2}{h^2})$ is larger than $-\frac{m+3}{2}\frac{d^2}{h^2}$, since the second term in Eq. (8) is larger than zero. Therefore we get $f_{\mathrm{L}}(d) > f_{\mathrm{g}}(d)$, as shown in Fig. 2. The difference between $f_{\mathrm{L}}(d)$ and $f_{\mathrm{g}}(d)$, as can be seen from Eq. (8) as well as Fig. 2, increases with d, resulting a larger mismatch in the tail of the illuminance distribution.

Now, in order to compensate for this difference, we propose another Gaussian approximation, denoted by $\hat{f}_{\mathrm{g}}(d)$, with a slightly larger variance $\hat{\sigma}^2 = \frac{2h^2}{m}$, i.e.

$$\hat{f}_{\mathrm{g}}(d) = \frac{(m+1)f_0}{2\pi h^2}\exp\left\{-\frac{m}{2}\cdot\frac{d^2}{h^2}\right\}, \tag{9}$$

which is also depicted in Fig. 2. It can be seen that, in general, $\hat{f}_{\mathrm{g}}(d)$ provides a better fit of $f_{\mathrm{L}}(d)$, and yet has the benefit of a simpler expression than $f_{\mathrm{g}}(d)$. Equivalently from the Taylor expansion, see Eq. (8), the approximation error is now compensated by $\frac{3}{2}\frac{d^2}{h^2}$. Note that here we only proposed a simple yet effective compensation for the Gaussian model. The discussion on the optimum compensation for $f_{\mathrm{g}}(d)$, which might exist for a given range of d and certain criterion of optimality, is however beyond the scope of this paper.

As introduced in the beginning of this section, the Gaussian approximation is proposed in this paper for analytical conveniences when computing the 2D Fourier transform. The illuminance distribution functions considered in this paper, namely $f_{\mathrm{L}}(d)$, $f_{\mathrm{g}}(d)$ and $\hat{f}_{\mathrm{g}}(d)$, are circularly symmetric. Therefore, we can easily obtain the equivalent expressions for these functions as $f_{\mathrm{L}}(x,y)$, $f_{\mathrm{g}}(x,y)$ and $\hat{f}_{\mathrm{g}}(x,y)$ in the 2D Cartesian coordinate system. Henceforth, the 2D Fourier transform can be applied to these functions, resulting in $F_{\mathrm{L}}(u,v)$, $F_{\mathrm{g}}(u,v)$ and $\hat{F}_{\mathrm{g}}(u,v)$, respectively. For the Gaussian approximations, $F_{\mathrm{g}}(u,v)$ and $\hat{F}_{\mathrm{g}}(u,v)$, we can get the analytical expressions as $F_{\mathrm{g}}(u,v) = f_0\frac{m+1}{m+3}\exp\left(-\frac{2\pi^2h^2}{m+3}(u^2+v^2)\right)$ and $\hat{F}_{\mathrm{g}}(u,v) = f_0\frac{m+1}{m}\exp\left(-\frac{2\pi^2h^2}{m}(u^2+v^2)\right)$ for any $m > 0$, no matter m is an integer or a non-integer. In order to evaluate the performances of the Gaussian approximations in terms of Fourier transform, we present some numerical results in Fig. 3. Here, we again look at the numerical data on a logarithmic scale by evaluating $10\log_{10}\frac{F_{\mathrm{L}}(u,v)}{F_{\mathrm{L}}(0,0)}$, $10\log_{10}\frac{F_{\mathrm{g}}(u,v)}{F_{\mathrm{g}}(0,0)}$ and $10\log_{10}\frac{\hat{F}_{\mathrm{g}}(u,v)}{\hat{F}_{\mathrm{g}}(0,0)}$, respectively. Moreover, we focus on an integer m such that we can numerically compute $F_{\mathrm{L}}(u,v)$ using Eq. (2). Furthermore, due to the symmetric property between u and v, and for the sake of convenience, we only show the values of the Fourier transform as a function of u at $v = 0$. It can be seen that both $F_{\mathrm{g}}(u,v)$ and $\hat{F}_{\mathrm{g}}(u,v)$ give good approximations of $F_{\mathrm{L}}(u,v)$. The accuracy of the approximations is higher for a larger m, i.e. when a light beam is narrow. Furthermore, $\hat{F}_{\mathrm{g}}(u,v)$ is closer to $F_{\mathrm{L}}(u,v)$ when $F_{\mathrm{L}}(u,v)$ is large, e.g. $10\log_{10}\frac{F_{\mathrm{L}}(u,v)}{F_{\mathrm{L}}(0,0)} > -10$ dB, where the major part of the signal energy lies.

2.2. *Impact of diffuse light*

In above discussions, we focus on the LOS path. In practice, light also propagates through one or more diffuse reflections to arrive at some location. Due to the nature of diffuse reflections, the light contribution from these non-LOS paths is almost uniformly distributed over the area of a room. A min-to-max variation in the illuminance of less than 3 dB is observed in the literature [13]. Moreover, the total received power from diffuse reflections is much smaller than

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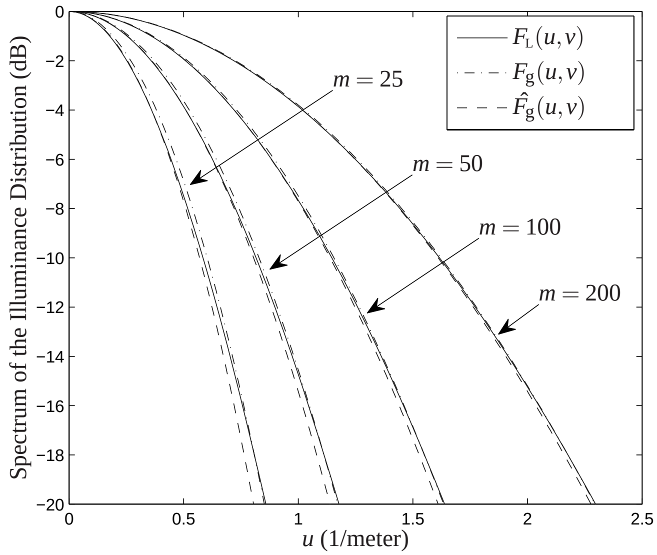

Fig. 3. The numerical values of the Fourier transforms, $F_{\mathrm{L}}(u,v)$, $F_{\mathrm{g}}(u,v)$ and $\hat{F}_{\mathrm{g}}(u,v)$, as function of u, at $h = 3$ meter, and $v = 0$.

that from the LOS path. In [7], a 10-20 dB difference is observed between the power from the diffuse paths and that from the LOS path at the center of the radiation beam. Since we focus on the illuminance distribution due to the LEDs with narrow beams, diffuse light mostly has to undergo at least two reflections before arriving at the location, unless the LED is located very close to a wall or other objects. Therefore the path loss is even higher and we can treat the effect of diffuse light reflections on illumination rendering to be negligible.

3. Concluding remarks

In this letter, we show that the illuminance distribution on a flat surface by a single LED with a generalized Lambertian radiation pattern can be well approximated by a Gaussian function. The approximation error is negligible for the LED with a narrow beam width, e.g. 10 $^{\mathrm{o}}$ to 5$^{\mathrm{o}}$. In addition to the analytical Gaussian model obtained, we also provide a modified Gaussian model which gives a better fit of the actual illuminance distribution. An application for this Gaussian model is that we can efficiently analyze the illuminance distributions for the illumination system consisting of a large number of LEDs.

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