Positioning System Mass Reduction due to Exchange of Structural Stiffness by Additional Actuators

model studies of beam and plate systems

A.M. van der Wielen
POSITIONING SYSTEM MASS REDUCTION DUE TO EXCHANGE OF STRUCTURAL STIFFNESS BY ADDITIONAL ACTUATORS

MODEL STUDIES OF BEAM AND PLATE SYSTEMS

PROEFSCHRIFT

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Summary

Positioning system mass reduction due to exchange of structural stiffness by additional actuators: model studies of beam and plate systems

Electromechanical positioning systems with high demands on accuracy are conventionally developed according to 'design rules'. Stiffness is obtained by the mechanical construction, actuators realize the desired motions in the different direction(s), sensors measure as close as possible at the location where position accuracy is needed and finally control realizes the preferred closed loop behaviour. In order to reach high stiffness the mechanical design can result in relatively heavy constructions. This leads to inherent limitations in the performance of the positioning system, e.g. as a result of limited actuation power. It was suggested that the application of lightweight constructions provided with additional actuators/sensors ("over-actuation") could offer significant mass reduction.

The configuration of the system, that is the spatial distribution of the required elements, affects the requirements of the above mentioned individual elements and thus the mass. The mass of the mechanical structure depends among others on the geometry. However, the optimum geometry depends on the configuration of the overall system. Despite the importance of the individual elements, mainly the effect of the configuration on the mass is researched in this thesis. When more parallel actuators are used, the mechanical stiffness, and thus the structural mass can be reduced. The reduced mass of the mechanical construction also implies that less force is required to accelerate the mechanical construction. The additional mass of the actuators required for proper operation, should however also be taken in account. The total moving mass of the actuators, which is attached to the mechanical construction, will change when an increasing number of actuators is used. The initial apparent mass reduction of the mechanical construction, as result of the additional actuators, might not always be present when the mass-effect of the actuators and other elements, such as the guidance, is also taken in account.

Different aspects of positioning systems with parallel actuators are studied and explained using a theoretical positioning system, where a beam is used as product carrier, and another, more realistic positioning system where a plate is used as product carrier. The performance is kept constant while the minimal required total moving mass is studied as function of the number of actuators.

and the location of the actuators. Product carrier mass aspects, such as material properties are researched in a static acceleration field. In addition sandwich structures that consist of metal foam as core material are considered as product carrier. After studying actuator and guidance aspects of the positioning system the dynamical behaviour is analyzed, with a focus on the transient behaviour.

The dynamical behaviour of the positioning system is the result of the complex interaction between the control-actuator system and the structural stiffness and mass distribution of the product carrier and guidance. Due to the complexity only two extreme cases are studied. In one case the applied control and guidance and actuator mass do not affect the dynamics of the product carrier. In the other case the location of the guidance and actuator mass and the control affect the dynamics to a large extent, and as result nodes of vibration modes are forced at the actuator locations. The same dynamical effects are studied when over-actuation is applied. The effect on the dynamics as result an increased number of parallel actuators and their locations are verified with experiments.

As result of an increased number of parallel actuators the structural mass can be reduced. Besides the structural mass, required to obtain stiffness, also mass is present that does not always reduce when the number of actuators is increased. The relation between the number of actuators and the mass depends among others on the configuration. In case the configuration requires a certain airgap in the Lorentz actuators the effective force of the actuators might decrease when smaller forces are required. The total mass of the elements present besides the structural mass, at a certain moment even increases due to over-actuation. Due to these effects an optimum in the number of actuators exists that results in the lightest positioning system. This optimum depends on the positioning task which implies: stroke, positioning time and a specified tolerance. The mass and stiffness of the product itself and the interaction with the positioning system affect the required number of parallel actuators as well, so does the initial ratio between structural mass and connected mass.

With the results of this research it is possible to determine the effect on the moving mass as result of an increased number of parallel actuators. Especially in the initial phase of the design process, thus even before optimizing the different elements, this knowledge is very useful to select an optimum configuration of the positioning system.
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## Symbols, acronyms and definitions

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<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>(a)</td>
<td>acceleration</td>
<td>(\text{m} \cdot \text{s}^{-2})</td>
</tr>
<tr>
<td>(A_m)</td>
<td>cross section of the magnet</td>
<td>(\text{m}^2)</td>
</tr>
<tr>
<td>(A_{\text{wire}})</td>
<td>cross sectional area of the wire</td>
<td>(\text{m}^2)</td>
</tr>
<tr>
<td>(\hat{a})</td>
<td>nodal acceleration direction vector (column vector)</td>
<td>[-]</td>
</tr>
<tr>
<td>(b)</td>
<td>width</td>
<td>(\text{m})</td>
</tr>
<tr>
<td>(B_{\text{air}}(r))</td>
<td>magnetic flux density in the airgap</td>
<td>(\text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-1})</td>
</tr>
<tr>
<td>(B_c)</td>
<td>magnetic flux density at radius (r_i + t_{\text{coil}}/2)</td>
<td>(\text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-1})</td>
</tr>
<tr>
<td>(B_m)</td>
<td>magnetic flux density in the magnet</td>
<td>(\text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-1})</td>
</tr>
<tr>
<td>(c)</td>
<td>stiffness</td>
<td>(\text{N} \cdot \text{m}^{-1})</td>
</tr>
<tr>
<td>(d_v)</td>
<td>viscous damping</td>
<td>(\text{N} \cdot \text{s} \cdot \text{m}^{-1})</td>
</tr>
<tr>
<td>(E)</td>
<td>Young's modulus of elasticity</td>
<td>(\text{N} \cdot \text{m}^{-2})</td>
</tr>
<tr>
<td>(f_{\text{fill}})</td>
<td>fill factor of the coil</td>
<td>[-]</td>
</tr>
<tr>
<td>(F)</td>
<td>force</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{act}})</td>
<td>effective force of an actuator</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{eff}})</td>
<td>effective force of the positioning system</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_q)</td>
<td>force amplitude of the (q)-th harmonic</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{EL}})</td>
<td>actuator force left actuator</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{ER}})</td>
<td>actuator force left actuator</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{EC}})</td>
<td>actuator force central actuator</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_L)</td>
<td>Lorentz force</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{L max}})</td>
<td>maximum Lorentz force of a given actuator</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{min}})</td>
<td>minimum required force</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(F_{\text{min}})</td>
<td>minimal required force for positioning</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(\hat{F})</td>
<td>nodal force vector ((\hat{F} \cdot \hat{F}))</td>
<td>(\text{N})</td>
</tr>
<tr>
<td>(h)</td>
<td>height</td>
<td>(\text{m})</td>
</tr>
<tr>
<td>(h_{\text{air}})</td>
<td>height of the airgap</td>
<td>(\text{m})</td>
</tr>
<tr>
<td>(H_{\text{air}})</td>
<td>magnetic field strength in the airgap</td>
<td>(\text{A} \cdot \text{m}^{-1})</td>
</tr>
<tr>
<td>(H_m)</td>
<td>magnetic field density of the magnet</td>
<td>(\text{A} \cdot \text{m}^{-1})</td>
</tr>
<tr>
<td>(I)</td>
<td>second moment of area, or area moment of inertia</td>
<td>(\text{m}^4)</td>
</tr>
<tr>
<td>(i_{\text{wire}})</td>
<td>current in the wire</td>
<td>(\text{A})</td>
</tr>
<tr>
<td>(i_{\text{coil}})</td>
<td>current in the coil</td>
<td>(\text{A})</td>
</tr>
<tr>
<td>(j)</td>
<td>(\sqrt{-1})</td>
<td>[-]</td>
</tr>
<tr>
<td>(J)</td>
<td>rotational inertia</td>
<td>(\text{kg} \cdot \text{m}^{-2})</td>
</tr>
<tr>
<td>(j_{\text{coil}})</td>
<td>current density in the coil</td>
<td>(\text{A} \cdot \text{m}^{-2})</td>
</tr>
<tr>
<td>(j_{\text{coil}})</td>
<td>current density in the coil</td>
<td>(\text{A} \cdot \text{m}^{-2})</td>
</tr>
<tr>
<td>(k_i)</td>
<td>generalized stiffness of mode (i)</td>
<td>(\text{N} \cdot \text{m}^{-1})</td>
</tr>
<tr>
<td>(K)</td>
<td>stiffness matrix</td>
<td>(\text{N} \cdot \text{m}^{-1})</td>
</tr>
<tr>
<td>(L)</td>
<td>length</td>
<td>(\text{m})</td>
</tr>
<tr>
<td>(L_m)</td>
<td>length of the magnet</td>
<td>(\text{m})</td>
</tr>
<tr>
<td>(m)</td>
<td>mass</td>
<td>(\text{kg})</td>
</tr>
<tr>
<td>(m_{\text{mov}})</td>
<td>mover mass of a single drive system</td>
<td>(\text{kg})</td>
</tr>
<tr>
<td>(m_{\text{con}})</td>
<td>conductive mass</td>
<td>(\text{kg})</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$m_{cup}$</td>
<td>cup mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{gui}$</td>
<td>guidance mass</td>
<td>kg</td>
</tr>
<tr>
<td>$M_{bm}$</td>
<td>beam mass per unit of area</td>
<td>kg · m$^{-2}$</td>
</tr>
<tr>
<td>$M_{pm}$</td>
<td>product mass per unit of area</td>
<td>kg · m$^{-2}$</td>
</tr>
<tr>
<td>$M_{int}$</td>
<td>internal moment</td>
<td>N · m</td>
</tr>
<tr>
<td>$m_p$</td>
<td>product mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{prod}$</td>
<td>product mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{dyn}$</td>
<td>dynamic mass of the system</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{car}$</td>
<td>product carrier mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{struct}(n_{act})$</td>
<td>product carrier mass dependent on the number of actuators</td>
<td>kg</td>
</tr>
<tr>
<td>$m_i$</td>
<td>generalized mass of mode i</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{ind}$</td>
<td>constant product carrier mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{con}(F_{L \text{ max}})$</td>
<td>mover mass dependent on the maximum required Lorentz force</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{fix}$</td>
<td>constant mover mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{stat}$</td>
<td>static mass of the system</td>
<td>kg</td>
</tr>
<tr>
<td>$mmf_{mag}$</td>
<td>magneto motive force of the magnet</td>
<td>A</td>
</tr>
<tr>
<td>$M$</td>
<td>mass matrix</td>
<td>kg</td>
</tr>
<tr>
<td>$M_L$</td>
<td>lumped mass matrix</td>
<td>kg</td>
</tr>
<tr>
<td>$n$</td>
<td>number of decoupled equations of motion</td>
<td>–</td>
</tr>
<tr>
<td>$n_p$</td>
<td>number of nodal diameters</td>
<td>–</td>
</tr>
<tr>
<td>$n_j$</td>
<td>number of nodal circles</td>
<td>–</td>
</tr>
<tr>
<td>$n_r$</td>
<td>number of rigid body modes</td>
<td>–</td>
</tr>
<tr>
<td>$n_c$</td>
<td>position index of a vector</td>
<td>–</td>
</tr>
<tr>
<td>$n_{act}$</td>
<td>number of actuators</td>
<td>–</td>
</tr>
<tr>
<td>$nrb$</td>
<td>number of rigid body modes</td>
<td>–</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure load</td>
<td>N · m$^{-2}$</td>
</tr>
<tr>
<td>$Q$</td>
<td>amplification at resonance</td>
<td>–</td>
</tr>
<tr>
<td>$q$</td>
<td>index of the $q$-th harmonic</td>
<td>–</td>
</tr>
<tr>
<td>$r_i$</td>
<td>internal diameter of the coil</td>
<td>m</td>
</tr>
<tr>
<td>$R$</td>
<td>radius of a circular plate</td>
<td>m</td>
</tr>
<tr>
<td>$R_i$</td>
<td>inner radius of the airgap</td>
<td>m</td>
</tr>
<tr>
<td>$R_o$</td>
<td>outer radius of the airgap</td>
<td>m</td>
</tr>
<tr>
<td>$R_p$</td>
<td>radius of the circular plate</td>
<td>m</td>
</tr>
<tr>
<td>$R_{airgap}$</td>
<td>airgap reluctance</td>
<td>[s$^2$ · A$^2$ · m$^{-2}$ · kg$^{-1}$]</td>
</tr>
<tr>
<td>$R_{yoke}$</td>
<td>remainder reluctance besides airgap reluctance</td>
<td>[s$^2$ · A$^2$ · m$^{-2}$ · kg$^{-1}$]</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace operator</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>period time of mode i (reciproque of frequency)</td>
<td>s</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>$t_i$</td>
<td>initial time</td>
<td>s</td>
</tr>
<tr>
<td>$t_c$</td>
<td>cycle time</td>
<td>s</td>
</tr>
<tr>
<td>$t_m$</td>
<td>motion time when treated as rigid body</td>
<td>s</td>
</tr>
<tr>
<td>$t_e$</td>
<td>time at which the specified position ends</td>
<td>s</td>
</tr>
<tr>
<td>$t_s$</td>
<td>time at which the position is specified</td>
<td>s</td>
</tr>
<tr>
<td>$t_{coil}$</td>
<td>thickness of the coil</td>
<td>m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$u_{0i}$</td>
<td>vibration amplitude of mode $i$ at $t &gt; t_m$</td>
<td>[m]</td>
</tr>
<tr>
<td>$V$</td>
<td>volume</td>
<td>[m$^3$]</td>
</tr>
<tr>
<td>$V_{coil}$</td>
<td>total coil volume</td>
<td>[m$^3$]</td>
</tr>
<tr>
<td>$V_{con}$</td>
<td>conductive volume in magnetic field</td>
<td>[m$^3$]</td>
</tr>
<tr>
<td>$V_{int}$</td>
<td>internal shear force</td>
<td>[N]</td>
</tr>
<tr>
<td>$w$</td>
<td>distributed load</td>
<td>[N · m$^{-1}$]</td>
</tr>
<tr>
<td>$W_C$</td>
<td>uniform distributed construction load</td>
<td>[N · m$^{-1}$]</td>
</tr>
<tr>
<td>$W_P$</td>
<td>uniform distributed product load</td>
<td>[N · m$^{-1}$]</td>
</tr>
<tr>
<td>$x(t)$</td>
<td>position dependent on time</td>
<td>[m]</td>
</tr>
<tr>
<td>$\ddot{x}_q$</td>
<td>acceleration amplitude of the $q$-th harmonic</td>
<td>[m · s$^{-2}$]</td>
</tr>
<tr>
<td>$\dot{x}(t)$</td>
<td>acceleration</td>
<td>[m · s$^{-2}$]</td>
</tr>
<tr>
<td>$X(s)$</td>
<td>transfer function representation of $x$</td>
<td>[m]</td>
</tr>
<tr>
<td>$x(y, z)$</td>
<td>nodal displacement vector (column vector) for the considered $(y, z)$ location</td>
<td>[m]</td>
</tr>
<tr>
<td>$\chi_i$</td>
<td>dimensionless error parameter of mode $i$</td>
<td>[-]</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>strain</td>
<td>[-]</td>
</tr>
<tr>
<td>$\Phi_{mag}$</td>
<td>magnetic flux</td>
<td>[kg · m$^2$ · s$^{-2}$ · A$^{-1}$]</td>
</tr>
<tr>
<td>$\Phi^T_i \cdot \hat{F}$</td>
<td>participation factor of mode $i$ (force excitation)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\Phi^T_i \cdot M_L \cdot \hat{a}/m_i$</td>
<td>participation factor of mode $i$ (base acceleration)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\Phi_i$</td>
<td>mode shape of mode $i$ (column vector)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>loss factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>magnetic permeability ($4 \pi \times 10^{-7}$)</td>
<td>[m · kg · s$^{-2}$ · A$^{-2}$]</td>
</tr>
<tr>
<td>$\mu_{air}$</td>
<td>relative magnetic permeability of air</td>
<td>[-]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>[kg · m$^{-3}$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
<td>[N · m$^{-2}$]</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>dimensionless time of mode $i$ ($\Omega_1/\omega_i$)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>angular natural frequency of mode $i$</td>
<td>[rad · s$^{-1}$]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>angular natural frequency of the excitation</td>
<td>[rad · s$^{-1}$]</td>
</tr>
<tr>
<td>$\Omega_1$</td>
<td>fundamental harmonic angular frequency</td>
<td>[rad · s$^{-1}$]</td>
</tr>
<tr>
<td>$\Omega_q$</td>
<td>$q$-th harmonic angular frequency</td>
<td>[rad · s$^{-1}$]</td>
</tr>
<tr>
<td>$\approx$</td>
<td>approximately</td>
<td>[-]</td>
</tr>
<tr>
<td>$\propto$</td>
<td>proportional to</td>
<td>[-]</td>
</tr>
</tbody>
</table>
acronyms

- COM: centre of mass
- DOF: degree(s) of freedom
- LSL: lower specification limit
- MMF: magneto motive force
- USL: upper specification limit

definitions

**Dynamic error:** The magnitude of the vibration amplitude outside the specified tolerance limit. Dynamic errors are caused by external sources such as floor- and acoustic vibrations, but also by internal sources, such as bearing defects and spindle error motion [82, 199, 271]. Varying inertial forces, such as acceleration induced dynamic forces in motion systems, are another source of dynamic errors. The acceleration induced dynamic force impacts on positioning accuracy, and increases as speeds rise. At higher positioning speeds, machine dynamics might even become the dominant source of positioning error, and form a barrier to rapid positioning.

**FEM-node:** Point that connects elements in a finite element model.

**Mode shape (eigenvector):** A function defined over a structure which describes the relative displacement of any point on the structure as the structure vibrates in a single mode. A mode shape is associated with each natural frequency of a structure [22].

**Node (vibration):** A point on a structure which does not deflect during vibration in a given mode [22].
Introduction

1.1 General introduction

1.1.1 Need for lightweight positioning systems

Moving and transporting all kinds of things from one place to another is a human need. Transportation systems such as trains, trucks, and elevators might be used to fulfill these tasks. Improved or new transportation systems for instance in terms of efficiency or pollution are appreciated. Even when some degree of excellence is reached, improved design solutions are thus often demanded. The growing realization of the scarcity of raw materials and a rapid depletion of conventional energy sources can for instance be translated into a demand for lightweight and efficient transportation means, as was first recognized by aerospace industry [86].

Improved performance might also shift, or overcome, limitations of already available design solutions. In systems where the weight of the structure contributes significantly in the load, the weight directly affects the performance. When as a result of new techniques lighter systems are possible, the performance might be positively affected e.g. due to the application of lighter materials higher buildings, larger bridges, faster accelerating systems, bigger airplanes etc. can be created. Reduced weight systems can, besides for improved performance, also lead to new design solutions. For instance due to the application of lightweight constructions less columns are required to support a bridge, or less engine power is required to accelerate a car. Besides application in known classes of problems also other or novel classes of problems might profit from improved design solutions. Direct or indirect weight reduction is often an essential criterium for novel applications. Improved electronics in mobile phones might for instance result in less battery weight.

A positioning system is used to change the relative distance (and orientation) between objects to an explicit defined value, a defined position. Besides position also a tolerance range is specified, and required for
value judgement of the positioning task [99]. Positioning of the lens in a DVD player with respect to the track on a DVD is an example of a positioning system in a consumer product. The importance of lightweight positioning systems might be motivated with the same reasons as mentioned for transportation systems.

The research is focussed on precision positioning systems used in manufacturing industry. To obtain the required accuracy often the machine is designed such that the mechanical stiffness is as high as possible. Structural stiffness is often obtained at the cost of structural mass. In case the positioning task mainly consists of acceleration phases, the mass often limits the positioning speed. As a result a tradeoff exists between mass that limits positioning speed and mass that is required for proper operation of the system. Prior to providing the research objective some design aspects of precision positioning systems in manufacturing industry are mentioned, e.g. tolerances, and axis configuration.

1.1.2 Tolerances in manufacturing industry

In many areas of consumer interest improvements can be made with tighter product tolerances, for instance reduced operating cost, increased performance, better portability through miniaturization, ease of repair through the use of fewer parts which are interchangeable [31, 188]. Tighter product tolerances might also be beneficial from manufacturing point of view; possible disadvantages will be mentioned further on. These tighter tolerances might for instance results in simpler products by eliminating added parts that would otherwise be needed to make adjustments or calibrate the final design. Also process or manufacturing steps might be eliminated when products with tighter tolerances are made. Due to the increased accuracy near-net-shape manufacturing might become possible that does not require finishing steps [27]. Furthermore the increased interchangeability of parts might be beneficial, and automatic assembly might be easier obtained. The manufacturing process might thus become more efficient.

Applications and products that require tighter tolerances, even to submicrometre accuracy, over a relatively large range, a few tens of millimetres or more, are for example [66, 151]:

- precision machining in metals, optics, laser cutting and diamond turning [134, 199]
- microscopy [219]
- data storage [35, 111, 139]
- semiconductor manufacturing [41, 182, 219, 281]
- lcd- and flat panel display manufacturing [120, 246]

As a result of tighter tolerances the relative tolerance, that is the ratio between tolerance and dimension, is decreased. In precision engineering,
manufacturing with high relative tolerances (in the order of 1 part in $10^4$, $10^5$ or larger) is concerned [150]. Due to the increased demand for products with tighter tolerances, the demand for more accurate manufacturing machines increases as well. Positioning of material or semi-manufactured products with respect to machine tools or other semi-manufactured products is often required to be able to execute the actual manufacturing processes, e.g. machining, welding, etching, tooling or assembling. The ratio between tolerance and dimension of the examples mentioned above require precision positioning systems. A precision positioning system is able to position with high accuracy in a relatively large workspace. In manufacturing industry, electromechanical servo systems are often applied in case of relatively moderate power and force (or torque), high environmental requirements (including high efficiency), high accuracy, high performance and maximum flexibility [82].

In contradiction to the mentioned benefits of simply tightening product tolerances, understanding of the design or re-design might also result in the required improvements. The production of parts with tighter tolerances is often thought to be more expensive [188]. Understanding of the (re-)design might result in better specified tolerances; not necessarily tightened compared to the original tolerances. The (re-)design might not be dependent upon tight tolerances. From manufacturing point of view the same advantages as indicated above, with tighter tolerances, can be motivated with increased understanding of the (re-)design. For instance the assembly process might be simplified due to the application of exact constraint design principles [21, 238], or complex solutions are replaced with simple ones as result of increased understanding of the problem [32]. A general best solution to all problems does thus not exist. Understanding of tolerances is however essential to obtain product improvements [188]. Sometimes the improvements can only be obtained with tighter tolerances.

The importance of understanding tolerances in manufacturing industry is clarified with the following example. Suppose a production machine has to be created that is able to manufacture more accurate products or parts than the parts of which the production machine is build. The production machine consists of different parts that are put together and as such form the production machine. Despite the uncertainty of the parts from which the machine is build are larger than the required machine accuracy, the accuracy is obtained by setting adjustments and calibration of the design. This is based on the principle that repeatability is achievable without accuracy, but accuracy is not achievable without repeatability [21]. The ability to compensate errors is however often limited. The benefits from the products, and their accuracy, created with the machine might be much larger than the amount of effort required to create this single machine. Knowledge about the benefits of improved products, or their performance, and the consequences for the production process might be used to change products or production processes. Knowing where to spend the, in general limited, available effort and money in order to obtain an efficient production process is a specialty on its own [60].
1.1.3 Design of precision positioning systems for manufacturing industry

The at that time, 1998, current status and trends in the design for precision is indicated by Schellekens et al. [223]. The static or quasi-static accuracy of some precision machines is obtained by minimizing or compensation of static errors that originate from thermal, kinematic or other environmental effects. This has been done by improving the quality of the hardware, including the use of materials with high structural stiffness, low-coefficient-of-thermal expansion materials, isothermal fluid bath, and by dedicated software [199]. In some cases, the hardware means is not even traceable and thus hard to improve. For example, the workpiece deformation caused by fixating forces cannot be avoided. A disadvantage of some of these methods to improve the quality is that it results in a relatively massive structure.

Besides high quality of manufactured products also cost effective production is required. Production machinery therefore needs to operate at high speed as well as with high accuracy. In general, as the operating speed of a machine is increased, the effects of the inherent flexibility of its structural elements and the dynamic response limitations of their drives become more significant, causing vibration problems. As a result the operating speed and precision of many machines are limited as the performance of the system is degraded in terms of reduced accuracy at higher speed. The decreased accuracy due to the operation speed is called a dynamic error. Dynamic errors refer to quickly varying errors compared to the motion time. The performance is limited even in the case when light-weight and highly stiff advanced materials such as composites and ceramics are used in its construction [201].

The mass used in precision positioning systems is thus a tradeoff between static and dynamic accuracy. The elements for quasi static accuracy are rather massive while this massive moving mass makes high speed actuation impractical. The maximum acceleration profits from a minimized moving mass. In precision engineering positioning systems the dynamic loading due to the acceleration of the mass contributes much in the loading of the system compared to machining or other disturbance forces. When the speed of the machine is increased the dynamic loading also increases. This may result in an increase of the dynamic position uncertainty, due to a vibration with certain amplitude [129]. As a result of the inertia and the limiting stiffness a dynamic performance barrier is present [222]. Higher operating speed and precision might be obtained in conventional ways as presented above. Via optimization of the geometry, or applied materials, is tried to increase the structural stiffness, and meanwhile decrease the structural mass, e.g. [239]. However, when the inertia forces form the main loading the achievable improvement is limited.

1.1.4 Recent developments in dynamics of precision positioning systems

The application of other, new, techniques, that indirectly profit from less moving mass might open new opportunities. The accuracy and speed of a single axis positioning system might for instance profit from a new design where the original single stage feed drive is replaced by a dual stage feed drive.
In the dual stage feed drive concept is in the direction of interest the base of a fine (high resolution) drive stage held by a coarse (large stroke) drive stage. The coarse drive stage can be used in the traditional role of transversal/feed and the fine drive stage could limit itself to high speed control of a less massive end effector over a smaller range [55]. This dual stage feed drive principle of a coarse and fine stroke system in series (also known as macro-micro system or long-short stroke system) is for instance applied in fast tool servos [116]. A fast tool servo on a precision lathe can for instance be used to produce aberration correction in an otherwise standard ophthalmic lens on a single machine [283].

In data discs storage systems the application of the dual stage feed drive resulted in higher data density compared to the case of a single stage feed drive. Due to the application of a dual stage feed drive the track pitch (distance between track) could be reduced. The ever increasing pressure for higher speeds called for a drastic improvement of the dynamic behaviour of the drives in positioning systems which led to the dual stage concept. The near net shape production of ophthalmic lenses could for instance not be obtained with the traditional single stage drives and traditional control techniques.

With the application of the dual stage concept both speed and accuracy were increased. Due to this new concept new opportunities arose that would not be possible with the conventional improvements. Either, the accuracy was too low or the required frequency too small. The required high frequency is obtained as a result of minimizing the moving mass. A dual stage feed drive is already in the design phase recognized to be used as dual stage feed drive. The opportunities of the interaction between control and the structural properties were in the design phase recognized. This differs from common practice where often first the structure of the machine and its drive elements are designed and then the controller and the control algorithms, together with the required motion planning and trajectory synthesis algorithms, are designed [200].

When the precision systems aim at accuracy in the sub-micrometre range, also vibration problems might become important [174]. Most precision machines have however only a small capacity to damp the internal elastic energy. Active elements, for instance piezos, have already shown their use as add-ons to vibrating structures in space and aircraft applications [52, 142, 190, 253]. The utilization of load bearing active structural elements, 'Smart Discs', with a particular focus on the damping properties is considered by Holterman and de Vries [106]. This 'Smart Disc' is based on a position actuator and a collocated force sensor, both consisting of piezoelectric material [105]. Although a position actuator is used, the focus in application of the 'Smart Disc' is on vibration control [104]. The 'smart discs' form a parallel actuation structure. Despite the small stroke such parallel actuation structure might be integrated in the dual stage concept to obtain new systems or improve performance. The dual stage principle in combination with parallel actuation is for instance sometimes applied in the manufacturing process of electronic devices [138, 180, 182, 219, 278].

The configuration applied in many positioning systems is often such that the minimum required number of actuators from kinematic point of view is used [21]. In the study of the kinematics, the parts are often assumed rigid,
thereafter the parts are dimensioned such that they behave as rigid. On one hand mass is required to obtain stiffness and accuracy, on the other hand, the mass should be as low as possible because this limits the operation speed. When the structural material available to support the payload is reduced or when a faster response of a given structure is demanded the limited flexibility becomes important [25]. The application of connecting more drives in parallel than strictly required from a kinematic point of view might be used to cope with the effects of flexibility and thus result in increased performance. The drives placed in parallel might be serial drives on its own, such as dual stage feed drives. Only limited knowledge is available about the limitations and opportunities that result from the application of more parallel drives than strictly required from kinematic point of view, and forms the major part of the research.

1.2 Literature overview and background information

1.2.1 Design and evaluation of novel machines

To remain effective and competitive a need for continuous improvement must be recognized [121]. The solution to increased performance specifications or new conditions might in many case be based upon the approach of adapting and improving existing machines. However, machines that have been around a long time and that have been improved by many designers reach a level of "perfection" that makes them difficult to improve upon [177]. To be able to meet the increased performance specifications it might be more sensible to design a novel machine entirely differing from the existing ones in the methods adopted [218].

Such basically novel machines, though not created very frequently, widen the designers choice of economically viable and promising solutions. The creation of a new machine leads directly to its evolution arising from the necessity to fit it to specific applications and manufacturing requirements [218]. As more scientific understanding becomes available, engineers are able to devise better solutions to problems. Literature on the design and evaluation of parallel (positioning) systems that deal with the effects of limited elasticity is scarce [173]. On the one hand, there is an extensive amount of literature on individual fundamental principles such as kinematics and elasticity in the design phase. On the other hand, there is quite some literature on practically applied theory, i.e. (analytic) descriptions of already existing designs [165].

Concerning the limited elasticity; it is common practice to assume that deflections are negligible and parts are rigid when analyzing a machine's kinematic performance. Fortunately, although all solids are flexible to some degree, machines are usually designed from relatively rigid material, keeping part deflections to a minimum. After the dynamic analysis when loads are known, the parts are designed so that the assumption of rigidity is justified [230].

A change in fundamental principles of a design might be beneficial for one particular aspect. However, a design is often the result of synergetic
combination of different aspects like the kinematic, thermo-mechanic, static, dynamic and control. Often a trade-off between these aspects exists, what complicates the design. To be able to evaluate the effect of fundamental changes to the overall design, the effect to all aspects should be understood. Literature that focuses on specific aspects is widely available for instance about the kinematic system [21], about dynamics, [198] and about the design of controller strategy [244]. Literature that considers fundamental aspects in order to design ‘good’ precision machines is:

- Van der Hoek [99], Rosielle and Reker [211] and Koster [128]: lecture notes for students of Eindhoven University of Technology, they consider principles and techniques for design of constructions and mechanisms while also dynamical aspects of positioning are taken into account in the design phase.
- Smith and Chetwynd [238] the foundations of ultra precision mechanism design;
- Slocum [236] consider all kinds of aspects of precision machine design;
- Renkens [203], Ruijl [216], Van Seggelen [226], Vermeulen [261] and Vermeulen [262]; design principles applied;
- Schellekens et al. [223] a paper which summarizes the aspects of design for precision;
- Bryan [32, 33] application of deterministic principles in design for precision.

Specific literature is referred to when specific topics are discussed. Despite, some literature about the design of electromechanical systems is provided below, because this literature is used throughout the whole thesis.

**Literature on the design of electromechanical systems**

Electromechanical positioning systems are widely applied in manufacturing industry. In electromechanical servo systems the dynamic behaviour is the result of the interaction between the electrical and the mechanical part of the system. The structural dynamics, the control, and the interaction between them are often of paramount importance. Literature that aims to achieve a design that uses the design freedom in these underlying principles optimal is: [46, 82, 130, 197]. The design of the physical system and the control structure are considered simultaneous in the design phase of the physical system. In that phase the physical system is still alterable. Integral knowledge of machine dynamics, control and the interaction between these topics is required in finding the optimal solution to design problems of electromechanical systems [197].
The fundamental principles discussed in the literature mentioned in the previous paragraph are related to the aspects described by Van der Hoek [99]. In that book the effect of the dynamic behaviour on the positional accuracy of mechanical systems, such as cam systems, is determined. Koster [126] provides more general rules to predict the dynamic behaviour of these purely mechanical systems, where the information source is physically connected to the power supply. In later literature similar aspects are dealt with, however the information source is in that case physically separated from the power supply, what resulted in servo systems [129].

The mentioned literature mainly discusses the dynamical aspects of serial systems. Literature about the dynamical aspects of parallel systems is scarce [56]. Often, only the kinematic aspects of parallel systems are treated. Rotor dynamics, and especially rotor dynamics with controlled bearing forces, are an example of parallel systems. Even though not used to change the position, the dynamical aspects of rotor systems show similarity with parallel positioning systems. A major difference between them is that in a positioning system forces are adapted to change position, while in rotor dynamics, forces are applied to maintain a certain position. Many different aspects of electromechanical rotor system are discussed in [225].

1.2.2 Positioning system configuration

Parallel and serial configuration

Positioning systems can be classified according to the arrangement of their drives: serial configuration, parallel configuration, or a combination of these two (hybrid). In a serial configuration the drives form a serial chain between end-effector and the ground; the base of a drive is held by another drive [227]. In a parallel configuration the drives form parallel chains between ground and end-effector (which can be a point or a platform). A combination of parallel and serial actuators is indicated with a hybrid configuration. Dependent on the application and the requirements different configurations can be applied. Selection is often based on criteria such as: workspace size, maneuverability, positioning accuracy, load carrying capacity, output torque/force capacity, speed, stiffness, and dynamic performance [25, 56, 61, 159, 172, 179, 209, 210, 270].

Via an analogy to the biological world the performance of a parallel, serial and hybrid configuration can easily be understood [57]. A human arm for instance has sweeping workspace and dextrous maneuverability. Two arms are used when increased load carrying capacity is required, the workspace is however decreased. For accurate positioning visual information obtained with the eyes might be used. Besides visual information also force information can be used, for instance to detect contact or reaction forces due to contact. Writing requires visual information for the required movement of the pencil point and force information to hold the pencil. While grasping an object with the fingers the fingers form a parallel configuration with the hand as common base. The hand, lower- and upper arm form a serial configuration. When the fingers, hand etc. are considered simultaneous they form a hybrid configuration.
The control of the drives and the sensor location also affects the performance of different configurations. When the accuracy is for instance considered, the measurement uncertainty accumulates when the sensors are placed in a serial configuration. Calibration, and hard- or software adjustments can be used to reduce the effects of repeatable errors, sensor uncertainty can however not be reduced. The configuration of the drives, their control and the sensor location determines the accuracy and uncertainty of the positioning system.

There has been a great amount of research on the kinematics of serial, parallel and hybrid configurations. The kinematic study is limited to the SI units time and length. To study the effect of forces the SI unit mass is added to the SI units of the kinematic study. Static force analysis of these systems, for instance to determine the required actuator forces are often performed. Literature on the dynamics of serial configuration is widely available [16]. A much smaller amount of literature is available on the kinetics of parallel systems [56, 80, 160, 251]. Kinetics relates the action of forces on bodies to their resulting motion. The bodies are assumed rigid in the kinetic study. When machines are operated with high speeds and accompanying accelerations it is necessary to make calculations based on the principles of dynamics [229]. The use of parallel systems for high-speed operation might be limited by vibrations that occur [131]. When the dynamics of a parallel system is described the manipulator is assumed to be made up of separate rigid bodies while the manipulator flexibility and dissipation is dominated in the connection between the platform and the base [280]. An example of such system is presented by Kozak et al. [131] where the ‘artificial’ flexibility and dissipation in each individual parallel actuator system is introduced by an independent PD-controller. In the study of rotor dynamics the distributed structural properties are taken into account [225]. Literature about the dynamics that takes into account the distributed stiffness and distributed mass of the platform in parallel (redundant) positioning systems, also known as Steward platform, is not known by the author.

**Redundant actuation**

Performance specifications and physical limitations of the drives might require more drives than required from kinematic point of view. Such configuration is often indicated as a redundant system [76, 164]. More drives than the bare minimum might be required to obtain the demanded performance. Besides improved accuracy or dynamic performance the additional actuators might as well be required to meet safety requirements or to provide pre-loading forces. The term redundant system is often only used to express the redundancy from kinematic point of view when rigid bodies are considered. The additional actuators are however required for proper operation and thus not redundant. The systems that are from kinematic point of view redundant when analyzed as rigid body can be classified as redundant serial- and redundant parallel system. A redundant serial system can reach a specified position with more than one configuration of the linkages [49]. The earlier mentioned fast tool servo, and the positioning of the lens in a DVD player, are examples of redundant serial systems. In these systems the relative small motion amplitudes contain the
high harmonic components while the relative large amplitudes contain the low
harmonic components. Thanks to application of the macro system, a large
motion range is possible. The macro system is as a result of the required
structural mass, not able to operate at high frequencies. Even when the stroke is
reduced to strokes smaller than the total motion range, the inertia of the system
remains often a limitation for high frequency operation. The micro drive is able
to operate at high frequency over a relative short stroke [55, 116, 284]. Actuators
that produce large forces at high frequencies over a small motion range, are also
well suited for the compensation of the effects of disturbances, as long as the
effects of disturbances remain within the frequency- and motion range of the
system [202].
In a redundant parallel system more drives form parallel chains, between
ground and end-effector (which can be a point or a platform), than strictly
required when the kinematics of the rigid bodies are considered. Due to the
additional drive two in principal different opportunities arise. Either the
control is such that an additional position is controlled (position control), or an
internal pre-load force is obtained independent of position (stiffness control).
The additional drive might assure mobility in case one or more chains are in a
singular position. The kinematic isotropy in its workspace might be improved.
This might result in an optimal load distribution among the actuators or
reduced power consumption. Thanks to reduced power consumption more
compact (efficient) actuators can be selected. The load carrying capacity and
the acceleration of the end-effector might be increased. The deformation
caused by elasticity can be decreased and backlash might be eliminated.
Furthermore lower, more even force on the environment might be exerted as
well as greater safety in case of breakdown of individual actuators can be
obtained [76, 124, 125, 168]. The kinematic- and dynamic accuracy as well
as the eigenfrequencies can be increased thanks to the application of parallel
redundant systems [80].
The design of redundant parallel systems where more positions are controlled
than required to describe the position and orientation of the rigid bodies is
complicated [39, 169, 256]. In that case the system is statically indeterminate,
the actuator forces cannot be obtained from static force balance equations
alone. Since in principle it would be possible to avoid this situation by
controlling only the minimum number of positions necessary to describe the
rigid body position and orientation, it should be convinced that controlling the
additional location is desirable and advantageous. The control of an additional
position results in additional stiffness. This additional stiffness due to the
additional controlled position of the redundant parallel system is regarded as
part of the external force distribution. The increased stiffness might allow a
reduction of the internal forces and moments in the structure. The decreased
internal forces and moments allow a lighter mechanical structure, that is able
to carry the same load, or can be accelerated faster.

**Overactuation**
The positioning system is a dynamic system where motion is possible to obtain
a required state. A kinematic study is performed to determine if each degree
of freedom in the positioning system is constrained. In a system that is from kinematic point of view exactly constrained, equilibrium equations can be used to determine the constraint forces. The system is in that case thus also statically determinate \cite{72, 99, 211}. Extended with the material properties also the deformation can be determined. In the design phase it is however common practice to assume that deflections are negligible and parts are rigid when analyzing a machine’s kinematic performance. Fortunately, although all solids are flexible to some degree, machines are usually designed from relatively rigid material, keeping part deflections to a minimum. After the dynamic analysis when loads are known, the parts are designed so that the assumption of rigidity is justified \cite{230}.

If the assumptions of rigidity are not justified the part deflections can not be neglected. Besides rigid body degrees of freedom, additional degrees of freedom appear to be present in the parts, known as internal degrees of freedom. If the number of additional degrees of freedom are constrained with an equal number of additional constraints, the system is still exactly constrained. With the additional constraints the system would be overconstrained when the assumption of rigidity is applied, i.e. from kinematic point of view. An overconstrained system is statically indeterminate, the constraint forces can no longer be determined from equilibrium equations alone. The equilibrium equations are therefore extended with equations that usually describe geometrical conditions associated with displacement and strain, known as compatibility and constitutive equations \cite{72, 273}. Equations describing other effects, for instance symmetry, might also be used as extension to the equilibrium equations.

In case the constraint forces can not be determined with equilibrium equations alone, the assumption of rigid parts, thus without internal degrees of freedom, leads to the conclusion that the system is overconstrained. If the positioning system has to change its state the constraint forces are changed or additional forces are applied. The required actuation forces can be determined open loop from equations that are assumed to describe the system sufficiently accurate, or closed loop from information obtained with sensors. In this thesis a system is considered to be overactuated if the actuation forces in a parallel redundant system are determined with either, additional equations to the equilibrium equations, or a sensor configuration that is able to discriminate between rigid body displacement and static elastic deformation.

### 1.3 Objective of research

The research objective was initially described in \cite{243}, and states that the mass reduction of positioning systems with conventional design methods is limited. In case the mass from an already optimized structure is reduced, the stiffness is reduced also. This mainly results in poorer dynamics and a larger positional error. The poorer dynamics might be overcome with forces from additional actuators. Using this principle it can be assumed that the application of redundant parallel drives in positioning systems results in mass reduction.
Thanks to the additional drive, less structural stiffness is required and as result the mass of the system can be reduced. In case the loading consists mainly of inertial forces the mass can even be reduced further because the loading decreases as well. Overactuation might thus provide a solution to break such mass spiral that is often present in high speed precision manufacturing machines, such as wafer scanners.

**Objective of the research:** Exploration of overactuation in electromechanical positioning systems in order to reduce mass.

Even though often relative lightweight products need to be positioned, precision positioning systems tend to be massive in order to obtain the required (static) accuracy. The operation speed is however often limited due to inertia forces generated during accelerating the payload. To obtain high operation speeds the moving mass is minimized. The design of a high speed precision machine is often a tradeoff in mass that affects both speed and precision. This thesis aims at a possible shift of this tradeoff in parallel positioning systems with high precision. Due to the application of overactuation less internal potential energy might be stored in the positioning system when accelerated. In first consideration only the dynamic behaviour is affected. Instead of increased precision or speed is aimed at reduction of the payload mass. When this can be reduced it is thought the required actuator force and thus mass can be reduced as well. However as a result of the mass reduction, initiated by the additional actuator, also the mass of other parts might be reduced. For instance the static actuator mass might be reduced. The reduced mass might furthermore result in reduced dissipated energy in the actuators; what might be beneficial for the precision of the positioning system.

In analogy with a comment in [88], a vital and tacit assumption is that mass is absolutely essential to create a positioning system.

In the design of electromechanical systems often mechanical-, electrical, and control engineers are involved, as was also the case in this research topic. In this thesis is focused on the mechanical aspects, and as such the SI units mass, length and time are the main topics. Physical effects requiring other SI units that might affect the mechanical system are considered when required. To understand the possible effect for the mechanical structure the other aspects are described with basic, relatively simple, models. In general the aspects are much more complicated than described by the models used in this thesis. The electro-mechanical part of the research is discussed by Makarovič [148] and the control part by Schneiders [224].

The system’s behaviour is a result of the combined performances of the drive system (motor, amplifiers), mechanical structure, sensors and control system, despite disturbances like (reaction) forces, thermal effects and so on [239]. This thesis is about the moving mass as result of overactuation for equal performance with respect to the dynamic accuracy. Despite geometry, material etc. are also important to realize lightweight systems the focus is on mass reduction resulting from overactuation. Mainly the effect of the number of parallel actuators is concerned. Understanding of the fundamental changes due to additional parallel drives can be used to estimate the opportunities and
limitations of redundant parallel configurations with first order approximations in particular applications. This can be used to select a configuration in the conceptual design phase, that does not fundamentally change when geometry and material are optimized afterwards.

1.4 Research method and scope

1.4.1 Use of benchmark problem

The mass, and possible mass reduction, of redundant parallel positioning systems is considered. The conclusions and recommendations should be based on the fundamental differences of the designs. Therefore the focus is on understanding the involved fundamental principles and differences. Understanding the fundamental principles might lead to novel designs that differ fundamentally from other designs. Despite optimization of particular systems might also lead to improvements, the fundamental principles remain in general unchanged. For this reason optimization of particular concepts is not considered. On the other hand it is not tried to obtain a "general purpose" positioning system [25].

Most problems in mechanical engineering design do not have a single right answer as a result of their complex nature [118]. Thus even when only one design problem is considered different solutions might be available, even using different fundamental principles. Selection criteria might then be used to select or motivate the best design. In order to limit the number of designs and only study the effect of interest, the effect of the number of actuators on the mass, a benchmark design problem is put forward. Despite many aspects, e.g. thermo-mechanic, might be important in real design problems, the dynamical aspects are considered most important in this study. The benchmark problem is formulated such that in the design solutions, beams and plates form an essential structural element of the positioning system. Despite beams and plates are not directly of practical use they serve as a benchmark system for many systems in engineering practice. Such as buildings, bridges, rotor systems, aircraft wings, side and bottom plate in the hull of ships, product carriers, car door panels, table tops, solar panels, printed circuits boards, floor slaps, truss etc.

Out of all structural components, beams are probably the most suitable for demonstrating various effects of different actuator locations. On one hand, beams are very simple systems which are idealized as one-dimensional continua, and thus the general principles involved are not obscured by computational complexities. On the other hand, most design problems can be demonstrated on beams because the members may be subjected to a variety of design requirements, [214]. As mentioned the dynamics of the beam has strong resemblance with rotor dynamics [225]. The limited bending stiffness results in deformations that have to remain within the specifications. The methods applied and insights obtained from the one dimensional beam system can be extended without principal changes to systems with more dimensions, such as plate like systems [37]. Plate like systems are for instance used in semiconductor manufacturing, i.e. wafers, and in lcd- and flat panel
manufacturing [41, 120, 182, 219, 246, 281]. Another advantage of the use of beams and plates is that the behaviour can be visualized relatively easy, what as a consequence, might result in better physical understanding of the different aspects.

These simplified design solutions that fulfil the benchmark requirements, have no other function than for analysis. Analysis can be performed on realized designs, however models can also be used to simulate effects. To prevent spending a lot of effort on realizing these designs, models of the designs are used for simulations. Experiments are performed to verify if the results from simulations match with practice. The conclusions and insight obtained from this thesis might be used to motivate design solutions when a design for a specific purpose has to be obtained.

Despite not much energy is spend on the design of a tailored solution, some aspects of design are mentioned here, because they are assumed to be of importance and affect the design solutions, even for these simplified designs. The opportunities and limitations of the actuator locations on the dynamic behaviour of systems are explained. The theory is explained with data obtained from simulations. The results of some simulations are compared with experimental results.

**Strategy of simulations and experiments**

The simulation strategy is designed in analogy with guidelines for design of experiments as described by [166]. The main difference between simulations and experiments is however that in simulations all input variables are known. Simulations are not affected by external sources of variability and thus robust. The performed simulations are even as most experiments iterative, [166]. The knowledge level increases while conducting simulations. As motivated further on, engineering knowledge is used to create an initial design. Models of this initial design are used in simulations. These simulations were not too comprehensive and only aim at increasing the knowledge level and possibly adapt the design. As the knowledge level increases also the relevance of simulations can be indicated. Engineering knowledge is used to select the simulations and experiments that are discussed in this thesis. The selected simulations represent reasonably extreme situations or posses properties that are discussed in detail in other research.

### 1.4.2 Design method, models, and evaluation criteria

**Design**

The conceptual beam and plate designs are obtained in a systematic way (engineering approach) instead of designs that have completely empirically evolved (craftsman approach) [69]. The designs are obtained with informal synthesis techniques because complete rational synthesis techniques were not available. Furthermore the design solutions are kept relatively simple [33].

In many design problems rational synthesis techniques are developed that offer a rather direct route to designs that lend itself well to automation. Rational synthesis might give easy recipes and methods for what [110] calls
foolproof design and might be used in a cookbook solution [236]. For a given design problem, theory might be available that describes the relevant phenomena. This theory is entirely analytical and can also be used in reverse for instance to adapt the initial design such that it meets the specifications. Very likely the design can be accurately optimized, converges systematically and predictably [110]. Rational synthesis methods are however restrictive when the validity of the analytical theory is limited. The method thus only exists for specific types of design problems and many practical problems do not fit within the available class of solutions. When the boundaries of the design problem are widened other solutions might become possible that require other theories to describe the relevant phenomena.

An alternative is to use informal synthesis, which is a methodology [264]. The basic procedure is to "guess" one or more design solutions, concepts, and then use analysis to check the resulting performance. Within each concept the boundaries of the possible solution are narrowed compared to the original design problem. When the boundaries are narrowed that far that analytical theory is available or can be created, analysis and rational synthesis can be applied to that particular concept. When during analysis the ideas are discovered to be flawed, iteration to at least the synthesis phase is required. The design is adjusted to attempt to match the performance specifications more closely, thereafter again analyzed. This process is repeated until an acceptable close match to the specifications is achieved [86, 264]. Knowledge and understanding of scientific principles is used to obtain a well motivated "guess". To predict the effect and to obtain systematically converging adaptations also knowledge and physical understanding is used. The better the initial estimate the less iterations are required [236]. The knowledge obtained from this informal synthesis might lead to rational synthesis techniques.

Accuracy and practicability of the used models

The different concepts can be analyzed with experiments on practical physical systems, or simulations using theoretic models, or both. The preferred method, or combination of methods, depends on several criteria such as accuracy, cost, time, safety and reliability. A physical system can be used to verify if the model governs the system behaviour while the model can be used for adaptations of the physical system. The focus is on the design, and analyses, of a positioning system as a whole. As result less effort is put in design and analysis of the individual parts of the system and only some of the major aspects are used in the overall model.

A model, a mathematical abstraction of the real world, is used to obtain good knowledge of the factors of influence by the relations that govern the system behaviour. The model is used to predict the behaviour of the designs, modify them, and test them prior to their actual construction [12]. Furthermore the implications of small changes can be easily implemented without having to build a multiplicity of new models. The information obtained from the models can be used to adapt the design, for instance when the model reveals that the performance depends on a delicate interaction of the involved properties. The information and knowledge obtained with the model can be used for either
1.4. Research method and scope

differentiation to, or for integration of, different parts and possibly involving other disciplines [234]. In differentiation better specifications can be provided, because the interaction with other parts is clear. The tolerances on mechanical parts for instance can be better specified when it is clear which tolerances to fight and which to relax [19]. Integration of parts might also benefit from knowledge and information obtained with the model. In order to benefit maximal from the interaction of the parts knowledge about the interaction might be useful. Limitations or problems that might arise when one part, or discipline, is considered on its own might for instance be easily overcome thanks to the interaction with other parts or disciplines. To determine the opportunities and limitations of the individual involved disciplines, knowledge about these disciplines remains required, e.g. about how hard it is to control a system [74, 276].

Initially only a limited amount of knowledge is present. To obtain this knowledge and to enhance the practicability in the design phase, the preferred model is the simplest model which still retains the essential features of the actual physical system [127, 156]. Moreover, a model with limited complexity can often be better understood than a complex model. Trends in key parameters are easier revealed and consequently better insight into the relevant properties can be obtained [63, 165]. Although in almost any practical situation the mechanical structure is non-linear and distributed, simplifying assumptions may very well lead to a description of the physical plant which describes the most dominant behaviour [78]. The results of the simulations can be compared with experiments to verify if the simplifying assumptions are justified. The model is thus a compromise between accuracy and practicability and the design of good models is quite an art [184].

This thesis aims initially at obtaining insight, and therefore the simplest model that still retains the essential features of the actual physical system will be chosen. After insight is obtained the limited quantitative accuracy of the model might be increased with more accurate models.

**Determinism in the design phase of the mechanical system**

The deterministic principle is assumed applicable in both the design and operation of the positioning system. Hale [89]:"The deterministic principle rests on the ability of physical laws to explain the behaviour of systems and processes. The deterministic philosophy instills a belief that all aspects of a system or process can be understood and ultimately controlled as desired. The systematic method used to identify root sources of error and to bring them under control has become known as the deterministic approach." Physical laws are also used to evaluate the effect of design modifications prior to realization in practice.

The tolerances on the mechanical parts are specified such that the design is robust for them. In case operation is also dependent on control, knowledge about the system might be incorporated in the controller in order to compensate effects a priori. The controller might also be adapted to the specific realized mechanical system in order to improve the system performance. Knowledge about the opportunities and limitations of the
control can also be used to adapt the design prior to construction. According to the deterministic philosophy in theory all aspects can be controlled. The opportunity to improve designs with more sophisticated control appears unlimited. However, in practice other constraints besides the technical specifications have to be taken into account in the design phase, for instance limited time and money. To guarantee an upper limit of the performance a realistic tolerance for the control is specified as well. Knowledge and information about the design and the involved physical effects might be used to specify this tolerance. Dependent on the tolerance assigned to control, the design might be changed, as well as the tolerances of the mechanical components.

The design is created such that an upper limit of the performance is guaranteed, while the constraints are taken into account. Often a wide variety of solutions exists for the same problem. Despite the application of the deterministic principle a tradeoff between the hardware requirements and control requirements is present in case control is required to operate the hardware. One option is to design the hardware such that the design lacks certain complicating effects and the control does not have to cope with them [74]. Another option is to allow some of these effects in the hardware and cope with them with control or add other parts to minimize the effect. An example of this tradeoff is shown in the different approaches to prevent or cope with friction in drives [6, 195]. Knowledge about both options can be used to obtain the best solution, and for specification of the tolerances assigned to hardware and control.

**Evaluation criteria in the design phase**

It doesn't help to generate concepts, unless evaluated. A great deal of analyses will inevitably be employed in the decision making process in order to be able to compare the benefits and drawbacks of the different concepts. Despite the design is obtained in a systematic way and should be able to fulfil the requirements, some design decisions are motivated with empirical knowledge and engineering judgement. Childs [43]: "Superior algorithms or computer codes will not cure a lack of engineering judgment." This is necessary due to the applied informal synthesis method. Dependent on the applied philosophy, as mentioned above, the selection from the possible solutions during the informal synthesis phase might result in different views upon robustness and stability in the design phase.

Knowledge about the limitations and opportunities of the different concepts with limited quantitative insight is often sufficient to select the most promising concept that will be designed in more detail. The concept might be a brute strength solution that directly addresses the heart of a problem with the intent to maintain the ultimate in simplicity. Bryan [32]: "System simplicity leads to better comprehensibility, reliability and maintainability, fewer parts, less space and weight. The alternative is a 'gadget' solution where gadgets are designed to get around the problem. This approach might introduce new problems. Even though aimed at the brute strength solution different components (or gadgets) are required, for instance for measuring position." The required components
and their arrangement depend on the concept.

The selection of the ‘best’ concept is based on a logic or non logic combination of several criteria differing from practical criteria such as size of the components, to criteria such as working conditions, environmental impact, costs, safety, robustness, etc. [60]. Knowledge and (physical) understanding of the problem and the concepts might be used to motivate the selection of the ‘best’ concept. Especially when more factors have to be compared, apparent more systematic approaches can be used to select the best concept, e.g. assignment of weighting factors to different criteria. (Physical) understanding might be used to assign the relative importance (weight) of the factors, what thus results in (engineering) judgement. Understanding is also required when realized that the amount and accuracy of quantitative information is often limited.

The available time, and money, form a constraint that should be taken into account in the design phase. As a result, a tradeoff between the number of concepts and the amount of details of the individual concepts is present. On one hand a large number of concepts is preferred in order to decrease the change of excluding the conceptual design that results in the ultimate lightweight structure. On the other hand the largest possible amount of information about the individual concepts is preferred in order to motivate design choices. Knowledge about opportunities and limitations obtained from understanding of the synergetic design might be much more important than increased accuracy of quantitative information of the individual parts. For instance how to deal with possible errors and tolerances, might be much more important than the magnitude of the error, or the tolerance.

Very often also costs are used to motivate design choices [25, 94]. The direct cost might profit from more accurate models and quantitative knowledge. From this it might be concluded that designs always profit from more accurate information. However, obtaining information also costs money [121]. The performance measures including, perhaps, speed or cycle time, range, accuracy or repeatability, payload mass might be related to cost. Also the technology required to enhanced performance and obey inequality constraints might be related to costs. When the benefits are also related to costs, weighing of the costs and benefits can be used to motivate design choices.

### 1.4.3 Description of the task of the positioning system

Positioning systems can be used for different tasks, such as point to point positioning, continuous path tracking or maintaining a certain position despite disturbances. In this thesis only point to point control is considered and as such the transient behaviour is most important [10]. Harmonic, modal and spectrum analysis are used to gain insight in the transient behaviour. In point to point motion the actual path of movement is assumed not to be critical as long as the target can be reached with maximum accuracy [82]. The tracking error (difference between desired and actual position) before time $t_s$ is of limited importance in this case. In practice also limitations can be prescribed
such as limited acceleration or velocity. This kind of prescriptions will however not be considered in this thesis. However, the acceleration is limited due to limited actuator force. From the analysis of point to point motion also insight in other positioning tasks involving transient behaviour can be obtained as well, for instance when a constant speed has to be obtained or repetitive tasks have to be executed.

The technical specification for a mechanical motion system will contain a description of the task it has to perform. This task is generally specified in terms of time and space \[46\]. The desired position of an object is always defined with respect to a reference. Specifying only the desired position is not complete, also borders should be indicated, as stated in ISO 14253-1 and \[99\]. Specifying the position of an upper specification limit (USL) and a lower specification limit (LSL) is sufficient. The difference between these limits form the total tolerance band. These limits are however often indicated as difference from a desired position. In this thesis the desired position is specified such that it is in the middle of the tolerance band.

![Figure 1.1: Positioning in general. The object starts at an initial position at time \(t_i\). Between time \(t_s\) and \(t_e\) the position (with its uncertainty) should be at the end position, within the specified lower (LSL) and upper limit (USL). The dashed lines are some possible positions as function of time that fulfil the requirements.](image)

Not only position and tolerance are specified but also time is often specified. At the specified time \(t_s\) the actual position (with its uncertainty) should be within the tolerances of the prescribed position. Often this prescribed position has to be ensured until a specified end time \(t_e\). In Figure 1.1 the basic requirements on positioning are visualized. The requirement is that the position of the object plus and minus its uncertainty is within the LSL and USL between time \(t_s\) and time \(t_e\).

To realize the specified position, a positioning system will be used. The positioning system should be able to guarantee that the actual position with its uncertainty is within the specified tolerance band. With an analysis of the important factors is tried to predict if the requirements can be fulfilled. The performance of the positioning system is thus predicted, Figure 1.2.
prediction consists of a nominal value plus an uncertainty due to the assumed parameter uncertainty in the models. The predicted uncertainty is often kept smaller than the required tolerance, which is an indirect safety factor.

![Figure 1.2: Visualization of a tolerance band with a desired position and a prediction of the performance of the designed positioning system (a predicted position with an uncertainty).](image)

Even in the conceptual design stage it must be clear which problems can appear when the system is build, and how these possible problems can be solved. Also it should be clear how to operate the manufactured design and how the manufactured design will react on the estimated disturbances. Because a design will always be a compromise of many design philosophies so certain errors remain. The accuracy of machines and instruments is mainly determined by five error sources, errors concerning: kinematic-, thermo-mechanic-, static-, dynamic-, and control system. To compensate some errors this can for instance involve that adjustments options are incorporated to overcome some tolerances as result of the manufacturing of the different parts of the positioning system. Furthermore information from sensors might be used to adapt the input of the actuators.

As discussed in this subsection only one point seems to be of interest in only one direction. In practice real objects have to be positioned thus a mutual position as well as mutual orientation are defined. For this purpose the whole area of the beam or plate is assumed to be positioned within the specifications. For simplicity only one direction is considered. Particular positioning systems might require positioning of only parts of the complete area at an instant time. Information and understanding from this thesis might as well be used to determine how to operate such particular positioning systems.

**Remark concerning errors**

In literature the term error is often used as synonym for deviation. There is however a difference between an error and a deviation. This is illustrated with the definition of the positional error and positional deviation in the next example. Between time \( t_e \) and time \( t_s \) the actual position, plus and minus its uncertainty, should always be in the specified tolerance band, see Figure 1.2.
The difference between the actual position and a desired position is the positional deviation. When the actual position is out of the specified tolerance band between time $t_s$ and $t_e$ the difference between the actual position and the nearest specified limit is the positional error. For consistency with most other literature the term deviation is in this thesis however replaced with the (in fact wrong) term error.

1.4.4 Benchmark specifications and assumptions

The benchmark is formulated such that positioning of a product with height $h = 0.5 \text{ mm}$ and density $\rho = 3000 \text{ kg/m}^3$ is required. Positioning in 'height'-direction over a stroke of $1 \text{ mm}$ with a tolerance band of $25 \text{ \mu m}$ on both sides has to be obtained within a motion time of $20 \text{ ms}$. In case of a beam system the other dimensions of the product are: length $L = 500 \text{ mm}$ and with $b = 20 \text{ mm}$. In case of a plate system a circular product is assumed with a radius $r = 150 \text{ mm}$. Even though the plate radius is equal to the radius of a currently often used wafer, these specifications might be regarded as arbitrarily chosen. The dimensions result in a flexible system in a particular direction, and is therefore well suited to obtain understanding of the physical principles involved in parallel (redundant) positioning systems.

Deformations are in the conceptual design phase assumed to remain elastic. Thereafter the parts are designed such that the deformations fulfil this assumption. Despite many other aspects affect the design of a real positioning system, focussed is mainly on the dynamic error that is obtained due to the positioning task. Furthermore a number of simplifying assumptions are made in order to determine the mass of different configurations with first order approximations in Chapter 6. The assumptions are:

- The product and product carrier are assumed to behave as one part. The product stiffness is assumed negligible, whatever the stiffness of the product carrier might be.
- Limited stiffness of the guidance is assumed not to effect the dynamic behaviour.
- A parallel system is assumed to be required.
- The 'ground' is assumed as base of the parallel drives and as result the effect of limited mass and finite stiffness of a practical machine base does not have to be considered.
- The individual drives of the (redundant) parallel positioning system are assumed to be single stage drives.
- Force control is assumed to be required. Therefore linear electric direct drives are assumed to be used, more specifically voice coil actuators.
- Disturbances are in the initial design phase not assumed to be present.
- In the considered initial design phase of the hardware, the different aspects can be modelled sufficiently accurate with linear models [78].
1.5 Outline of the thesis

In order to obtain insight whether mass reduction of the product carrier itself is possible, the deflection of the product carrier in a constant acceleration field is researched in Chapter 2. Specifically the case with a beam as product carrier is researched. The effect on the deflection of the support location, and the number of supports is determined. Also the effect of several material parameters and their relation to the deflection is determined. Sandwich materials are a specialty on its own and only briefly mentioned.

The guidance and drive aspects of the product carrier are treated in Chapter 3. The kinematical configuration determines the required stroke of guidance and drive systems. Some basic propulsion principles of linear motors are mentioned to motivate the choice for a voice coil actuator. The voice coil actuator is thereafter discussed in more detail. General guidance aspects are discussed using the parallel actuated beam to which voice coils are applied as drive system. The effect of mass on the deformation is initially calculated for a constant acceleration without transient system behaviour, a 'static' response. In Chapter 4 the dynamic response is described and the response of any arbitrary force can be calculated.

Chapter 4 outlines the theoretic models underlying the dynamical aspects of parallel actuation systems. Because only limited knowledge is available about the dynamic limitations and opportunities of parallel positioning systems, the methods available for the design and analysis of serial systems are adapted such that they are applicable to parallel systems. The effect of different actuator locations is thereafter discussed. The minimum number of parallel actuators required from kinematic point of view of rigid bodies are used. The dynamics are the result of the interaction between the control of the actuator forces and the structural mass and stiffness of the system. To cope with an initial unknown control structure and unknown actuator mass the situation is first discussed where the actuators are assumed massless and the forces are obtained from open loop control. Thereafter the other extreme is discussed where the actuator location is assumed to form nodes of vibration modes as result of actuator mass and control. In practical cases the dynamic behaviour will be somewhere between these extremes as will be mentioned. Considering the mentioned extremes is used to obtain understanding about opportunities and limitations of different actuator locations.

The aspects mentioned in Chapter 4 are applied to parallel redundant systems in Chapter 5. Experiments of an overactuated plate positioning system are used to verify the results from simulations. In Chapter 6 the elements that contribute in the moving mass of the system are discussed. Not solely the mass of the product carrier contributes to the total moving mass but also the mass of drive and guidance system. If overactuation result in mass reduction depends on the mass ratio of the different parts of the system. A mass reduced product carrier does not necessarily result in a mass reduced drive system. Conclusions are drawn and recommendations are given in Chapter 7.
Chapter 2

Mass aspects of the product carrier

2.1 Introduction

The different elements of the positioning system have to be arranged in such a way that the positioning system is able to fulfill the requirements. The configuration is dependent on the individual elements and vice versa, the individual elements depend on the configuration. With the aim to obtain a low moving mass, a lightweight product carrier seems essential. In this chapter the properties of the product carrier are studied. Besides material aspects such as, mass and stiffness also the effect of the number of supports and their location is studied. The magnitude of the support forces is in this chapter not considered, but might be limited because the support forces result from actuators, as will be explained in later chapters.

To be able to position a system forces have to be applied on the product, respectively the product carrier. In an idealized, kinematic, situation the product carrier is rigid, has infinite stiffness, and therefore only rigid body displacements are obtained. In practice however every object has finite stiffness which means the object is elastic [223]. The applied forces are in equilibrium with the inertia forces. The inertia forces result in a storage of energy in the elastic product carrier and increase as result of increased accelerations. In positioning system used in manufacturing industry the positioning task consists often mainly of acceleration phases. At high speed operation the inertia force is often the most dominant force and is limiting the operation speed. The elastic energy stored in the structure changes as result of changed accelerations. This stored energy must be handled with care because it can result in vibrations and result in dynamic positioning errors.

A relation exists between the position uncertainty as result of the system dynamics and the applied force in the motion time in Figure 1.1, as shown in [46, 89, 99, 197]. In this chapter only the effect of a static acceleration field is studied. The relation between the force profile and the dynamics is in this chapter neglected, but will be discussed in Chapter 4. The deformation during the con-
stant acceleration is related to the vibration amplitude after a positioning task, as will be shown in Chapter 4. The actuator location is in this chapter regarded as pinned support. The complex interaction between controlled actuator force and product carrier is treated in Chapter 4. The ratio deformation to acceleration can be calculated, which is in case of bending deflection related to the ratio stiffness over mass. This ratio is a measure of the efficiency of the used material, the higher this ratio the more efficient the material has been used. An optimal material distribution might be obtained from optimization procedures of a fixed layout, e.g. dimensioning the cross sectional size. Besides cross sectional size, the layout comprises also topology, i.e. the spatial sequence of members and joints, and geometry, i.e. the location of the joints [215]. The potential savings affected by the structural layout are generally more significant than optimization of a fixed layout [123]. In this thesis the effect of the number of actuators and their location on the mass, is considered the most important issue. These aspects are an important part of the initial design process. Afterwards optimization, of the fixed layout might be possible. In this chapter the effect on the product carrier is determined as result of the number of supports and their location. Besides the unknown support condition, the unknown load as result of density dependent body forces is complicating the problem [30]. In both the beam and plate positioning system equivalent aspects are important. Despite, the plate is more often used as product carrier than a beam, the beam is studied to obtain insight and understanding in parallel positioning systems. After the aspects are known for the beam system an extension to plate systems is made, and the similarity with the beam system is indicated.

2.2 Product carrier material aspects

The requirements for the product carrier depend, among others, on positioning task and product specifications. As was mentioned, in this thesis positioning of a flexible product is assumed to be required. Furthermore the product is assumed to contain only mass and does not contribute in the stiffness. A requirement for the product carrier is therefore that it should have enough stiffness to support the product. For high speed operation the mass should however be as low as possible. In this initial design phase the product carrier is assumed to completely support the product. The dimension in two directions is thus specified by the product dimensions and can not be changed. In case of the beam positioning system the width and length can not be changed, respectively the radius of the circular plate positioning system cannot be changed. The height of the product carrier is the only dimension that can be varied. Despite the height can be varied over the area, in this thesis a uniform height of the product carrier over the area is assumed. As result of the homogeneous product specifications and the homogeneous product carrier height the inertia forces can be modelled as a distributed load, or pressure.

The effect of the product carrier material properties, Young’s modulus of elasticity ($E$) and the density ($\rho$), is in this section discussed even as the effect of
changing product carrier height. As performance measure the ratio deformation to acceleration of the product carrier is used. The deformation is the result of stiffness and inertia loading, which is related to the mass distribution and acceleration. When accelerated in the weakest direction, largest ratio deformation to acceleration, the bending stress is assumed to contribute most in the total stress. This assumption is valid when the dimensions of the product carrier thickness height / length ratio is larger than 1/20, and assumed valid for both the beam and plate systems. First the beam is considered in a positioning system as schematically presented in Figure 2.1.

![Figure 2.1: Beam positioning system as case to explain theories](image)

The effect of acceleration in transversal \( (x-) \) and lateral \( (y-) \) direction of the product carrier in Figure 2.1 are considered. Only translational motions are considered and therefore the resultant force vector is assumed to go always through the centre of mass (COM). Furthermore the guidance is minimal loaded when the resultant actuator force vector passes through the centre of mass. Initially the mass and stiffness of the product carrier are assumed to dominate the system behaviour. As result the product is in the calculations initially neglected. The validity of this assumptions is thereafter verified, and when necessary the product is included.

### 2.2.1 Acceleration of a beam in lateral direction

Independent of the number of forces or the location of those forces, every setup of forces applied to accelerate the beam in transversal direction can be simplified to a basic situation shown in Figure 2.2. The beam, or a part of it when simplified, has the following properties: length \( (L) \), width \( (b) \), height \( (h) \) and density \( \rho \). The beam will be accelerated with constant acceleration \( (a) \). A force \( (F) \) is put on the beam to obtain the required acceleration.

\[
F = m \cdot a = \rho \cdot V \cdot a = \rho \cdot b \cdot h \cdot L \cdot a
\] (2.1)

Every point on this beam has to be positioned simultaneously in y-direction. The largest deviation from the nominal position appears on the right side of the beam. The deformation will be determined between this point and the location where the force is exerted. The shortening of the beam \( (\Delta L) \), as result of the acceleration, is calculated. First the stress in the beam is calculated. The
normal stress \( (\sigma_y) \) in the beam is a result of the inertia force. This normal stress has a maximum at \( x=0 \) and decreases with coordinate \( y \). At location \( y \) the stress is \( \sigma_y \) at location \( y + \Delta y \) the stress is reduced to \( \sigma_y - \Delta \sigma_y \).

\[
\Delta \sigma = a \cdot \rho \cdot \Delta y \Rightarrow \lim_{\Delta y \to 0} d\sigma = a \cdot \rho \cdot dy
\]  

(2.2)

The strain \( (\varepsilon) \) is also maximal at location \( y = 0 \) the change in strain \( \Delta \varepsilon \) changes with coordinate \( y \).

\[
d\varepsilon = \frac{d\sigma}{E} = \frac{a \cdot \rho \cdot dy}{E}
\]

(2.3)

The total strain \( \varepsilon \) is:

\[
\varepsilon = \int_{y=0}^{y=L} d\varepsilon = \frac{a \cdot \rho \cdot L}{E} = \frac{\Delta L}{L}
\]

(2.4)

The shortening of the beam during constant acceleration is equal to.

\[
\Delta L = \frac{a \cdot \rho \cdot L^2}{E}
\]

(2.5)

The beam is simplified as a one mass spring system; assuming the spring does not have mass and the mass does not have stiffness. This is visualised in Figure 2.3.

The shortening of the beam should be \( \Delta L \). In this model it is proposed that the location where the force is excerpted on the beam is equal to the left end of the spring. The right end of the beam is equal to the right end of the spring.
The equilibrium equations for this system are:

\[ F = c \cdot \Delta L = m \cdot a \]  \hspace{1cm} (2.6)

From Equation 2.5 and 2.6, the steady state shortening during the constant acceleration is.

\[ \frac{\Delta L}{a} = \frac{m}{c} = \frac{\rho \cdot L^2}{E} \]  \hspace{1cm} (2.7)

The material properties, present in Equation 2.7 are the Young’s modulus and the density. The deformation per acceleration depends on these parameters as:

\[ \frac{E}{\rho} \]  \hspace{1cm} (2.8)

This ratio is also called the specific stiffness. The larger this ratio of the material the less mass is required to obtain the specified stiffness. The height of the beam is not present in Equation 2.7 and thus does not affect the deformation per acceleration. In theory the height can thus be reduced to zero when only the beam itself is considered. This conclusion is however the result of the assumptions that external loads, such as due to product inertia, can be neglected. In practice the external load is present and should thus be taken in account.

The effect of a distributed load over the length can be determined, via modification of the above presented equations. The mass and stiffness of the product are in that case incorporated in the beam properties. This aspect is not discussed in detail. Despite the deformation in lateral direction might be important, the deformation in transversal direction is researched in detail. Vibration amplitudes and deformation in transversal direction are in general larger than in lateral direction.

### 2.2.2 Acceleration of a beam in transversal direction

A positioning system with only a single force applied to obtain the required acceleration in transversal direction is shown in Figure 2.4. The material properties and dimensions are the same as in case with the previously discussed case where the acceleration in lateral direction is treated. Additional forces might be present at arbitrary locations, the resulting force should however pass through the centre of mass in order to maintain pure translations. In practice a guidance system can be used to prevent unwanted motions, such as rotations. The guidance system will be discussed in more detail in the next chapter. In order to minimize the loading of the guidance, the nett force should still pass through the centre of mass. In first consideration the product mass is neglected.

The inertia forces are in equilibrium with the nett applied force. Because the beam is homogeneous the inertia can be considered as a constant distributed load. The inertia load plus the resulting deflection of the beam are presented in Figure 2.5. As assumed every point on this beam has to be positioned, the deformation at every point during acceleration is considered. However the purpose is to guarantee and upper limit for the performance thus mainly focussed
on those locations where the effect of the deformation is largest. In case the acceleration of the beam in Figure 2.4 is considered, the largest deviation from the nominal position appears on the sides of the beam. The deformation will be determined between this point and the location where the force is exerted. The deformation will again be determined during a constant acceleration, only the steady state response is taken in account.

The inertia force depends on the mass and acceleration. In case of a homogeneous beam this results in a uniform distributed load \( w \):

\[
w = a \cdot \frac{m}{L} = \frac{F}{L}
\]  

(2.9)

The deflection at the side \( (\Delta x) \) is:

\[
\Delta x = \frac{w(L)}{8E \cdot I} = \frac{a \cdot m \cdot L^3}{128E \cdot I}
\]  

(2.10)

The deflection at other locations can also be determined, they will however be smaller than the deflection at the side. With the purpose to guarantee an upper limit for the deformation it is in this case sufficient to only consider the side.

The second moment of area \( (I) \), also known as area moment of inertia, depends
on the height. The height of a homogeneous beam depends on the mass as:

\[ h = \frac{m}{\rho \cdot b \cdot L} \quad (2.11) \]

The second moment of area related to mass instead of the height is:

\[ I = \frac{b \cdot h^3}{12} = \frac{m^3}{12\rho^3 \cdot b^2 \cdot L^3} \quad (2.12) \]

Equation 2.12 combined with Equation 2.10 can be used to relate the deflection of a beam to the other beam dependent parameters and the acceleration. The ratio deflection to acceleration is:

\[ \frac{\Delta x}{a} = \frac{12}{128} \cdot \frac{\rho^3 \cdot b^2 \cdot L^6}{E \cdot m^2} \quad (2.13) \]

This Equation is not directly dependent on the height, but indirect via the mass. From Equation 2.13 it can be observed that as result of reduced mass the deflection (\( \Delta x \)) increases. The other parameters are assumed unchanged. The deformation in x-direction between two points can be modelled in the same way as presented in Figure 2.3. However be aware that in Figure 2.3 the equivalent stiffness and mass are used.

**Beam mass without external load**

In transversal direction the influence of the material is not directly seen in Equation 2.13, because the density affects the mass of the beam. Equation 2.13 is therefore transformed to an equation that does not depend on the total mass but solely on the dimensions and the Young’s modulus of elasticity and the density.

\[ \frac{\Delta x}{a} = \frac{12}{128} \cdot \frac{b^2 \cdot L^6}{m^2} \cdot \frac{\rho^3}{E} = \frac{12}{128} \cdot \frac{b^2 \cdot L^6}{(\rho \cdot b \cdot h \cdot L)^2} \cdot \frac{\rho^3}{E} = \frac{12}{128} \cdot \frac{L^4}{h^2} \cdot \frac{\rho}{E} \quad (2.14) \]

The same ratio \( \frac{E}{\rho} \) as in transversal case is used to calculate the deflection. From Equation 2.14 the required beam height can be determined as function of the material properties \( E \) and \( \rho \), in case the deflection per acceleration is given and the length and width are fixed.

\[ h = \sqrt{\frac{12L^4 \cdot a}{128\Delta x}} \cdot \sqrt{\frac{\rho}{E}} \quad (2.15) \]

The mass of the beam is:

\[ m = \rho(b \cdot h \cdot L) = b \sqrt{\frac{12L^6 \cdot a}{128\Delta x}} \cdot \sqrt{\frac{\rho^3}{E}} = \left( \frac{12b^2 \cdot L^6 \cdot a}{128\Delta x} \right)^{\frac{1}{2}} \cdot \left( \frac{\rho}{E^{\frac{3}{2}}} \right)^{\frac{3}{2}} \quad (2.16) \]

In this way the beam mass is related to the Young’s modulus of elasticity and density and parameters that are assumed fixed as result of design constraints,
2.2. Product carrier material aspects

e.g. acceleration and some dimensions. The first group in Equation 2.16 contains the parameters that are specified by the design constraints. The second group contains the material parameters.

Beam mass with external load only

When the uniform distributed external load is large, compared to the load due to the inertia forces of the beam itself, the selection criterium might change. In the cases above the product was not assumed to contribute to the loading, in practice it does. The constantly distributed inertia loading as result of accelerating the product \( w_p \) can be summed with the inertia load of the product carrier \( w_c \).

The distributed loading on the beam without mass in Figure 2.6 is thus the summation of the inertia due to the product \( w_p \) and the inertia of the construction (beam) itself \( w_c \). Now it is assumed the inertia of the product is much larger than the inertia of the product carrier. The distributed load \( (w) \) in Equation 2.9 is the result of the external inertia and not related to the beam height. The height is only considered in the second moment of area \((I)\). The product mass per unit of area is contained in variable \( M_{pm} \). The total product mass is obtained by multiplication of the mass per unit area with the length and width \( (m_p = M_{pm} \cdot b \cdot L) \).

\[
\Delta x = \frac{a \cdot M_{pm} \cdot b \cdot L \cdot L^3}{128E \cdot I} \quad (2.17)
\]

The required beam height can be deduced and is:

\[
h = \left( \frac{12a \cdot M_{pm} \cdot L^4}{128\Delta x} \right)^{\frac{1}{3}} \cdot \left( \frac{1}{E} \right)^{\frac{1}{3}} = M_{pm}^{\frac{1}{3}} \cdot \left( \frac{12L^4 \cdot a}{128\Delta x} \right)^{\frac{1}{3}} \cdot \left( \frac{1}{E} \right)^{\frac{1}{3}} \quad (2.18)
\]

In this way the beam mass is related to the Young’s modulus of elasticity and density and parameters that are assumed fixed as result of design constraints,
e.g. acceleration and some dimensions. When only the height of the beam can be changed the required beam mass is:

\[
m = \rho (b \cdot h \cdot L) = (b \cdot L \cdot M_{pm})^{\frac{1}{3}} \cdot \left(\frac{12b^2 \cdot L^6 \cdot a}{128\Delta x}\right)^{\frac{1}{3}} \cdot \left(\frac{\rho}{E^{\frac{1}{3}}}\right)
\]  
(2.19)

The first group in Equation 2.19 contains the external mass. The second group contains parameters that are specified by the design constraints, and the third group the material parameters of the product carrier.

**Beam mass with external load and self weight**

In practice the loading will be the result of external loading and the own inertia. The importance of the ratio between the Young’s modulus of elasticity and the density depends on the ratio of the external and internal mass, Equation 2.16 and 2.19. When the required beam height has to be calculated it is an iterative calculation. Started is with a certain loading condition to determine the required beam height. The calculated beam height affects the loading and is used for a more precise calculation of a new beam height. When started with Equation 2.18 the mass of the beam can be incorporated by the addition of the beam mass per unit of area unit of area \((M_{bm} = h \cdot \rho)\) to the product mass per unit of area. This new loading, as result of beam height and external mass is used to calculate a ‘new’ beam height in the same way as with Equation 2.18.

\[
h = \left(\frac{12a(M_{pm} + M_{bm})L^4}{128\Delta x}\right)^{\frac{1}{3}} \cdot \left(\frac{1}{E}\right)^{\frac{1}{3}} = (M_{pm} + M_{bm})^{\frac{1}{3}} \cdot \left(\frac{12L^4 \cdot a}{128\Delta x}\right)^{\frac{1}{3}} \cdot \left(\frac{1}{E}\right)^{\frac{1}{3}}
\]  
(2.20)

When the inertia increases the external load can at a certain moment be neglected \((M_{pm} \ll M_{bm})\). The height can in that case be directly determined with replacing \(M_{bm}\) in Equation 2.20 with \(\rho \cdot h\). This is equal to the height determined in Equation 2.15.

### 2.2.3 Conclusion with respect to material properties

The required mass of the product carrier depends on the design constraints, such as geometric requirements for length and width, but also the allowable deformation per unit of acceleration. Furthermore, the mass depends on the material properties: Young’s modulus of elasticity and the density. In lateral direction the ratio \(E/\rho\) should be considered. This ratio is also known as specific stiffness sometimes used to characterize dynamic behaviour of materials (the square root of this parameter is equivalent with the speed of sound through the material).

In transverse direction the ratio \(E/\rho^3\) should be considered to determine what material results in the lightest construction, [8, 89]. An equal specific stiffness of materials \((E/\rho)\) results in equal required amount of mass when loaded in lateral direction and the deformation is specified. For instance Steel and Aluminum both have a specific stiffness of \(E/\rho \approx 27 \frac{MPa\cdot m^3}{kg}\). However when
these materials are used in a system that results in transversal loading the required amount of mass to obtain equal deflection per acceleration is different. Furthermore the required amount of mass also depends on the ratio of external forces and inertia forces of the system itself. When the lateral direction is considered and the load is mainly due to body dependent forces, such as inertia Equation 2.16 is used:

\[ m_{\text{aluminum}} \propto \left( \frac{\rho}{E\pi} \right)^{\frac{3}{2}} = 0.53 \quad m_{\text{steel}} \propto \left( \frac{\rho}{E\pi} \right)^{\frac{3}{2}} = 1.52 \quad (2.21) \]

When the body forces are the most important loading an aluminum construction weights thus three times less than a steel construction for the same deflection.

When the external forces are large compared to the body dependent forces, the mass of different materials can be determined with Equation 2.19 and depends on the Young's modulus of elasticity and the density as:

\[ m_{\text{aluminum}} \propto \frac{\rho}{E\pi} = 0.66 \quad m_{\text{steel}} \propto \frac{\rho}{E\pi} = 1.32 \quad (2.22) \]

The use of Aluminum results in a twice as light structure for the same deflection compared to the use of steel.

In this thesis the effect of mass reduction will be researched further, what happens if the mass is reduced or material with another specific stiffness \((E/\rho)\) is used, and as a result the dynamic properties change. How to compensate the resulting poorer dynamics by adding actuators.

### 2.3 Product carrier mass due to increased number of actuators

The number of actuators and their location also affect the requirements for the product carrier. As performance requirement still the deformation per acceleration is specified. As observed in the previous section the required mass was related to the Young's modulus of elasticity and the density besides parameters that are assumed fixed as result of design constraints, e.g. acceleration and some dimensions. In this section the design constraints remain present, only the effect of an increased number of actuators and their location is determined. The location is varied such that symmetry is maintained, in Chapter 3 asymmetric actuator locations are discussed.

The length is assumed to be much larger than the height of the beam. As result the deformation per acceleration in lateral direction, Equation 2.7, is assumed much smaller than the deformation per acceleration in transversal direction, Equation 2.14. In the remainder of this thesis only the effect of transversal deformation is studied for this reason. Still bending deformation is assumed most important, as will be founded when sandwich beams are considered.
2.3.1 The number of actuators and their locations

The product still requires support over the length \( L \) and width \( b \). These properties cannot be changed. The height however can be changed. The location of the actuators is determined while the deflection per acceleration is kept constant. The force of the actuators should always pass through the centre of mass to assure positioning in x-direction is obtained, while rotation around the z-axis is prevented without loading the guidance. Dependent on the setup of the system it could be possible that only one actuator is used to position in the required direction, e.g. Figure 2.5. The force of that actuator goes ideally through the centre of mass in order to minimize the loading of the guiding system of the fixed degrees of freedom. When the maximum deflection \( \Delta x \) is specified and the acceleration is known the required beam height can be calculated.

Figure 2.7: A coordinate system, indicated with indices \( o \) is attached to the beam. The origin of the \( y_o \) axis is the left side of the beam. The origin of the of the other axis (\( x_o \) & \( z_o \)) are in the middle of the width and height of the beam. The orientation of this coordinate system is equal to the principal axes of the not deformed beam. The beam position is indicated with respect to another coordinate system, indicated with indices \( r \). The orientation of that coordinate system is equal to the orientation of the beam system. Positioning in x-direction is required. In the other figures the coordinate system is only used to indicate the orientation.

In practice also setups with two or more actuators in parallel can be used. A kinematic representation of a system that can be used to position in x-direction is shown in Figure 2.8. Acceleration in the transversal direction is required. The connections to the ‘world’ represent constraints that constrain four degrees of freedom. The requirement that the nett forces passes through the centre of mass, should constrain the rotation around the z-axis. The only degree of freedom that remains is as required: the rigid body motion in x-direction. The number of actuators to position the system can be increased, see Figure 2.9. The ratio between the forces should remain such that the nett forces passes through the centre of mass, otherwise the system rotates around the z-axis, which is not allowed. From this requirement the force ratio can be determined, as will be shown in Chapter 5.

Two actuators used to position the beam

Different actuator locations are possible while the nett force still passes through the centre of mass. In this chapter only symmetric setups are discussed. As
2.3. Product carrier mass due to increased number of actuators

Figure 2.8: Four degrees of freedom are constrained to the ‘world’ the fifth degree of freedom is constrained by the requirement that the nett force passes through the centre of mass. Positioning in the free dof \((x)\) is obtained by applying force in that direction.

Figure 2.9: Three forces are used to exert the acceleration forces. At least two actuators are required to constrain rotation around the z-axis.

mentioned in Chapter 1, engineering knowledge is used to determine actuator locations that are considered realistic. Two possible configurations using two actuators are presented in Figure 2.10 and Figure 2.11.

Figure 2.10: The maximum difference in x-location \((\Delta x)\) during a constant acceleration depends on the location of the supports.

Figure 2.11: The actuators are both moved over a distance \(d_{\text{act}}\) to the centre of the beam. As a result the difference in x-location \((\Delta x)\) during a constant acceleration changes.

Rigid body equilibrium equations, for the (rotational) inertia, can be used to determine the force ration of the actuators. The total actuator force in Figure 2.10 and 2.11 is in equilibrium with the inertia force of the system in a similar way as presented in Figure 2.6. Equilibrium conditions of forces and moments still hold when the system is considered in more detail. The beam can for instance be cut at any arbitrary location as shown in Figure 2.12. Only the forces in x-direction and moments around the z-axis are shown because they are much larger than the other forces and moments when the beam has to be accelerated in x-direction.

As result of the acceleration inertia forces appear. As result of the inertia forces
Figure 2.12: When the beam is cut at any arbitrary location the forces \((V_{int})\) and moments \((M_{int})\) should be in equilibrium.

Figure 2.13: The slope of the beam axis \((\varphi)\) can be determined at any arbitrary location.

and the finite stiffness the beam deforms. The deformation results in a different y-direction of the points on the beam. When the actuators are located at the outside of the beam as in Figure 2.10, the difference in x-direction is equal to the deflection of a simply supported beam with a distributed load. The difference is:

\[
\Delta x = \frac{5(w_p + w_c)L^4}{384E \cdot I}
\]  

(2.23)

This is 40% smaller than in case only one actuator was used on the same beam as in Figure 2.5. To calculate the deflection in case one actuator is used the term \(w\) in Equation 2.10 has to be replaced with \((w_p + w_c)\). It is expected that the actuator location also affects the deformation in x-direction. When the actuators in Figure 2.10 are moved towards the centre of mass, as in Figure 2.11, the x-difference between all points will first decrease. The difference in x-direction during constant acceleration decreases until the the actuators are located at 22.31% from the sides \((d_{act} = 0.2231 L)\) [62]. The deformed shape of the beam, dependent on the actuator location, is illustrated in Figure 2.14. The difference in x-direction is in Figure 2.14 minimal. If the actuators are moved even more to the centre of mass than in Figure 2.14 the difference in x-direction start to increase. In Figures 2.14 and Figure 2.15 the deformed shape is plotted, however the difference in deflection is not drawn.

Figure 2.14: Deformed shape when the actuators are located for minimal deflection. The actuators are located at 22.31% from the sides.

Figure 2.15: Deformed shape when the actuators are moved more inwards than the location in figure 2.14. The x-difference \((\Delta x)\) increases, however this cannot be observed from this figure.
Three actuators used to position the beam

By using three actuators to position the beam in x-direction, overactuation is applied. In this case is not only the location of the actuators is variable, but also the ratio of forces between the actuators. The system is still symmetric, therefore one actuator is always located at the centre of the beam. The other two actuators are located symmetric to the centre of mass. The difference in x-direction as result of two possible locations is illustrated in Figures 2.16 and 2.17.

![Figure 2.16: The deformed shape in a setup with three actuators.](image1)

![Figure 2.17: Deformed shape when the actuators are located for minimal deflection.](image2)

To determine the required force ratio for minimal deflection the elasticity of the beam should be taken into account. In Chapter 4 will be explained how to calculate the required force ratio. In this chapter is assumed the actuator forces are such that the locations in x-direction is equal for all actuators. Thus independent on the actual required actuator force, the position of the actuator location in x-direction is known.

The minimum deflection might however appear in a situation where the x-location of the actuators is not necessarily equal. In advance of Chapter 4 another method is described, that is based on a minimum contribution of subsequent vibration modes. This optimum (minimal peak valley value in x-location) might be found by consequently passing through all values of the prescribed actuator locations. The force ratio is calculated such that subsequent vibration modes are not excited. When the actuator location is at 14.23% from the sides the minimal deflection is obtained. The actuators $F_{EL}$ and $F_{ER}$ each exert 31.88% while the actuator at the centre exerts 36.24% of the required force [217]. A first order approximation can however be obtained from the assumption that the x-location of the actuators remains equal.

2.3.1.1 Adding more actuators, an approximation

The exact actuator locations and forces can be calculated for a large number of actuators. The beam is considered solid, not rigid, and thus elastic deformations are present. Before calculating the optimal actuator location for every number of actuators used it might be useful to indicate a trend line. The trend line might indicate whether it is useful to really perform the calculation of the location and force of each actuator for every arbitrary number of actuators that
are used. The trend line should indicate how much mass reduction is possible as result of using more actuators.

A simplification is introduced to approximate the effect of adding an extra actuator. An error is made but the general tendency as result of using more actuators can however be deduced. The system with two actuators that are located at the optimal location, $2.14$, can for instance be split at the locations where the slope is zero, see Figure 2.18.

![Figure 2.18](image)

**Figure 2.18:** When two actuators are used and the actuators are located at their optimal location, figure 2.14, the deformed beam is split at the locations where the slope is zero.

The split at the location where the slope is zero is not necessary equal to the actuator location. This split is used to calculate the effect of adding actuators. The different parts in Figure 2.18 can be modelled as the systems in Figure 2.19 and Figure 2.20. When three actuators are used and they are located at their optimal locations the same method can be performed. The system in Figure 2.17 is split in the same way. The sides can be modelled as the system in Figure 2.19. For the central part two system as in Figure 2.19 are used.

![Figure 2.19](image) ![Figure 2.20](image)

**Figure 2.19:** Standard system that can be used to model the side parts in figure 2.18  
**Figure 2.20:** Standard system can be used to model the central part in figure 2.18

When the actuators are not in their optimal location other characteristic systems can be used. The system in Figure 2.16 can for instance be modelled as two of the systems in Figure 2.21:

### 2.3.1.2 Characteristic length as function of the number of actuators

When actuators are added and the peak valley value has to be reduced, the characteristic length reduces. The change in characteristic length is calculated
2.3. Product carrier mass due to increased number of actuators

Figure 2.21: Standard system that can be used to model a part in Figure 2.16 when more actuators are used.

for the case that the actuators are located in their optimal position. The systems shown in Figures 2.19 and 2.20 are used to represent the systems where an increased number of actuators is applied. The characteristic length $L_A$ can be calculated as function of the number of actuators ($n_{act}$).

\[
2L_A + 2(n_{act} - 1)L_B = L \quad \Rightarrow \quad L_B = \frac{L/2 - L_A}{n_{act} - 1} \quad (2.24)
\]

The length $L_A$ is determined from the requirement that the deflection of the systems in Figures 2.19 and 2.20 is equal. The equations describing the deflection in these figures are:

\[
\frac{3w \cdot L_A^4}{24E \cdot I} = \frac{w \cdot L_B^4}{24E \cdot I} = \frac{w(L/2 - L_A)^4}{24E \cdot I} \quad (2.25)
\]

With this equation $L_A$ can analytical be expressed as function of the number of actuators.

\[
L_A = \frac{L/2}{\left(3^{\frac{1}{4}}(n_{act} - 1) + 1\right)} \quad (2.26)
\]

Figure 2.22: Relative characteristic length as function of the number of actuators

With Equation 2.26 also the relative length change as result of an additional actuator can be determined. The relative length decreases at an increased num-
ber of actuators. I.e. the relative length change is larger when an additional actuator is added to a system that originally consisted of a small number of actuators.

Now the characteristic length is known as function of the number of actuators the deflection, and thus peak valley value can be calculated this is:

$$\Delta x = \frac{w \cdot L_A^4}{8E \cdot I} \quad (2.27)$$

The reduced length $L_A$ as function of the number of actuators can be used to reduce the height. The peak valley value $\Delta x$ should however be constant. Even as in section 2.2.2 the possible mass reduction depends on the ratio between the body dependent forces and the external forces.

**Effect of product mass and structural mass**

When the deflection due to the load in Equation 2.27 is considered. The only variable that can be reduced is the height of the beam. The stiffness depends on the height as given in Equation 2.12. The distributed load depends on the body dependent forces of the beam ($w_c$), proportional with the height, and an external load ($w_p$) that is constant.

$$w = w_p + w_c = w_p + \rho \cdot a \cdot b \cdot h \quad (2.28)$$

The density of the beam is present by parameter $\rho$, the acceleration by $a$, the width by $b$ and the height by $h$. How the height influences the deflection depends on the ratio between the values of $w_p$ and $\rho \cdot a \cdot b \cdot h$. When $w_p \ll \rho \cdot a \cdot b \cdot h$.

$$\Delta x(L, h) \propto \frac{w \cdot L^4}{E \cdot I} \propto \frac{h \cdot L^4}{h^3} \propto \frac{L^4}{h^2} \quad (2.29)$$

When $w_p \gg \rho \cdot a \cdot b \cdot h$.

$$\Delta x(L, h) \propto \frac{w \cdot L^4}{E \cdot I} \propto \frac{L^4}{h^3} \quad (2.30)$$

When an actuator is added the characteristic length is reduced. As result of the change in length the difference in $x$-direction is reduced. This result in an allowed reduction of stiffness, in this case the height. The height reduction incorporates mass reduction. The height reduction has a smaller effect in case $w_p \ll \rho \cdot a \cdot b \cdot h$ than in case $w_p \gg \rho \cdot a \cdot b \cdot h$. The possible mass reduction in case $w_p \ll \rho \cdot a \cdot b \cdot h$ is thus larger than in case $w_p \gg \rho \cdot a \cdot b \cdot h$.

**Mass of the beam as function of the number of actuators**

For a solid beam, where only the height is varied the mass is proportional to the height. The total length and width are kept constant. Using more actuators influences the characteristic length as for instance expressed in Equation 2.26. Thus when $\Delta x$ is kept constant in case $w_p \ll C \cdot h$ the characteristic length $L$ depends on the number of actuators ($n$). The height as function of the charac-
2.3. Product carrier mass due to increased number of actuators

teristic length \((L)\), and indirect the number of actuators, is:

\[ h \propto \left( \frac{L^4}{\Delta x} \right)^{\frac{1}{2}} \propto m \]

(2.31)

When \(w_p >> \rho \cdot a \cdot b \cdot h\) the height as function of the characteristic length \((L)\), and indirect the number of actuators, is:

\[ h \propto \left( \frac{L^4}{\Delta x} \right)^{\frac{1}{3}} \propto m \]

(2.32)

As can observed from Equation 2.26 the relative length change as result of additional actuators decreases. The relative effect on the length of additional actuators also affects the mass as determined with Equation 2.31 and 2.32. From the decreased relative mass reduction when an increased number of actuators is used it might be concluded that is not interesting to use a large number of actuators.

\textbf{Beam example}

In the previous Chapter an example is defined. In case the beam has to be positioned over a stroke of \(1 \, mm\) within a motion time of \(t_m = 20 \, ms\) the required acceleration and deceleration is \(a = 10 \, m/s^2\). Then it is assumed during the first half of the motion time the system is accelerated and the second half of the motion time the system is decelerated. For the allowed static deflection the specified tolerance is used, thus \(\pm 25 \mu m\).

Despite the importance of the product properties the calculation method is used that initially neglects the mass of the product. When the height of the beam is determined it is verified if this assumption is valid. In case the assumption is not valid, a new height is determined. First is the height of a beam with two actuators at the outsides calculated. The theoretical case that only the
beam has mass and thus inertia. $w_p \ll \rho \cdot a \cdot b \cdot h \rightarrow w = \rho \cdot a \cdot b \cdot h$. The height is determined from the equation that describes the deflection at the centre:

$$\Delta x = \frac{5 w \cdot L^4}{384 E \cdot I} = \frac{12 \cdot 5 \cdot a \cdot \rho \cdot b \cdot h \cdot L^4}{384 E \cdot b \cdot h \cdot h^2} = \frac{60 a \cdot \rho \cdot L^4}{384 E \cdot h^2} = 25 \mu m$$

(2.33)

The height is thus:

$$h = \sqrt{\frac{60 a \cdot \rho \cdot L^4}{384 E \cdot 25 \times 10^{-6}}} = \sqrt{\frac{60 \cdot 10 \cdot 0.5^4}{384 \cdot 25 \times 10^{-6}}} \cdot \sqrt{\frac{\rho}{E}}$$

(2.34)

When Aluminum or Steel are used, the specific stiffness is $E/\rho \approx 27 \frac{MPa \cdot m^3}{kg}$ the required height is 12.1 mm. Because it is a solid beam the mass is proportional with the height, Equation 2.16. The beam mass per unit of area ($M_{bm}$) is obtained by multiplication of the height with the density:

$$M_{bm} = \rho \cdot h = 62.5 \cdot \left(\frac{\rho}{E^2}\right)^{\frac{3}{2}}$$

(2.35)

When steel is used with a density of, $\rho = 7850 \ kg/m^3$ the beam mass per unit of area is $M_{bm} = 95 \ kg/m^2$. When Aluminum is used $\rho = 2700 \ kg/m^3$ the beam mass per unit of area is $M_{bm} = 33 \ kg/m^2$. The total beam mass is 950 g for the steel beam, respectively 370 g for the aluminum beam. The product mass of 15 g is relatively small compared to the beam mass, and the initial assumption, neglecting the product mass, might therefore be applied.

The two actuators are now located at their optimal locations. The system in Figure 2.19 is used, so Equation 2.27 can be used to calculate via the deflection the required height:

$$h = \sqrt{\frac{12 \cdot 10 \cdot L_A^4}{8 \cdot 25 \times 10^{-6}}} \cdot \sqrt{\frac{\rho}{E}}$$

(2.36)

The beam mass per unit of area in $M_{bm} = 14.6 \ kg/m^2$ when steel is used and 5.0 kg/m² when Aluminum is used. The total mass of the beam is respectively: 146 g and 50 g. As a result of the mass reduction the magnitude of carrier mass is approaching the magnitude of the product mass. When the product mass is incorporated in the iterative calculation the beam mass is respectively 150 g and 59 g.

### 2.4 Adaptations to solid cross sections

Solid beams can be used to meet the required specifications. However when the weight or efficiency of material use is a concern, other cross sections should be used. Weight reduction from the solid beam to a wide flange or I-beam is for instance possible, see Figure 2.26. If a large external load is present thick facings of the flanges might be required and also be most economical. When however the load is small and only thin facings are required buckling of the flanges will probably occur. In that case the sandwich concept is the best concept[20].
2.4. Adaptations to solid cross sections

Figure 2.25: Beam mass as function of the number of actuators. The actuators are located such that the deflection is minimized. The deflection per acceleration is in all cases constant and the effect of the product mass is incorporated.

Figure 2.26: Different cross sections of the beam. From left to right a solid section, a wide flange beam and a sandwich beam.

The load is a result of inertia of the product, the inertia of the structure and the acceleration. The weight of the structure is assumed to contribute significant to the load. In that case a sandwich beam is lighter than an I-beam, [20]. Different sandwich concepts exists, for instance structural cores such as honeycomb core or foam cores such as from metal or other foams [5]. Optimization of sandwich panels is a specialty on its own. In this section only metal foam cored panels are shortly considered. An advantage of foams is isotropy, which simplifies the calculation process [9]. Metal foams posses a relatively large shear rigidity, compared to other foam materials often used in sandwich panels. Furthermore metal foam is more easily applied as core material in complex shaped products than for instance honeycomb core material. The theory as used here originates from Ashby et al. [9] and Gibson and Ashby [79].

The goal is to find the lightest sandwich panel. The core thickness \((s_c)\) flange thickness \((s_l)\) will therefore be optimized such that the weight is as low as possible. The maximum deflection is prescribed \((\Delta x)\). The pressure load \((p)\) is
assumed to be independent of the weight of the sandwich panel. The weight of the sandwich panel can afterwards be taken in account. First the panel is calculated for the external load; thereafter the load is changed such that the weight of the panel is included, and a modified panel can be calculated. This iteration has to be performed a few times.

Remark on shear deformation

A major difference with solid beams is that beside bending deflection the shear deformation contributes significantly in the total deformation of sandwich beams. The internal shear force is obtained from the integration of the distributed load. Using material and shape parameters the local shear strain can be determined. Integration of the shear strain over the corresponding length results in the deformation due to shear [228]. The internal moment is obtained from integration of the internal shear force. The deformation due to bending is obtained by double integration of the internal moment. The unknown constants result from the continuity of slope and deflection.

The effect of the number of actuators and their location on the deflection is considered more important than on shear deformation. The optimal actuator location for minimum bending deformation is assumed to be more sensitive to actuator location changes than it is for minimum shear deformation. In this thesis the effect of the actuator location on the bending is therefore initially considered most important. When the effect of shear deformation cannot be neglected with respect to the effect of bending deformation, first only the effect of the bending deformation is determined. Thereafter the corresponding shear force and the resulting shear deformation can eventually be added. This method is also applied when the effect of the number of actuators and their location on the mass has to be determined.

2.4.1 Sandwich beam actuated only at the end points

A sandwich beam under a pressure load \((p)\) is considered, see Figure 2.27. The length of the beam is \(L\), the width, \(b\), the core thickness, \(s_c\), and the face thickness, \(s_t\). The largest deflection is denoted by \(\Delta x\) and appears in Figure 2.27 at the centre. The material properties of the sandwich beam are indicated with the normal parameters as Young's modulus, density and shear modulus, however subscripts are added. The subscript 'f' refers to the faces, the subscript 'c' to the foamed core and the subscript 's' to the solid from which the core is made. The face sheet material is this thesis the same as that of the fully dense core material.

The core moduli, Young's \((E_c)\) and shear \((G_c)\) modulus, are assumed to vary with the foam density \(\rho_c\) as:

\[
E_c = C_E \cdot E_s \left(\frac{\rho_c}{\rho_s}\right)^2 \\
G_c = \frac{E_c}{2(1 + \nu_c)} = \frac{C_E}{2(1 + \nu_c)} \cdot E_s \left(\frac{\rho_c}{\rho_s}\right)^2
\]  (2.37)

Parameter \(C_E\) is a quality factor that can be used to express for instance the homogeneity of the foam structure. \(C_E = 1\) applies to a core material with
relative inferior properties and $C_E = 4$ to a material with properties somewhat better than material available in the year 2000, according to [9]. In this thesis the value $C_E = 1$ is used in order to guarantee an upper performance limit. For the poisson ratio of the cellular core $\nu_c$ the value $\nu_c = 1/3$ is used [9].

The equivalent flexural rigidity $(EI)_{eq}$ and the equivalent shear rigidity $(AG)_{eq}$ of the beam are used to simplify the calculation. The flanges are assumed to contribute only to the bending stiffness the core only to the shear stiffness. The equivalent rigidities are:

$$(EI)_{eq} = \frac{E_f \cdot b \cdot s_t \cdot s_c^2}{2} \quad (AG)_{eq} = b \cdot s_c \cdot G_c$$  \hspace{1cm} (2.38)

When a pressure load $p$ is applied the deflection is the sum of the bending and shear components:

$$\Delta x = \Delta x_b + \Delta x_s = \frac{p \cdot b \cdot L^4}{B_1(EI)_{eq}} + \frac{p \cdot b \cdot L^2}{B_2(AG)_{eq}} = \frac{2p \cdot L^4}{B_1 \cdot E_f \cdot s_t \cdot s_c^2} + \frac{p \cdot L^2}{B_2 \cdot s_c \cdot G_c}$$  \hspace{1cm} (2.39)

$B_1$ and $B_2$ depend on the load and support condition [9]. For a simply supported beam with distributed load: $B_1 = \frac{384}{5}$ and $B_2 = 8$.

The weight of the beam has to be minimized for a given deflection $\Delta x$, load $p$ and length. The equation to be minimized is:

$$m = (2\rho_f \cdot s_t + \rho_c \cdot s_c)b \cdot L$$  \hspace{1cm} (2.40)

The minimization problem is equivalent to minimizing the mass per area $(A)$:

$$\frac{m}{b \cdot L} = \frac{m}{A} = 2\rho_f \left( s_t + \frac{\rho_c \cdot s_c}{\rho_f} \right)$$  \hspace{1cm} (2.41)

The free variables in this optimization process are $s_t$, $s_c$ and $\rho_c$. First the optimization procedure is done with a fixed core density $\rho_c$. The thickness of the
face sheets is determined using Equation 2.39:

\[ s_t = \frac{2p \cdot L^4 \cdot B_2 \cdot G_c}{B_1 \cdot E_f \cdot s_c} \left( \frac{1}{B_2 \cdot \Delta x \cdot G_c \cdot s_c - p \cdot L^2} \right) \] (2.42)

This face thickness can now be filled into Equation 2.41. Which is minimized to the only variable the core thickness \( s_c \). By setting \( d(m/(b \cdot L))/ds_c \) equal to zero the optimal core thickness is determined. With the optimum core thickness the optimum face thickness can be determined with Equation 2.42. After the optimum parameters of the sandwich beam are determined, it should be verified if the stress in the different parts remains within the failure limits [9, 249]. The parameters as presented below fulfill this requirement.

The optimum dimensions of the sandwich beam are in this thesis determined using a graphical method. Therefore the objective function in Equation 2.41 is inverted to:

\[ s_t = \frac{m}{b \cdot L} \cdot \frac{1}{2\rho_f} - \frac{\rho_c}{\rho_f} \cdot \frac{s_c}{2} \] (2.43)

This gives the relation between the thickness of the two variables; the face thickness \( s_t \) and the core thickness \( s_c \). This is plotted for different values of \( m/(b \cdot L) \) in Figure 2.28. The required flange thickness as function of the core thickness can be calculated with Equation 2.42. The same specifications as used before are used thus the specified tolerance is used for the allowed deflection \( \Delta x = 25 \mu m \) and the length and with are fixed \( L = 0.5 \, m \, b = 20 \, mm \).

In the solid beam case Aluminum was used. This is also assumed to be used as material for the core and faces \( E_f = E_s = 70 \, GPa \). The relative density of the core is assumed \( \rho_c/\rho_s = 0.1 \).

The mass of the product is \( M_{pm} = 1.5 \, kg/m^2 \) as initial mass for the beam is used the mass required when solid material is used. This mass is \( M_{bm} = 37 \, kg/m^2 \) as was calculated before. The assumed load is obtained by multiplication with the acceleration. As in the solid beam case an acceleration of \( 10 \, m/s^2 \) is assumed which results in a pressure load \( p = 385 \, Pa \). When this calculation is performed a new beam mass is obtained for the sandwich beam \( (M_{bm} = 10 \, kg/m^2) \). This can be used as new loading condition to determine the mass more accurate. In Figure 2.28 the face thickness is plotted as function of the core density when the beam itself is assumed to have a mass of \( M_{bm} = 5.6 \, kg/m^2 \). This value is obtained after a few iterative calculations.

From Figure 2.28 can be concluded that the minimum mass of \( \approx 5.6 \, kg/m^2 \) is obtained when the core thickness \( s_c \approx 14 \, mm \) and a face thickness of \( s_t \approx 0.34 \, mm \). These dimension correspond with a beam mass per area of \( M_{bm} = 5.6 \, kg/m^2 \) when calculated with Equation 2.41. Further iteration is thus not required.

### 2.4.2 Optimal actuator location and increased number of actuators

The support location also effects the deflection of the sandwich beam. The optimum location for the bending contribution is previously determined. The optimum location for only the shear deformation is obtained by minimizing...
Figure 2.28: Graphical method of optimization. The thickness of the face sheets as function of the core thickness is plotted. The core and face thickness can be determined for the different $m/A$ values. Two supports are located at the outside, while the loading is the result of an acceleration field of $10 \text{ m/s}^2$. The optimum beam dimension has a core thickness $s_c \approx 14 \text{ mm}$ with a face thickness $s_t \approx 0.34 \text{ mm}$. The beam mass per area is $m/A \approx 5.6 \text{ kg/m}^2$.

The length of each beam part. Optimized for minimum shear deformation results in a beam where the length of the overhang is half the length between the actuators. To obtain an impression about the effect of the number of supports on the mass the characteristic length as determined for solid beams is used, however than the contribution of shear to the deformation is approximated too small. To guarantee an upper limit of the deflection the characteristic length is therefore increased as is optimal for minimal shear deformation. The optimum actuator location might be different, however an upper limit is guaranteed.

Figure 2.29: Characteristic length for the sandwich beam. In case two actuators are used the length is one fourth of the total length.

With Equation 2.39 the deflection is determined. The deflection of cantilever loaded with a distributed load as presented in Figure 2.29 is determined with Equation 2.39 where the values $B_1 = 8$ and $B_2 = 2$ have to be used [9]. As initial loading the product mass plus the mass of the sandwich beam with two supports at the outsides is used. After a few iterations the sandwich beam mass per unit of area is $M_{bm} = 1.4 \text{ kg/m}^2$. No more iteration is required because when this is used as loading the required sandwich mass does not change, Figure 2.30.
Figure 2.30: Determination of the optimum beam properties for a cantilever beam with a length of 1/4 of the original length. This represent the deformation with two supports. The optimum beam has a core thickness $s_c \approx 3.2$ mm with a face thickness $s_t \approx 0.10$ mm. The beam mass per area is $M_{bm} \approx 1.4$ kg/m².

The characteristic length is 0.5/6 m when three actuators are used. The values of the optimum are $s_c \approx 1.9$ mm with a face thickness $s_t \approx 44$ µm. The beam mass per length is $M_{bm} \approx 0.75$ kg/m². The beam mass is 7.5 g.

Despite the calculation of sandwich beam mass can be extended to much more actuators without mathematical limitations, practical aspect might limit the production of such beams.

2.5 Extension to plate structures and optimization

2.5.1 Plates

The equations required to describe the bending deformation of solid beams under different loading and boundary conditions are relatively simple. The height to length ratio is assumed smaller than 1/20 and therefore the Bernouille-Euler theory was applied [254]. The deformation ($x_m$) was dependent on the y-coordinate ($x_m(y)$). The governing differential equation for the bending of a homogenous beam with height $h$ is:

$$\frac{d^2}{dy^2} \left( \frac{d^2 x_m}{dy^2} \right) = p \cdot b \cdot \frac{1}{E \cdot I} = p \cdot \frac{12}{E \cdot h^3} = \frac{p}{D_b} \quad (2.44)$$

The flexural rigidity of the beam is expressed with parameter $D_b$, the other parameters are already explained. A plate can be considered as an extension of the beam in another dimension. The deformation ($x_m$) is now also dependent on the z-coordinate ($x_m(y, z)$). For a plate the governing differential equation for the bending of a homogeneous plate with height ($h$), or thickness, is:

$$\frac{\partial^4 x_m}{\partial y^4} + 2 \frac{\partial^4 x_m}{\partial y^2 \cdot \partial z^2} + \frac{\partial^4 x_m}{\partial z^4} = p \left( \frac{1 - \nu^2}{E \cdot I} \right) = p \left( \frac{12(1 - \nu^2)}{E \cdot h^3} \right) = \frac{p}{D_p} \quad (2.45)$$
Where $D_p$ is the flexural rigidity of a plate. The plate manifests greater stiffness than the beam by a factor $1/(1 - \nu^2)$ [254]. The material in a beam is free to expand sideways, while in a plate it is not free to expand sideways. As result the anticlastic curvature that appears in beams is prevented in plates, and this results in greater stiffness [11].

From literature the analytical solution for some specific cases and some boundary conditions is available, for instance the solution for a round plate simply supported around the whole circumference. More complicated is the analytical solution for point supported plates. In that case solutions obtained with numerical methods can be faster, for instance by using finite element methods.

In the differential equation for plates and beams the same material parameters are used. And the stiffness depends in both cases in the same way on the height. When the mass has to be determined also the same parameters are used. A fixed area and a variable homogeneous height are multiplied with the density to determine the mass. The dependency of the plate mass on the material is therefore the same as was for beams. Despite the stiffness of plates contains the factor $1/(1 - \nu^2)$ it is assumed not to affect the material selection criteria as was used for beams, because this effect is only a minor aspect compared to the other aspects such as Young’s modulus and density. The poisson ratio for most metals is approximately $\nu \approx 0.3$ and thus the factor is only slightly increased.

The bending of honeycomb plates is often expressed using the plate Equation 2.45 and replacing the flexural rigidity ($D_p$) by an equivalent flexural rigidity ($D_{honeycomb}$) [258].

$$D_{honeycomb} = \frac{E \cdot h_{eq}^3}{12(1 - \nu^2)} \tag{2.46}$$

From the honeycomb structure an equivalent plate height for a solid plate ($h_{eq}$) is determined. This equivalent plate height depends on the properties of a honeycomb structure. A similar rigidity might be used for sandwich panels.

For the bending deflection of beams a characteristic length was used, to approximate the effect of an increased number of actuators. Equivalent with this method a characteristic area might be defined in case of plates. As was mentioned for Equation 2.26; the characteristic length change decreases at an increased number of actuators. Similar it is expected that the characteristic area change decreases at an increased number of actuators. In other words the percentage of change of the characteristic area is larger when an actuator is added to a system that exists of a small number of actuators when compared to a system that exists of a large number of actuators.

The ratio between the dimensions of the beam and the plate are such that shear can be neglected in case of a solid cross sectional area. When a sandwich material or honeycomb structure is used the contribution of the shear might be in the same range as the bending deflection, and should thus also be taken in account. For solid material the beam and plate show similarity to material parameters. This is therefore also assumed for sandwich beams and plates. Optimizing sandwich and honeycomb plates to particular cases and especially
with different number of point support locations is a specialty on its own and considered beyond the scope of this thesis.

### 2.5.2 Configuration optimization

For a given number of actuators, their location and a specified loading the optimal spatial distribution of material might be optimized to obtain a minimum mass product carrier \[71, 98, 181, 185, 215, 259, 282\]. The sandwich panels might for instance be converted to bridge like shapes. The expectation is however that only a small amount of core material can be removed as result of this conversion. The core material is lightweight so not much mass savings are expected. [196] mentions: "To say an optimizer has improved a structure is one thing, to ask how much improvement there is still to go is a vastly more demanding question. In structural mechanics the analyst is not able to say how far away the structural design is from being perfect."

In this thesis the effect of the number of actuators and their location is considered. The focus is thus on the effect of the configuration. Despite important, optimizing the spatial material distribution is however not considered in more detail in this thesis. Also specialized solutions to obtain minimum mass structures, such as by the application of sandwich material, is thus not considered in more detail.

### 2.6 Conclusion

In a high speed precision positioning systems acceleration of the inertia forms a major load of the system. The error depends among other on the structural stiffness. In order to determine the effect of additional parallel actuators a quasi static acceleration field was used. The mass and the stiffness of the product carrier affects the amount of energy stored during a constant acceleration. The ratio deformation over acceleration is used as initial measure of the elastic energy stored in the structure while accelerated. The dynamic aspects of parallel actuation are considered in Chapters 4 and 5.

The relation between the material parameters that affect structural mass and stiffness of the product carrier depends, besides the Young's modulus of elasticity and the density, also on the ratio of the external load with respect to the inertia force of the product carrier. In this thesis is focussed on the moving mass change as result of overactuation. As such the material parameters that affect the quasi static performance are indicated. In practice much more material parameters of the product carrier might be important for application in high speed precision positioning systems.

Besides material influences also the number of actuators and their location affects how much elastic (potential) energy is stored in the system during a constant acceleration. By spreading the forces required for acceleration less energy is stored during that acceleration. As a result, a more flexible product carrier can be used, what results in a mass reduced structure. Due to the reduced product carrier mass the total actuator force required to accelerate the product
and product carrier is reduced. Nevertheless the mass of the actuators might increase, for instance when the actuators become less efficient. Besides less efficient actuators also other aspects might limit the possible mass reduction, for instance the requirements for the production of the product carrier. Furthermore the product carrier incorporates often not solely material used for stiffness but also other mass that does not contribute to the structural stiffness such as the moving mass of the actuators in case direct drives are applied. Also guidance mass and mass required to connect the product might be present that does not depend on the number of actuators. Some of these aspects are mentioned in the next chapters.
Chapter 3

Actuator and guidance system: general aspects

3.1 Introduction

The forces required to position the system are generated in the drive system. The drive system includes a power source, possibly a guidance system and if necessary a transmission. In the previous chapter movement of the product carrier in only one direction was considered. The other degrees of freedom were implicitly assumed to be constrained. A constraint is used to reduce the number of degrees of freedom of the object (relative to the reference object) [21]. Besides the degree of freedom in motion direction, the other degrees of freedom are from kinematic point of view, exactly constrained [21, 89], or in other terms statically determinate [99]. The practical requirements for the kinematic constraints depend on the system specifications, such as stroke, accuracy and presence of disturbance forces. In other positioning systems positioning in more directions might be required, what also affects the guidance requirements. The drive requirements are also dependent on these aspects. Despite a particular case was specified as benchmark system in Chapter 1 the focus of this thesis is to obtain general insight in the mass effects of overactuation. The mass of the guidance and the drive depends on the configuration of the system. General aspects of drive and the actuator system are therefore treated in this chapter with the aim to obtain insight. Especially the consequences of the configuration are mentioned. The insight might be used when overactuation is applied to more complicated positioning systems, than the benchmark systems. Different electromechanical drives are briefly mentioned from which the selection of a voice coil actuator, that was decided by the project team at an early stage in the project, is comprehensible. The other drives might however be applied in systems with other requirements e.g. stroke. A single voice coil drive is studied in more detail. Specifically the scaling of the copper mass and the guidance of the mover are discussed. Thereafter the design as-
pects of the assembly of the drive and product carrier to a positioning system will be discussed. Mainly the principal differences of several guidance systems are treated.

3.2 Drive system

The positioning system is used to position a product. The function of the drive is to provide forces or torques such that the required accelerations are obtained. Torque or force is in general obtained by the conversion of energy through hydraulic, pneumatic or electrical media [251]. Groenhuis [82] made a comparison between these operation principles while also a distinction is made in open- and closed loop (servo) application. In this case, with relative moderate power and force, high accuracy, high performance and maximum flexibility requirements, an electric servo drive is considered the best choice. The use of other media is assumed in this case to result in relative massive inefficient structures.

An initial approximation of the required actuator force is obtained by assuming the system has to be positioned as rigid body. In this stadium, the design phase, only feedforward is considered to obtain appropriate specifications for the actuator and mechanical system. Disturbances or system dynamics are not taken in account only the movement as rigid body is considered. The minimal required acceleration is obtained by applying a constant acceleration during the first half of the settling time and consequently a constant deceleration the second half of the settling time. This is referred to as bang-bang control [207, 252]. The position is obtained by integrating the acceleration twice and can thus be described with parabolic functions. This input type is therefore also known as a second degree input function [46, 82]. The combination of required stroke and settling time affect the requirements for the acceleration and velocity, and can be used to select an appropriate amplifier and actuator [46, 195]. The requirements are discussed in more detail in Chapter 4, however the bang-bang criterium is used in this thesis to obtain an initial approximation of the drive specification. In Chapter 4 the drive specification based on a force that is a factor $\pi/2$ larger than the force obtained from the bang-bang criterium is motivated.

3.2.1 Direct electric drive

Linear direct drives are particularly suitable for machine tools where high accuracy, high speed, short settling times and thus high accelerations are required [29, 261]. Linear direct drive concepts are more and more replacing the conventional electromechanical concepts that use a transmission.

Detrimental effects of indirect drives

In indirect drive systems many mechanical elements with finite stiffness are present in the structural loop. The structural loop is formed by all structural components in the path of the load that connect the reaction force to the action
fore. Newton's laws of motion and d'Alembert's principle [176, 230] are used to calculate the force in case of open loop force paths, for instance as result of inertia or weight. The transmission ratio value is in general larger than one, i.e. the required speed at the motor side of the transmission is increased compared to the required speed at the load side of the transmission. On the other hand in case of a transmission ratio larger than one; the required force, or torque, at the motor side of the transmission is reduced compared to the required force, or torque, at the transmission side of the transmission.

Direct drive systems are gaining popularity because of advantages that are related to the lack of mechanical elements such as ease of maintenance, little clearance and friction (no transmission), fewer parts, high reliability, no dead zone and friction induced backlash, small volume [34, 40, 195, 197]. Another disadvantage of indirect drive systems is the finite stiffness of the mechanical elements. When they are subjected to loading, elastic energy is stored in the mechanical elements. This stored elastic energy should be handled with care, because it can be exchanged to kinetic energy, which means vibrations. Therefore the mechanical elements in the structural loop should have sufficient stiffness. Often the transmission is the limiting factor in the structural loop because it contributes most to the compliance and determines often the first dominant frequency [269]. The mechanical elements applied in an indirect drive are the result of the tradeoff between mass and stiffness. Increased stiffness often results in increased mass. Increased mass on the other hand means increased inertia. Even when the transmission is carefully tuned to the total mechatronic system the compliance of the transmission often remains the limiting element.

**Direct drive**

Direct drives might be used to overcome some of the limitations of indirect drives. Despite still mechanical elements with finite stiffness are often present in the structural loop, their stiffness is often not limiting the performance. Direct electric drives have besides the effect that no transmission is used, also the possibility to obtain a position independent force. When arranged in such a way that a position independent force is obtained there is no mechanical connection between the frame and the end-effector in the considered direction. This effect might also be used to prevent transmission of possible frame vibrations to the end-effector via the drive[174, 269].

In direct drive actuation or semi-direct-drive actuation with low transmission ratio, a tradeoff between position- and force control is present. The transmission ratio in a direct drive equals one. This affects the tradeoff between speed and position control [206]. Compared to indirect drives with a transmission ratio greater than one, the reduced transmission ratio might result in improved quality of force control. Output torque or force can be sensed using motor currents [170, 210]. However, in case of a reduced transmission ratio, the controller settings should be raised with intent to improve the position control behaviour (for instance to maintain the stiffness). Increased controller settings might however not be sufficient to obtain the required stiffness. The maximum force for instance is still limited by the actuator. To achieve the same torque with a smaller transmission ratio in rotary electromagnetic mo-
tors the size of the motor has to be increased; thus at the expense of high motor mass [122]. Stability aspects of a serial direct drive concept are discussed by Rankers [197].

3.2.2 Linear electric motors

Types of linear electric motors

The force in electric drives can originate from different principles, see [28, 48, 59, 92, 130, 135, 165, 174]. In a piezoelectric material for instance, force is generated as result of static charge. Moving charge in a magnetic field can also result in force generation. The electromechanical drive systems are classified according to the effects that contribute most to the force generation. The classification is based on the classification as presented by Jufer [117] and extended with 'strain' actuators:

1. Strain systems. Force results from the strain of piezoelectric, electrostrictive or magnetostrictive material. Piezoelectric material expands or contracts when a voltage is applied. Electrostrictive and magnetostrictive material expands or contracts as the result of changes in the electric respectively magnetic field in which the material is placed [51, 145].

2. Reluctance systems; not containing permanent magnets and also lacking force as result of mutual flux interaction, or mutual inductance. Force is generated as result of position dependent self inductance, inversely proportional to reluctance.

3. Electromagnetic systems; Mutual flux interaction results in a force when a current carrying conductor is placed in a magnetic field, created with magnets and ferromagnetic material, e.g. a voice coil actuator.

4. Electromagnetic systems; the system comprises a magnet and a ferromagnetic circuit with a coil. Force is generated as result of the reluctance force of the magnet, and mutual flux interaction. The effect of coil inductance is much smaller. A magnetic potential well is created.

5. Polarized reluctance systems; characterised by an electric circuit, a magnet and ferromagnetic material. Force is generated as result of self inductance, reluctance, and mutual flux. The magnitude of force thanks to the mutual flux is in the same range as the force magnitude of the self inductance.

6. Electric machines; force results from the mutual inductance between two or more 'coils' located on different sides of the airgap. These currents can be directly supplied (synchronous and direct current machines) or induced (asynchronous machines).

A major difference between the strain systems and the other drive types is the mechanical connection. In a piezoelectric, electrostrictive and magnetostrictive drive a mechanical connection is present because their operation is dependent on mechanical deformation (strain). In the other drive types the force
is transferred without mechanical connection [59]. The absence of a mechanical connection can be desirable in zero-stiffness applications, e.g. in vibration isolation systems.

The relative displacement of piezoelectric material is approximately $1/1000$ of its length [189]. A motion over a distance of $300 \mu m$ is considered a long stroke when a piezoelectric drive has to be applied [66, 199]. The stroke of electrostrictive and magnetostrictive actuators is also in the order of $\mu m$ [59].

In precision positioning applications voice coils are typically used in case the stroke of the mentioned strain systems is insufficient. According to Devos [59], voice coils are the logical choice in precision positioning applications when the stroke in high accuracy direct drive precision machines is more than $1 mm$. The following voice coil drive properties are often considered to be an advantage: good linearity, low electric time constant, and single phase [117, 148]. A disadvantage of voice coil actuators is observed when a constant force, such as gravitational force and prestress force is required. A constant force might lead to significant heat dissipation [59].

Reluctance, electromagnetic and polarized reluctance systems can be realized in different ways. The force depends among others on the airgap length; as a result the operation range of the actuator is limited to relative small strokes, i.e. in the order of millimetres. The stroke of these drive systems can be extended via the creation of stepper motors. Remarks about e.g. power efficiency, non-linearity, negative stiffness, cogging, inertia or the electric time constant are mentioned in [59, 130, 146, 165, 174, 225].

Electric machines are regarded appropriate as direct drive for large machines [136]. For small electric machines, the constant current carrying coil might be replaced with a permanent magnet. Such machine should than however be classified as: electromagnetic machine or as polarized reluctance system [117]. When the asynchronous machines and synchronous electric machines are compared, a clear trend towards using synchronous drive types instead of induction (asynchronous) drive type is observed [195]. One of the main reasons is the larger feed force per unit of weight and size of the synchronous drive. Furthermore the induction force is either slow, or power inefficient [165].

An additional category of `electric’ actuators are shape memory alloy actuators. With electricity the temperature can be changed and thus change the state of the shape memory alloy. Due to their small motion range or the large time constants compared to the other strain based actuators, as mentioned above, the shape memory alloy is not considered in detail. More about the application of memory alloys is presented by Crawley [51], Peirs et al. [187].

**The drive concept used in this thesis**

In this thesis, positioning systems that use contactless direct drives are considered. The voice coil motor, or Lorentz actuator, is considered most suitable as electric linear direct drive for the benchmark system, see also [148]. Voice coil drives are in the required stroke and force range, the most commonly used electromechanical linear motors [59].

The allowed temperature rise as result of the generated heat in the drive, is
in this thesis considered limited by element failure, e.g., the isolation of the wires. Deformations due to temperature changes are not taken into account even though this might affect the accuracy. In practice the produced heat is a major point of concern in precision engineering applications.

3.3 Guidance

The positioning system should be able to move in the required direction. A rigid body 'floating', thus not connected to other objects, in 3D space has three translational and three rotational degree of freedom (dof). In the direction that requires positioning the degree of freedom is constrained as result of the interaction between the force of the drive system, measurement information and control software. The remaining dof need to be constrained as well. This can be realised with variable constraints (thus also drive systems) or fixed constraints referred to as guidance. Some relevant properties of the guidance system are:

- low or no stiffness in motion direction;
- low (moving) mass;
- high natural frequency
- (price, preferably cheap)

Kinematic design is applied to make sure every dof is properly constrained. The condition where the dof that need to be constrained, are constrained only once, is called exactly constrained [21]. Often the kinematic design is based on idealised components. Although all solids are flexible to some degree, machines are usually designed from relatively rigid material, keeping part deflections to a minimum. After the dynamic analysis when loads are known, the parts are designed so that the assumption of rigidity is justified [230, 261]. A similar assumption is used when kinematic constraints are applied on certain locations. The constraints are in the kinematic modelling assumed to be applied as point contacts. In practice this is translated into a relatively small contact area between the constraint and the solid that needs to be constrained compared to the other dimensions of the solid.

**Kinematic modelling**

A schematic representation of a guidance system is often used to visualize the constraints and degrees of freedom, for instance the system in Figure 3.1. This symbolic representation is also used in [21, 89, 99, 211]. In the schematic drawing of Figure 3.1 the lines 1 to 5 represent constraints that prevent motion in one direction [89]. A constraint line represents a constraint that possesses much larger stiffness in axial direction than in transversal direction [21]. In this way the only motion allowed by the constraints is perpendicular to the constraint lines. By connecting the lines in a certain orientation the object degrees of freedom are constrained. The rotation around the z-axis (ϕ) is for instance constrained as result of lines 1 and 2. The arrangement of
Figure 3.1:  Symbolic representation of constraining degrees of freedom (dof). The rigid object is only able to move in x-direction, the other dof are constrained. A drive system can be used to position in x-direction.

lines in Figure 3.1 form a guidance that constrains all degrees of freedom except the x-direction. The required guidance stiffness depends among others on the configuration, as will be explained.

In practice the constraint lines can be created in different ways, e.g. with the use of bearings but also with an elastic guidance. In case of an elastic guidance the guidance should be designed such that in particular directions the rigidity of the guidance is used to constrain degrees of freedom. Meanwhile should the elasticity of the elastic guidance allow motion in the required motion direction. In case of wire springs the elasticity in transversal direction of is used to allow motion in that direction. The elastic guidance is discussed in more detail in the next section. Also bearings can be used as constraints. The construction with bearings should however be in such a way that the force can be considered as if it acts on a contact point. If this is not the case rotation stiffness will not be obtained. To obtain the required bearing stiffness pre-loading is often applied. This means however a more bulky system.

Stiffness and mass of the guidance

The guidance is used to constrain degrees of freedom. In the direction of the dof that needs to be constrained the guidance should possess stiffness. In the required motion direction the guidance should be as flexible as possible, otherwise additional actuator force is required to overcome this stiffness. Another reason for very low or no stiffness in the motion direction is that a border for disturbances can be created. If the guidance has only little mechanical stiffness in the direction of motion disturbances in that direction are filtered out. When no stiffness at all is present in the direction of motion a full decoupling can be created.

When airbearings are used no stiffness in motion direction is added. An elastic guidance is based on the relative large elasticity in the required direction of motion. The elasticity is much smaller in the other dof that have to be constrained. An elastic guidance can be build from different elements [13, 64, 99, 144, 211]. Standard elements for an elastic guidance are blade flexures and wire flexures as for instance shown in Figure 3.2 and 3.3. There is thus elasticity in the directions that do not need to be constrained. This additional stiffness in motion direction has to be overcome by additional actuator force. In some cases the
3.3. Guidance

additional stiffness of the elastic guidance in motion direction is compensated by adding negative stiffness [226].

Not only the stiffness of the guidance is important but also its mass. The guidance consist of stationary and moving parts. The moving mass should be kept as small as possible, because mass means additional inertia and thus extra actuator force may be needed to obtain the required acceleration.

Figure 3.2: A wire flexure, with relative high axial stiffness compared to the transversal stiffness, constrains the axial direction between the object and the world. Due to the relative small flexural stiffness relative movement in other direction is possible. Elastic deformation is distributed. This system is more sensitive for buckling than the system in Figure 3.3.

Figure 3.3: The same function as in Figure 3.2 is fulfilled but the flexural stiffness of the central part is increased. The central part can be considered rigid. The flexural stiffness in the constriction is low, these act as hinges. Elastic deformation is localised in the hinges [183].

Another aspect that has to be taken in account when an elastic guidance is used is the natural frequency and damping of the guidance. The mass and stiffness distribution should be such that the natural frequency of the guidance does not introduce disturbances. For instance the flexure in Figure 3.2 is used to constrain one degree of freedom. Not only buckling might be a problem but also the natural frequency might not be sufficient [89, 174].

When the guidance vibrates in its natural frequency, referred to as slinky modes [58], the actual positioning system is disturbed. The effect of this disturbance depends on the impedance ratio between the guidance and the disturbed system. Slinky modes, when present, should be well damped because the stiffness in the direction that needs to be constrained is reduced when the guidance vibrates in its natural frequency [261]. The natural frequency of the guidance in Figure 3.2 can be increased by for instance another material distribution, for instance as in Figure 3.3. Dependent on the material used, the stress in the material and the number of motions fatigue might occur in the elastic guidance.

The connection of different elements in the positioning system determine the configuration. The arrangement requires some mutual degrees of freedom, while others need to be constrained. The stiffness requirements for the guidance depends on the configuration, for instance the required stiffness might increase due to misalignment. The mass of the guidance system depends also on the configuration. These aspects will be discussed in more detail in the next sections. In the guidance elements considered further on the guidance is as-
sumed to be designed in such a way that the stiffness in motion direction can be neglected, even when elastic elements are used. It does thus not contain a spring effect in motion direction.

3.4 Voice coil actuator

Lorentz actuators are suitable when the maximum required force and the power dissipation are not limiting their application. Otherwise other drive types, such as mechanical or pneumatic drives, should be used. In general, voice coil drives are popular in applications that need proportional or tight servocontrol in order to obtain high accuracy. A major disadvantage of voice coil actuators is often the generated heat as result of the dissipation of electrical energy due to the wire resistance. For this reason voice coil actuators are in general not applied to generate static forces in case the generated heat is a problem.

The type of voice coil actuator that is considered in this thesis is a cylindrical symmetric voice coil actuator with a central circular magnet magnetized in axial direction, see Figure 3.7. Due to cylindrical symmetry a full 3D model of the actuator is not required. It is sufficient to define a 2D cross section of the actuator. Other types, for instance a hollow disc peripheral magnet can also be used, see Figure 3.6. The design of the stator affects the field strength in the airgap. Magnets may be placed next to the coil, e.g. Figure 3.5. They may also be located remotely, with flux carried to the coil through steel pole pieces. The latter permits higher flux density, increasing the output force or permitting a smaller and lighter coil, at the cost of increased volume, complexity, cost and fluxleakage [245]. The flux of the relative large central circular magnet in Figure 3.4 is focussed into the relative small airgap in Figure 3.7.

As part of a multidisciplinairy research project the design, control and optimization of voice coil actuators is not discussed in large detail in this thesis [243]. Only some basic aspects are mentioned that are required to obtain
insight. In voice coil actuators a force is generated as result of current flowing in a coil that is placed in a magnetic field. Besides the electric and magnetic aspects also other aspects might be taken in account when a voice coil actuator is designed. For instance elements to prevent collision between mover and stator, or elements to connect the mover to the actual payload. The name mover is a substitute of the name rotor in case of rotational motors. The mentioned aspects of the actuators depend on the positioning systems as a whole, the configuration, and will be discussed in this thesis.

3.4.1 Mechanical aspects of a voice coil actuator

The drawing in Figure 3.8 represents a voice coil actuator. The coil is placed in the airgap in which a magnetic field is present. When electric current flows through the coil windings a so called Lorentz force is generated in x-direction. The force generated by the voice-coil actuator is used for accelerations in x-direction. The accelerations are needed for positioning the system. A guidance system is required to ensure only relative movement in x-direction between stator and mover is possible. The guidance system should prevent stator and mover touch. Collision might result in unwanted disturbances as friction but also the electric isolation of the coil can be damaged. The required alignment accuracy between the mover and stator depends on the clearance in the airgap between stator and mover.

The force of a particular electromagnetic motor depends on the magnetic field distribution in the airgap between the mover and the stator [122]. The clearance between mover and stator affects the required airgap. The mover and stator should be designed such that enough clearance is present between them. The airgap size affects the magnetic field in the airgap. The smaller the clearance, the smaller the airgap and thus the reluctance of the airgap. A smaller reluctance results in a larger magnetic flux density and thus a larger force. However, the smaller the clearance, the more accurate the mechanical system must be to prevent the mover and stator from touching. Because the flux density and current density are limited, the amount of force can only be changed by the amount of current carrying conductive material in the airgap. Besides, to
obtain a small reluctance, a small airgap is also beneficial for the heat transfer from the coil to the stator [96].

A symbolic representation of a guidance system is presented in Figure 3.9. Like in Figure 3.1 the lines are used to represent a constraint in axial direction. In practice the bars are for instance air bearings or an elastic guidance (this depends, among other things on the required stroke). The stator is assumed to be connected rigid to 'the world', while the mover is connected to 'the world' such that it has only one degree of freedom. As result of this guidance system only relative movement in x-direction is possible. In practice the mutual dof between stator and mover are fixed, the 'world' is in this case used as intermediate body. Fixating the five degrees of freedom between stator and mover is sufficient. Due to rotation symmetry around the x-axis this dof needs not necessarily be constrained.

The guidance is not directly connected to the coil but often a cup is used as intermediate body. The mover of an actuator consists not only of elements that are purely used to generate the static force ($F_L$) but also of a mechanical structure. In this case the mechanical structure consist of the elements cup, coupling and guidance. The functions of these elements are mainly:

- cup: provide stiffness for the coil and transfer the force generated in the coil to other elements;
- coupling: transfer the Lorentz force to the carrier;
- guidance: guide the mover relative to the stator in order to prevent contact between coil and yoke parts.

**Figure 3.8:** Cross section of a schematic Lorentz- or voice coil actuator. By applying a current in the coil the coil assembly moves in x-direction relative to the field assembly. In order to prevent collision between stator and mover a clearance ($\Delta_c$) is present.

**Figure 3.9:** Schematic representation of a mechanical guidance system that has only 1 dof in x-direction. The five dof between the two parts in Figure 3.8 should be connected like this.
The coil is in general used as mover, because it has the lowest mass compared to the magnet. (The cooling of the coil might however be more easy when the coil is used as stator.)

The wires of the coil are held together with the isolation material, an epoxy resin [36]. As a result of this construction the coil obtains mechanical rigidity. The mechanical rigidity of the coil is however limited and can be increased when the coil is wound on a thin bobbin [36]. This thin bobbin is in this thesis referred to as cup. The coil can be attached to the cup in different ways, for instance on one side as in Figure 3.8 or the cup might enclose the coil as in Figure 3.22. Due to the mechanical rigidity deformations of the coil, for instance as result of Lorentz forces are prevented. As mentioned the coil itself also exhibits some rigidity, even when no cup is used. In that case the coil might be connected direct to the structure [90, 194]. In both cases, with or without cup often an intermediate structure is used to transfer the Lorentz force to the structure. The requirements for this connection depend on the total construction and are discussed in more detail in section 3.6.

The cup material must be carefully selected. Beside the mass and stiffness of the cup material also other material properties are important. If the cup is made from ferromagnetic material the mover and yoke will attract each other as result of the Maxwell stress. Ferromagnetic material is therefore not used as cup material. The electric conductivity of the cup material is also important. In conductive cup material eddy currents are generated as a result of the change in magnetic flux density [257, 279]. The magnetic flux density changes as a result of current variations in the coil. The eddy currents in interaction with the magnetic flux density in the gap results in a force opposite to the Lorentz force of the coil. The magnitude of this damping force increases with an increased frequency of the current change. The eddy current induced damping is dependent on speed, and as a result, besides the frequency, also dependent on the amplitude of the (harmonic) motion [73]. Thermal conductivity of the cup material might also be important because the cup can be used to transfer the in the coil generated heat. When damping due to eddy currents is a problem nonconductive materials such as for instance plastics or pertinax can be used as cup material. These materials possess however, lower stiffness than for instance Aluminum.

### 3.4.2 Electric and magnetic aspects of a voice coil actuator

The design of electromagnetic actuators, such as voice coil actuators, is a specialty on its own and not discussed in this thesis. The effect of airgap size changes is briefly discussed, in order to determine if possible limitations for the Lorentz force exist, for instance current- or the magnetic flux limitations. The Lorentz force formula is used to consider aspects of the current carrying part. However the part of the actuator where the magnetic field is created has to be considered as well. That part not only affects the magnitude of the magnetic field but also the volume where the magnetic field is present. These aspects are considered with formulas that are only used for qualitative insight.

A circle symmetric actuator is considered. The airgap between the poles is a
cylinder with an inner diameter indicated \((R_i)\), outer diameter \((R_o)\) and height \((h_{air})\). The height of the airgap is assumed to be equal as the pole height. The different aspects are considered to be uniform over the height and leakage is not assumed to be present. First the reluctance of the airgap is calculated. Therefore a magnetic flux is assumed to be present which is indicated with \(\Phi_{mag}\). The flux density in the airgap at a radius \((r)\) is \(B_{air}(r) = \Phi_{mag}/(2\pi r \cdot h_{air})\). The magnetic field strength in the airgap is \(H_{air} = B_{air}/(\mu_0 \cdot \mu_{air})\). The magnetic field strength multiplied with the distance over which it acts results in the magnetomotive force. The magnetomotive force over the airgap is obtained by multiplication of the field strength with a small length \(dr\) and thereafter integration from the outer- to the inner radius. The reluctance of the airgap is obtained by dividing by the present magnetic flux \(\Phi_{mag}\) and is \([36]\):

\[
R_{airgap} = \int_{r=R_i}^{r=R_o} \frac{1}{\mu_0 \cdot \mu_{air} \cdot 2\pi r \cdot h_{air}} \cdot dr = \frac{\ln \frac{R_o}{R_i}}{\mu_0 \cdot \mu_{air} \cdot 2\pi h_{air}} (3.1)
\]

The magnetic flux \(\Phi_{mag}\) depends on the total reluctance of the system and the sources of magnetomotive force. For this static magnetic field holds that the summation of magnetomotive force \((mmf)\) over a closed path is zero \(\oint H \cdot dL = 0\) \([28]\). As result of the magnetic flux in the airgap and the reluctance of the airgap a magnetomotive force is created that counteracts the magnetomotive force of the magnet. In the other components also magnetomotive force is created, for instance due to reluctance of the iron. For simplicity the magnetic flux is considered for a static operation, without current carrying conductors. The only available source of magnetomotive force is a magnet and only reluctance is present, what results in a magnetic flux \(\Phi_{mag} = \frac{mmf_{mag}}{\sum reluctance}\). The reluctance of the airgap is indicated with \(R_{airgap}\), the reluctance of the other parts, including the magnet itself, with \(R_{yoke}\). The magnetic flux is:

\[
\Phi_{mag} = \frac{mmf_{mag}}{R_{yoke} + R_{airgap}} = \frac{mmf_{mag}}{\frac{\ln \frac{R_o}{R_i}}{\mu_0 \cdot \mu_{air} \cdot 2\pi h_{air}}} (3.2)
\]

The value of \(mmf_{mag}\) is however not constant and depends on the operation point of the magnet. The demagnetization curve of the used magnet and the crossing with the load line define the operation point \([48, 233]\). The load line can be obtained from Equation 3.2. The magnetic flux density in the magnet \(B_m\) is determined by the flux \(\Phi_{mag}\) divided by the cylindrical cross sectional area of the magnet \((A_m)\). Parameter \(mmf_{mag}\) is replaced with the length of the magnet \(L_m\) and the magnetic field strength \(H\); \(H\) depends now on the operation point. In the magnet the direction of the magnetic field and the flux density are of opposite sign. The load line of the magnetic structure is:

\[
B_m = \frac{\Phi_{mag}}{A_m} = \frac{-H \cdot L_m}{A_m \cdot R_{yoke} + A_m \cdot \frac{\ln \frac{R_o}{R_i}}{\mu_0 \cdot \mu_{air} \cdot 2\pi h_{air}}} (3.3)
\]
Lorentz force is generated as result of the current in a coil that is located in the airgap. The radial clearance on the inner and outer diameter of the coil are assumed equal. The amount of coil volume in the airgap \( V_{con} \) is assumed constant. In this case is assumed the coil length is larger than the airgap height and therefore only Lorentz force is assumed to be created over the height of airgap \( h_{air} \). The current density in the coil is indicated with \( j_{coil} \). The Lorentz force generated in a cylinder with small thickness \( dr \) and diameter \( r \) is:

\[
dF_L = j_{coil} \cdot B_{air}(r) \cdot 2\pi r \cdot h_{air} \cdot dr = j_{coil} \cdot B_{mag} \cdot dr \tag{3.4}
\]

Integration of this force over the radius result in the total generated force. The inner radius of the coil is indicated with parameter \( r_i \) and the thickness of the coil with \( t_{coil} \). Integration of Equation 3.4 results in the Lorentz force:

\[
F_L = j_{coil} \cdot B_{mag} \cdot t_{coil} \tag{3.5}
\]

The magnitude of the Lorentz force that can be generated is limited. The current density \( (j_{coil}) \) is limited due to heat generation as will be shown further on. The thickness of the coil \( t_{coil} \) is limited by the airgap size. The magnetic flux \( (\Phi_{mag}) \) is limited, not only by the magnet but also by the design of the actuator, for instance the airgap size. As result of an increased airgap size the available volume is increased, however the magnetic flux \( \Phi_{mag} \) changes as well. An increased outer radius for instance results in a decreased slope of the load line, Equation 3.3. The result of a decreased slope of the load line is a decreased magnetic flux density in the magnet and thus the flux in the airgap.

Equation 3.5 is transformed to a more commonly known formula, using the coil volume in the airgap. The magnetic flux \( \Phi_{mag} \) in Equation 3.5 is replaced with the magnetic flux density at the average coil radius \( (r_i + t_{coil}/2) \) indicated with parameter \( B_c \). The area at that radius is \( 2\pi (r_i + t_{coil}/2) \cdot h_{air} \). The Lorentz force is then:

\[
F_L = j_{coil} \cdot B_c (2\pi (r_i + t_{coil}/2) \cdot h_{air}) \cdot t_{coil} = j_{coil} \cdot B_c \cdot V_{con} \tag{3.6}
\]

In Equation 3.6 parameter \( V_{con} \) is used for the coil volume in the airgap.

The airgap is not completely filled by the coil. Clearance between the mover and stator is available to prevent contact, or collision, what might result in damage of the system. The clearance is assumed equal on both sides of the airgap and indicated with parameter \( \Delta_c \). The cross sectional area of the coil not only consists of conductive volume but also other material, for instance air or isolation material. The amount of current carrying with respect to the total cross sectional area is indicated with the fill factor \( f_{fill} \). The current in the wire is for instance:

\[
i_{wire} = f_{fill} \cdot j_{coil} \cdot A_{wire}.
\]

In practice often iron is used for the magnetic structure in order to minimize the reluctance. Despite the relative low permeability of iron compared to some other materials it still contributes to the total reluctance. When the reluctance is taken into account, and assumed constant, only the slope of the load line in Equation 3.3 changes. When also saturation of the iron is taken into account
the load line is no longer a straight line and the design is more complicated. The current carrying coil is also a source of magnetomotive force and might affect the load line [36, 165]. The design of a voice coil actuator is thus far more challenging than it appears at first sight from the formulas provided above. Some of the aspects that complicate the design of voice coils are [245]: "variations in the static magnetic field, flux-leakage, nonlinearities in the B-H curve of the pole steel, field variations caused by DC coil current, other effects caused by the rate of change of flux, effects on the drive electronics caused by coil motion, changing resistance due to heating, changing inductance and other problems". For evaluation and optimization of the layout of the magnetic circuits of new actuators a more accurate description is necessary to overcome potential pitfalls of these simplified formulas [165]. Nevertheless the simplified equations as presented above can be used to reveal trends of airgap changes.

### 3.5 The mass and effective force of an actuator

The mass of the moving part of the actuator is important because it has to be accelerated as well, that mass is attributed to the mover, Figure 3.9. The components that contribute direct in the mover mass are the coil that consist of electrical conductive and insulation material. The mass of the insulation material is negligible compared to the conductive mass. The coil mass is indicated with $m_{\text{con}}$. This mass is related to the amount of conductive volume in the airgap ($V_{\text{con}}$), but also to the stroke. The mass of the cup is indicated with $m_{\text{cup}}$. Only a certain amount of guidance mass contributes to the mover mass. The guidance consist of moving parts and static parts. To a certain extent the moving parts of the guidance contribute to the mover mass. The mass of the guidance that contributes to the mover mass is indicated with $m_{\text{mgd}}$. The mass required to couple the mover to another system might be incorporated in the cup or the guidance mass. The mover mass $m_{\text{mov}}$ is:

$$m_{\text{mov}} = m_{\text{con}} + m_{\text{cup}} + m_{\text{mgd}}$$  \hspace{1cm} (3.7)

The actuator is used to exert forces on a product carrier such that the required accelerations are obtained. A part of the Lorentz force generated in the actuator is required to accelerate the mover itself. The mover mass is kept as small as possible so more force is left to accelerate the actual payload. In the next part the theory will first be explained qualitative, thereafter an example is used to demonstrate the quantitative properties.

**Scaling of the voice coil actuators**

In a voice coil actuator the performance depends on the physical size scale. In order to determine how the physical size scale affects the performance scaling theory is used. In scaling theory a set of equations is used that describe the system. The independent variables are assumed to be constant or vary in proportion to a known power of scale, e.g. as presented by Kamerbeek [119]. This variation with scale can either be due to physical law and geometry or due to
man made design requirements (such as design rules or tolerances). The scale dependency of the variables can then be computed. Often the rate at which a physical quantity changes when the 3D dimensions change proportionally is determined \[28\]. This assumes that the geometry of the magnetic circuitry can be scaled uniformly. In this thesis non-uniform scaling is however required; the stroke of the actuator has to be maintained. Practical aspects might limit the uniform scaling and are also taken in account. Tolerances might for instance be constant as \[107\]. The relative importance of tolerances thus increases at smaller scale. Soemers \[240\] mentions:" The manufacturing of coils and strong magnets becomes more difficult when scaled to the micro-domain." While Devos \[59\] states:"The producibility of small voice coil actuators becomes more difficult because of the number of components and need for assembly."

### 3.5.1 Mover mass as function of the Lorentz force

The Lorentz force depends on the amount of electric charge moving in the magnetic field. Due to variations in the parameters in Equation 3.6 the Lorentz force can be varied. In a realized system the coil volume \(V_{con}\) and the average magnetic flux density in the airgap \(B_c\) are fixed. During operation variation in the Lorentz force is obtained from variation of the current density in the coil \(j_{coil}\). Because the allowed current density in the coil is limited the maximum obtainable Lorentz force \(F_{L\ max}\) is limited. Instead of variations in the Lorentz force during operation phase in this section focussed on the design phase of the actuator. In the design phase the amount of conductive mass and the magnetic flux density are still variable. The actuator is designed such that it is able to provide the maximum required force \(F_{L\ max}\), while the mass is minimized. When during operation smaller forces are required the current density in the coil \(j_{coil}\) can be varied.

Without considering practical aspects an increased amount of conductive material results in an increased Lorentz force when the current density and magnetic flux density are assumed constant, see Equation 3.6. In case the same amount of conductive material is used the mover mass increases as result of an increased amount of conductive volume, see Equation 3.7. The force that can be used to accelerate the actual payload might change as well. Because the actuator force changes as well as the actuator mass an optimum configuration of the actuator might exists. The design and optimisation of voice coil actuators is a specialty on its own \[67, 148, 279\].

In this case scaling is discussed starting from an available voice coil actuator that is able to meet the requirements. The initial actuator is assumed to be designed and optimized for its purpose. The effect of scaling this actuator is then considered over a limited range. The stroke of the actuator has to be maintained while the amount of conductive material in the airgap \(V_{con}\), and thus the maximum obtainable Lorentz force in Equation 3.6, is varied when scaling the coil. The magnetic flux density \(B_c\) in the airgap is assumed to be constant. Thus only the amount of conductive volume scales. The assumption of constant magnetic flux density \(B_c\) might affect the requirements for the stator,
such as a changing diameter, changing height of coil and airgap, these stator effects are however not taken in account. The mover mass is initially the main concern.

The mass of the conductive material depends directly on the maximum required Lorentz force \( (F_{L\ max}) \). The diameter of the coil and thus the mass of the cup depends also direct on the maximum required Lorentz force \( (F_{L\ max}) \). However diameter changes are not considered. The mass of the cup is thus assumed to be constant. The dependency of the guidance and coupling mass on the amount of Lorentz force is harder to determine. The size and mass of the guidance depends for instance mainly on the required stroke and the accuracy in other directions. Because changes in Lorentz force are considered over a limited range only, the mass of the coupling and guidance is assumed constant.

The mass of the mover can then be calculated from:

\[
m_{\text{mov}}(F_{L\ max}) = m_{\text{con}}(F_{L\ max}) + m_{\text{cup}} + m_{\text{mgd}} = m_{\text{con}}(F_{L\ max}) + m_{\text{fix}} \tag{3.8}
\]

As mentioned a change in the maximum required Lorentz force \( (F_{L\ max}) \) results in a change of conductive mass in the airgap and thus mover mass. The maximum Lorentz force \( (F_{L\ max}) \) depends on the size of the actuator. The linear scale of the coil is indicated with parameter \( S \). The mass of the mover expressed in Equation 3.7 is in Equation 3.8 related to scale. Different scaling laws are known in literature, they depend on the assumed principle of heat removal from the coil. The amount of conductive volume scales as \( S^3 \).

The current density in the coil is limited as result of thermal aspects. Due to the electrical resistance of the conductive material energy is dissipated in the coil when current flows \( (P_{\text{dis}} = i_{\text{wire}}^2 \cdot R) \propto j_{\text{wire}}^2 \cdot S^3 \). The dissipated energy result in a temperature rise of the coil. The allowed temperature of the coil is however limited by for instance the melt temperature of the insulation of the wires [96]. The generated power might also be limited because it could result in (thermomechanic) stress and deformation in the structure. Failure of the actuator due to high temperatures is considered to be the fundamental performance limit. Therefore the scaling is applied such that the temperature is constant [91].

The heat is removed from the coil by the physical principles: conduction, radiation and convection. These macroscopic effects of heat removal are considered most important. The required stroke of the actuator is in the range of millimetres and thus allow the application of macroscopic theories. Heat transfer due to radiation is normally small compared to the other effects and therefore neglected, [186]. The effect of conduction and convection on the scaling of the Lorentz force are considered separately. Conduction can be assumed when actuators are in close contact with larger objects that act as heat sink [187]. When a constant temperature difference is assumed, the heat removal due to conduction scales as \( P_{\text{cond}} \propto S^4 \) [187]. Equating the produced heat and the removed heat results in a current density in the coil that scales as \( j_{\text{wire}} \propto S^{-1} \). Because the amount of conductive mass scales as \( S^3 \), the maximum Lorentz force according to Equation 3.6 scales as \( F_{L\ max} \propto S^2 \) [91, 187, 250].

For large objects the heat removal due to convection scales as \( P_{\text{conv}} \propto S^2 \) [187].
The scaling of the current density is then $j_{\text{wire}} \propto S^{-1/2}$. What results in a maximum Lorentz force that scales as $F_{L \text{ max}} \propto S^{5/2}$ [91, 187, 250]. According to Peirs et al. [187]: “Natural convection is driven by density gradients, thus by mass-related forces while viscous friction forces counteract them. When reducing size, mass goes down much faster than viscous friction such that natural convection is disfavoured by miniaturization. The result is that for small sizes convection is replaced by conduction through the gas.” As result the natural convection scales proportional to size $Q \propto S$ for very small sizes of the actuator. The maximum Lorentz force scales as $F_{L \text{ max}} \propto S^2$, [59, 187].

The scaling of the maximum Lorentz force is thus assumed to scale with $F_{L \text{ max}} \propto S^{zp}$. All values of parameter $zp$ found in literature are $zp \leq 2.5$. The mass of the coil scales with $m_{\text{con}} \propto S^3$ thus $S \propto (m_{\text{con}})^{1/3}$. Often the maximum force per mass is calculated dependent on the size. This scales as:

$$\frac{F_{L \text{ max}}}{m_{\text{con}}} \propto \frac{S^{zp}}{S^3} \propto S^{zp-3} \tag{3.9}$$

Because parameter $zp \leq 2.5$, a smaller size results in a larger maximum Lorentz force per mass. From physical point of view miniaturization has a favourable effect.

### Limits on miniaturization

Acceleration of the coil alone, as in Equation 3.9, is in a positioning system not a purpose in itself. In a positioning system a load with mass $m_{\text{load}}$ has to be positioned and therefore accelerations of the load are required. Furthermore consists the actuator of more elements that contribute in the mass than only the conductive mass in the airgap. Dependent on the stroke and the configuration also conductive material might for instance be present outside the airgap, see Figure 3.6 and 3.7. The total coil volume ($V_{\text{coil}}$) thus not only depends on the amount of conductive volume in the airgap $V_{\text{con}}$, but also on the stroke. The in the airgap generated force is thus also used to accelerate the conductive material that does not contribute to force generation at that instant time. The stroke dependency complicates the scaling. In this first order approximation the requirement of maintained stroke, the coil mass is split in a part that scales with the generated force and a part that is constant. The coil mass ($V_{\text{coil}}$) is related to the amount of conductive mass in the airgap ($V_{\text{con}}$), and they are assumed to scale in a similar way. The preservation of stroke of the actuator is incorporated by a constant mass, even though the actuator is scaled to small sizes. In Equation 3.8, this effect is incorporated by the mover mass dependent on the maximum Lorentz force and a fixed mass. The Lorentz force generated by the actuator is used to accelerate the moving parts of the actuator and the load.

$$F_L = a (m_{\text{load}} + m_{\text{mov}}) \tag{3.10}$$

The maximum Lorentz force depends on the amount of conductive volume. The mass of the mover also depends on the amount of conductive volume, see Equation 3.8. The mass that has to be accelerated is $m_{\text{load}} + m_{\text{mov}}$. The maximum Lorentz force scales with the amount of conductive volume in
the airgap as $F_{L_{max}} \propto C_1 \cdot S^{zp}$ where $zp$ depends on the assumed cooling principle. For parameter $zp$ is stated $zp \leq 2.5$. The mass of the conductive volume scales with the size of the coil as: $m_{con} \propto C_2 \cdot S^3$. The acceleration is:

$$a \propto \frac{F_{L_{max}}}{m_{load} + m_{cup} + m_{mgd} + m_{con}}$$ \hspace{1cm} (3.11)

When the mass $m_{load} + m_{cup} + m_{mgd}$ is assumed constant and indicated with the parameter $m_{fix}$ the acceleration depends on the size of the coil as:

$$a = \frac{C_1 \cdot S^{zp}}{m_{fix} + C_2 \cdot S^3}$$ \hspace{1cm} (3.12)

This function is now examined for different values of $m_{fix}$ compared to $C_2 \cdot S^3$. At one extreme the value of $m_{fix}$ is much larger as the mass of the coil $m_{fix} >> C_2 \cdot S^3$ the acceleration $a \propto S^{zp}$. The larger the size of parameter $C_1 \cdot S^{zp}$ compared to $m_{fix}$, the larger scale the larger the acceleration. This is represented in the left side of Figure 3.10.

![Figure 3.10: Trend of Equation 3.12 that shows the acceleration as function of the scale parameter S. Independent of the value of the fixed mass ($m_{fix}$) an optimum exists. The lines intersect at a scale value $S = 1$. The largest acceleration is obtained within limits of the scale parameter.](image-url)

At the other extreme when $m_{fix} << S^3$ the acceleration $a \propto S^{-3}$. Because $zp \leq 2.5$, the smaller the coil size the larger the acceleration. This is represented in the right side of Figure 3.10.

From evaluation of Equation 3.12 in Figure 3.10 it is clear that an optimum exist in the acceleration as function of the amount of conductive volume that is used. To determine the exact location of the optimum an actuator should be optimized. The purpose of this thesis is not the optimization of one particular actuator but to discus the possible favourable effects of overactuation on the moving mass. The exact optimum is therefore not required.

The aim of a positioning system is to position an object. Therefore accelera-
tions are required. Always a certain amount of mass \((m_{\text{fix}})\) should thus be accelerated. At certain values of \(S\) the mass \((m_{\text{fix}})\) cannot be neglected compared to \(C_2 S^{2p}\). From the left part of the graph in Figure 3.10 can thus be concluded that always a minimal amount of conductive volume is required to obtain a certain acceleration of the load \((m_{\text{fix}})\). On the left side of the graph in Figure 3.10 the force increases faster than the mass when the amount of conductive volume is increased.

Equation 3.9 describes only the right side of the graph in Figure 3.10 and can thus only be used until a certain extent of miniaturization \((m_{\text{fix}} < C_2 S^{2p})\). On the right side of the graph in Figure 3.10 the mass increases faster than the force when the amount of conductive volume is increased. The Lorentz force for instance scales with size as \((F_{L \text{ max}} \propto S^{5/2})\) when the heat is removed from the coil due to convection. Because the mass of the conductor increases with \(m_{\text{con}} \propto S^3\). The conductive mass is related to the maximum Lorentz force as: \(F_{L \text{ max}} \propto m_{\text{con}}^{5/6}\). When the amount of conductive mass \((m_{\text{con}})\) is increased the increase in force is less than the increase in mass. The acceleration \(a \propto F_{L \text{ max}}/m_{\text{con}} = m_{\text{con}}^{-1/6} = S^{-1/2}\) thus decreases with increased amount of conductive mass.

An existing actuator is used as point of departure when the amount of conductive material of an actuator is scaled. This actuator is assumed to be designed for optimum performance, maximum force per actuator mass (acceleration). The amount of force can be changed by in- or decreasing the amount of conductive mass. In practice the changed amount of conductive material has consequences for the required airgap size and clearance. Therefore scaling over a limited range is considered, when the range is extended other actuators might be used as initial actuator.

**Actuator scaling example**

An example is provided to indicate the effect of scaling. The parameters of the initial, not scaled, actuator correspond to a scale value \(S = 1\). For this initial actuator the following properties are assumed: Magnetic flux density in the airgap \(B = 0.6 \, T\); total mass of the copper coil: \(5 \, g\); volume \(V_o = 562 \, mm^3\); allowed current density \(j = 30 \, A/mm^2\). The requirements of stroke etc. are assumed such that in the original situation all current carrying mass attributes to force generation, such as presented in Figure 3.4 or 3.5.

The value of \(C_1\) and \(C_2\) depend on this assumption. They are determined from the original, not scaled, actuator \((S = L_{\text{new}}/L_{\text{old}} = 1)\) and are respectively \(C_1 = V_o \cdot j \cdot B = 10 \, N\) and \(C_2 = \rho_{\text{cu}} \cdot V_o = 5 \, g\). In case only the copper has to be accelerated the maximum acceleration is determined with \(a_{\text{max}} = j \cdot B/\rho_{\text{cu}} = 2000 \, m/s^2\), see [47]. In this example a fixed mass of 10% of the original copper mass is assumed present; thus \(m_{\text{fix}} = 0.5 \, g\). The acceleration of the mover is determined with Equation 3.12. In Figure 3.11 the maximum acceleration of the mover is plotted as function of the used copper mass. The shape of the curves plotted in Figure 3.11 depend on the initial assumed ratio between the different parameters.
Figure 3.11: Maximum acceleration of the mover, cup + coil, as function of the used copper mass. The maximum acceleration is at a large amount of copper mass limited by the heat transfer. When a small amount of copper is used the fixed mass \(m_{\text{fix}}\) limits the acceleration.

### 3.5.2 Effective actuator force

The mass of the mover is in general kept as small as possible because the Lorentz force \(F_L\) has to be used to accelerate both the mass of the load \(m_{\text{load}}\) as well as the mass of the mover with an acceleration \(a\), Equation 3.10. The force that is effectively used to accelerate the load is \(F_{\text{act}}\). This force is:

\[
F_{\text{act}} = a \cdot m_{\text{load}} = F_L - a \cdot m_{\text{mov}}
\]  
(3.13)

The effective force of the actuator \(F_{\text{act}}\) in Equation 3.13 depends on the Lorentz force \(F_L\) and the inertia of the mover mass \(a \cdot m_{\text{mov}}\). The effective actuator force \(F_{\text{act}}\) is obtained by substraction of the acceleration dependent inertia force from the generated Lorentz force.

The effective actuator force is used to accelerate the mover mass and the actual payload, Equation 3.10. When the effective force of the actuator is negative it means the Lorentz force is not able to accelerate the mover with the required acceleration value. If that system would be operated the obtained acceleration is less than the required acceleration. Even though the required actuator force might be small, still a certain amount of Lorentz force might be required to accelerate the actuator itself.

On the left side of the graph in Figure 3.10 the effective actuator force increases with increased size. The Lorentz force increases faster than the mass. On the right side of the graph in Figure 3.10 the effective actuator force decreases with increased size. The increase in mass is larger than the increase in Lorentz force. When the heavier field assembly (with magnets) is for instance used as mover more force and thus a more powerful actuator is required to obtain the same effective actuator force.

The actual mass of the mover consists of the amount of conductive volume, the
mass of the cup, the coupling and the guidance as shown in Equation 3.8. Dependent on the requirements for the stroke in axial direction and the accuracy in radial direction a voice coil drive is designed. The amount of conductive volume depends on the required Lorentz force. The mover mass as function of the force \( F_{\text{act}} \) can not be given in a general function. The mass of the conductive material depends also on the allowable heat generation. The Lorentz force of the actuator depends on the magnetic flux density.

### 3.5.3 Example

To quantify the theory an example will be used. As initial actuator a commercial available actuator from BEI-Kimco Magnetics Division [14] is used that is able to provide a continuous force up to \( F_L = 2 \, \text{N} \). The stroke is 2 mm and the weight of the coil assembly is 8 g. The effect on the Lorentz force and acceleration is examined when the amount of conductive volume is reduced. The coil assembly consists of material used to conduct the current and material for other functions. The conductor mass is approximated to be 3 g, the other mass that does not scale is then 5 g.

![Figure 3.12](image1.png) ![Figure 3.13](image2.png)

**Figure 3.12:** Conductive mass as function of the actuator force. The mass of the cup and guidance of the actuator is assumed to weight 5 g.

**Figure 3.13:** Effective actuator force \( F_{\text{act}} \) as function of the mover mass when a certain is required.

In Figure 3.12 the mass of the actuator is plotted as function of the Lorentz force. Convection is considered the most important heat removal mechanism. The conductive mass scales with Lorentz force as \( m_{\text{con}} \propto F_L^{6/5} \). When the total mover mass is 8 g the actuator is able to generated a force of 2 N. This is according to the specification of the actuator that is used for the scaling. In Figure 3.13 the effective actuator force is plotted for different accelerations. When larger accelerations are required the effective force reduces. When the fixed amount of mass is increased, for instance when a guidance is connected, the effective force changes as well. This effect will be presented in Chapter 6.

Scaling the actuator is possible over a large range. However, the reliability of the
outcomes at relative small or large scaling values should be verified. At small or large scale values the original actuator can probably better be replaced with another actuator that is designed for that force range. That actuator can then be scaled.

### 3.6 Application of actuators in a parallel positioning system

Some qualitative aspects of the guidance and drive system are presented to obtain insight in the relation between the different aspects. The connection between the different elements, e.g. movers and product carrier, depends on the total setup of the system. The constructive aspects of the system are briefly mentioned. The parallel systems are designed according to the kinematic design based on idealised components, see page 56.

In Chapter 6 the mass of these systems is quantified. The beam positioning system is used to explain the different aspects of parallel actuation. These aspects are assumed to be also present in other parallel positioning systems. Parallel actuation is assumed to be required due to the dimensions of the system. Rotation stiffness around the z-axis in Figure 3.14 is for instance difficult to obtain with the guidance system due to the small height of the beam. The rotation stiffness around the z-axis in Figure 3.14 can be obtained from feedback control of the two actuators.

Different types of bearings can be used to constrain degree of freedom. In this thesis only airbearings and elastic 'bearings' are considered. Despite particular aspects of these bearings are important, only the general aspects are mentioned. Aimed is at obtaining insight and the considered bearings are for that purpose considered sufficient for instance to show the effect of stroke limitations or pre-loading requirements.

#### 3.6.1 Beam positioning system using two actuators

Positioning of the beam in transversal direction is assumed to be required. Due to the relative small height of the beam, two actuators are used. Stiffness around the z-axis is in Figures 3.14 and 3.15 realised with feedback control of the two actuators.

The location of the actuators in y-direction can be chosen arbitrary. An increased distance results in larger rotation stiffness around the z-axis. To limit the amount of required force the y-location of the actuators is such that they are located at different sides of the centre of mass. A symmetric location results in equal required force, and thus equal actuators can be used. To limit the amount of force in such a way that the forces acting on the system when accelerated result in minimal deformation of the product carrier. The requirements for the guidance of both systems in Figure 3.14 and 3.15 are discussed.
3.6. Application of actuators in a parallel positioning system

3.6.1.1 Constraining individual DOF of the movers

In Figure 3.14 only one DOF between each mover and the product carrier is constrained. The other five degree of freedom of the mover need to be taken into account by individual mover guidances. The DOF of the product carrier need to be constrained as well. The guidance of the product carrier is discussed in the next section, where the carrier is combined with the mover. The same guidance is assumed to be applicable to the product carrier in this setup and therefore not mentioned here.

Elastic guidance

Because the required stroke in x-direction is only a few millimetres an elastic guidance can be used for both the carrier and the mover. The different elements of the elastic guidance should be strong / stiff enough in the direction that is constrained. Also buckling should be prevented when loaded. The guidance should be as compliant as possible in motion direction. The deformation of the elements of the elastic guidance should remain in the elastic range. When plastic deformation appears the guidance might fail or otherwise the properties of the guidance are changed.

Elastic guidance of the movers in Figure 3.14 can for instance be made as shown in Figure 3.16 and 3.17 [73, 90]. Applied to the voice coil actuator result in a system as shown in Figure 3.18. The shown membranes are used to explain some principal differences between them. The possible stroke in x-direction is with the use of the membrane in Figure 3.17 larger than in case the membrane in Figure 3.16 is used. The rotation around the x-axis of the membrane in Figure 3.17 is less compared to the membrane in Figure 3.16. A principal difference is the stiffness in y and z-direction. The membrane in Figure 3.16...
the stiffness depends mainly on the one dimensional stiffness of the connection between inner and outer circle. In Figure 3.17 the stiffness depends also on bending and rotational stiffness of the rods. The additional corners in the membrane in Figure 3.17 might act as elastic hinges that reduce the stiffness in y- and z-direction.

Two of these membranes are used to constrain the five DOF between mover and stator in Figure 3.18. The system is overconstrained, from rigid body point of view. In practice the membranes are solid, but not rigid. The elasticity limits the internal force as result of the overconstrained state. Despite the overconstrained state, the internal forces are assumed limited due to use of equal membranes that are placed parallel. The applied membranes in Figure 3.18 constrain five degrees of freedom. However as result of movement in x-direction rotation around the x-axis appears. The connection between the mover and the product carrier allows this rotation, Figure 3.14. This rotation is also allowed by the mover itself, because the system is circle symmetric. The system is shown in Figure 3.18 fulfils the requirements.

An advantage of symmetry of the membranes is that the centre line remains in position when homogeneous deformations appear due to thermal expansion. The material used for the membranes should not short circuit the magnetic field from the centre to the outer part. The magnetic field in the airgap has to be maintained. When nonmagnetic materials are used for the membranes care should taken with the application in the yoke. The construction should be such that the extra reluctance due to membranes is low. Despite in this thesis only membranes are mentioned, other elastic guidances constructions, for instance based on wire flexures, can be created as well.

Airbearings
Air-bearings can also be used as guidance but in general pre-loading is required to obtain sufficient stiffness. This pre-loading might results in a bulky system,
however due to the circle symmetric mover preloading is easily obtained. A possible guidance of the mover using airbearings is presented in Figure 3.19. The presented solution can however only be used to indicate the bearing principle, manufacturability is for instance not taken into account. The bearing gap is in the order of $\mu m$ [103]. The stiffness depends on the bearing gap height and is therefore often minimized. Thermal expansion is assumed not to result in failure of the bearing.

![Figure 3.18: Cross section of a circle symmetric actuator. Two membranes as shown in Figure 3.16 or 3.17 are used for the guidance. Rotation around the x-axis appears when moved but is a function of the x-position, [211].](image1)

![Figure 3.19: Cross section of a circle symmetric actuator. An airbearing is used to guide the mover. Two bearings are used to suppress rotation around the y and x-axis. Rotation around the x-axis is free, this is however no problem.](image2)

In Figure 3.19 two bearings are used to suppress rotation around the y and x-axis. Rotation around the x-axis is free, this is however no problem due to the rotation symmetry. The carrier is guided also with air-bearings or elastic bearings. The operation of the magnetic actuator might be affected by the application of ferromagnetic guiding material and this effect should thus be taken into account. The bearing gap size is in the order of $\mu m$, while the airgap is often much larger. As result the reluctance over the airbearings might be much smaller than the reluctance of the airgap where the Lorentz force has to be created. Using ferromagnetic bearing material might short circuit the magnetic flux.

**Required guidance stiffness**

In the considered case, with individual exactly constrained movers as shown in Figure 3.14, the connection between mover and carrier should only posses stiffness in x-direction. If the coupling has also stiffness in the other directions the constraints of the carrier are connected to the constraints of the mover. This may result in failure when the mover or carrier is disturbed or when mover and carrier do not move along the same axis. Even when no external disturbances
are assumed present the guidance should possess stiffness to obtain a robust system. The mover of the actuators should not touch the stator, because that might affect the system performance. Stiffness is required to overcome force in y- and z-direction. The required stiffness of the mover guidance depends on the forces $F_D$ and the clearance. In Figure 3.20 a schematic coupling between mover and carrier is presented that is used to transfer force $F_A$ from mover to the product carrier. As result of misalignment of mover guidance and carrier guidance the force on the coupling is not parallel with the axial direction of the coupling. The difference in angle between force $F_A$ and axial direction of the coupling is indicated with the parameter $\alpha$. As result of the misalignment a disturbance force $F_D$ acts on the mover. This force is counteracted as result of guidance stiffness with the aim to prevent collision between mover and stator. In Figure 3.21 the mover is presented even as the force $F_D$. This force loads the bearing system. The stiffness of the bearing system should be stiff enough to generate the force while the system keeps functioning.

The applied guidance, either the elastic membranes or the airbearing, should designed such that they posses the required stiffness. The stiffness of the membrane might be calculated using finite element methods; buckling should be considered as well. The stiffness of the airbearing can also be determined. For reference the stiffness of an axial air-bearing $\otimes 20\ mm$ that consist of 8 holes with a diameter of 0.4 mm, an airgap of 15 $\mu$m and is fed with 4 bar ato (atmospheric overpressure) has a stiffness of 3.5 $N/\mu$m [102, 167]. The rotation stiffness also depends on the distance between the bearings ($d_1$ in Figure 3.21). The system in Figure 3.19 can thus be realized while the required stiffness are obtained.
3.6. Application of actuators in a parallel positioning system

3.6.1.2 Constraining DOF of the carrier combined with the movers

In Figure 3.15 all six DOF between each mover and the carrier are constrained. The carrier is guided with either an air bearing guidance or an elastic guidance. The guidance should be designed to match the system requirements. The guidances mentioned above where designed for rotation symmetric setups. The product carrier is not rotation symmetric and other designs might be used. Preloading of the air-bearings remains required. In this case the location of the movers in the stator is determined by the product carrier. Care should be taken in account when the movers and stators are aligned. The mutual difference between both movers should be equal to the mutual distance between both stators. While building the system the alignment between the movers and the stator is thus more important than in case separate guidances are used. In operation this system is more sensitive for disturbance due to for instance different thermal expansion between both movers and both stators. Also rotation of the carrier around the x-axis might lead to problems in the actuators.

Deformation of the product carrier affects the relative position between the two movers. As a consequence the movers might touch the stators. Because the system in Figure 3.15 is more sensitive for disturbances the clearance between mover and stator \( (\Delta_c) \) is in Figure 3.15 increased compared to the clearance in Figure 3.14. As result of the increased clearance also the airgap and thus the reluctance is increased. The magnetic flux density decreases, when the same magnet is used. Less Lorentz force can thus be generated. To obtain the same actuator force, that can be used to accelerate the carrier, either the magnetic flux density or the amount of current carrying conductive material in the airgap should be increased. The system is assumed to operate at its maximum performance where thermal limitations do not allow an increase of the current density. Increased magnetic flux density can be obtained by adapting the stator, the mover does thus not change. For increased current carrying conductive material in the airgap both the mover as well as the stator should be adapted. As a result the mover mass will increase.

![Figure 3.22: Cup attached to the carrier. The carrier is drawn in the upper limit of the stroke. The distance d should be larger than the total stroke otherwise the elements touch. The cup encloses the coil deformation of the coil due to the Lorentz forces is therefore reduced.](image-url)
The cup of the mover can be connected to the carrier as shown in Figure 3.22. The carrier is drawn at a limit of the stroke. The system should be able to move over the total stroke therefore distance d in the Figures should be larger than the stroke. The material of the carrier might affect the operation of the actuator. When the carrier is for instance made from ferromagnetic material it can be attracted by the magnet. Despite the amount of conductive volume in the airgap scales with the required maximum Lorentz force the stroke requirement has to be fulfilled. Even when a minimum mass mover is used a connection between the conductive mass used to generate force at an instant time, and the actual payload is required. The cup in Figure 3.22 is used for this purpose. Beside the cup also conductive mass might be present outside the airgap in order to be able to fulfil the stroke requirements, for instance the coils shown in Figure 3.6 and 3.7. As result of the stroke requirements, the mover mass can not be scaled to very small values. Some mass remains required for operation.

3.6.1.3 Comparison between the systems

An important difference between the systems in Figure 3.14 and 3.15 is the clearance between the mover and the stator. To increase the efficiency of the actuator, and thus reduce conductor mass, the clearance is in general as small as possible. To prevent touch between stator and mover the clearance required in Figure 3.14 should in general be larger than the clearance required in Figure 3.15. A larger clearance means a larger airgap, and as result an increased reluctance, see Equation 3.1. Due to the increased reluctance the magnetic flux density is decreased, Equation 3.2. When the same coil is used the maximum Lorentz force is decreased as result of the increased clearance, Equation 3.5. To maintain the magnetic flux density in the airgap the stator has to be adapted when the clearance is changed, Equation 3.2.

The mutual alignment of the stators should be more accurate in the system of Figure 3.14 than the system in Figure 3.15. Also is the system in Figure 3.14 more sensitive to disturbances such as rotation of the carrier around the z-axis and thermal expansion of the carrier. The clearances between mover and stator in Figure 3.15 do not change due to these disturbances.

A difference between the centre line of the coil and the centre line of the stator has no effect on the generated force. When a force difference is present it is only important when the system is operated in feedforward control. However as result of the separation between force- and measurement loop in the actuator, feedback control is always assumed to be present.

3.6.2 When to use actuators with or without own guidance

The actuators used in the positioning system might have their own guidance as shown in Figure 3.14 and described in paragraph 3.6.1.1. Another method is a rigid connection between the carrier and the movers as shown in Figure 3.15 and described in paragraph 3.6.1.2. Aspects of building the systems and operation of the system are described. Most effort is in first case put in obtaining a system that is able to fulfil the requirements. The application of either a system
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with or without own guidances of the movers is therefore first motivated from functional perspective. The positioning system should be able to meet the specifications. Touch between mover and stator is assumed to have a devastating effect on the system performance. It might result in unwanted disturbances such as friction, but also the electric isolation of the coil can be damaged. An implicit requirement is thus that touch between stator and mover should in all cases be prevented. The required clearance between mover and stator depends on the guidance system. When the guidance of the product carrier is sufficient to allow a reasonable clearance between each stator and mover the movers can be directly attached to the product carrier as shown in Figure 3.15. Even when the system is disturbed the guidance of the product carrier should be able to prevent touch between each mover and stator.

In case the guidance of the product carrier is not accurate enough or not able to deal with disturbances in an appropriate way a larger clearance between stator and mover is required. Individual guidances between stators and movers are used to obtain a more accurate guidance such that the required clearance can be decreased. This is shown in Figure 3.14.

Due to the increased number of elements required in positioning systems with an own guidance of each mover, Figure 3.14, the system without own guidances, Figure 3.15, may be preferred. The specifications for the mutual alignment of the different elements should be realistic. Also the effect of disturbances should be known. When the system without own guidances is not able to meet the requirements systems with own guidances between mover and stator have to be used. Another reason for the use of system with own guidances between each mover and stator is for instance when the actuator and its guidance are an individual (separate) module in the positioning system.

The system that results in the lowest mass depends on the requirements that the system has to meet. On one hand results a large clearance in a smaller Lorentz force. To cancel this effect more conductive volume (and thus mass) is required. On the other hand when individual guidances between mover and stator are used the amount of conductive volume might be reduced. However the mass is increased as result of the extra guidance. To accelerate this additional mass, additional force and thus conductive volume is required.

3.6.3 Beam positioning system to evaluate the dynamic behaviour

An example is used to illustrate different aspects of the design. This system was build for research purposes and is also used by Makarović [148]. A very flexible product with dimension $500 \times 20 \, mm^2$ is assumed to require positioning over a stroke of $1 \, mm$ within $20 \, ms$, as was mentioned in Chapter 1. A steel beam with the specifications as mentioned on page 106 is used as product carrier. In order to prevent a constant actuator force to overcome the gravity force that acts in negative z-direction, the system is orientated such that the guidance bears the weight, as shown in Figure 3.24. As guidance elements in Figure 3.24 wire springs are used. Elastic elements are used as guidance in Figure 3.24. Rods of a $0.6 \, \mu m$ diameter and $60 \, mm$ length are used. In axial direction they
are stiff enough to bear the weight and also to prevent buckling. In transversal
direction the rods are flexible such that the system is able to move over the
required stroke in x-direction. The parasitic motion in y-direction is assumed
to remain within the tolerances.
The required actuator force is determined by assuming the beam moves as rigid
body and bang-bang operation. The acceleration should be 10 m/s² when a
second order profile should be followed. The system weights ≈ 0, 2 kg thus the
total required actuator force is ±2 N. Commercial available actuators are used
that are able to deliver 2 N continuous force each. This is somewhat overdi-
mensioned but it is ensured the actuators are able to provide the required force,
and the system is able to fulfil the requirements. The coils are attached to the
beam such that all mutual DOF are constrained, see Figure 3.24.

**Figure 3.23:** rotation of the beam due to the finite stiffness of the beam around the y-axis.

**Figure 3.24:** Schematic representation of the beam positioning system that shows the actuators, sensors and guidance. The movers are directly coupled to the beam. Elements A-E are wire springs.

In Figure 3.24 four DOF are constrained by five wire flexures, thus overconstrained from kinematic point of view. Initially the rotation around
the y-axis was constrained by creating a certain distance between guidance elements B and C. The distance between the guidance elements is created by
attaching a small plate perpendicular to the beam. For symmetry reason this piece is also added on the other side of the beam. Guidance element B and C inter-
act with guidance element D and E to constrain the z-direction and rotation
around the x- and y-axis. Four constraints are used to constrain three de-
gree of freedom. This extra constraint is used for reasons of symmetry. Due to
the limited torsional stiffness of the beam torsion might appear when the sys-
tem is moved in x-direction. Rotation on one side is prevented by the guidance
while the other side prefers to rotate as result of the angle of the wire spring, as shown in Figure 3.23. The stiffness of the slender rods can be calculated with
the methods provided by Bos [26]. With the extra guidance element the rotation
around the y-axis is actually constrained twice. In this case this does not
lead to problems due to symmetry of the elements B,C with the elements D and E. Differences in guidance symmetry result in axial loading of the wire springs and a torque on the beam around its y-axis. The differences in symmetry are assumed to be small.

Torsion of the beam is only introduced when wire flexures as shown in Figure 3.3 are used. To prevent this drawback an overconstrained system is created. When three wire flexures as presented in Figure 3.3 are used torsion is not introduced due to movement in x-direction. A combination of the different wire flexures result in rotation around the x-axis when moved in x-direction. The system as shown in Figure 3.24 is assumed to fulfil the requirements, despite overconstrained.

The clearance between the stator and the mover is 0.4 mm on each side. The stators are fixated to the world. The alignment of both movers with respect to both stators should be such that they do not touch. The y-location of the actuators can be varied over a spatial grid of 20 mm. The location of the actuators is discussed in more detail in the next chapter.

3.6.4 Plate positioning system to evaluate the dynamic behaviour

The plate positioning systems is regarded as an extension of the beam positioning system. The same aspects as mentioned for the beam system are also valid for the plate system. This system is created and designed by de Ruijter [21]. When instead of a beam a plate positioning system is used, constraints are removed from the beam system and replaced with an actuator, as shown in Figure 3.25.

The location of the three actuators form a triangle. In this way rotation stiffness around the y- and z-axis can be obtained by feedback control of the actuators. The guidance element that constrains rotation around the y-axis in the beam setup in Figure 3.14 and 3.15 is therefore no longer required. The motion in y- and z-direction as well as the rotation around the x-axis is still constrained by the guidance elements A,B and C in Figure 3.25. The guidance elements A and C are placed in a way that the constraint lines are parallel and intersect in infinity. When these constraint lines intersect an instant rotation centre is created which is unwanted [21, 89].

The system as presented in Figure 3.25 is also build and used for the experiments in Chapter 5. A brass plate with the specifications as mentioned on page 114 is used as product carrier. The location of the actuators is discussed in more detail in the next chapter. The actuators are placed symmetric with respect to the centre of the plate, as shown in Figure 3.26. The actuators should be at a certain distance from the centre not only as result of space limitations but also to obtain rotation stiffness. By placing the actuators symmetric equal rotation stiffness with the feedback control can be obtained. Also the centre of mass of the system remains in the centre. Another advantage of the symmetric setup is that each actuator contributes the same amount of force when positioning in x-direction.

The radius of the actuators depend on the requirements. To obtain the highest rotation stiffness around the y and z-axis the distance to the centre should be
Figure 3.25: A schematic representation of a plate positioning system where all mutual DOF between each mover and the carrier are constrained. The translation in $y$- and $z$-direction is even as rotation around the $x$-axis constrained with guidance elements $a, b$ and $c$. The remaining DOF are constrained with closed loop control.

Figure 3.26: Location of the actuators with respect to the plate. The actuators are used to position the plate in $x$-direction. Stiffness around the $z$- and $y$-axis is obtained as result of closed loop control of the actuators. The rotation around the $x$-axis and translation in $z$- and $y$-direction are constrained with the flexures. The sensors measure as close as possible in the mover axis. A photo of the realized system is shown in Figure 5.24.

as large as possible. The actuators are in that case located at the largest possible radius. Other actuator locations are mentioned in the next chapter. In Figure 3.25 the actuators measure on the top side of the product carrier. In practice this is not always possible and sometimes forced to measure at the underside of the carrier.
The experiments presented in Chapter 5 are obtained from this plate system. More details about this system are described by de Ruijter [217].

### 3.7 Conclusions

Different aspects of drive systems have been discussed. The applied type of actuator, the voice coil, is discussed in more detail. Miniaturization of the drive system is limited. If the load does not change the benefits of miniaturization are questionable.

The connection between the product carrier and the actuators is discussed. The actuators can be connected in different ways to the product carrier, with or without their own guidance. In case the actuators have their own guidance the alignment between stator and mover is always maintained. When no separate guidances are used the mutual distance between the stators is determined by the object to which the stators are connected. The mutual distance between the movers is determined by the product carrier. The alignment between each mover and the corresponding stator is as result more sensitive to disturbances.

The actuator construction that results in the lowest mass depends on each specific application. When small forces are required the guidance weights relatively much compared to the conductive mass. Preferably systems with as less as possible components are used. In case of the beam example the system in Figure 3.15 has less components than the system shown in Figure 3.14. Some specifications are however only possible with a configuration that requires individual guidances of the movers, e.g. when the clearances in Figure 3.15 are smaller than the thermal induced displacement of the coils relative to the stator.

Separate guidances of the movers result in smaller clearances between mover and stator, therefore less conductive material is required. As result of the guidance the mass and thus inertia of the mover is however increased, therefore more conductive mass is required. When no separate guidances are used for the movers the required clearance is larger and thus more conductive mass is required. Due to the absence of separate guidances no additional inertia is present. In case the guidance aspects are related to cost, the calculation should not be based on material used alone, e.g. not related to its mass only.
Chapter 4

Dynamical aspects of parallel actuation

4.1 Introduction

The actuator locations affect the dynamic behaviour of the system. The dynamic behaviour of the system is however not only affected by mass and stiffness distribution, but moreover also by the forces generated in the actuators. These forces depend on the control system. In a closed loop control system information from the mechanical system is used to generate the force of the actuators. Under influence of the control the elastic system changes itself, because the dynamic behaviour is the result of the interaction between the mechanical system and control of the actuator forces. Thanks to the application of contactless drives parallel systems are created with constant dynamic properties in the workspace. The theory in this chapter is applicable to systems with dynamical behaviour that is fixed over the workspace. The dynamics as result of other drive types is more complicated since the dynamic properties vary over the workspace [131].

In this thesis mainly the dynamic behaviour of the system is discussed without the complex interaction between closed loop control and mechanical structure. Possibilities and limitations of the dynamic behaviour are explained for different actuator locations based on mechanical and physical principles. With the obtained insight the mechanical system, including the actuator locations, can be designed such that the requirements are fulfilled best. However, a general best solution does not exist and therefore this chapter hints only at trade-offs and possible strategies to overcome different problems. The basic tradeoff in control theory between performance and robustness mentioned by Schweitzer et al. [225], is for instance also applicable in selecting the best actuator location. After the hardware is built, the location of the sensors, and control of the actuator forces, might be adapted in order to realise improvements [23, 224].
4.2. Modelling the dynamical behaviour

To determine the response of a structure as a result of applied forces, a spatial model of the system is made. This spatial model is a description of the structure's physical characteristics in terms of its mass, stiffness and damping properties. Linear equations are in this thesis assumed a reasonable compromise between accuracy and practicability. In the spatial model linear compatibility and constitutive equations are therefore used. The response to forces is obtained via a transformation of the spatial model to a modal model \([70, 77, 204]\). This modal model is a description of the structure's behaviour as a set of decoupled second-order differential equations of motion. Each differential equation can be thought of a single mass-spring (damper) system. These equations of motion are solved in modal domain. Thereafter the response of the spatial system is obtained via an inversion of the transformation previously used to transform from spatial to modal domain.

In order to simplify the calculation the actual structure is replaced by a finite number of elements, each of which is assumed to behave as a continuous structural member. The elements are assumed interconnected at points that are indicated as FEM-nodes. The solution of this approximate model might be easier to determine than the exact solution. The idea is that the finite element solution can be made to converge to the exact solution as the element size is reduced \([198]\). In the finite element model the mass, stiffness and damping are collected in individual square matrices of size \(n\). The value of \(n\) depends on the number of FEM-nodes and the degree of freedom of each node. Via transfor-
The transformation to modal domain $n$ decoupled equations of motion are obtained. The transformation from spatial to modal domain and back is based on the non-trivial solution of the spatial mass ($M$) and spatial stiffness matrix ($K$):

$$ (K - \omega_i^2 \cdot M) \Phi_i = 0 \quad (4.1) $$

The $n$ eigenvectors ($\Phi_i$) contain the nodal coordinate values of the vibration mode shapes. The $n$ eigenvalues $\omega_1^2, \omega_2^2, \ldots, \omega_n^2$ correspond to the angular natural frequencies squared. The mode shapes ($\Phi_i$) and angular natural frequencies ($\omega_i$) are used to show the effect of different actuator locations on the vibration amplitude. Positioning is required thus not all rigid body degrees of freedom are constrained. The number of rigid body modes is indicated with parameter $n_r$, their angular natural frequency is zero $\omega_i = 0$. The required translational rigid body mode is assigned index one ($i = 1$).

A parallel positioning system is used to position in x-direction. The forces are therefore also directed in x-direction. The dynamic behaviour in x-direction depends however on each specific location $x = x(y, z)$, which is called the nodal displacement vector. In order to fulfil the required positioning task the parallel actuators exert forces on the mechanical structure, contained in vector $F$. A fixed ratio between the parallel actuator forces is assumed in the design phase of the mechanical components of a positioning system. Thanks to this constant force ratio, the force vector can split in two components ($F(t) = F(t) \cdot \hat{F}$). The magnitude of the force and the dependency on time ($t$) is expressed in scalar $F(t)$. This scalar is expressed in Newton. $\hat{F}$ is a dimensionless direction vector in which the constant actuator force ratio is represented. The summation of the elements in the force vector $\hat{F}$ equals one, see Equation 4.10.

Every arbitrary force profile can be expressed in frequencies via application of Fourier theory. The frequency response function is determined via the steady state response to a sinusoidal force with frequency $\Omega$. The magnitude of the frequency response function is obtained by dividing the steady state output magnitude of the system (with equal frequency $\Omega$) with the magnitude of the input forcing function. The dynamic behaviour of the complete system with a fixed ratio between the forces is transferred to modal domain where the equations of motion are solved and thereafter transferred back to spatial domain. The frequency response function is obtained by the summation of the individual modal contributions, [197]:

$$ \frac{X(y, z, \Omega)}{F(\Omega)} = \sum_{i=1}^{n} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F}}{-m_i \cdot \Omega^2 + k_i} = \sum_{i=1}^{n} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F}}{m_i(-\Omega^2 + \omega_i^2)} \quad (4.2) $$

In Equation 4.2 $\Phi_i^T$ is the transposed of the mode shape vector of mode $i$. $\Phi_i(y, z)$ represents the mode shape value of mode $i$ that corresponds to the location of interest $y, z$. The multiplication $\Phi_i^T \cdot \hat{F}$ is in this thesis referred to as participation factor of mode $i$. The generalized- mass and stiffness are represented with the parameters $m_i$ and $k_i$. They are the $i$-th diagonal element of the matrix: $\Phi^T \cdot M \cdot \Phi$, respectively $\Phi^T \cdot K \cdot \Phi$. The mode shapes are in this thesis
normalized with respect to the mass matrix \( (\Phi^T \cdot M \cdot \Phi = I) \). As a result the
generalized masses are \( m_i = 1 \) kg. The generalized stiffness are then equal to
the angular natural frequencies squared multiplied with the generalized mass \( k_i = m_i \cdot \omega_i^2 \) N/m, [132]. Even though the magnitude of the generalized mass \( |m_i| = 1 \) kg it is still included, because of the dimension assigned to the general-
ized mass. In this way the mode shape vectors do not require a SI dimension.

The frequency response representation can only be used to describe the re-
response in frequency domain. In order to describe transient inputs, as present in
positioning systems, a description in time domain is required. The transfer
function representation is used which is unique and is very useful for directly obtai-
ing insight into the properties of a system. It is defined as the Laplace trans-
form of the impulse response. For the case of the constant force ratio \( (\hat{F}) \) the
transfer function is obtained via the summation of the individual modal contribu-
tions:

\[
\frac{X(y, z, s)}{F(s)} = \sum_{i=1}^{n} \frac{\Phi_i(y, z) \Phi_i^T \hat{F}}{m_i s^2 + k_i} = \sum_{i=1}^{n} \frac{\Phi_i(y, z) \Phi_i^T \hat{F}}{m_i (s^2 + \omega_i^2)} \tag{4.3}
\]

The rigid body behaviour is also present in Equation 4.3, the stiffness is then
zero \( (k_i = 0 \) N/m). The sequence of the considered additional modes is equal
to the numbering of modes with increased frequency. Due to finite stiffness all
systems are elastic and rigid systems do not exist in practice. The frequency
response function in Equation 4.2 is directly obtained from the Laplace trans-
fer function, Equation 4.3, by replacing \( s = j \cdot \Omega \), [235].

The number of modes that should be included depends on the mutual impor-
tance in for instance the vibration amplitude. Some modes should also be in-
cluded because their frequency is near the frequency of potential external dis-
turbance forces. The contribution of modes in the dynamic behaviour is rela-
tive. In this section is started to consider first the low order modes due to their
relative large contribution in the total amplitude. Thereafter the effect of higher
order modes is considered.

**Application of force**

The dynamic behaviour to a step force with magnitude \( F \) \((F(t) = F \cdot 1(t))\) is
solved via Laplace domain where \( F(s) = F/s \). With the assumption of initial
conditions, position and speed, equal zero, the dynamic response to this tran-
sient input is:

\[
X(y, z, s) = \sum_{i=1}^{n} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F}}{m_i (s^2 + \omega_i^2)} \cdot \frac{F}{s} \tag{4.4}
\]

In this step force case care has to be taken for the \( n_r \) rigid body modes when
splitting the transfer function in parts. For the rigid body modes \( i = 1, 2, \ldots n_r \),
Chapter 4. Dynamical aspects of parallel actuation

holds $\omega_i = 0$. The response is:

$$X(y, z, s) = \sum_{i=1}^{i=n_r} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot F}{m_i \cdot s^3}$$

$$+ \sum_{i=n_r+1}^{n} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot F}{m_i \cdot \omega_i^2} \left( \frac{1}{s} - \frac{s}{s^2 + \omega_i^2} \right)$$

(4.5)

When a sinusoidal force is applied: $F(t, \Omega) = F \cdot \sin(\Omega t)$ which is in Laplace domain $F(s, \Omega) = F \cdot \Omega / (s^2 + \Omega^2)$ the response is:

$$X(y, z, s) = \sum_{i=1}^{n} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot F}{m_i (s^2 + \omega_i^2)} \cdot \frac{F \cdot \Omega}{s^2 + \Omega^2}$$

$$= \sum_{i=1}^{n} \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot F}{m_i (\omega_i^2 - \Omega^2)} \left( \frac{\Omega}{s^2 + \Omega^2} - \frac{\Omega}{s^2 + \omega_i^2} \right)$$

(4.6)

**Incorporating damping in the modal domain**

Damping is the mechanism by which the vibrational energy is gradually transferred into heat or sound. In spatial models it is often modelled as one or more of the types: coulomb, viscous or hysteretic damping \cite{18, 50, 53, 54, 143, 153, 237, 260, 261, 277}. In dry friction, or coulomb damping, the force is constant in magnitude but changes its algebraic sign when the velocity does. As result virtual play might be created that limits the accuracy. Viscous damping is a retarding force proportional to velocity. Hysteretic, structural or viscoelastic damping is a force that opposes velocity, but is modelled proportional to displacement.

Different methods are available to cope with the second order equations that are coupled via damping. A representation that does not restrict the distribution of the damping forces is obtained when the system is lightly damped. In this approach the damping forces are assumed to be an order of magnitude smaller than the inertial and stiffness forces, and the damping-induced perturbations of the eigenfrequencies and mode shapes are also assumed small \cite{147, 277}. Another way to deal with damping is the assumption that the damping in the structure is distributed according to the spatial- stiffness- and mass- distribution \cite{38, 143}. This allows the spatial damping matrix to be diagonalized simultaneously with the mass and stiffness matrix \cite{2}. To this kind of damping is in this thesis referred to as proportional damping \cite{3}. In case of general linear viscous damping the $n$ second order equations are recast into the form of $2 \cdot n$ first order equations which allows exact calculation, but which loses much of the intuitive immediacy of the familiar treatment via normal modes. A major advantage of the light and proportional damping methods lies in the fact that the damped frequencies and mode shapes, can be deduced readily from the undamped ones \cite{277}.

In the models used in the initial design phase of the positioning system the effect of damping is considered according to the magnitude of the damping force with respect to the magnitude of the inertia- stiffness force. The un-damped
4.2. Modelling the dynamical behaviour

Mode shapes are used to incorporate the effect of relative small damping forces, that are modelled via the assumption of light- or proportional- damping. These assumptions are not valid for damping forces generated with supplemental viscous dampers that are able to produce large localized damping forces. These viscous damping forces, that might as well be created with actuators, have the potential to perturb the mode shapes [147]. As a result the effect of such supplemental dampers on the dynamic behaviour is modelled in another way. Often it is desirable to optimize the placement and sizing of these supplemental dampers to maximize vibration mitigation. In order to tune these supplemental dampers efficiently an accurate representation of the influence of the dampers on the dynamics of the structure is required.

Instead of the exact method, to describe the general linear viscous damping, an approximate solution, described by Main and Krenk [147], might be used to facilitate the search for optimal placement and sizing of added dampers in structures. The approximate solution is based on a simplified two component representation of the dominant, damped, vibration mode. The two components of the representation are, the modes of the structure without damping, and with the actuator location fully locked. The two component representation is exact in the limits of zero and infinite damping gain. In these limits the open and closed loop poles, as result of different damper locations are known [192]. The two component representation can be used to obtain an expression for the natural frequency of the damped vibration mode, and thereby to a parametric expression for the corresponding damping ratio [101]. This approximation is particular useful in the initial design phase to determine the opportunities and limitations of supplemental viscous dampers without the need to solve many complex eigenvalue problems.

In a parallel positioning system the actuators can also be used to generate relatively large damping forces. In order to approximate the opportunities and limitations of damping created with the actuators in the initial design phase of the parallel positioning system, the approximate solution as described by [147] is used. In advance of the used control structure, in this thesis the only type of damping generated with the actuators is viscous damping, where the damping force is in opposite phase with velocity [100]. A difference exist however, between a physical viscous damper and a viscous damper created with an actuator. The magnitude of the viscous damping created with the actuator is limited as result of the limited actuator force [206]. Furthermore, also the frequency of a controlled viscous damper is limited as result of limited control bandwidth. The effect of actuator damping on the modes with larger frequencies than the bandwidth is thus limited and the vibration modes of the uncontrolled system can be used to represent the dynamic behaviour. The undamped modes can be used to represent the response at low frequencies and small amplitudes that do no result in large damping forces. In case the damping force is much larger than the stiffness and inertia forces. The dynamic behaviour at that frequency range can be described in modal domain by replacement of the viscous dampers with nodes at the actuator location.

In this thesis the dynamic behaviour is described using the transformation to modal domain and back. The effect of damping is initially neglected to describe
the dynamic behaviour. When light- or proportional damping are assumed
valid damping models, the effect might simply be added to the undamped sys-
tem. In case damping is created with the actuators, the two component de-
scription is used, and thus the undamped system can be used to describe the
dynamic behaviour. In this way an upper bound of the performance of the po-
sitioning system can be guaranteed in the design phase. Other models might be
used to describe the dynamic behaviour of specific positioning systems more
accurate. However, in this chapter dynamical aspects of parallel positioning
systems in general, are treated. Nevertheless some remarks about damping are
made.

4.2.1 Homogeneous solution for an elastic beam

For a solid beam the mass and stiffness are assumed to be spatial distributed
according to the Bernoulli-Euler beam theory. This theory can be used with-
out problems when the thickness length/width ratio is larger than 1/20, [254].
Rotation inertia and shear are in this model neglected. The beam can be con-
sidered as a one dimensional system, and the dynamic behaviour therefore de-
pends on only one location parameter y (x = x(y)).
The first two modes are the rigid body modes that are not constrained with
the guidance. Higher numbered modes are elastic modes with an increasing
natural frequency. For instance eigenvector 3 to 6, (Φ³ . . . Φ⁶), correspond
to the first four elastic vibration mode shapes and are presented in Figure 4.1.
The angular natural frequencies (ωᵢ) due to the elasticity of a rectangular beam
are, [22]:

\[ \omega_i = \frac{\lambda_i^2 \cdot h}{L^2} \cdot \sqrt{\frac{E}{12 \cdot \rho}} \quad i = 3, 4, 5, \ldots \]  

(4.7)

Parameter λᵢ depends on the boundary conditions. For a free, not constrained,
beam parameter λᵢ is determined from the equation that fulfils

\[ \cos \lambda_i \cosh \lambda_i = 1. \]

The lowest values of λᵢ that fulfil the requirement are:

\[ \lambda_3 = 4.73, \quad \lambda_4 = 7.85, \quad \lambda_5 = 11.00 \] [22].
The mode shapes of this continuous beam system can be expressed with ana-
lytic formulas, [22, 77]. Instead of the analytic mode shapes formulas the lowest
numbered mode shapes are visualized in Figure 4.1.

4.2.2 Homogeneous solution for a circular elastic plate

In the engineering analysis of plates, global characteristics, e.g. deflections,
eigenfrequencies, etc., can be calculated by very simple theories based on kine-
matical assumptions. The mass and stiffness are assumed to be spatial distrib-
uted according to the Kirchhoff-Love plate model. For solid plates this theory
can be used without problems when the thickness length/width ratio is larger
than 1/20, [254]. Rotation inertia and shear are in this model neglected. The
plate is a two dimensional system and the system behaviour therefore depends
on two orthogonal location parameter y and z (x = x(y, z)). In case of a round
4.2. Modelling the dynamical behaviour

(a) mode 1 (translation rigid body mode)

(b) mode 2 (rotation rigid body mode)

(c) mode 3 (1\textsuperscript{st} elastic mode)

(d) mode 4 (2\textsuperscript{nd} elastic mode)

(e) mode 5 (3\textsuperscript{rd} elastic mode)

(f) mode 6 (\ldots elastic mode)

Figure 4.1: First six mode shapes of the beam. The number of nodes increases with increased mode number. E.g. mode 4 contains 3 vibration nodes, while mode 5 contains 4 vibration nodes.

plate the orthogonal parameters y and z are used interchangeably with a coordinate system based on angle (\(\theta\)) and radius \(r\).

The homogeneous solution of the differential equations can be solved by means of a power series known as Bessel functions, \([114, 152]\). The first three modes are rigid body modes. Higher numbered modes are elastic modes with an increasing natural frequency. The natural frequency of a solid homogenous plate with radius \(R_p\) is, \([22, 114]\):

\[
\omega_i = \frac{\lambda_i^2 \cdot h}{R_p^2} \cdot \sqrt{\frac{E}{12 \cdot \rho(1 - \nu^2)}} \quad i = 4, 5, 6 \ldots
\] (4.8)

Parameter \(\lambda_i\) depends on the boundary conditions and the poison ratio \(\nu\) and can be determined from equations presented in \([4]\). For a free, not constrained, plate with poison ratio \(\nu = 0.375\) the lowest values of \(\lambda_i\) are: \(\lambda_{4,5} = 2.26, \lambda_6 = 3.02, \lambda_{7,8} = 3.45\).

The mode shapes of a circular plate that is not constrained in space, free, can be expressed using the two independent parameters of the polar coordinate system. In the following equation radial dependency is indicated in the first part, while in the angular dependency is expressed in the second part.

\[
X(r, \theta) = \text{Bessel function}(r) [\sin (n_p \cdot \theta) + \cos (n_p \cdot \theta)]
\] (4.9)

In Equation \(4.9\) is the number of nodal diameters indicated with parameter \(n_p\). Because the sine and cosine are orthogonal the mode is split in two modes, one comprising the sine and the other the cosine term. The natural frequency of these modes is equal. The shape of some Bessel functions used to describe mode shapes are for a specific plate represented in Figure 4.22. The coefficients of the appropriate Bessel functions as well as the frequency parame-
The first 701 mode shapes of uniform, circular free-edge plates made of material with a poisson ratio $\nu = 0.33$, are provided by [114]. Despite the Bessel function($r$) can be expressed in much more detail, this is considered as a specialty on its own. The Bessel function is a second order differential equation. The value of the coefficients depends beside the specific properties such as dimensions also on the boundary conditions. This is considered out of scope and therefore not described in this thesis. Detailed information about Bessel functions and the application to circular elastic plates can be found in [114, 152].

The first three modes are rigid body modes ($\omega_{1,2,3} = 0$). They represent a displacement in x-direction, a tilt around the y-axis and a tilt around the z-axis. The mode shapes of the circular plate are presented in Figure 4.2 to 4.3.
4.2. Modelling the dynamical behaviour

Figure 4.3: Seventh to twelfth mode shapes of the plate. For an explanation, see the caption of Figure 4.2. The number of nodal diameters in mode 7 or 8 is \( n_p = 3 \). Mode 9, or 10, represents one nodal circle \( (n_j = 1) \) in combination with one nodal diameter \( (n_p = 1) \). Other modes can be expressed in the same way.

4.2.2.1 Modes with equal natural frequency

The circular plate is unique because the angular orientation of the modes is in theory arbitrary. The sine and cosine in Equation 4.9 are orthogonal, thus if the orientation of the mode belonging to the sine term is known, the orientation of the cosine mode is also known. The sine and cosine term belong to two different vibration modes with equal frequency. Two modes with equal natural frequency can combine into one vibration mode of which the orientation seems prescribed. [268].

When for instance a force \( (F) \) acts on the plate, mode four is excited as shown in Figure 4.4. The angular dependency of mode shape four is \( X = \cos(2\theta + \beta_1) \). The angular dependency of mode shape five, Figure 4.5, with the same natural frequency as mode four is \( X = \sin(2\theta + \beta_1) \). Parameter \( \beta_1 \) represents an arbitrary orientation. The real dynamic behaviour is a combination of modes. Due
to the equal natural frequency of mode four and five only one mode appears to be present when the mode shapes are combined, as shown in Figure 4.6.

When the force is located on another position the orientation of the ’new’ mode is changed. Also when more forces are used the orientation might change. The combination of two modes with equal frequency only affects the orientation of the nodal lines. One mode seems to be present while the other is not. When the plate is not perfect, for instance due to additional mass or not perfectly round the orientation of the modes is no longer arbitrary.

4.2.3 Homogeneous solution for other structures

The dynamic behaviour can be described using the mode shapes and natural frequencies of a system. Mode shapes and natural frequencies consist of the non-trivial solution of Equation 4.1. In some particular cases, such as a beam and plate, the solution can be determined via analytical methods using continuous models [63, 75, 77]. When more complex systems are considered analytical solutions might be difficult to obtain. Numerical methods, for instance finite elements, can also be used to obtain the mode shapes and natural frequencies.

Analytical solutions to known problems can be used to verify the results obtained with the numerical methods. In the conducted research, the results obtained from finite element program ANSYS® are used. Despite ANSYS® provides a large number of benchmark problems to show the accuracy of the solutions obtained with ANSYS®, an additional verification was performed. The dynamic behaviour of a plate obtained via ANSYS®, is compared with the results obtained with the analytical method provided by [114]. Despite the calculation in ANSYS® itself is quite straightforward, errors might be introduced by the user during modelling or implementing the results. The results of both, the analytical as well as the numerical, methods were equal. In the other considered cases the results obtained from ANSYS® are therefore also assumed to be correct and reliable.
In the design phase a sufficiently accurate model is used to provide insight and to evaluate different design concepts. A transparent model that results in physical understanding, and which is simple in mathematical manipulation, is therefore preferred. Despite analytical models such as the Timoshenko model for a beam and the Mindlin-Reissner or Föppl-Von Karman model for the plate, [248], can be used to model systems even more realistic, they are not preferred at the initial design phase. The mode shapes of continuous systems might be expressed analytical [37, 63]. In this thesis the numerical solutions from a model that is described with a finite number of elements is used. In this way the method is applicable to other structures modelled with finite elements. The mode shapes are obtained from the non-trivial solution to Equation 4.1 using the finite element program ANSYS ®.

A vector notation is in this thesis used to cope with the parameters of multi-dimensional problems [158]. Physical perception is in this thesis considered important as well. For that purpose the geometric meaning of the vector is often emphasized by a sketch of the corresponding vector polygon. This method is also applied when different dynamic aspects are treated. Mode shape sketches are initially used while less attention is directed towards the mathematical details [110, 157]. After understanding the physics of the design problem, accuracy is obtained using the mathematical expression.

4.3 Positioning system with parallel actuators

In the beam and plate positioning system discussed in Chapter 3 the actuators are placed in parallel. The number of actuators considered is in first case equal to the number of degrees of freedom of the rigid body. Systems that use more parallel actuators will be discussed in the next chapter. The dynamical behaviour of the system is the result of the structural system, mass and stiffness distribution, and the interaction with the forces generated in the actuators. The effect of the actuator forces depends on the location of the forces and thus the actuator location. Even though the aspects of an infinite number of different actuator locations can be researched the number of considered actuator locations is in thesis limited. Only actuator locations are considered that will reasonable be applied in engineering practice as motivated in Chapter 1. This is often a compromise between maximum ability to control the structure and minimum elastic deformation. When a certain actuator location is discussed the effect of small changes in actuator locations will be mentioned shortly when the control is discussed.

First the structural system is discussed as if it does not change as result of different actuator locations. The actuators are in first consideration assumed massless and only apply forces to the structure. After the best actuator location is determined for massless actuators the effect of additional actuator mass is taken in account. Practical aspects of the guidance used to constrain degrees of freedom are discussed in Chapter 3. The additional mass or stiffness of the guidance is in first consideration not taken into account.
Positioning with minimum excitation of consecutive modes
The summation of the vibrations should remain within the specified limits and in theory all modes should thus be considered. However, the lowest numbered modes possess the lowest stiffness, and as a result often dominate the total vibration amplitude when a step force profile is applied. The excitation of the modes depends however also on the location of the forces and the ratio between them. Even though the required position might be a step profile, the applied force profile is initially considered. The force profile is not a step profile, as will be mentioned later. The vibration amplitude of the individual modes depends as a result also on the interaction between the force profile and the system dynamics. Nevertheless the lowest numbered modes often dominate the vibration amplitude and the effect of the forces on the lowest numbered modes will therefore be researched first.

A nett force acts on the system which is used to fulfil the positioning task. The actuator forces are present in vector $\hat{F}(t)$. The vector $\hat{F}$ is a direction vector and should be such that the summation of the elements, sum along column, is one:

$$\sum_{n_c=1}^{n} \hat{F}(n_c) = 1$$  \hspace{1cm} (4.10)

The forces in vector $\hat{F}$ are assumed to be realizable in practice. The required force of each actuator is calculated with $F(t) \cdot \hat{F}$. Where $(\hat{F})$ is the force ratio of the actuators and the magnitude of $F(t)$ is such that the required rigid body acceleration is obtained. The mass normalized translational rigid body mode has value $\Phi_1 = 1/\sqrt{m}$. The value and dimension of the mode, in this without dimension, depend on the applied normalization, see page 88. Excitation of the first mode is required to be able to position the system. The excitation of the first mode is therefore $\Phi_1^T \cdot \hat{F} = 1/\sqrt{m}$. How to deal with variations in the mutual forces (robustness for limited accuracy and uncertainty) is considered later. In order to minimize the required actuator force during positioning the actuator forces should not counteract each other.

Besides the required rigid body movement the other modes are excited as well. The excitation of the other modes depends on the actuator location and the exerted force. Independent on the location, the individual actuator forces are defined such that consecutive higher numbered modes are not excited. The required force of each actuator is determined with Equation 4.10 and a number of equations that prevent excitation of consecutive modes. So,

$$\Phi_i^T \cdot \hat{F} = 0$$  \hspace{1cm} (4.11)

When for instance two actuators are used the force should be such that mode two is not exited ($\Phi_2^T \cdot \hat{F} = 0$). Mode two in case of the beam is the rotational rigid body mode. When three actuators are used, e.g. in the plate system, the forces should be such that both the second and third mode are not excited ($\Phi_2^T \cdot \hat{F} = \Phi_3^T \cdot \hat{F} = 0$). In the design phase errors of the actuator location should be considered, what might lead to other force ratios during operation. In this thesis the required force ratio ($\hat{F}$) is assumed to be realized without er-
rors. If errors in force ratio should be considered, closed loop operation might be applied to deal with them, as discussed further on.

Application of the force ratio as determined with Equation 4.11 during operation, results in an uncontrollable system in case no feedback control is applied. Controller design and the operation of the system is a specialty on its own and discussed by [224]. The force ratio calculated in Equation 4.11 is in this thesis however only used as initial start for the design of the system. The limitations and opportunities of different actuator locations, for instance to affect modes, will be discussed further on.

The force $\hat{F}$ is thus such that consecutive modes are not excited independent on the actuator location. The actuator location can however be optimized such that some higher mode or combination of higher modes is excited as less as possible. The excitation of the higher numbered modes is an effect of the actuator location and the corresponding force ratio. This method of minimum excitation of consecutive modes is demonstrated with the beam and the plate example as discussed further on. In case of the beam system with two actuators the force vector is such that positioning is possible, Equation 4.10. The force ratio $(\hat{F})$ is such that mode two is not excited ($\Phi_2^T \cdot \hat{F} = 0$) independent on the location. When the two actuators are located in the nodes of the third mode, this mode is excited as less as possible during a positioning task ($\Phi_3^T \cdot \hat{F} = 0$).

The dynamic behaviour of the rigid body mode, mode 1, plus one elastic mode (mode $i$) can be described by

$$X(y, z, s) \frac{F(s)}{F(s)} = \frac{\Phi_1(y, z) \cdot \Phi_1^T \cdot \hat{F}}{m_1 \cdot s^2} + \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F}}{m_i (s^2 + \omega_i^2)}$$

$$= \frac{1}{m \cdot s^2} + \frac{\Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F}}{m_i (s^2 + \omega_i^2)}$$

(4.12)

The mode shape representing the rigid body mode $\Phi_1$ does not depend on the $y$ and $z$-location and the value is every where equal to $\Phi_1(y, z) = 1/\sqrt{m}$, in case the mode shape is normalized to mass.

**Location of interest**

As mentioned before the dynamic behaviour in $x$-direction depends on the $y$- and $z$-location. Instead of describing the dynamic behaviour at every location of the beam only the dynamic behaviour at a few well chosen locations is presented. These locations are selected using the recommendations provided by Montgomery [166]. The dynamic behaviour at other locations can be determined and described in a similar way, i.e. without fundamental changes of the used method.

As shown in Equation 4.3 the dynamic behaviour depends on the mode shape value at the locations of interest. The location of interest are chosen such that they correspond to maximum and minimum values of the mode shape. Perhaps unnecessarily; the dynamic behaviour of other points on the system is between these extremes.
Force profile designed for positioning
To obtain the required displacement of mode one, forces have to be exerted on the system. Not only a position is specified but also an upper and lower limit, as shown in Figure 1.1. This specification holds for the whole structure. The required actuator force is determined with the most stringent requirement for the positioning system. In this positioning case it is assumed that the most stringent requirement is positioning over the largest required stroke \( h_m \) within a certain motion time \( t_m \).

The force profile is designed such that the speed of the rigid body is zero at the motion time \( t_m \). The momentum of the system as result of the impulse has to be removed from the system to obtain zero speed. For this purpose the force profiles considered in this thesis are anti-symmetric in \( t_m/2 \). The required force furthermore depends on the applied force profile. The maximum required actuator force is often minimized, because that results in general in the smallest actuator and minimum actuator mass as discussed in Chapter 3. The required amount of actuator force is minimum when the acceleration of the rigid body mode is minimized. The minimal acceleration is obtained with a force profile referred to as bang-bang force profile, because it switches (or: bangs) back and forth between its maximum and minimum value, \([207, 252]\). This profile is shown in Figure 4.7. During the first half of the motion time \( t_m/2 \) the system is accelerated while at the remaining second half of the motion time the system is decelerated. The minimal required actuator force \( (F_{min}) \) is \([7, 82, 99]\):

\[
F_{min} = m \cdot a = \frac{m \cdot h_m/2}{1/2 \cdot (t_m/2)^2} = \frac{4m \cdot h_m}{t_m^2}
\]

Figure 4.7: A bang-bang force profile that results in minimum required actuator force.

Also other bang-bang force profiles, for instance with a central zero force can be used. The rigid body motion during the central zero force part is a constant speed. Such force profiles can be used in positioning systems when the maximum speed is limited, e.g. by the induced electromotive force in the coil, or eddy current induced damping in the conductive material moving through the magnetic field. Also scanning or printing systems might require a force profile with a constant speed \([46, 232]\). Another motivation for a constant speed part in positioning systems is minimal temperature variation in the
4.3. Positioning system with parallel actuators

The dynamic behaviour of the system is known as function of the forcing frequency $\Omega$, a FRF, Equation 4.3. The response to an applied force profile can be determined when the frequencies in the force profile are known. Using Fourier theory any arbitrary, periodic force profile can be expressed as an infinite number of harmonic frequencies. A windowed Fourier transform is used for representation of the nonperiodic signal within a single positioning task [137]. For the task of the system considered here, positioning, a rectangular (dirichlet) window is used which starts simultaneous with the first non-zero value of the force and ends at the motion time $t_m$. The fundamental angular frequency of the considered time interval from zero to $t_m$ is $\Omega_1 = 2\pi/t_m$. Parameter $q$ is used to express the $q$-th harmonic. The angular frequency of the $q$-th harmonic is a multiple of the fundamental frequency as: $\Omega_q = q \cdot \Omega_1$. The frequency content of the force profile that result in minimum required force, bang-bang, contains only the odd numbered harmonics and is:

$$F(t) = \frac{4F_{min}}{\pi} \sum_{q=1}^{\infty} \frac{\sin (q \cdot \Omega_1 \cdot t)}{q} \quad (q \text{ odd number}) \quad (4.14)$$

The force ($F_{min}$) is the minimum required force as calculated with Equation 4.13. The window was selected such that only the forced response was considered. The dynamic behaviour after motion time $t_m$ depends on the force after motion time $t_m$ and the initial conditions, at motion time. Within a single position task, no force is exerted after motion time. As a result the free response is considered after motion time. The position and speed at motion time $t_m$ resulting from the forced response, are the initial conditions of the free response.

The motion cycle mentioned here treats only one positioning task. This is equivalent to only the time period of either rise or fall as mentioned in the analysis of cam systems. In the analysis and design of cam systems the rise or fall period is also often discussed separately from the complete motion program. However also the complete motion program, that is a combination of rise, fall and eventually dwell periods should be considered as well, when consecutive tasks are considered [178, 274].

**Transient response as result of an applied force profile**

In positioning systems or systems where a constant speed has to be obtained, such as scanning and printing systems, the transient response is important. The transient behaviour of positioning or scanning systems depends on the applied force and the dynamics of the system. As result of system dynamics and the interaction with the applied force to obtain the required rigid motion, a deviation from the ideal value might be present, [46, 82, 99, 126, 129, 211].

The relation between the natural frequency of mode $i$ ($\omega_i$) and the relation to the fundamental harmonic angular forcing frequency $\Omega_1$ is now discussed. The positional behaviour of the rigid body mode plus the higher modes is obtained by application of this force profile in Equation 4.6. Instead of the steady state response the transient response is considered. At the motion time $t_m$ in this case
t_m = 2\pi/\Omega the position should be obtained within the specified limits. When the force from Equation 4.13 is used and the ratio between natural frequency (\omega_i) and the fundamental harmonic frequency (\Omega_1) of the forcing profile is defined as:

\[ \tau_i = \frac{\Omega_1}{\omega_i} = \frac{1}{f_i \cdot t_m} \quad (4.15) \]

The position and speed just after motion time form the initial conditions that have to be fulfilled by the equations that describe the dynamic behaviour after motion time. In other literature similar ratios are used, e.g. in [10, 154, 231]. The position after the motion time can be described using [99]. The force profile is created such that the positioning task is fulfilled, a rigid body displacement over distance h_m in motion time t_m. Using Fourier theory the frequency content of the windowed force profile is determined. A Dirichlet window is applied to select only the forced response. Each individual harmonic is then expressed as \( F_q \cdot \sin (q \cdot \Omega_1 \cdot t) \). Just after motion time the rigid body mode displacement is obtained to which the contribution of the elastic modes is added.

\[
x(y, z, t > t_m) = h_m + \sum_{q=1}^{\infty} \sum_{i=n_r+1}^{n} \left[ \frac{2q \cdot \tau_i}{q^2 \cdot \tau_i^2 - 1} \cdot \sin \left( \frac{\pi}{\tau_i} \right) \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot F_q \right] \cdot \cos \left( \omega_i \cdot t - \frac{\pi}{\tau_i} \right) \quad (4.16)
\]

This is a vibration in the natural angular frequency \( \omega_i \) with a phase shift of \(-\pi/\tau_i\). The vibration amplitude depends on the mode shape values, present in \( \Phi_i \). If the actuator location is in the nodes of the vibration mode or the node location is considered the vibration amplitude would be zero. In practice it is almost impossible to locate the actuators in the node of the vibration mode, and therefore it is assumed the mode will always be excited. The amount of excitation varies. The effect of different actuator locations will be discussed in following paragraphs.

When a relatively large amount of damping was present in the system the initial conditions, position and speed, for the dynamic behaviour after motion time are changed. If a vibration is completely damped during motion time the amplitude is half the amplitude as presented in Equation 4.16 [99]. However using the amplitude as presented in Equation 4.16 the maximum amplitude is obtained. This maximum amplitude is used in the design phase in order to guarantee an upper bound of the performance.

Instead of the method presented above, the transient response to any arbitrary force profile can also be determined with transient analysis simulations. The method presented here might appear complicated, however aimed is at obtaining physical understanding and insight in the transient dynamic response when a force profile is applied to execute a positioning task. Physical understanding might be used to predict the effect of changes on the response prior to actual adaptation or calculation, and as a consequence direct the search for an optimum. Of course can physical understanding also be obtained from time domain simulations. However, in that case the number of simulations should
be large in order to cover a wide range of parameter variations, and meanwhile not to exclude the existence of local minima and maxima. For this reason the method presented in this thesis is used. In some particular cases time domain simulations are used to compare results.

**Transient response force profile containing the fundamental harmonic**

The dynamic behavior is now discussed in case the forcing frequency contains only one harmonic frequency. In this way the physical aspects are easier explained because the response is not blurred as result of the addition of other harmonics. Equation 4.16 is then simplified because it contains only a single harmonic. The single harmonic force profile can later be extended with other frequencies and thus adapted to other profiles, as will be discussed later. The motion time \( t_m \) is maintained while the effect of different harmonics is considered. The positioning task needs to be fulfilled, thus the required distance \( h_m \) needs to be obtained in motion time \( t_m \) independent of the considered harmonic. When only a single harmonic is used the force profile during motion time is:

\[
F(t) = \frac{q \cdot 2\pi \cdot m \cdot h_m}{t_m^2} \cdot \sin \left( q \cdot \frac{2\pi}{t_m} \cdot t \right) = \frac{q \cdot \pi \cdot F_{\min}}{2} \cdot \sin (q \cdot \Omega_1 \cdot t) \tag{4.17}
\]

The acceleration obtained from Equation 4.17 integrated twice and taking into account the initial conditions, result in the required distance \( h_m \) at motion time \( t_m \). The amplitude of the force is \( F_q = q \cdot m \cdot h_m \cdot \Omega_i^2 / (2\pi) \). Using Equation 4.15 the factor \( 2F_q/\omega_i^2 \) in Equation 4.16 can be replaced with \( q \cdot \tau_i^2 \cdot m \cdot h_m / \pi \) such that the dynamics are directly related to the specified positioning task. In case only one harmonic is used the position after motion time is:

\[
x(y, z, t > t_m) = h_m + \sum_{i=n_r+1}^{n} \left[ \frac{\tau_i^3}{\tau_i^2 - 1/q^2} \cdot \frac{h_m}{\pi} \cdot \sin \left( \frac{\pi}{\tau_i} \right) \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot m \right] \cdot \cos \left( \omega_i \cdot t - \pi / \tau_i \right) \tag{4.18}
\]

The vibration amplitude after motion time results in a deviation from the specified position. The number of rigid body modes is indicated with parameter \( n_r \). The deviation should be within the specified tolerance. The response is still obtained from the summation of the modes. The vibration amplitudes of the modes might interfere and therefore a maximum error is obtained by the summation of the amplitudes of all modes, which is:

\[
u_0 = \sum_{i=n_r+1}^{n} \left[ \frac{\tau_i^3}{\tau_i^2 - 1/q^2} \cdot \frac{h_m}{\pi} \cdot \sin \left( \frac{\pi}{\tau_i} \right) \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot m \right] \tag{4.19}
\]

In Equation 4.17 can be observed that the force amplitude increases when higher harmonics are used. Therefore the case is considered when the force profile consists of solely the fundamental frequency \( q = 1 \). The force ampli-
tude from the minimum required force profile, $F_{\text{min}}$ in Equation 4.13, has to be increased with 57% to be able to fulfil the positioning requirements.

The vibration amplitude of each individual mode after motion time ($u_{0i}$) is now considered in more detail. Especially the effect of local minima due to interaction between motion time $t_m$ and the angular natural frequency of the mode ($\omega_i$) is considered. These parameters are related with Equation 4.15, and expressed in variable $\tau_i$. From Equations 4.19 and 4.18 it can be observed that $u_{0i}$ depends on parameter $\sin \left( \frac{\pi}{\tau_i} \right)$. The interaction between motion time and natural frequency of mode $i$ results in magnitude of $\sin \left( \frac{\pi}{\tau_i} \right)$ from zero to one. Parameter $\tau_i$ is however also present in the first fraction of Equation 4.18. The parameters that relate the forcing frequency $\Omega$ to the natural frequency of each mode ($\omega_i$), in the remaining vibration amplitude after motion time, are combined in parameter $(\chi_i)$ according to:

$$\chi_i = \left| \frac{\tau_i^3}{\tau_i^2 - 1} \cdot \frac{1}{\pi} \cdot \sin \left( \frac{\pi}{\tau_i} \right) \right|$$

(4.20)

Parameter $\chi_i$ is a dimensionless error parameter of mode $i$, and can be used to replace parameters in Equation 4.19. The absolute value is used because negative or positive vibration amplitudes are regarded equally important, and only the magnitude of the vibration amplitude is of concern. The creation of this error parameter is analogy with the method used by Van der Hoek [99].

The creation of the dimensionless error parameter $\chi_i$ is not a purpose on itself. The parameter can however be used to obtain insight in the system performance due to changed motion time without simulating each arbitrary motion time. The dimension less error $\chi_i(y, z)$ can also be used in reverse. When the allowed vibration amplitude of mode $i$ after motion time ($u_{0i}$) is specified, as well as the stroke and the motion time, the requirements for the structural stiffness can be calculated. In practice however a tolerance is specified that contains the contribution of all modes. The specified tolerance can be used as vibration amplitude of mode $i$ after motion time to obtain insight in the initial design phase. The lowest numbered modes contribute in general most to the total vibration amplitude, see Equation 4.19. After the first initial design attempt the relative contribution of each mode is known. This can be used to divide the specified tolerance in different vibration amplitudes of the modes after motion time.

The error factor $\chi_i$ is plotted in Figure 4.8. Due to factor $\sin \left( \frac{\pi}{\tau_i} \right)$ the error parameter has several minima at $\tau_i = 1/2$ $\land$ $1/3$ $\land$ $1/4$ $\land$ $1/5$ $\ldots$. In Figure 4.8 local minima exists. These local minima can be used to change the motion time such that the contribution of a certain mode in the total error is minimized. Due to the changed motion time the contribution of the other modes might change as well. The time optimal motion time can be determined for single modes. However, the dynamic response is the result of the combination of modes. The total vibration amplitude can be minimized when the motion time is tuned to the lowest mode that contributes most in the error. The residual vibration amplitudes of the other modes remain however present.

In the design phase of the hardware it is often unknown if these local minima can be obtained in practice. As observed from Equation 4.15, $\tau_i$ relates motion
time to natural frequency. Especially at small values of \( \tau_i \) the local minima of the dimensionless error become more sensitive for changes in the dimensionless time \( \tau_i \). To be independent of local minima in the design phase an envelope curve is used. When designed in more detail the real curve might be used. However in this thesis an envelope is used for values of \( \tau_i < 2/3 \).

\[
\begin{align*}
\tau_i \leq \frac{2}{3} & \Rightarrow \chi_i = \left| \frac{\tau_i^3}{\tau_i^2 - 1} \cdot \frac{1}{\pi} \right| \\
\tau_i \geq \frac{2}{3} & \Rightarrow \chi_i = \left| \frac{\tau_i^3}{\tau_i^2 - 1} \cdot \frac{1}{\pi} \cdot \sin \left( \frac{\pi}{\tau_i} \right) \right|
\end{align*}
\]  

(4.21) (4.22)

For values of \( \tau_i < 2/3 \) the factor \( \sin (\pi/\tau_i) \) is set to its maximum magnitude namely 1. The envelope should cover all local minima. The envelope can however only be used to values of \( \tau_i \) smaller than \( \tau_i < 1 \) because that value of \( \tau_i \) would result in an infinite value when the covering formulae is used. At a value of \( \tau_i = 2/3 \) the value of \( \chi_i \) is equal and thus a continuous function is obtained. At increased forcing frequencies above \( \tau_i > 1/2 \) the vibration amplitude will increase fast. When the ratio of \( \tau_i = 1 \) a limit case exists that results in \( \chi_i = 1/2 \). At even higher forcing frequencies the error parameter will approach 1. The envelope function as presented in Equations 4.22 and 4.21 is plotted in Figure 4.8.

Using the envelope of parameter \( \chi_i \) the error of each mode can be calculated with:

\[
u_{0i}(y, z, t > t_m) = \chi_i \cdot \frac{h_m \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot \hat{F} \cdot m_{mi}}{m_i} \cdot \cos (\omega_i \cdot t - \pi/\tau_i)
\]

(4.23)

In Equation 4.23 parameter \( u_{0i} \) is the vibration amplitude of mode \( i \) after the motion time, see Equation 4.19. The stroke is represented with \( h_m \), the mass with \( m \), the mode shape value of the mass normalized mode \( i \) at location of interest with \( \Phi_i(y, z) \). The generalized mass \( m_{mi} = 1 \text{ kg} \) due to normalisation of the mode shapes. Parameter \( \Phi_i^T \cdot \hat{F} \) is the excitation of mode \( i \), where \( \Phi_i^T \) is
the mass normalized mode shape and \( \hat{F} \) is the unit force vector with the forces such that the specification in Equation 4.10 is fulfilled. \( \tau_i \) is the ratio between the fundamental harmonic \( \Omega_1 \) and the natural frequency of mode \( i \) (\( \omega_i \)), see Equation 4.15. Using the envelope, a design is created that is independent on local minima as result of the interaction between motion time and natural frequency.

The sensitivity for motion time changes is with the use of the fundamental harmonic frequency force profile much smaller than in case a bang-bang force profile is used [99, 155, 171]. Compared to the minimum required force \( F_{min} \) the force amplitude of the fundamental harmonic force profile is increased for equal motion time. The sensitivity for motion time changes is however reduced.

**Fundamental force frequency and higher harmonics with respect to natural frequency**

The error after a single positioning task is relatively large for large values of \( \tau_i \). The value of \( \tau_i \) reduces with increased natural frequency of the considered modes, see 4.15. The response furthermore depends on the amount of excitation of the modes \( (\Phi_i^T \cdot \hat{F}) \). In general the low frequency modes result in a relative large contribution in the vibration amplitude after motion time, compared to the contribution of the higher modes. For positioning purposes the frequency of the force profile (\( \Omega \)) should be much smaller than the natural frequency that contributes most in the vibration amplitude after motion time. For large values of \( \tau_i \) the error factor \( \chi_i \) is large and the applied force is more regarded as shock loading than a force profile used for positioning, see also [10].

In case the force profile is created with a higher harmonic than the fundamental harmonic, the required force increases. The maximum required force is for instance \( 3\pi F_{min}/2 \), when the third harmonic is used, see Equation 4.17. Besides the maximum required force is larger compared to the case where only the fundamental frequency is used, also resonant response might develop during the positioning task. For both these reasons, positioning with a force profile that consists of the lower harmonics is preferred. In order to reduce the maximum force of the force profile during positioning, higher harmonics might be added to the fundamental forcing frequency. Preferably lower harmonics are added, because the number of periods available to develop resonant response is then also limited. A combination of the fundamental harmonic with the third harmonic can be used to create a force profile with limited maximum force and limited maximum jerk [84, 99]:

\[
F(t) = \frac{15\pi F_{min}}{32} \left( \sin \left( \frac{2\pi}{l_m} \cdot t \right) + \frac{1}{3} \sin \left( \frac{6\pi}{l_m} \cdot t \right) \right) \quad (4.24)
\]

The maximum force is \( \sqrt{2/3} \) compared to the maximum force in case only the first harmonic is used, while the jerk is increased with \( 3/2 \). Other amplitude ratios between the fundamental forcing frequency and the third harmonic frequency result in an increased maximum force.

Force profiles which minimize velocity, jerk, energy or power, see e.g. [7, 61, 207], might also be used. When however higher harmonics are available in the
force profile resonant response might develop, even though the amplitude of
the harmonics is small. In steady state resonant response the vibration am-
plitude is only limited by the damping in the system. The selection of a force
profile is often a tradeoff between different aspects, e.g. maximum force or jerk.
Optimizing one performance criteria implies often sacrificing some other per-
formance criteria.
Despite a step in the jerk is required when the force profile contains only the
fundamental harmonic frequency profile, this profile is preferred in this thesis
due to the simplification of Equation 4.16. To design a robust system that is
independent on local minima due to the interaction between system dynamics
and the force profile only the error envelope as described in Equations 4.21
and 4.22 is used. These assumptions provide reasonable approximations about
the system performance, the required forces, and the dynamics in interaction
with the force profile over a certain motion time.

4.3.1 Beam positioning system with two actuators

The same beam as in subsection 2.3.1.2 is used. However, a real system may
look like the positioning system represented in Figure 3.24, at this moment
only the beam is considered. The dimension of the steel beam are: length \( L =
500 \text{ mm} \), width \( b = 20 \text{ mm} \) and height \( h = 2 \text{ mm} \). The density is \( \rho = 7850 \text{ kg/m}^3 \)
what result in a mass of \( m = 0.157 \text{ kg} \). The material properties are furthermore
Young’s modulus \( E = 210 \text{ GPa} \) and the Poisson ratio \( \nu = 0.33 \). The whole beam,
thus all y-locations in Figure 3.24 and Equation 4.3, have to be positioned in x-
direction.
The mass normalized mode shapes of the beam are presented in Figure 4.10.
The value of mode 1, the translational mode and mode 2, the rotational mode,
is determined via d’Alemberts principle applied to translations and rotations.
The value of mode 1 is \( \Phi_1 = 1/\sqrt{m} \). The value of mode 2 depends on the
y-location and the rotational inertia around the centre \( J = mL^2/12 \) and is
\( \Phi_2(y) = (y - L/2)/\sqrt{J} \).

Figure 4.10: Mass normalized mode shapes of the steel beam. The coordinate system as
presented in Figure 3.15 is used to define the axes directions.
The actuator location is indicated with expressing the shortest distance to the outside of the beam. The considered actuator locations are chosen in first case symmetric. In accordance with Equation 4.11 the forces of the two actuators are such that mode two, rotational rigid body mode, is not excited $\Phi^T \cdot \hat{F} = 0$. The forces should however fulfil Equation 4.10 as well. Due to the symmetric actuator location the force ratio is constant, independent on the location of the actuators. The ratio between the actuator is: $\hat{F}^T = [1 \ldots 1]/2$.

Besides the required translational rigid body mode, the first mode considered is mode three (the first elastic mode). The largest difference between the mode shape values of the third mode appears between the outsides of the beam and the centre. The points of interest are therefore the outside and the centre. Additionally the dynamic behaviour of the beam at the actuator locations is also considered.

### 4.3.1.1 Forces at the location of the anti-nodes of the elastic mode

When the forces are located in the anti-nodes of the elastic mode the excitation of that mode is maximal.

![Figure 4.11: Two mass less actuators are attached at from the outsides of the beam.](image)

In this beam example the actuators are located at the outsides. In Figure 4.12 the dynamic behaviour of mode three is shown when the actuators exert a step force. The transient behaviour to a step force profile is described in Equation 4.5. Due to the impulse of the force the momentum of the system increases. Besides a rigid body mode displacement also the other modes are excited. Only the lowest numbered excited elastic mode is initially considered, in this case mode three. Initially mode three is a straight line. When a step force profile is applied the straight line deforms towards its steady state deformation, Figure 4.12. When however the steady state deformation is reached, still kinetic energy is present in the mode. Due to the inertia the beam shoots through the steady state shape to a shape where only potential energy is stored in the beam.
and the kinetic energy is zero. As result of the potential energy the system goes back towards the steady state shape. The steady state shape is again passed as result of the inertia. In other words the mode vibrates around the steady state shape. The frequency is the natural frequency of mode three \( f_3 = 42.5 \text{ Hz} \). As can be seen in Figure 4.12 the vibration amplitude depends on the \( y \)-location on the beam.

The relation between actuator force and vibration amplitude on every location on the beam will now be considered in more detail. The rigid body behaviour is equal for every location. The contribution of mode three in the dynamic behaviour depends on the value of the mode shape and the angular natural frequency. The mode shape value of mode three has a maximum at the outside of the beam \( \Phi_3(0 \cdot L) = 5.0 \) and a minimum at the centre \( \Phi_3(L/2) = -3.1 \), see Figure 4.10.

The dynamic behaviour to a step force is expressed in Equation 4.5. The first term represent the movement of the rigid body, the second term the vibrational behaviour of mode three with an angular frequency of \( \omega_3 = 267 \text{ rad/s} \). When the step force \( F = 1 \text{ N} \) the amplitude of the vibration mode at the side is \( x_3(0 \cdot L) = 5.0 \cdot 5.0/267^2 = 3.6 \times 10^{-4} \text{ m} \). At the centre the amplitude is \( x_3(0 \cdot L) = 5.0 \cdot -3.1/267^2 = -2.2 \times 10^{-4} \text{ m} \).

The transient behaviour to a step force is determined above. Even though the transfer functions provide sufficient information, it is useful to understand how the dynamic behaviour is affected by different design choices. The frequency response behaviour is studied as well in order to examine the behaviour at different forcing frequencies. In a bode plot the modulus and argument of the compliance in the frequency response function are plotted as functions of the forcing angular frequency \( (\Omega) \), see Equation 4.2. The bode plots of different locations are represented in Figure 4.13. The transfer function in Equation 4.12 is a combination of the first mode and the first elastic mode, in this case mode 3.

![Figure 4.13: Modulus and argument of \( x/F \) for different locations. The actuators are located at the sides. (The magnitude of the force is thus \( F = 1 \text{ N} \).)]
In Equation 4.6 the transient response to a sine shape input force is determined. The response consists of vibrations with frequencies equal to the forcing frequency ($\Omega$), and vibrations with the natural frequencies ($\omega_i$) of the system. The vibration in the natural frequency is the result of the transient response. When damping is available, even in the smallest amount, the transient response is damped when enough time has elapsed. The frequency response contains only the steady state response.

Thanks to the applied force the elastic modes are, besides the required rigid body mode, also excited. The natural frequency of the elastic modes is nonzero $\omega_i \neq 0$. Starting at a forcing frequency much lower than the natural frequency ($\Omega \ll \omega_i$), the amplitude of the vibration is dominated by the angular natural frequency ($\omega_i$), see Equation 4.6. The vibration amplitude at the side is for instance $x_3(0 \cdot L) \approx 3.6 \times 10^{-4} \text{m}$ at low frequencies. This amplitude is equal to the amplitude of a step force profile as determined before. When the forcing frequency is equal to the natural frequency ($\Omega = \omega_i$) the vibration amplitude is in theory infinite. In practice the vibration amplitude is at that frequency limited by damping or other not included effects, e.g. non-linear stiffness. The resonance frequency of mode three is 42.5 Hz, what results in a peak in the frequency response plot, Figure 4.13. The magnitude of the vibration at the side ($y = 0$) is zero at a forcing frequency of $\Omega = 19 \text{ Hz}$. This frequency is also known as an anti-resonance frequency, or transfer function zero [163]. The anti-resonance frequency depends on the location of interest. In case consecutive positioning tasks are performed such that an harmonic force profile with a frequency of 19 Hz is applied, the side location is in theory, without regards to internal damping, not moving. This frequency is in this case, with the actuators at the outside considered the performance limiting frequency for consecutive positioning tasks.

### 4.3.1.2 Forces in the nodes of the elastic mode

The forces are now exerted in the nodes of the elastic mode. The rigid body mode behaviour remains the same as described before. The behaviour of the elastic mode changes as result of the smaller excitation of that mode. In the ideal case the mode is not excited. Small deviations in the actuator position with respect to the nodes result however in excitation of these modes. This effect is incorporated in the calculation by the assumption of the presence of a small alignment deviation from the nodes.

The nodes of the elastic mode are located at $22.4\% \cdot L$. The mode shape value is then $\Phi_3(\text{act}) = 0.0$. The actuators are in this particular case however assumed to be connected with a position deviation of 8 mm and are therefore located at 104 mm from the sides. The mode mode shape value of mode three at the actuator location is $\Phi_3(\text{act}) = 0.33$. The natural frequency of mode three and the mode shape value at the side and centre remain still the same.

Due to the small excitation of mode three the amplitude at the side is $x_3(0 \cdot L) = 5.0 \cdot 0.33/267^2 = 0.23 \times 10^{-4} \text{m}$ when a step force of 1 N is applied. The amplitude in the centre is $x_3(0 \cdot L) = -3.1 \cdot 0.33/267^2 = -0.15 \times 10^{-4} \text{m}$. These values are drawn exaggerated in Figure 4.14.
4.3. Positioning system with parallel actuators

Figure 4.14: Theoretic behaviour of mode three when a sinusoidal force is applied. This shape appears at all frequencies except at $42.5 \text{ Hz}$ due to a small misalignment.

In Figure 4.15 a resonance peak is present at a forcing frequency of $\Omega/(2\pi) = 42.5 \text{ Hz}$. Even though the excitation of mode three is very small the amplitude will peak near $\Omega/(2\pi) = 42.5 \text{ Hz}$, the denominator in Equation 4.6 approaches zero at that frequency. Anti-resonances are also present in Figure 4.15. The excitation of the modes depends on the actuator location. Minimal excitation of mode three results in a reduced numerator in Equation 4.6. Compared to other actuator locations the summation of the amplitudes becomes zero at smaller values of the denominator in Equation 4.6. Small values of the denominator are obtained when the forcing frequency equals the natural frequency. The anti-resonance appears therefore near the resonance frequency of mode three, as observed in Figure 4.15. Compared to the anti-resonance in Figure 4.13 at $19 \text{ Hz}$ the lowest anti-resonance frequency is increased in Figure 4.15. Due to minimal excitation of the third mode the accuracy of a single positioning task within the same motion time can be increased. When the tolerance specification is maintained the motion time can be decreased. Another option is to

Figure 4.15: Modulus and argument of $x/F$ for different locations. The actuators are located $0.208 \cdot L (= 104 \text{ mm})$ from the sides, while the nodes of the third mode are located at $112 \text{ mm}$ from the sides. The force is $F = 1 \text{ N}.$
reduce mass because less structural stiffness is required. This will be treated in Chapter 6.

Because mode three is only slightly excited, the summation of the amplitudes of the vibration modes is zero near the natural frequency. As result of small position errors of the actuator location, the first elastic mode will be excited. Dependent on the amount of excitation and the considered location an anti-resonance frequency appears. The amplitude of the first resonance and anti-resonance depends among others on the amount of excitation and the amount of internal damping. When the first elastic mode is only slightly excited the anti-resonance frequencies are located near the resonance frequency, see Figure 4.15. When no internal damping is present the lowest anti-resonance frequency is considered to be the performance limiting frequency for consecutive positioning tasks.

4.3.1.3 Actuators at other locations

In the previous part the dynamical behaviour is mentioned in case the elastic mode is excited maximal, while in the other case the excitation is minimized. When the actuators are located in other positions the dynamic behaviour is between these extremes.

The actuators are now located between nodes and antinodes. They are located at 15/100 · L which is in this case 75 mm from the sides, see Figure 4.16. The natural frequency and the mode shape values remain the same because the actuators are assumed massless. The mode shape value of mode three at the actuator location is \( \Phi_3(\text{act}) = 1.6 \). With the known mode shape values from the previous section the dynamic behaviour at other y-locations can be determined with Equation 4.6.

![Figures 4.16 and 4.17](image_url)

**Figure 4.16:** Two mass less actuators are attached at 15/100 · L from the outsiders of the beam.

**Figure 4.17:** Vibration of mode three when a step force is applied. The actuators exert an equal constant force.

The vibration amplitude as result of the transient response and the steady state response of mode three are shown in Figure 4.17. The vibration amplitude can be calculated with Equation 4.5 and is at the sides \( x_3(0 · L) = 5.0 \times 1.6/267^2 = 1.1 \times 10^{-4} \) m when a step force of 1 N is applied. The magnitude of the transient response and equilibrium shape in Figure 4.17 are smaller when compared to Figure 4.12.
The frequency response is plotted in Figure 4.18. Compared to the frequency response in Figures 4.13 and 4.15 the resonance peak remains at the same frequency. The anti-resonance frequency at the side and centre are however shifted to other frequencies. As a result the performance limiting frequency for consecutive positioning tasks is also shifted.

![Figure 4.18: Modulus and argument of \( x/F \) for different locations. The actuators are located at \( 15/100 \cdot L \) from the outsides \((= 75 \text{ mm})\), the force is \( F = 1 \text{ N} \).](image)

The actuators are now both moved further inwards. They are located in between the nodes of mode three. The location is selected such that the mode shape value of mode three is equal but of opposite sign as the case discussed above. In the beam case the location is at \( 31/100 \cdot L \), which is in this particular example at \( 155 \text{ mm} \), from the sides, see Figure 4.19. The mode shape value of mode three at the actuator location is \( \Phi_3(\text{act}) = -1.5 \). The natural frequency of mode three and the mode shape value at the side and centre remain the same.

![Figure 4.19: Two mass less actuators are attached at \( 31/100 \cdot L \) from the outsides of the beam.](image)

![Figure 4.20: Vibration of mode three when a step force is applied. The actuators exert an equal constant force.](image)

The vibration amplitude at the side when a step force of 1 N is applied is \( x_3(0) = -1.0 \times 10^{-4} \text{ m} \), Equation 4.5. The magnitude of the vibration amplitude is equal to the magnitude of the vibration amplitude at the sides when the actuators are
located at \(15/100 \cdot L\). The sign is however opposite. This result also in different frequency response functions when Figure 4.21 and Figure 4.18 are compared. The difference is caused by the phase difference of the actuator location.

![Figure 4.21: Modulus and argument of \(x/F\) for different locations. The actuators are located at \(31/100 \cdot L\) (= 155 mm from the outsides), the force is \(F = 1\ N\).](image)

**Higher numbered modes**

The dynamic behaviour is the summation of modes, Equation 4.3. The contribution of the modes depend on the forcing frequency \((\Omega)\), the natural frequency \(\omega_i\) and the mode shape values. In the above mentioned cases, the dynamics of the beam are described with the modes up-to mode three. Higher numbered modes can be added to describe the dynamic behaviour more accurate. The contribution of mode four is however zero \(\Phi T_4 \cdot \hat{F} = 0\), due to symmetry. Mode five is after mode three the first mode that contributes in the dynamic behaviour, and thus the vibration amplitude.

The dynamic behaviour to a step force is expressed in Equation 4.5. The angular natural frequency of mode five is \(\omega_5 = 230 \cdot 2\pi \text{ rad/s}\), Table 4.1. When the actuators are located at the side the value of the mode shape at the actuator location is 5.0, Figure 4.10. When the step force \(F = 1\ N\) the amplitude of the vibration mode at the side is \(x_5(0 \cdot L) = 5.0 \cdot 5.0^2 / 2090^2 = 0.1 \times 10^{-4}\ m\). This contribution is smaller than the contribution of mode three which is \(3.6 \times 10^{-4}\ m\). The contribution of mode six is zero \(\Phi T_6 \cdot \hat{F} = 0\), due to symmetry. Due to symmetry all even numbered modes are not excited. The contribution of mode seven can be determined. Due to the increased natural frequency the contribution in the static deformation will be small compared to mode three and five. When a dynamic force is applied the modal contribution peaks when the forcing frequency is equal to the natural frequency \((\Omega = \omega_i)\). In practice a limited frequency range is considered. The dynamic behaviour can be described sufficiently accurate by taking in account the modes that are in the frequency range of interest. This are in general only a few low numbered modes.
Asymmetric actuator location
Hitherto only a symmetric actuator location was considered. This resulted in no excitation of all antisymmetric, even numbered, modes. When an asymmetric actuator location is used the ratio between the actuator forces changes. The forces should remain such that mode two, the rotational rigid body mode, is not excited $\Phi_T^2 \cdot \tilde{F} = 0$. Due to the asymmetric actuator location and the corresponding force ratio mode three will always be excited. Mode four, and all other even numbered modes, will also be excited and contribute in the dynamic behaviour. When the force ratio is changed, for instance to minimize the excitation of mode three, mode two will as result of the force ratio be excited what result in other dynamic behaviour. For minimum contribution of the consecutive numbered modes a symmetric actuator location is preferred.

Conclusion open loop operated beam system
Minimal excitation of the third mode, due to the actuator location results in increased accuracy of a single positioning task within the same motion time. When the tolerance specification is maintained the motion time can be decreased. Another option is to reduce mass because less structural stiffness is required. This will be treated in Chapter 6. For consecutive task damping might be required. This should then be incorporated in the system, because the actuators are controlled open loop. Closed loop operation will be mentioned further on.

4.3.2 Plate positioning system with three actuators
A circular plate has been chosen as subject of study. However a circular plate is used other shapes of plates can be used as well [140, 265, 266, 267], the actuator location is chosen with the same methodology as used for the beam case.
A circular plate with radius $R_p = 150$ mm, thickness $h = 480$ µm, made of brass with density $\rho = 8430$ kg m$^{-3}$ is used. The Young’s modulus and Poisson ratio depend as well as the density on the used materials. In this case the brass contains 71% copper. The properties are Young’s modulus $E = 110$ GPa and the Poisson ratio $\nu = 0.375$ [1, UNS C26000]. In this brass only one material phase is present. A texture is present because the material is rolled once, this is however not assumed to result in elastic anisotropy [24]. The mass normalized mode shapes of the plate are presented in Figure 4.22. The value of mode 1, the translational rigid body mode and mode 2 and 3, the rotational rigid body modes, is determined via d’Alemberts principle. The value of mode 1 is $\Phi_1(r, \theta) = 1/\sqrt{m}$. The value of mode 2 and 3 depends on the radial location and the rotational inertia around the centre ($J = mR_p^2/4$) as $\Phi_{2,3}(r) = r/\sqrt{J}$.
Instead of indicating the different modes with an identification based on the number of nodal circles and nodal diameters as in [212, 265], in this theses the modes are indicated by the assigned number that increases with natural frequency. The two modes with a natural frequency of 19.5 in Table 4.2 are for instance referred to as mode 4 and 5, see also Figures 4.2 and 4.3.
For the plate positioning system, as shown in Figure 3.25, three actuators are used to fulfil the required task, i.e. positioning in x-direction. In Figure 3.26 a
Figure 4.22: Radial values, see Equation 4.9, of a few mass normalized mode shapes of the brass plate ($\nu = 0.375$). The complete mode shapes of a circular plate with this poison ratio are present in Figures 4.2 and 4.3.

<table>
<thead>
<tr>
<th>$n_p$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>no.</td>
<td>$f$ [Hz]</td>
<td>no.</td>
<td>$f$ [Hz]</td>
<td>no.</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>35.00</td>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>1</td>
<td>2-3</td>
<td>78.56</td>
<td>9-10</td>
<td>22-23</td>
</tr>
<tr>
<td>2</td>
<td>4-5</td>
<td>134.5</td>
<td>15-16</td>
<td>30-31</td>
</tr>
<tr>
<td>3</td>
<td>7-8</td>
<td>201.6</td>
<td>20-21</td>
<td>37-38</td>
</tr>
<tr>
<td>4</td>
<td>11-12</td>
<td>279.3</td>
<td>26-27</td>
<td>45-46</td>
</tr>
<tr>
<td>5</td>
<td>13-14</td>
<td>367.2</td>
<td>33-34</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>18-19</td>
<td>175.2</td>
<td>43-44</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>24-25</td>
<td>235.1</td>
<td>49-50</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>28-29</td>
<td>303.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>35-36</td>
<td>379.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>41-42</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Natural frequencies $f$ of the brass plate ($\nu = 0.375$). $n_p$ represents the number of nodal diameters, $n_j$ the number of nodal circles. The nodal diameters and circles can be observed in the mode shape figures as plotted in Figures 4.2 and 4.3.

possible actuator location is presented. The effect of different actuator locations is now discussed from a dynamic point of view. In first consideration the actuators and guidance are assumed not to possess mass. The effect of actuator and guidance mass will be taken into account later. The guidance constrains the movement in $y$ and $z$-direction as well as the rotation around the $x$-axis.

Three actuators are required to prevent excitation of the rotation rigid body modes during positioning. The three actuators are equally spaced along the circumference of a concentric circle. The angle between the actuators is thus 120°. The larger the radius the larger the possible effect on the rotational rigid body modes. The actuators are therefore first located at the side of the plate. The actuator forces are such that mode 2 and 3, the rotational rigid body modes, are not excited $\Phi_2^T \cdot \hat{F} = \Phi_3^T \cdot \hat{F} = 0$. The ratio between the actuator forces due to
the location and the requirements is: \( \hat{F}^T = [\ldots 1 \ldots 1 \ldots 1 \ldots ]/3. \)

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{actuator_location.png}
\caption{Three actuators located at equi-spaced points of equal radii.}
\end{figure}

The excitation of modes is determined by multiplication of the mode shape values with the force vector. The actuators are located at an equal radius thus the dependency on the radius is equal for the three actuators. The radial value of the mode shape of mode 4-5 at the radius of the actuators is indicated with parameter \( B_{4,5} \) and presented in Figure 4.22. The mode shape value at the actuator location furthermore depends on the angular location. The angular actuator locations are in this case \( \theta = 0, 2\pi/3, 4\pi/3 \). Mode four is described with \( B_{4,5} \cos (2\theta) \). Mode five is described with \( B_{4,5} \sin (2\theta) \). Mode six does not depend on the angle of the actuators only on the radial value of the actuators. The excitation of the first three elastic modes, mode 4-6, is.

\[
\begin{align*}
\Phi_4^T \cdot \hat{F} &= B_{4,5} \left[ (\cos (2 \cdot 0) + \cos (2 \cdot 2\pi/3) + \cos (2 \cdot 4\pi/3)) / 3 \right] = 0 \\
\Phi_5^T \cdot \hat{F} &= B_{4,5} \left[ (\sin (2 \cdot 0) + \sin (2 \cdot 2\pi/3) + \sin (2 \cdot 4\pi/3)) / 3 \right] = 0 \\
\Phi_6^T \cdot \hat{F} &= B_6 \left[ 1 + 1 + 1 \right] / 3 = B_6
\end{align*}
\]

The radial component \( (B_6) \) of mode six is zero at two third of the plate radius \( B_6(r \approx 102 \text{ mm}) = 0 \), see Figure 4.22. Besides mode six, the first mode that is excited during positioning is mode pair 7-8. Furthermore mode 17 and mode 18-19 will be excited.

The location of interest is indicated as a radial coordinate in mm and an angular coordinate in radians. In case the actuators are located at the side \( r=150 \) and a step force with a magnitude of \( F = 1 \text{ N} \) is applied. The locations of interest are chosen such that the vibration amplitudes are maximal at that location. Due to the angular orientation modes with zero, three, six, nine etc. nodal diameters are excited, (modes that correspond to \( n_p = 0, 3, 6, 9, \ldots \) in Table 4.2). The mode shape values in Figure 4.22 and natural frequencies in Table 4.2 are used to calculate the amplitude to the step force. Mode six with a frequency of \( f_6 = 35 \text{ Hz} \) vibrates at the side with an amplitude \( x_6(r = 150, \theta) = -2.8 \cdot -2.8/(35 \cdot 2\pi)^2 = 0.16 \text{ mm} \). The amplitude of some vibration are presented in Table 4.3. Also higher numbered modes are excited, however the contribution in the total vibration amplitude decreases due to the higher stiffness.

When the actuators are not located at the side but located at a radius of \( r = 102 \text{ mm} \) mode 6 is excited less compared to the situation where the actuators
Table 4.3: Vibration amplitude of the modes when the actuators are located at the outer radius at equi-spaced points. The magnitude of the applied step force is 1 N.

<table>
<thead>
<tr>
<th>location</th>
<th>mode 6</th>
<th>mode 7-8</th>
<th>mode 17</th>
<th>mode 18-19</th>
</tr>
</thead>
<tbody>
<tr>
<td>r [mm]</td>
<td>(35 Hz) [mm]</td>
<td>(46 Hz) [mm]</td>
<td>(147 Hz) [mm]</td>
<td>(175 Hz) [mm]</td>
</tr>
<tr>
<td>150 0</td>
<td>0.16</td>
<td>0.27</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>0 θ</td>
<td>-0.22</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.4: Vibration amplitude of the modes when the actuators are located at a radius r = 102 mm at equi-spaced points. The magnitude of the applied step force is 1 N.

<table>
<thead>
<tr>
<th>location</th>
<th>mode 6</th>
<th>mode 7-8</th>
<th>mode 17</th>
<th>mode 18-19</th>
</tr>
</thead>
<tbody>
<tr>
<td>r [mm]</td>
<td>(35 Hz) [mm]</td>
<td>(46 Hz) [mm]</td>
<td>(147 Hz) [mm]</td>
<td>(175 Hz) [mm]</td>
</tr>
<tr>
<td>150 0</td>
<td>-0.006</td>
<td>0.11</td>
<td>-0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>0 θ</td>
<td>0.008</td>
<td>0</td>
<td>-0.014</td>
<td>0</td>
</tr>
</tbody>
</table>

The absolute values of the amplitudes presented in Tables 4.3 and 4.4 are plotted in Figure 4.24.

Figure 4.24: Vibration amplitude at the centre (r = 0) and at the side (r = 150 mm) when three actuators are used that exert a step force profile with a total magnitude of F = 1 N. In one case the actuators are located at the side (r = 150 mm), in the other case the actuators are located at (r = 102 mm) see Figure 4.23.

The bode plot of the frequency response at the side (r = 150 mm), at an angle of 0° is plotted in Figure 4.25. A peak can be observed at the resonance frequency of mode 6 (35 Hz). This mode is thus in both case excited. When the actuators are located at the outer radius the excitation of mode six is larger compared to the situation where the actuators are located in the nodal line of mode six. In case the actuators are located at the outer radius the amplitude and phase of
the elastic modes 6 and mode 7-8 contribute at low frequencies much to the overall response, see Figure 4.24. The total response is the summation of the different modes, including the rigid body mode. As a result of this summation an anti-resonance frequency appears at 12 Hz when the side location \((\theta = 0^\circ)\) is considered. In case the actuators are located at a radius of 102 mm mode six is still slightly excited. This results in a resonance and anti-resonance frequency at 35 Hz, see Figure 4.25. In that case the lowest anti-resonance is present at 24.6 Hz. The vibration amplitude at that frequency is besides the rigid body mode, dominated by mode 7-8, see Figure 4.24. In both cases mode 4-5, with a resonance frequency of 19.5 Hz was not excited. In case errors in actuator locations are made, the force \(\hat{F}\) is adapted such that consecutive modes are not excited, see Equation 4.11. As result mode 4-5 is then excited. This results in a resonance peak at 19.5 Hz. Dependent on the amount of excitation and the location of interest an anti-resonance appears as well. The anti-resonance frequency of 12 Hz is considered a performance limiting frequency for consecutive positioning tasks in case the actuators are located at the outer radius. When the actuators are located at the nodal radius of mode 6, the resonance-anti resonance frequency of mode 4-5 (19.5 Hz), is considered the performance limiting frequency for consecutive positioning tasks. When the location of one of the actuators is not exactly as required and the force ratio remains the same, mode 2-3 are the first modes that are excited during positioning. Mode 1 is required, that is the positioning task. In order to prevent excitation of mode 2-3 the force ratio between the actuators is changed. As result of the error in actuator location and the changed force ratio other modes are excited as well. When the error of the location of the actuators is in radial direction mode 4-5 and 6 are the first modes that are excited during positioning. When the error is in angular direction mode 6 is not affected only mode
4-5 are excited. The force ratio can not be adapted to minimize excitation of
the modes 4-5 during positioning because that also affects the lower numbered
modes. The forces of the three actuators can be such that mode 2 and 3 are not
excited during positioning (excitation of mode 1). When mode 4 or higher is ex-
cited during positioning it can not be compensated for, without neglecting the
put forward requirement for the force ratio to not excite the lower numbered
modes.
The location of the actuators and the phase of the mutual actuator force will not
be as ideal as assumed. If the three actuators exert a harmonic force of \(19.5\) Hz,
the amplitude of mode 4-5 will increase to infinite values when no material
damping is present. In practice the amplitude is limited because damping is
present. However, the possible instability at the frequency of mode 4-5 should
be taken into account.

### 4.4 Practical aspects and the dynamic behaviour

In the previous sections idealized beams and plates were discussed. In practice
actuators need to be connected, that comprise mass. As observed in Chap-
ter 2 the degrees of freedom of the other directions need to be constrained, and
thus guidances are required. Furthermore stability and robustness might be
required. All these aspects affect the dynamic behaviour. The mode shapes for
instance change when another mass and stiffness distribution is obtained. Also
the interaction of the actuator force and the structural dynamics due to closed
loop control affect the dynamic behaviour. The possibility to cope with limited
model accuracy, parameter uncertainty and disturbances also depends on the
actuator location and is shortly considered. However the actuator locations are
in this thesis discussed for positioning purposes.

**Robust, stable and deterministic design**

The deterministic principle can be used in the design- and operation phase
of positioning systems. The required actuator forces can be determined prior
to construction or actual operation in case the system behaves according to
the used models. In common engineering practice a difference exists between
theoretic models and practice. The limited model accuracy might be the result
of limited validity of the model describing the physical phenomena or due to
parameter uncertainty.

In the design phase it is often uncertain if all aspects are taken in account in
the models. Unmodelled disturbances might for instance be present. The goal
of the models used in the design phase is to guarantee an upper bound on the
performance. In the design phase, increased model accuracy might be used to
adapt and improve the design. However, the sensitivity for parameter vari-
ations and know how to cope with them might even be more important. The
performance might for instance depend on a delicate balance of forces and di-
mensions [19]. Tolerances need to be specified in the design phase, which can
be incorporated as parameter uncertainty in the models [31, 81, 188]. The para-
meter uncertainty in the models prior to actual construction is in general larger
than in the models describing realized systems. Furthermore the validity of the models used during the design phase can only be verified after the design is realized. The system should therefore be designed such that it is robust for limited model accuracy, and parameter uncertainty. These effects deal in principle with robust design [255]. Easy to manufacture tolerances might for instance be specified when large parameter uncertainty is allowed. A robust design is also required to cope with changing operation conditions for instance due to disturbances. The system may change with time, e.g. components may age, or their parameters may vary with temperature. Also the operating conditions may vary e.g. load changes and disturbances [46]. Stability is, besides robustness, another issue that has to be concerned in the design of the physical system. With stability is in this thesis meant that a (quasi) static equilibrium state is obtained for static inputs or disturbances. Furthermore should be returned to the original equilibrium state after the perturbation has ended. Also should bounded (dynamic) inputs and disturbances result in bounded outputs. The result of stability and robustness should be such that the performance remains within specifications.

In order to improve the performance after the system is created, the actuator input might be adapted using more accurate models. While making use of deterministic philosophy the design can in theory be modelled exact and the required actuator input can be determined without direct actual information [33]. Some undesired effects might be compensated for, by adapting the actuator input. Measurement information from sensors can be used to detect changes. The results from these measurements can be related to effects appearing at other locations and eventually affect the actuator input. A closed information loop between sensor and actuator is obtained when the information from the sensor is used to adapt the actuator input and the sensor is able to detect the effects of the changed actuator input. Due to the closed information loop an accurate model is not required. High gain feedback is applied. Despite a lot of effects can be modelled using the deterministic principle, the time and money available to realize the design is assumed limited. To cope with uncertainty and limited model accuracy a tradeoff between the application of the deterministic philosophy and closed loop control structures is assumed present. Formulae providing more accuracy and less uncertainty might be available but are often accompanied by higher computational effort [272]. Despite the designs are based on models with limited accuracy it is in the design phase clear how to deal with expected uncertainty and deviations. In the design phase the goal is to guarantee an upper bound on the performance, in this case the absolute value of the positional error of the end effector after a reference path has reached the end point [46].

### 4.4.1 Application of feedback control

The dynamic behaviour when feedback control is applied, is the result of the complex interaction between the control of the actuator forces and the elastic mechanical system. The design of the control structure and the actuators is a specialty on its own, see [224]. Only some of these aspects are briefly con-
Chapter 4. Dynamical aspects of parallel actuation

Considered in order to determine opportunities and limitations that result from different actuator locations. In this thesis is assumed the actuator and sensor have perfect dynamics, see [193]. In the previous part of the chapter the system was operated open loop [74]. I.e. no measure of the system output being controlled, was used to compute the control action to take. Now closed loop operation is considered, where a measure of the controlled output signal is fed back for use in the control computation. In case it is possible to anticipate on either a track to be followed or, a disturbance to be rejected, this information might be feed forward as an early warning to the control system. Feedback and -forward might be combined to create an optimal controlled structure, but they are in this thesis considered separate. Known cause and effect relations might be incorporated via feedforward. The aspects of solely feedforward control are regarded similar to the aspects previously treated in this chapter. The feedback is in this thesis used to cope with systems with unknown-, or limited accuracy of-, cause and effect relationships [33]. The effect on the dynamic response is in the remainder of this section determined in case only feedback control is applied. Determining the cause and effect relationships more accurate and possibly incorporate this in the control structure, e.g. in the feedforward, is thus not aimed for.

Sensor configuration and type of control

The feedback control is designed according to the guidelines of Schweitzer et al. [225]. This means that decentralized control of each individual actuator is applied. I.e. each actuator has its own sensor and controller structure. Another guideline is to locate the sensor as close as possible to the actuator location. In this thesis the actuator and sensor are assumed to be located at the same position, i.e. collocated [193]. The control structure of each control loop is furthermore restricted to a basic proportional and differential controller. Without regards to limited actuator force nor limited bandwidth, can this type of controller initially be replaced with their mechanical analogons: linear springs and dampers. The use of this analogy is regarded appropriate in the initial design phase of the mechanical system. Afterwards it should be verified if the actuators are able to generate the required force and, if the corner frequency of the individual elements in the control loop which have low pass filter characteristics, is well above the natural frequencies of the considered vibration modes, e.g. as performed in [232]. An example where limited bandwidth and limited actuator force is taken in account is the rotor system discussed by Schweitzer et al. [225]. The electrical resistance of voice coils with respect to frequency is mentioned by Wright [279].

The location of the nodes of the vibration modes will change due to the settings of the feedback compensator and the location of the actuators. A changed location of a node implies other mode shapes and also other natural frequencies. When no feedback control is applied the node location is determined by the stiffness and mass distribution of the mechanical system, in the other extreme, with tight actuator force control, the actuator locations form the nodes of the vibration modes. Tight control is used here to refer to the principle of high gain feedback control, that is able to result at any time in zero deviation between
actual and required position at the actuator location. Even though, collocated actuator sensor locations can not always be realized in practice, it is initially assumed possible. A wide range of other control structures and different sensor locations are possible that might improve the performance. The ability to observe modes can be changed when using other sensor locations, what as a consequence might result in a different control structure to obtain the required actuator forces [193]. The required actuator force depends, among others, on the actuator location. The ability to affect modes with actuator force depends on physical aspects of the actuators, such as the location and maximum force. These physical aspects are independent of the control structure. The effect of the actuator location on the performance, based on decentralized control, will therefore not change due to other control structures. The aspects of other sensor locations and other control structures is a specialty on its own, and discussed by Schneiders [224].

4.4.2 Actuator and guidance mass

Hitherto only forces acting on elastic systems were discussed. In practice always mass is required to generate the forces. In this thesis Lorentz type actuators are used, where one part of the actuator is located on the static world, while the other part has to move with the system. Due to the additional mass of the actuator the mass distribution of the system changes and thus also the natural frequencies and mode shapes change. As result of the additional moving mass a larger force is required to obtain the same acceleration. The acceleration of the beam should still be such that the beam is positioned within the motion time. The effect of the actuator mass depends on the amount of mass and the location of the mass. The actuator masses are considered to be point masses without any stiffness, also the rotational inertia of the actuator is neglected. When the actuators are located in the nodes of a mode, that mode and the natural frequency are not affected by the actuator mass. The effect of the actuator mass on the vibration amplitude in Figure 4.14 is minimal, because the actuator location does not affect the location of the nodes of the third mode. Other modes and frequencies are however affected by the actuator mass. A real system also contains a guidance to constrain the degrees of freedom in the direction that does not require positioning. The guidance also implies additional mass, and as such affects the mode shapes and natural frequencies in the direction that requires positioning. The stiffness of the guidance in the direction that should be constrained might depend on the guidance location. For optimal rotational stiffness two constraints should be located with the largest possible mutual distance between them. For optimal performance in the required positioning direction the guidance is best located at the location of the actuators used to position. At that location the guidance is assumed to be stiff enough to fulfil the stiffness requirements in the direction that should be constrained. As result of this location the effect of the guidance mass can then be incorporated in the actuator mass. To obtain the required acceleration, extra force is required to accelerate the additional
4.4.3 Positioning with forced nodes at the actuator location

The additional mass of the actuator and guidance, and the application of feedback control result in changed dynamics of the system. An upper limit of the deviation between actual- and required position after motion time is determined, despite the complicated interaction between the different elements. Therefore the case is considered where a feedback compensator is applied such that the system at the actuator location exactly follows a reference path. A reference path is applied in order to limit the required actuator force for the positioning task. As result of tight control nodes are created at the actuator location of the system, and this location follows the reference path without deviation. The deviation at other locations is considered as well, and forms an upper limit for the deviation. More sophisticated control should result in a smaller deviation. The practical aspects required to force nodes at the actuator location of the system, are considered further on.

The dynamic response of a structure under the action of any time-dependent load can be determined with transient dynamic analysis. This method can also be used to calculate the dynamic error as result of the positioning task. The dynamic error is the result of the response to the reference profile. The acceleration as result of the actuator force and the inertia result in the generation of inertia forces. As result energy is stored in the system that might result in deviations when it is released. The deviation just after motion time is determined. This depends on the system dynamics and the accelerations that result from the reference profile. Many, time consuming, transient analysis simulations might be required to reveal the relevant aspects of the reference profile on the response. A frequency domain analysis method is initially used to obtain understanding, thereafter transient analysis can be used to determine the response exact.

The acceleration profile and the inertia are used to determine the loading and the response. Via Fourier theory the frequency content of the acceleration of any arbitrary reference profile $x_{\text{ref}}$ can be determined. The acceleration profile within a single positioning task is however nonperiodic. Therefore the frequency content of the windowed signal is determined in a similar way as previously described for the open loop forced response. The rectangular window starts simultaneous with the first non-zero value of the acceleration profile and ends at the motion time $t_m$, thus contains only the forced response. In this analysis a reference profile is used of which the windowed acceleration profile contains only a single frequency. In order to ensure the windowed acceleration profile consists of only a single frequency, and no static component, the windowed acceleration profile is selected and integrated twice to obtain the reference profile. When the initial conditions are fulfilled, a smooth reference profile is obtained that does not contain steps in speed or position, while the requirements, positioning over distance $h_m$ before motion time $t_m$, are fulfilled. The acceleration magnitude ($\ddot{x}_q$) of the reference profile created with only the q-th harmonic frequency ($q \cdot \Omega_1$) of the motion time is $\ddot{x}_q = q \cdot h_m \cdot \Omega_1^2/(2\pi)$ =
q \cdot h_m 2\pi / t_m^2$. The reference profile obtained via integration of the windowed acceleration profile is a step type function with a cycloidal front:

\[ x_{ref}(t \leq t_m) = h_m \cdot \frac{t}{t_m} - h_m \cdot \frac{q \cdot \Omega_1}{2\pi} \sin(q \cdot \Omega_1 \cdot t) \]  

(4.26)

The mode shapes and natural frequencies of the system, with nodes at the actuator location, are determined with modal analysis. Even though mode shapes and natural frequencies are determined, Equation 4.3 is only applicable for the response of external forces, but not for base motion. Spectrum analysis can however be used to determine the response and amount of excitation of individual modes, to harmonic base excitation. Despite the windowed step type reference function with cycloidal front itself is not a harmonic motion profile, the windowed acceleration profile is. Because the response is determined via the reaction forces, the negative acceleration amplitude ($-\ddot{x}_q$) of the motion profile is used, see [44]. Even though the acceleration at the actuator location is positive, the sign of the acceleration profile is negative because the response is determined via the reaction forces. The reaction forces are assumed to vary in a similar way as the applied acceleration profile. This assumption is validated later.

The amount of excitation of the individual modes is indicated with the term: modal participation factor. This is obtained from the mass matrix premultiplied by the mass normalized eigenmode times a vector representing the excitation direction [45, 113]. The base acceleration ($a(t)$) is therefore split in a vector representing the excitation direction ($\hat{a}$), and a parameter containing the time dependency of the acceleration $\ddot{x}(t)$. The elastic response to an acceleration ($a(t) = -\ddot{x}(t) \cdot \hat{a}$) is thus determined. The direction ($\hat{a}$) is incorporated in the modal participation factor, and the magnitude $-\ddot{x}_q$ is defined as input for the spectrum analysis. The participation factor for base motion is determined with $\Phi_i^T \cdot M_L \cdot \hat{a} / m_i$. Parameter $M_L$ represents the lumped mass matrix, and is obtained from the consistent spatial mass matrix $M$ by adding on the diagonal all translational contributions and neglecting rotational terms [191, 204]. The lumped mass matrix has to be used in case the vectors with the nodal degrees of freedom ($\Phi_i$) contain also rotations. The eigenvector, with nodes at the actuator location, is represented by parameter $\Phi_i^T$. The participation factors can also be determined with a finite element program, e.g. ANSYS®; where mass normalized mode shapes are used in the calculation.

Up to time $t_m$ the windowed acceleration profile is used to determine the response of the initial part of the positioning task. The response to the constant part of the reference path after motion time is considered separate from the previously windowed acceleration profile. This response depends on the initial conditions at time $t_m$. The vibration amplitude after motion time, that results from base acceleration, is determined in a similar way as in Equation 4.16. For completeness the response is expressed for the case where the acceleration profile consists of different harmonics ($-\ddot{x}_q \cdot \sin (q \cdot \Omega_1 \cdot t)$). The definition of $\tau_i$ in Equation 4.15 is used to relate the acceleration profile to the natural fre-
quency and results in:

$$x(y, z, t > t_m) = \sum_{q=1}^{\infty} \sum_{i}^{n} \left[ \frac{2q \cdot \tau_i}{q^2 \cdot \tau_i^2 - 1} \cdot \sin \left( \frac{\pi}{\tau_i} \right) \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot M_L \cdot \hat{a} \cdot -\ddot{x}_q \cdot \frac{\Phi_i(y, z)}{m_i \cdot \omega_i^2} \cdot \cos (\omega_i \cdot t - \pi/\tau_i) \right]$$  \hspace{1cm} (4.27)$$

During the application of the acceleration profile the rigid body mode response should be added to obtain the complete response. In case only a single harmonic is used to create the acceleration profile the magnitude of $-\ddot{x}_q$ is directly related to the required rigid body motion, as mentioned before, and the response after motion time is:

$$x(y, z, t > t_m) = \sum_{i=1}^{n} \left[ \frac{\tau_i^3}{1/q^2 - \tau_i^2} \cdot \frac{h_m}{\pi} \sin \left( \frac{\pi}{\tau_i} \right) \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot M_L \cdot \hat{a} \cdot \frac{\Phi_i(y, z)}{m_i} \cdot \cos (\omega_i \cdot t - \pi/\tau_i) \right]$$  \hspace{1cm} (4.28)$$

Now the acceleration profile is considered to be build with only the fundamental frequency $(q = 1)$ because that results in the lowest acceleration. The reference profile obtained from double integration of this acceleration profile is referred to as ramped sinusoid[62, 99]. The error of each mode can be determined in the same way as in Equation 4.23 and is:

$$u_{0i}(y, z, t > t_m) = \chi_i \cdot \frac{h_m \cdot \Phi_i(y, z) \cdot \Phi_i^T \cdot M_L \cdot \hat{a}}{m_i} \cdot \cos (\omega_i \cdot t - \pi/\tau_i)$$  \hspace{1cm} (4.29)$$

For parameter $\chi_i$ the envelope, as specified in Equations 4.21 and 4.22, can be used to obtain a result independent on local minima.

**Actuator force in relation to base acceleration frequency**

The required actuator force to maintain nodes at the actuator location depends on the frequency of the base acceleration with respect to the natural frequencies of the system. However, when only a single period of the base motion is performed the time for interaction between acceleration profile and natural frequency of the system is limited. As a result the required actuator force is limited as well, even in case the acceleration frequency is equal to a natural frequency. In case a single positioning task is performed with the application of higher harmonics in the base acceleration profile, the required actuator force to maintain nodes at the actuator locations might increase to large values. This is the case when the frequency of the base acceleration is equal to a natural frequency of the system and sufficient time is available to develop a resonance response. If there are only a few cycles at the resonant frequency the build-up of resonant response may not be significant at all. In a resonance response the required actuator force, to force nodes at the actuator location, increases over
the consecutive periods of the harmonic base acceleration. In order to limit
the required actuator force the frequency content of a single positioning task
is therefore preferably created with the fundamental harmonic. In order to re-
duce the maximum required actuator force for positioning of the rigid body,
higher harmonics might be added to the acceleration profile created with the
fundamental harmonic. However, the force required to create nodes at the ac-
tuator location increases when the frequency of a higher harmonic is equal to
a natural frequency.

The required actuator force to create nodes at the actuator location should also
be concerned when consecutive positioning tasks need to be performed, even
though the fundamental frequency is used for the base acceleration. With the
actuator locations as nodes, resonant response develops when the frequency
of a harmonic base motion is equal to a natural frequency of the system. The
steady state vibration amplitude, and the actuator force required to create
nodes at the actuator location, of a system in resonant response depends on
the damping in the mechanical system. The base acceleration frequency for
consecutive positioning tasks is therefore in general limited by the natural fre-
quency that results in vibration amplitudes outside the specified tolerance
band, or when large actuator forces are required to create nodes at the actuator
location with respect to the force required for positioning. In general the first
natural frequency is limiting the fundamental frequency of the base motion
for consecutive positioning tasks. However, in case the participation factors of
the first mode, or modes, is almost zero and sufficient damping is present, the
fundamental frequency of the base motion for consecutive positioning tasks is
limited by a higher natural frequency. Attention should be payed to guarantee
the performance of this special case.

The restriction for consecutive positioning tasks results in a relatively low fre-
quent base motion with respect to the first natural frequency that limits the
base motion. Also the acceleration profile of a single positioning task is prefer-
ably created with the lower harmonics. In order to transfer sufficient momen-
tum to the system during the motion time in case of a single positioning task,
the fundamental harmonic in the acceleration profile should also be smaller
than the performance limiting frequency. Otherwise positioning in the speci-
fied motion time is not performed, and the acceleration profile is experienced
as shock loading, see also [10]. As a result of these restrictions for the assump-
tion that the reaction forces vary in a similar way as the acceleration profile
provides a good initial indication of the dynamics in interaction with the refer-
ence profile. Equations 4.23 and 4.29 can be used to approximate the vibration
amplitude after a single positioning task via the acceleration profile. This ap-
proximation is used to obtain insight, required for engineering judgement [43].
More accurate responses for particular cases, can be determined via a transient
analysis.

**Required actuator force**

The collocated decentralized control constitutes an attractive means to deter-
mine the performance of a positioning system in the initial design phase. As
a result of the collocated control the individual control loops are no longer af-
fected by mechanical flexibility between each sensor and actuator. This is ben-
eficial for the stability of each control loop and reduces the dependency on ac-
curate system models [100, 193]. The lower numbered modes with nodes at the
actuator location are limiting the performance. Nodes of the lower numbered
modes at the actuator location can be created with limited gain and frequency
of decentralized proportional controllers. This feedback control initially affects
the lowest free vibration modes most [112]. The roll-off frequency of the ac-
duator force, e.g. as result of controller settings, or hardware limitations should
be larger than the performance limiting frequency. Limitations might result
from different sources Eppinger and Seering [68], e.g. from limited sampling
rate [108] or saturation limits [206]. High frequency roll-off might be used to
accommodate the actuator and power amplifier dynamics, and prevent ampli-
ification of high frequency noise [193]. The control gain and actuator force are
selected such that the deviation at the actuator location is small when a sta-
tic acceleration field is applied with a magnitude of four times the magnitude
of the acceleration profile used during positioning. With a small deviation is
in this case referred to: small with respect to the specified tolerance limit of a
single positioning task. The factor four is a margin, used to incorporate the in-
creased support force in case the fundamental frequency of a single positioning
task is equal to a natural frequency of a vibration mode.

The collocated decentralized proportional controlled actuator force can in a
finite element program be modelled with its mechanical analogon, a linear
spring, that is placed in between the structure and the base that is accelerated.
The spring represents the proportional controller actuator force however only
in the frequency range below the roll-off frequency of the controlled actuator
force. Furthermore, the physical limited actuator force should be taken into
account when the proportional controlled actuator force is represented with
a spring. The vibration amplitude and the frequency are assumed to require
actuator force that remains within the saturation limits of the controlled actu-
ator force [206]. When these aspects are both taken into account, the response
of a system with proportional control can still be determined with the method
presented above. Compared to the infinite controller gain, a reduction of the
controller gain results in other natural frequencies. For simplicity only the low-
est natural frequency with nodes at the actuator location is considered. When
nodes are not forced at the actuator location the number of degrees of freedom
is increased compared to the case where nodes are forced. As result of reduced
controller settings, compared to infinite gain, the original first vibration mode
is split in a mode with lower frequency, as result of a rigid body mode, and a
mode with higher frequency compared to the frequency with nodes at the ac-
tuator location.

In advance of the damping created with these actuators, this aspect is shortly
mentioned here. Even though the natural frequency is reduced the stability,
and the transient response, of the system might be improved when both modes
can profit from damping created with the actuator. In case of a serial sys-
tem both modes profit optimal from the damping realised with the actuator
in case of an inertial match [82, 129, 197]. The inertial match is obtained with a
mass and stiffness distribution such that the motor mass is equal to the payload
mass. In a parallel positioning system the excitation of modes is affected by the actuator location. Whereas the first mode in a serial system is always excited, the amount of excitation in a parallel system depends on the actuator force ratio and the location. This complicates the damping created with actuators in a parallel positioning system. Too little effect of the damping leaves the structure virtually undamped, while too much damping simply locks the structure at the actuator location. In both cases mitigation of the vibration amplitude after a single positioning task should be obtained from other damping elements than the actuators used for positioning.

Even though the limited control bandwidth and actuator force result in a reduced positioning performance at the actuator location, the global performance might increase. A low pass filter is however created for positioning tasks due to hardware limitations, e.g. motor saturation places a limit on the saturation bandwidth \[206, 232\]. The acceleration profile of a second order reference path contains a large number of harmonic frequencies. The lower harmonics contain in general sufficient energy to perform the actual positioning task. Resonant response might develop as result of higher harmonics, however these harmonics are not transferred to the system.

**Positioning a single point of the surface**

In case only a part of beam or plate needs to be positioned, an additional sensor can be used to decrease the effect of (quasi) static errors. With quasi static error is referred to an error with a frequency much smaller than the performance limiting frequency. The quasi static frequency range is far from the frequency range that is regarded as shock loading, e.g. such as defined by Aspinwall \[10\].

In Figure 4.26 a simplified modal model of a parallel positioning system is shown. This model incorporates a rigid body mode and a vibration mode. Initially a control loop over the motor mass is created, thereafter an additional control loop over the position of interest is added.

![Figure 4.26](image)

**Figure 4.26**: Simplified modal representation of a parallel positioning system representing the rigid body mode and a vibration mode. Initially only position \(x_e\) is used to control the individual force. When \(x_m\) requires positioning quasi static errors can be reduced with measurement of \(x_m\) and adapting the force. With quasi static is referred to a frequency much smaller than the natural frequency in case a node is created at the actuator location \(x_e\).

A simplified control structure of the system shown in Figure 4.26 is presented in Figure 4.27. In case accurate positioning of parts of the beam or plate is required while the static error should be excluded, the positioning task itself should be quasi static. In order to ensure only quasi static inputs enter the
Chapter 4. Dynamical aspects of parallel actuation

4.4.4 Damping of vibrations

Vibration damping is considered important in many systems, e.g. precision machines [263], but also stay-cable bridges. Even though the nominal requirements of the positioning system are specified in Chapter 1, in- or external forces might result in vibrations, and as a result deteriorate the performance of the positioning system. Internal perturbation forces might originate from, for instance process forces, but also from the actuators used for positioning, e.g. as result of parameter uncertainty and its bounds. Furthermore the performance of the system might be required to operate in a ‘polluted’ environment, e.g. floor vibrations and pressure variations might be present. Appropriate precautions have to be taken against these disturbance sources in case they affect the performance of the system. The opportunities and limitations to cope with disturbances that introduce vibrations, depends on the designed system, but also on the source of vibrations. For the purpose to indicate different disturbance...
sources and possible solutions to damp vibrations, a simplified system with parallel actuators is presented in Figure 4.28. Only vibrations in x-direction are considered.

**Figure 4.28:** Simplified representation of a flexible system with parallel actuators that might be used to assign different disturbance sources and possible solutions to damp vibrations.

Remaining within the focus on the dynamics of positioning systems, only some disturbance sources that result in vibrations of the system, are briefly considered. In case the system is excited in one or more of its natural frequencies vibrations appear. Vibration damping is a solution to limit the vibration amplitude in case of harmonic excitation in a natural frequency, or to reduce the initial vibration amplitude in time of transient response. In positioning systems the settling time of transient response to impulsive loads can be reduced with damping. In this subsection first some aspects that are in general important in high-precision machines, are mentioned. Thereafter is focussed on the need, and the opportunities and limitations, of vibration damping in parallel positioning systems.

**Isolation for varying external accelerations**
In precision machines the relative vibration amplitude is often of major importance. Besides damping also isolation might be required. In high precision machines accelerations changes might limit the accuracy. The structure with, limited stiffness, is as result of the varying accelerations, subjected to varying
inertia forces. The resulting vibration amplitudes might limit the accuracy. To exclude the effect of external acceleration changes the system might be isolated from them with an appropriate isolation system [58]. Disturbance frequencies above the natural frequency of the isolation system are filtered. In order to profit maximal from the isolation direct connections between the 'polluted' environment and the isolated machine are not preferred. When damping forces are required they are from isolation point of view preferably generated internally, thus without connection to the world. In Figure 4.28 the isolation for varying external acceleration of the base is represented with the two springs $c_b$.

As a result of the isolation the system is split in a clean body and a dirty 'world'. In case disturbances act on the clean body and the resulting vibrations need to be damped, this should be done in such a way that the isolation is not perturbed. In order to profit optimal from the isolation, the damping should be obtained internally, without connection to the 'world'.

**Process and disturbance forces acting on parallel system**

The parallel system shown in Figure 4.28 is not able to represent the complicated problems and solutions for specific machines. Nevertheless, some of these specific aspects can be assigned to the simplified general representation shown in Figure 4.28.

The actuator force of the parallel actuators is represented with $F_a$. Different vibration sources to parallel positioning systems can be assigned to parameter $F_d$ or $F_p$ in Figure 4.28:

- **Static and dynamic process forces that require a closed structural loop**, are represented by parameter $F_d$, e.g. cutting forces while machining. To limit the deformation as result of static process forces, mainly stiffness is required. The flexible system and the control should be designed such that the system is able to cope with the specified range of dynamic process forces.

- **Unbalance forces in rotor systems** can be represented with parameter $F_d$. This is in that case a speed dependent disturbance force for the magnetic bearings. The force from the magnetic bearings might in Figure 4.28 be indicated with the parallel actuators. The control might be such that the amplitude of resonant response is limited while spinning up the rotor speed and passing through natural frequencies [225].

- **Pressure variations in high precision machines** can also be represent with parameter $F_d$. In order to obtain a required (deterministic) thermal behaviour a conditioned airflow might be applied. This airflow might introduces micro vibrations that limit the performance [106, 193, 208]. In case a relative position with respect to an external reference has to be maintained this distance can in Figure 4.28 be indicated with parameter $x_c$.

Via the control of the parallel actuator forces $F_a$ the dynamic properties of the parallel system can be affected, as well as the vibration amplitudes in resonant response. In case the control is designed according to the recommendations
of Schweitzer et al. [225], both actuator forces $F_a$ are controlled with collocated decentralized proportional and differential control. The sensors $m$ in Figure 4.28 are used for this control. Without regards to limited actuator force nor limited bandwidth, can this type of controller be replaced with their mechanical analogons: linear springs and dampers. The dynamic behaviour of the system is now the result of the complex interaction between feedback control of the actuator forces and the flexible system. As a result the response to disturbance and process forces can be affected. For static performance the proportional gain might be set as high as possible. However, for dynamic performance the actuator might also be used to damp vibrations of resonant response. To be able to profit from actuator damping, motion should be allowed at the actuator location. The stiffness created with the controller should allow this motion. Thereafter the optimal damping ratio can be achieved for vibration modes in a limited frequency range. The damper will be effectively too rigid in the high frequency range and too compliant in the low frequency range. In designing for damping, it is necessary to select the frequencies range in which optimal damping is desired, and the damping performance will be suboptimal in other modes. However, when the limited bandwidth of the actuator forces is taken into account the mode shapes at high frequencies correspond to the free mode shapes.

**Need for damping in parallel positioning systems**

Despite external introduced vibrations might be damped with the parallel actuators, in this thesis is focussed on parallel positioning systems. Two reference frames are introduced to identify the positioning task. The position, and its time derivatives, of objects with respect an external defined reference is referred to as absolute. In case the position, and its time derivatives, of an object is defined with respect to a reference frame attached to another object this is referred to as relative. A positioning system might be used to change the relative position to a specified value. Initially only the kinematics are considered; the positioning system should be able to fulfil the kinematic specifications. Thereafter the effect of force and inertia is incorporated in the dynamical study. The positioning task results in absolute acceleration variation, what as a result of the inertia result in loading of the structure with time dependent inertia forces. The loading and the limited stiffness of the structural elements, in this case the product carrier, might result in residual vibrations after the positioning task no longer requires absolute acceleration variations. The relative vibration amplitude, a dynamic error, depends on the required absolute acceleration variation and thus the operation speed. In high precision machines this dynamic error might limit the operation speed and forms a dynamic performance barrier [222]. In the rest of the text the position is defined as a relative measure.

The vibration amplitude after a single positioning task, force- or acceleration profile consisting of a single harmonic, can be determined with Equations 4.19 and 4.28. Using the envelope of the error, as presented in Figure 4.22, the error is independent on local minima. The error is however determined for the two extreme cases, open loop operation and closed loop operation that results in nodes at the actuator location. The transient response might profit from
damping created in the actuator. The ability to damp vibrations depends on the actuator location with respect to the elastic modes. The excitation of these modes, and thus the vibration that needs to be damped, to perform a positioning task however also depends on the actuator location. In a serial positioning systems, as for instance discussed in [82, 129, 162, 197], the actuator excites the elastic mode, but might as well be used to damp energy from the elastic mode. In a parallel positioning system it is possible not to excite modes during positioning. As a result damping of these modes is not required after the positioning task has ended. The damping capabilities of the parallel system for other disturbances might also be limited.

To guarantee an upper performance limit in the design phase of a parallel positioning system, independent on the complex control structure, or the unknown actuator mass the extreme case where nodes are forced at the actuator location is considered. As a result the parallel actuators used for positioning are not able to damp vibrations. When damping is required it should thus be obtained in other ways. In the initial design phase the natural frequencies of the positioning system are unknown. In case subjected to disturbance sources with a narrow frequency band the system should be designed such that it is not susceptible to them. When it is not possible to shift the natural frequencies out of the disturbance frequency range, damping is required to limit the amplitude of resonance. In resonant response the energy of the system builds up cycle by cycle, until an equilibrium is obtained between the added- and dissipated energy during each cycle. The required amount of damping depends on the energy in the disturbance frequency, and the allowed vibration amplitude. Even though the structural damping parameters might depend on the amplitude of vibration, as this grows to large values in resonant response, the system behaviour is in this thesis still described from a departure that assumes linearity of the components [78].

**Methods to improve damping in parallel positioning systems**

The damping present in materials is often modelled with a hysteretic model. The loss factors for steel and brass plates are: $(\eta < 10^{-3})$ [17, 53, 260]. The amplification at resonance $(Q)$, also known as the storage coefficient, is the reciprocal of the loss factor $(Q = 1/\eta)$. In practice the loss factor of materials also depend on amplitude and frequency, see e.g. [50]. The damping in the product carrier can be improved by:

1. addition of layer(s) of viscoelastic material [87];
2. addition of piezo's on the surface [42, 51, 52, 145];
3. use of positioning actuators that are able to generate torque [148];
4. additional parallel actuator.

Ad 1.) The addition of an unconstrained layer of viscoelastic material is simple in implementation economic and can be incorporated at either the design phase itself or as a corrective measure [213, 247]. In precision machines this material can best be used to create composite structures, in order to maintain
sufficient rigidity and prevent long term creep problems [83, 275]. The damping achieved by an unconstrained layer remains however relatively small. As rule of thumb: to be effective, the mass of the damping layer has to be greater than 20% of the structural mass [175]. Constrained damping layers can be used instead of unconstrained layer dampers when significant weight increase is unacceptable. As rule of thumb, the mass of a constrained layer required to provide the same amount of damping as an unconstrained layer is 10% or less of the structural mass. With properly matched constrained layer dampers, loss factors of $\eta = 0.4$ can be obtained, [237, 260].

Ad 2.) Piezo's added on the surface damp the structure in a comparable way as an unconstrained damping layer. They are located at positions such that when the system vibrates, strain is introduced in the piezo. The strain rate of the piezo can be used to damp energy. The piezo add in general only a relative small amount of mass.

Ad 3.) When the parallel actuators used for positioning, are replaced with the actuators designed by Makarović [148] torque can be generated. This torque can be used to dissipate energy. The excitation of modes during a positioning task can be reduced as well. The efficiency of the torque during a positioning task depends on the location of the parallel actuator.

Ad 4.) An additional parallel actuator can be used to damp energy. Instead of a physical damper, an actuator that exerts force in opposite phase with velocity is used, because the main function of the system is positioning. Optimal damping can be achieved for vibration modes in a limited frequency range. Besides for damping, this actuator can also be used to prevent excitation of modes during positioning. This aspect will however be discussed in the next chapter.

Tailored dampers, such as damped auxiliary mass dampers, might be used to dissipate energy, but are not considered practical for application in the positioning systems discussed in this thesis. In such damping systems, damping is created due to the relative velocity with an apparent fixed point in space [93, 133]. Despite these dampers might be useful to limit steady state vibration amplitudes, they are considered not applicable in positioning system, because the damping force opposes the required rigid body motion as well.

4.4.5 Sensitivity of the dynamic behaviour for actuator location variations

The excitation of modes depends on the actuator location. In order to avoid excitation of the lowest numbered modes during positioning, the actuators are located at defined locations, with a specified force ratio between the actuator forces. First the effects of the system without mass of the actuators and feedback compensator is discussed, thereafter the situation with feedback compensator and the actuator location forms the nodes. The eigenvectors change only gradually as a result of gradual changes in stiffness- and mass distribution [86]. Therefore the calculated eigenvectors can be used as approximation for the eigenvectors of the slightly changed actuator location.

Mode shapes not affected by actuator mass and control

For positioning purposes the actuators are located in the nodes of a free vi-
The rate of change is determined when the actuator location is slightly changed. The ratio between the forces to prevent excitation of the lower modes also changes, this is however small. The changed excitation of the mode shape due to a small error depends on the change of the mode shape. The mode shape change is approximated with a first order Taylor series. The effect is present for a change in one direction, $\Delta y$ in y-direction, the same method hold for the z-direction. This is however, not presented. For small perturbations $\Delta y$ the use of the first derivative is sufficiently accurate. The relative rate of change is:

$$\Psi_i = \frac{\Delta(\Phi_i^T \hat{F})}{\Phi_i^T \hat{F}} = \frac{\Delta y \cdot \partial \Phi_i \hat{F}}{\Phi_i^T \hat{F}}$$  \hspace{1cm} (4.30)$$

The relative rate of change depends on the derivative. When the actuators are located in the nodes the value of the mode shape is zero. The rate of change is as a result infinite. However, aimed at an actuator location in the nodes, the effect of misalignment should be taken in account in the design phase.

**Sensitivity of the dynamic behaviour of the beam positioning system**

The derivative of the mass normalized modes of the beam are plot in Figure 4.29. The relative rate of change ($\Psi_i$) for the beam is plotted in Figure 4.30. As example mode three of the beam is considered and the actuators are located at the side. When the left actuator is considered to posses an alignment error of $\Delta y = 10$ mm. The relative derivative at the left side is $\frac{d(\Phi_i^T \hat{F})}{\Phi_i^T \hat{F} dy} = -9.3$, Figure 4.30. The relative change in mode excitation is $\Psi_3 = -9.3 \cdot 10^{-3} = -0.093$. The excitation of mode three is thus reduced with 9%. This can be verified with the mode shape values. In the old situation the mode shape value is $\Phi_3(y = 0) = 5.05$ in the new situation the mode shape value is $\Phi_3(y = 0) = 4.58$. This is also a change of 9.3%.

The beam has a mode shape value of zero at 22.4% from the sides (112 mm).
The tolerance on the actuator location is assumed to be \( \pm 8 \) mm. The amplitude of the modes is determined. The mode shape value of mode three is \( \Phi_3(104 \text{ mm}) = 0.33 \) and \( \Phi_3(120 \text{ mm}) = -0.31 \) at the maximum tolerance of the actuator location. Mode three is maximal excited when the actuators are located at the maximum of the tolerance band and at equal phase of the mode. For mode three this is the case when the actuators are placed symmetric, e.g. both at 104 mm or both at 120 mm from the sides. As result of symmetry the force ratio between the actuators, in order to prevent excitation of mode two, remains constant. The amplitude to a step force is calculated before on page 109. Mode four is not excited, while mode five is. When the actuators are located in the range \( 112 \pm 8 \text{ mm} \) the value of mode five is in the range \( \Phi_5 = -3.24 \) to \( \Phi_5 = -3.34 \). The amplitude of mode five at the side is in that range maximal \( x_5(0 \cdot L) = 5.0 \times -3.34/1445^2 = 8.0 \times 10^{-6} \text{ m} \). The amplitude of higher modes is smaller.

**Sensitivity of the dynamic behaviour when nodes are forced at the actuator location**

The actuators are located in the nodes of a free vibration mode. In practice a tolerance is specified where the actuators are located. An upper limit of the vibration amplitude is obtained when assumed that due to mass and control the actuator location will form the node of the vibration mode. When not exactly located in the nodes of the free vibration mode the natural frequency and mode shape of the lowest mode will change only a little bit. The excitation of the modes might however changes much more.

The free mode shapes determine the preferred dynamic behaviour of the system. The kinetic and potential energy are exchanged within the system, without exerting forces on other systems. The exchange between potential and kinetic energy is not affected by the magnitude of the acceleration field. Acceleration field changes might however affect the dynamic behaviour, because it affects the potential energy stored in the system. In the nodes of the free vibration mode, acceleration field changes do not excite that particular mode. In case the
actuator location forces nodes at other locations than the preferred location of the free system, the mode is excited during acceleration field changes. In Figure 4.31 the actuators are located in the nodes of the first elastic free mode. When the control and mass of the actuators forces nodes at the actuator location, the first mode does not change. Base acceleration does also not excite that mode. When another actuator location is used, the actuators are no longer located in the nodes of the free system. During base acceleration the lowest mode is then excited.

**Figure 4.31**: Mode shapes when the actuators are located at 112 mm from the sides and form the nodes. The frequencies are $f_1 = 42.5 \, Hz$, $f_2 = 80 \, Hz$, $f_3 = 137 \, Hz$, $f_4 = 312 \, Hz$, $f_5 = 567 \, Hz$. The mode shape and frequency of the first mode is equal to the first elastic mode of the beam presented in Figure 4.10

**Actuator sensitivity for the beam positioning system**

In case the actuators are located at 112 mm from the sides, they are located in the nodes of the first elastic mode. When the vibration modes and natural frequencies of this system are determined when the actuator location forms the nodes, the first mode is not excited. In Figure 4.32 the vibration amplitudes are presented when a step in acceleration is applied of $\ddot{x} = 1 \, m/s^2$. The step acceleration profile corresponds in this homogeneous beam case with a pressure load of $p = \rho \cdot h \cdot \ddot{x} = 15.7 \, N/m^2$, or a distributed load of $w = b \cdot p = 0.314 \, N/m$. Mode 1, an elastic mode with a natural frequency of 42.5 Hz, is excited however the nett excitation is zero. Mode three has the main contribution in the vibration amplitude. The steady state shape in Figure 4.32 represent the summation of the modes one to five. The steady state deformation at the side is approximately equal to the deflection in a static acceleration field as determined from Figure 2.19 and the first part in Equation 2.25.

The tolerance on the actuator location is again assumed to be 8 mm. The vibration amplitude is calculated when both actuators are located 8 mm to the outside of the nodes of the free mode (at 104 mm). Due to actuator mass and control the node is formed at the actuator location. The natural frequency of the first elastic mode is (42.5 Hz). The values of the first mode shape are
4.4. Practical aspects and the dynamic behaviour

also slightly changed. The nett vibration amplitude of this mode is however changed. Due to the small generalized stiffness of this mode, a small excitation results immediately in a relative large contribution of the vibration amplitude. The vibration amplitudes of the modes are presented in Figure 4.33 when a step force of 1 N is applied. Due to excitation of mode 1, the total vibration amplitude changes much. The contribution of mode two and higher remains of the same order. In the same way the excitation of modes can be determined when both actuators are located at 120 mm. Or in case one actuator is located at 104 mm from the side, while the other is located at 120 mm from the side. In many systems stiffness and mass are often distributed such that nodes are located at the actuator location. It is in that case not useful to determine sensitivity of other actuator locations, because the system is not designed for that purpose. In case mass and stiffness are not distributed such that nodes are formed at the actuator location, such as with beam and plates, the actuators can be attached to the structure at other locations than the nodes. Even when misalignment the system has to fulfil the requirements. This aspect should be incorporated in the design phase in order to obtain a design that is robust for the specified tolerances. More stiffness and thus mass is required to meet the specifications when the actuators are misaligned. When the actuators are mis-

**Figure 4.32:** Vibration amplitude when the actuators are located at 112 mm from the sides and a base acceleration of $\ddot{x} = 1 \text{ m/s}^2$ is applied (which corresponds to a distributed load of $p = \rho \cdot h \cdot \ddot{x} = 15.7 \text{ N/m}^2$). The actuator location forms the nodes.

**Figure 4.33:** Vibration amplitude when the actuators are located at 104 mm from the sides and exert a step acceleration of $\ddot{x} = 1 \text{ m/s}^2$. The actuator location forms the nodes.
aligned improvements might be possible after the system is built. The applied force to position and the control of the actuators can be adapted to the specific case. Another option is affecting the location of the nodes and the excitation of modes during positioning with hardware, for instance by the addition of mass at certain locations. The sensitivity of the natural frequency on the location of two rigid supports is also mentioned in [241].

4.4.6 Optimization for positioning or disturbance reduction

In serial systems the lowest mode is always excited, and a compromise between performance and stability might be obtained [46, 82, 129, 197]. The remaining vibration amplitude depends on the interaction between the force profile and the system dynamics. In the parallel system the excitation of the modes can be affected by the actuator location. In case the positioning task is assumed most important the actuators are best located in the nodes of consecutive free vibration modes. Despite the system might be operated open loop, closed loop operation is also considered in the initial design phase. In order to guarantee an upper bound of the performance in the initial design phase the performance is determined in case nodes are forced at the actuator location. The positioning error of the two extreme case, open loop operation and tight control operation, can be determined with Equations 4.23 and 4.29. Improved control after realization might result in a performance increase. This is considered as additional performance improvement, the main aim is to guarantee a certain performance in the initial design phase.

The opportunity to damp vibrations with the parallel actuators is completely excluded when the design method with the two proposed operation modes is applied. This apparent disadvantage is however closely related to the requirements of the positioning system. Positioning is in this thesis regarded as the main task. For minimum excitation of the lowest modes, at maximum speed, the actuators are best located in the nodes of the free vibration modes. The excitation of the higher modes is then a given fact that can not be changed. When movement of the actuator location is possible for these higher modes, they might be damped with the actuators also used for positioning. The opportunity to realize damping with the actuators used for positioning should not be excluded, despite Franklin et al. [74] mention: "It may be much easier to meet the specifications by altering the structure of the plant through the addition of stiffening members or by passive damping than to meet them by control strategies alone." The conceivably realized damping with the actuators used for positioning, should however not result in a deteriorated positioning performance. Even though another actuator location seems better to damp certain vibration modes, the actuator location should not be shifted immediately. When shifting the position the lower modes are excited during positioning and this will destroy the positioning performance significantly. The damping should then be focussed on the lower mode, because that will contribute much more in the total vibration amplitude. Applied to the beam system this means the actuators are for positioning purpose best located in the nodes of the third mode, the first elastic mode. When a positioning task is performed, the fifth free mode,
the next relevant elastic mode, is excited. As long as the actuators do not form a
node for that mode it can be damped. Despite another actuator location might
be better to damp mode five, mode three will be excited during positioning at
other actuator locations. Mode three contributes in general much more to the
vibration amplitude than mode five. When the actuators are not located in the
nodes of mode three the damping should concentrate on damping of mode
three instead of mode five.

In case the parallel actuators are located and controlled in such a way that cer-
tain modes or a specific frequency range are optimally damped, the damping
performance of other modes will be limited. The damping will be effectively
too rigid in the high frequency range and too compliant in the low frequency
range. When the damping is created from controlled actuator force, the limited
bandwidth and actuator force should also be incorporated. In case of a specific
application, the system should be able to cope with the specified disturbance
frequency range. Such system can be created in many ways. In case a re-
nance amplitude need to be damped this does not necessarily require damp-
ing from the parallel actuators. After the system is designed for the intended
purpose, the susceptible disturbance frequencies can also be identified. This
information can of course also be used to show the limited robustness of the
system. However, when made robust for that particular frequency range other
susceptible frequencies can be identified. In this thesis is focussed on the po-
positioning performance of parallel positioning systems.

4.4.7 Closed loop operation of the plate system

The methods and practical aspects mentioned above were mainly applied to
the beam system. They can be applied to the plate system as well. This is how-
ever not performed in large detail. The development of the analysis techniques
and obtaining insight, as obtained from the beam system, is considered more
important than the application of the analysis techniques to arbitrary systems.
Some annotations about the plate system are given.

As was shown in the beam example the support stiffness affects the mode
shapes when the actuators are not located in the nodes of the vibration modes.
Modes that are not affected by the actuator stiffness can also not be damped
with the actuators. Some modes are not influenced due to the actuator lo-
cation. Modes with high frequency are not affected due to the roll-off of the
actuator force, \[225\]. As result of the support stiffness the first mode is a com-

bination of one of the three rigid body modes with one of the modes 7 and 8
and mode 6. In this case mode 7 is assumed, that means that mode 8 is not
affected by the actuators stiffness (and damping).
The dynamic behaviour of a system in interaction with stiffness and damping
of the supports is the result of the complex interaction between supports and
the elastic structure \[225\]. The change in natural frequency of the beam shows
strong resemblance with rotor dynamics where a flexible shaft is in interaction
with the bearings. In a similar manner the dynamic behaviour of a circular
plate supported at three equally spaced points along the circumference of a
concentric circle is discussed by [112].

### 4.5 Conclusion

A minimum mass parallel positioning system is obtained when the actuators are located in the nodes of consecutive free vibration modes. The force ratio and the location of the actuators is such that during the application of force, required to realize the rigid body motion, the lowest numbered consecutive modes are not excited. The envelope of the vibration amplitude after a single positioning task is as a result minimal. In operation, feedback control might be applied in order to obtain, for instance robustness, stability, etc.. The dynamic behaviour is then the result of the complex interaction between the feedback control of the actuator forces and the elastic mechanical system. In order to guarantee an upper limit of the positioning performance in the initial design phase in case feedback control is applied, the performance is determined with the actuator locations as nodes of vibration modes. This corresponds to the application of relatively stiff supports compared to the structural stiffness, or to a fully locked damper at the actuator location. An apparent disadvantage of this method is that the parallel actuators can not be used to damp vibrations. When damping is required it should be obtained in other ways. Even though damping created with parallel actuators might be regarded as a stringent requirement in order to damp a large range of disturbance frequencies, its effect is limited. In case a disturbance frequency range is identified the created parallel positioning system should not be sensitive for it. The error envelope after a single positioning task, with the actuators as nodes of the vibration modes, is determined in case the acceleration of the reference profile contains only the fundamental frequency. Due to the application of this profile the required actuator force remains limited, even though a step in a jerk is required. This design method can be used to guarantee an upper limit of the dynamic performance of parallel positioning systems in the initial design phase. The obtained information can be used to motivate conceptual design choices, with a limited amount of information from detailed design solutions.
Overactuated positioning systems

5.1 Introduction

Parallel redundant systems are in this chapter discussed. In the previous chapter the dynamic aspects of parallel actuated system were discussed. The number of actuators was equal to the number of rigid body modes. When an additional parallel actuator is used, a parallel redundant system is created. The required actuator force ratio can then no longer be determined from the rigid body modes alone. The minimum number of sensors required for closed loop operation of decentralized control, increases with the addition of actuators. Each decentralized control loop is assumed to contain at least an individual position sensor. The position sensors are furthermore assumed to measure at different locations. The information of the minimum number of sensors can be used to discriminate between rigid body displacement and static elastic deformation.

The parallel redundant system in combination with the method to determine the required actuator force results in an overactuated positioning system. In the previous chapter the number of parallel actuators was equal to the number of rigid body modes. Equilibrium equations alone were sufficient to determine the force ratio for open loop operation. The addition of actuators as will be discussed in this chapter requires the addition of equations, for instance, symmetry, compatibility, and constitutive equations [72, 273]. In the previous chapter the information of the minimum number of sensors was not sufficient to discriminate between rigid body displacement and static elastic deformation. In closed loop operation of the parallel redundant system additional degree(s) of freedom appear to be present, which are known as internal degree(s) of freedom. The assumption of rigidity, or the application of equilibrium equations alone, results in an indeterminate system.

The methods presented in the previous chapter to evaluate different aspects of parallel positioning systems such as: positioning performance, damping of vibration modes, sensitivity of the actuator location on the performance, and the
sensitivity for disturbance frequencies are also applicable to parallel redundant systems. The theory in the previous chapter has been described such that it is easy extendable to parallel redundant systems as mentioned in this chapter. Similar to the aspects in the previous chapter, the effect of the actuator location on the dynamic behaviour is determined. The actuator force remains such that consecutive free vibration modes are not excited, see equation 4.11. However, in this chapter the force ratio is such that besides the rigid body modes also elastic modes are not excited. The same beam and plate, as used in the previous chapter, are used to show the effect of overactuation. The effect of the additional actuators on the dynamic behaviour is discussed, while the properties of the product carrier, beam and plate, remain unchanged. Most aspects are however not discussed in large detail, because the aspects of parallel positioning system are already explained in the previous chapter.

5.2 Overactuated beam and plate positioning system

The effect on different aspects are discussed in the same way as in Chapter 4 thus first massless actuators, symmetry etc. Thereafter all aspects as mentioned in the previous chapter can be discussed for every system, such as the effect of control, actuator and guidance mass, actuator location sensitivity, etc. Despite in the previous chapter analysis methods are presented that can be used to study the dynamic effects in large detail, the amount of detail in the analysis is in the specific design solutions limited. Development of analyse techniques and revealing trends is considered more important, than application of these techniques to specific problems. The aim of this thesis is to determine mass changes due to overactuation and therefore the amount of details about specific design solutions will be limited. If required the design solutions might be analyzed in much more detail using the techniques mentioned in the previous chapter.

The number of possible systems and number of possible actuator locations is large. As mentioned in Chapter 1 about the strategy of simulations only a few typical cases will be analyzed.

5.2.1 Overactuated beam positioning system

The actuator location is initially selected such that a symmetric system is obtained. Later on the effect of an asymmetrical system will be discussed. Symmetry means in this case with three actuators that one actuator is always located at the centre of the beam. The two other actuators are located symmetric with respect to the centre and exert a mutual equal force, that might differ from the force of the central actuator.

To obtain the required acceleration the summation of the elements in the force vector is 1, see Equation 4.10. The force \( \hat{F} \) is such that consecutive modes are not excited \( (\Phi_2^T \hat{F} = \Phi_3^T \hat{F} = 0) \), see Equation 4.11. Due to the assumed symmetry mode two is not excited during acceleration, \( (\Phi_2^T \hat{F} = 0) \). The force can be calculated from the requirement that mode three is not excited \( (\Phi_3^T \hat{F} = 0) \).
Because three forces are applied, the excitation of mode three is calculated via the following formula:

\[
\Phi_3^T \hat{F} = [\Phi_{3, \text{left act}} \Phi_{3, \text{centre act}} \Phi_{3, \text{right act}}] \begin{bmatrix} F_L \\ F_C \\ F_R \end{bmatrix} = 0 \quad (5.1)
\]

In Equation 5.1 the parameter \( \Phi_{3, \text{left act}} \) is used to indicate the mode shape value of mode three at location of the left actuator. Because massless actuators are assumed and the same beam as in the previous chapter is used the mode shape values can be obtained from Figure 4.10. \( F_L \) is the actual force of the left actuator, see Figure 5.2. The same analogy holds for the other parameters. The aim of the equation is to determine the required actuator force in order not to excite mode three.

The dynamic response at any arbitrary location \((x(y))\) on the beam is described with Equation 4.3. Excitation of the fourth and higher numbered modes should be considered, dependent on the actuator location. The force ratio is such that mode two and three are not excited. As result of symmetry mode four is not excited, as well as all higher even numbered free vibration modes. The dynamic behaviour of mode five, and consecutive odd numbered modes, is therefore considered. The error after a single positioning task can be determined with Equation 4.23.

**Two actuators at the outsides**

The outer actuators are attached at the sides, while one actuator is located in the centre, see Figure 5.1. The forces between the central actuator and the side actuators is calculated with Equation 5.1. The mode shape values are obtained from Figure 4.10, the actuators are still assumed massless. At the actuator locations at the side, the value of mode three is \( \Phi_3(y = 0) = 5.0 \). The mode shape value of mode three at the centre is \( \Phi_3(y = L/2) = -3.1 \). When these values are filled in in Equation 5.1 and symmetry is taken in account the following ratio is
the excitation of mode seven is 

\[ \Phi_7^T \hat{F} = 0 \Rightarrow 3.1 \cdot F_C = 5.0 \cdot F_L + 5.0 \cdot F_R = 10.0 \cdot F_R \Rightarrow \frac{F_C}{F_R} = \frac{10.0}{3.1} \quad (5.2) \]

The force of the central actuator and the other two actuators is known. Combined with the magnitude requirements of Equation 4.10 the force vector is:

\[ \hat{F}^T = [F_L \ F_C \ F_R] = [0.19 \ 0.62 \ 0.19] \quad (5.3) \]

In case a step force profile is applied \((F(t) = F \cdot \epsilon(t))\) by the actuators the modes are excited. Due to the impulse of the force the momentum of the system increases. Besides a rigid body mode displacement also the other modes are excited. However, as a result of the force ratio \((\hat{F})\) mode three, with 42.5 Hz is not present in this ideal case, as well as all even numbered modes. The vibration amplitude is therefore dominated by the fifth free mode that vibrates with 230 Hz in a mode shape as shown in Figure 5.2. The excitation of mode five depends on the mode shape values at the actuator location. For the actuator at the central location this is \(\Phi_5(y = L/2) = 3.6\). At the other actuator locations, the value is \(\Phi_5(y = 0) = 5.0\). With the force vector as presented in Equation 5.3 the excitation of mode five is \(\Phi_5^T \hat{F} = 4.1\).

The dynamic behaviour depends also on the location of interest, see Equation 4.3. The location of minimum and maximum values of mode shape five are considered because they represent two extremes of the dynamic behaviour. The mode shape values are obtained from Figure 4.10. Mode five has a maximum at the side, \(\Phi_5(y = 0) = 5.0\), and a minimum at \(110 \ mm\), \(\Phi_5(y = 110 \ mm) = -3.3\). Also mode seven is taken into account this has a maximum at the side, \(\Phi_7(y = 0) = 5.0\) and a minimum at \(y = 70 \ mm\) from the sides and is \(\Phi_7(y = 70 \ mm) = -3.3\). The mode shape value of mode five at \(y = 70 \ mm\) is \(\Phi_5(y = 70 \ mm) = -1.9\), while at the minimum of mode five, \(y = 110 \ mm\) the mode shape value of mode seven is \(\Phi_7(y = 110 \ mm) = -0.4\). For completeness also the dynamic behaviour of the centre is considered \((\Phi_7(y = L/2) = -3.6)\). With the forces as presented in Equation 5.3 the excitation of mode seven is \(\Phi_7^T \hat{F} = -0.3\).

The dynamic behaviour of the modes to a step force is presented in Equation 4.5. When a step force \(F(t) = 1 \ N\) is applied the vibration amplitude of mode five and seven at the sides is calculated with Equation 4.5:

\[ x_5(0) = \frac{\Phi_5(0) \Phi_5^T \hat{F} F}{m_5 \omega_5^2} = 1.0 \times 10^{-5} \ m \quad \& \quad x_7(0) = \frac{\Phi_7(0) \Phi_7^T \hat{F} F}{m_7 \omega_7^2} = -1.2 \times 10^{-7} \ m \quad (5.4) \]

The dynamic response to a sinusoidal force, Equation 4.6, is presented in Figure 5.3. At forcing frequencies much smaller than the natural frequency of mode seven the total vibration amplitude is dominated by node five. In case three actuators are used, as shown Figure 5.1, the vibration amplitude of mode five at the side is: \(x_5(0) = 0.1 \times 10^{-4} \ m\). In case two actuators are used, as shown in Figure 4.11, mode three was excited what resulted in a vibration amplitude at the sides of: \(x_3(0) = 3.6 \times 10^{-4} \ m\).
As mentioned in this ideal case the even numbered modes are not present due to symmetry. Mode three is not present due to the applied force ratio ($\hat{F}$). The first mode that contributes in the vibration amplitude is therefore mode five, in this case with a frequency of 230 Hz, see Table 4.1. The low frequent behaviour is dominated by the rigid body motion. The modulus of $x/F$ is determined, the magnitude of the force is thus $F = 1$ N. At low frequencies the rigid body amplitude in Figure 5.3 is equal to the amplitude in Figure 4.13. The actuators posses no mass thus the low frequent amplitude should be equal.

The dynamic behaviour of this beam case with three actuators can be compared with the beam systems where two actuators are used, as discussed in the previous chapter. The frequency responses of Figure 5.3 is compared to the frequency response in Figure 4.13, 4.15 and 4.18. Mode three with a frequency of 42.5 Hz is in Figure 5.3 not excited while it is excited in case two actuators are used. In Figure 4.15 the two actuators were located near the nodes of the third mode. Despite the actuators were preferably located in the nodes of the third mode, location errors should be taken into account. As result of actuator location errors, the possible excitation of mode three should be taken into account when two actuators are used. In case of two actuators, the force ratio was tuned such that mode two was not excited when forces are applied for positioning ($\Phi^T_2 \hat{F} = 0$). The excitation of mode three was, in that case, minimized by locating the two actuators in the nodes of the third mode. In Figure 5.3 three actuators are used to exert force on the structure. The force ratio $\hat{F}$ is tuned for mode two and mode three; and the actuator location is such that higher numbered modes are minimal excited during positioning. In this case deviations in the realized actuator location do not lead to excitation of mode three, because this mode is prevented from excitation during positioning with an adapted force ratio such that $\Phi^T_3 \hat{F} = 0$. The accuracy of the applied force ratio ($\hat{F}$) defines the excitation of mode three. As mentioned in the comments
about Equation 4.11 the force ratio is assumed to be determined exact according to the requirements. Mode three is therefore not present in Figure 5.3, despite possible errors in actuator location.

Despite not present in the frequency response of Figure 5.3 mode four is the lowest frequency that is excited when errors in the actuator location are present. The frequency of mode four is 117 Hz, see Table 4.1. As result of symmetry mode four is not excited. When the symmetry is lost, the force ratio remains such that mode two and three are not excited. Due to the changed force ratio mode four is excited. When it is only slightly excited a resonance peak appears at the natural frequency of mode four. Dependent on the amount of excitation of the vibration modes and the location of interest, the frequency of the anti-resonance is affected. In case mode four is only slightly excited, the anti-resonance frequencies will appear near the resonance frequency of the fourth mode.

As result of the application of three actuators the lowest anti-resonance and resonance frequency are increased, compared to the case with two actuators. Because mode three is not excited, its amplitude does not contribute to the dynamic error. The accuracy of a single positioning task within the same motion time can be increased. When the tolerance specification is maintained the motion time can be decreased. Another option is to reduce mass because less structural stiffness is required. This will be treated in the next chapter.

Outer actuators moved inwards

The actuators are now attached in the nodes of the fourth mode, see Figure 4.10. This means in this case the two actuators are attached at 66 mm from both sides, while one actuator remains at the centre, see Figure 5.4. The

forces of the central actuator and the side actuators are now calculated. At the actuator locations at the side the mode shape value is $\Phi_3(y = 66 \text{ mm}) = 2.0$, Figure 4.10. The mode shape value of mode three at the centre is still $\Phi_3(L/2) = -3.1$. The mode shape values do not change as result of the actuator location or the number of actuators because the actuators are assumed the posses no mass. In the same way as before, Equation 5.1 is used and symmetry is taken in account what results in a force ratio $F_C/F_R = 4.0/3.1$. Combined with the

![Figure 5.4: Actuators attached in the nodes of the fourth mode, see Figure 4.10. One actuator is attached at the centre the other two are attached at $y = 66 \text{ mm}$.](image)

![Figure 5.5: Vibration of mode five when a step force is applied. The forces of the central actuator and those at the sides is fixed.](image)
requirements of scaling, Equation 4.10. The scaled force vector to prevent excitation of mode three corresponding to this actuator location is:

\[
\hat{F}^T = [F_L \ F_C \ F_R] = [0.30 \ 0.40 \ 0.30]
\] (5.5)

In case a step force is applied by the actuators, the modes are excited. Due to the impulse of the force the momentum of the system increases. Besides a rigid body mode displacement also the other modes are excited. Only the lowest numbered excited elastic mode is considered, in this case mode five. This mode starts to vibrate with 230 Hz in a shape as shown in Figure 5.5. Mode three, with 42.5 Hz is not present. The mode shape value of mode five at the central location is as mentioned \(\Phi_5(L/2) = 3.6\). The mode shape value of mode five at the outer actuator locations is \(\Phi_5(y = 66 \text{ mm}) = -1.6\). The excitation of mode five is \(\Phi_5^T \hat{F} = 0.4\). Mode seven at the central location \(\Phi_7(L/2) = -3.6\) at the actuator location \(\Phi_7(y = 66 \text{ mm}) = -3.3\) The excitation of mode seven is \(\Phi_7^T \hat{F} = -3.4\).

The vibration amplitude at the side when a step force \(F(t) = 1 \text{ N}\) is applied, Figure 5.5, is calculated as in Equation 4.5 and is \(x_5(0) = 1.2 \times 10^{-6} \text{ m}\). The vibration amplitude at the side of the seventh mode is \(x_7(0) = -1.4 \times 10^{-6} \text{ m}\). Mode seven thus contributes almost equal in the total vibration amplitude as mode five. The frequency response of this configuration and fixed force ratio (\(\hat{F}\)), see Equation 4.6, is presented in Figure 5.6. The locations of interest remain the same as in the previous case with the actuators at the outside. The minima and maxima of mode shape five and seven and the centre. For the chosen actuator location and the calculated forces of the actuators the bode plot is presented in Figure 5.6.

![Figure 5.6: Modulus and argument of \(x/F\) at several locations. Two actuators are located at 66 mm from each side, one actuator is located in the centre. In the legend the location of interest is indicated with the amount of mm from the sides.](image)

The frequency response with two actuators at the outside and one in the cen-
tre, as presented in Figure 5.3 is compared with the frequency response in Figure 5.6. Due to minimized excitation of mode five, the lowest anti-resonance in Figure 5.3 is shifted towards the resonance frequency of the fifth mode, see Figure 5.6. As result of minimum excitation of the fifth mode, the accuracy of a single positioning task within the same motion time can be increased. When the tolerance specification is maintained the motion time can be decreased. Another option is to reduce mass because less structural stiffness is required. This will be treated in the next chapter.

A physical explanation of the anti-resonance near \(331\) Hz in Figure 5.6 is obtained when the vibration amplitudes and phase of mode one, five and seven are studied in more detail. The phase and the vibration amplitude are at \(331\) Hz, such that no nett amplitude is present, see Equation 4.6. The phase and the vibration amplitudes at other locations, might be such that no anti-resonance is present at the considered frequency range.

In case the actuators are not located at the preferred position, the force ratio \(\hat{F}\) should be adapted. When symmetry is lost, mode four is also exited during positioning. In that case this mode will show up in the frequency response as additional resonance frequency at \(117\) Hz. The additional anti-resonance frequency depends on the amount of excitation of mode four and the location of interest. The amplitude in the mentioned resonance and anti-resonance frequencies depends, besides on the amount of excitation of mode four, on the amount of internal damping. Even though not present in the frequency response shown in Figure 5.6, the anti-resonance / resonance as result of mode four forms a frequency limitation for successive positioning tasks.

**Other actuator locations**

In the examples above two possible actuator locations were discussed. In the last example the actuators were located in the nodes of the fourth mode, however due to symmetry that mode would not be excited during accelerating the system. When located in the nodes of the fourth mode the excitation of mode five is \((\Phi_5^T\hat{F} = 0.4)\). This excitation can be minimized by another symmetric actuator location. Mode two and mode four are as result of symmetry still not excited. Due to the actuator in the centre the free vibration mode five, Figure 4.33, is always excited. The excitation of mode five can be reduced by locating the outer actuators more to the centre than the nodes of mode four. The magnitude of mode five increases and the contribution of the central actuator is therefore reduced, what results in reduced excitation of mode five. The force ratio \(\hat{F}\) should however remain such that mode three is not excited, see comments about Equation 4.11.

When the actuators are all located within the nodes of the third elastic mode and the requirements of positioning and meanwhile no excitation of mode three has to be fulfilled, some of the forces in \(\hat{F}\) become negative. Because negative actuator forces are present the magnitude of the actuator forces increases in order to fulfil the requirement of Equation 4.10. Increased actuator force results in actuators that contain more mass and are therefore not wanted. The location of the actuators is therefore chosen such that the forces in vector \(\hat{F}\) are all positive. In this beam case this means nodes of mode three have to be
present between the actuator location.
The equations used to calculate the actuator forces, Equation 4.10 and 4.11, are also valid when an asymmetric actuator location is chosen. An asymmetrical actuator location results however in excitation of the even numbered free vibration modes. To prevent excitation of mode two and three the first mode that contributes in the vibration amplitude is mode four. The vibration amplitude increases and the frequency of mode four forms a speed limitation for consecutive positioning tasks. Another force ratio that results in preventing mode four during acceleration, implies excitation of mode two or three or both. Thus in that way the performance of the positioning system becomes worse. This is in analogy with the comments mentioned in section 4.3.1.

**Other aspects of the positioning system**
The aspects mentioned in section 4.4 are also applicable in the overactuated case. The control structure remains decentralized, thus each actuator has its own sensor and controller structure. With the three controlled forces the location of the nodes, and thus the system dynamics can be affected. Beside the control of the actuators does affect the location of the nodes the node location is also affected by the actuator mass. The assumption of tight control for the actuator location results in nodes of the vibration modes at the actuator location. The actuator mass has the same effect. As explained in the previous chapter a performance indicator is the vibration amplitude when a ramped sine is used as reference path and the actuator location form the nodes. The vibration amplitude of the modes is calculated with Equation 4.29.

**Figure 5.7:** Mode shapes when the actuator location form the nodes of vibration modes. The actuators are located at 66 mm from the sides and one in the centre. The frequencies are $f_1 = 117 \text{ Hz}$, $f_2 = 160 \text{ Hz}$, $f_3 = 277 \text{ Hz}$, $f_4 = 335 \text{ Hz}$, $f_5 = 646 \text{ Hz}$.

The location of the actuators in Figure 5.7 is the same as presented by Dresig [62]. However mode five is excited when the actuators are located at that position.
The aspects about optimization for positioning or disturbance reduction as discussed in section 4.4 are also applicable here. Either the actuator forces are used to remove energy from modes when they are excited by disturbances, or the modes are excited as less as possible by actuator forces when a positioning task is performed. In order to damp modes the actuators should not be located
5.2. Overactuated beam and plate positioning system

Figure 5.8: Vibration amplitude when the actuators are located at 66 mm from the sides and are forced to form the nodes of vibration modes. A step profile base acceleration is applied with a magnitude of $\ddot{x} = 1 \text{ m/s}^2$. The inertia of the homogeneous beam corresponds to a pressure load of $p = \rho \cdot h \cdot \ddot{x} = 15.7 \text{ N/m}^2$. The odd numbered modes in Figure 5.7 are not excited. Mode two and mode four are the lowest modes that are excited due to a base acceleration.

in the nodes of vibration modes. When two actuators are located at the sides and one in the centre of the beam, and the control does not force nodes at the actuator locations, the actuators can be used to damp energy of several modes. The damping capability is however limited. At elevated damping parameters, or speed, the motion of the actuator location is restricted by the damping force and the actuator location starts to behave as node.

The vibration amplitude after motion time with a ramped sine as reference path and actuator locations as nodes can be determined with Equation 4.29. The highest resonance frequency is obtained when the actuators are located in the nodes of the fourth free mode, see Figure 5.7. Even though not present in the bode plots, in Figures 5.3 and 5.6 a resonance frequency is present at a frequency of $f_4 = 117 \text{ Hz}$. In the bode plots the first resonance frequency is that of mode five at $f_5 = 230 \text{ Hz}$. When symmetry is broken the mode with a frequency of 117 Hz will be excited. When the actuators are located in the fourth free mode, not only the highest natural frequency is obtained, but also the highest possible anti-resonance frequency. At the resonance or anti-resonance frequency the positioning task is not fulfilled. The positioning performance is no longer limited by mode three of the free system with a frequency $f_3 = 42.5 \text{ Hz}$ but limited by mode four of the free system with $f_4 = 117 \text{ Hz}$.

At relative low speeds the amplitude mainly depends on the free mode five and might thus benefit from minimum excitation of mode five. In Figure 5.9 the outer actuators are located such that in case of open loop excitation mode five is excited as less as possible during positioning. The actuators are located at 72 mm from the sides and are assumed to form nodes due to the control or actuator mass. The natural frequencies and the mode shapes are only slightly changed. Compared to Figure 5.8 mode two is no longer excited when a base acceleration is applied. Despite the natural frequency is reduced compared to the situation when the actuators are located in the nodes of the fourth free mode, the positioning performance in terms of steady state deformation is increased.
Chapter 5. Overactuated positioning systems

Figure 5.9: Vibration amplitude when the actuators are located at 72 mm from the sides and exert a step profile base acceleration of $\ddot{x} = 1 \, \text{m/s}^2$. The actuator locations form the nodes. The natural frequencies are $f_1 = 116 \, \text{Hz}$, $f_2 = 151 \, \text{Hz}$, $f_3 = 263 \, \text{Hz}$, $f_4 = 332 \, \text{Hz}$, $f_5 = 676 \, \text{Hz}$. Only mode four is excited due to the base acceleration. The steady shape deformation is reduced compared to the shape in Figure 4.32.

The steady state shape in Figure 5.9 represents the summation of the modes one to five. The steady state deformation at the side is in the same range as the deformation determined from Figure 2.19 and Figure 2.20 and Equation 2.25.

The optimization of the actuator location for the higher modes is the result of the assumed symmetry. Initially the actuators are located such that consecutive modes are not excited. In this case with an odd number of actuators some modes are not excited due to symmetry. When however the symmetry is broken, the lowest mode is excited during positioning, and the effect of that mode should be taken in account. The lowest natural frequency is 116 Hz, which forms a speed limitation of the closed loop system. In the closed loop case proper gains of each controller have to be selected such that an optimum between damping and stiffness is obtained [220, 224]. The amount of damping obtained from the actuator is however limited. The excitation of the lower and higher numbered modes is affected by the control. The overall performance might for instance increase due to reduced compensator settings. As result nodes of the higher modes might no longer be located at the actuator location. The reduced compensator settings affect however also the lower numbered nodes. As result of the lower modes phase delay might destroy the performance. The frequency of the fourth free mode 117 Hz remains a limitation for the open loop system. That mode is excited during open loop operation and slight mis-alignment of the actuators. The force ratio can be changed to compensate possible misalignment of the lower modes, however then the free mode four is excited.

The methods presented in the previous chapter to evaluate damping of vibration modes, sensitivity of the actuator location and the sensitivity for disturbance frequencies are also applicable to overactuated beam system. These aspects are however not discussed in more detail here. The aim is to determine the possible mass reduction due to overactuation. Tailored systems might be created for specific problems.
5.2. Overactuated beam and plate positioning system

Increased number of actuators
In case another extra actuator is used the number of actuators is even. The force ratio of the actuators is such that consecutive modes are not excited. For minimum excitation of higher numbered consecutive modes the four actuators are best located in the four nodes of the fifth free vibration mode, Figure 4.10. When the actuators are located in these nodes at $53 \, mm$ and $184 \, mm$ from the sides the vector is $\hat{F}^T = [0.24, 0.26, 0.26, 0.24]$. The excitation of mode five is $\Phi_5^T \hat{F} = -10 \times 10^{-3}$ while the excitation of mode seven $\Phi_7^T \hat{F} = -79 \times 10^{-3}$. The even numbered modes are still not excited. In case the actuator locations forms the nodes the frequency of the fifth free mode is limiting the performance.

An odd number of actuators are used when another actuator is added. They are best located in the nodes of the sixth free mode. However due to symmetry mode six is not excited. The actuator location is then slightly changed such that mode 7 is excited as less as possible. This shows similarities with the use of three actuators.

Conclusion for the beam system
Minimum excitation of consecutive free vibration modes during positioning is advantageous to decrease the vibration amplitudes. The location of the actuators is such that consecutive numbered vibration modes are excited as less as possible. Initially the actuators are therefore located in the nodes of the higher numbered free vibration mode. However due to symmetry the lowest vibration mode might be prevented from excitation due to base acceleration. Due to slight changes of the actuator location the excitation of higher numbered modes can be minimized. The frequency of the lowest free mode to which the force ratio is not tuned during open loop excitation remains an upper speed limitation. Due to the actuator mass the mode shapes are affected. Errors in the location of the actuators result in a changed force ratio. As result of an increased number of actuators, consecutive modes are thus not excited. This results in decreased vibration amplitudes at low frequency input signals. With low frequent input signals is meant a frequency much smaller than the lowest natural frequency of the first free vibration mode. In the extreme a step input profile is considered. The accuracy of a single positioning task within the same motion time can be increased. When the tolerance specification is maintained the motion time can be decreased. Another option is to reduce mass because less structural stiffness is required. This will be treated in the next chapter.

From the frequency response functions can be concluded that: preventing the lowest modes from excitation during positioning, results in an increase of the lowest anti-resonance or resonance frequency when an increased number of parallel actuators are used. The lowest resonance, or anti-resonance frequency, forms a frequency limitation of subsequent positioning tasks. Positioning of the whole system is required. Excitation in the resonance frequency results in amplitudes, only limited by material damping. Excitation at a frequency that corresponds to an anti-resonance results in no movement at the considered location. For positioning of the whole system, rigid body motion is required. An increase of the number of actuators can thus be used, to decrease the time
of consecutive positioning tasks. Another option is to allow a reduction of the natural frequency. Which means the ratio stiffness and mass can be changed to lower values. In other words mass can be reduced.

### 5.2.2 Plate positioning system using four actuators

The same plate positioning system as discussed in Chapter 4 is used to mention the effect of additional actuators to a plate system. In the previous chapter equilibrium equations, that describe the rigid body behaviour, were used to determine the force ratio of the actuators ($\hat{F}$). These equations are extended with equations that describe the excitation of the elastic modes. For positioning purposes, the force vector ($\hat{F}$) should still fulfil Equation 4.10. Furthermore is the requirement of minimum excitation of consecutive modes during positioning, extended with an additional mode, see Equation 4.11. The original requirement of minimum excitation of the rigid body modes ($\Phi^T_2 \hat{F} = \Phi^T_3 \hat{F} = 0$) is extended with the requirement of minimum excitation of an elastic mode during positioning.

The actuators are located such that consecutive higher numbered modes are excited as less as possible. In first case a symmetric actuator location is discussed, later on the effect of an asymmetrical actuator location will be mentioned. Maintaining symmetry has a positive effect on minimal excitation of the lowest modes that contribute in general most to the vibration amplitude. Even though certain actuator locations can result in less excitation of higher order modes, the focus is on preventing the excitation of lower order modes. Two symmetric configurations are considered. The situation where one actuator is located in the centre while the other three actuators are located at equi-spaced points of equal radii will be discussed further on.

#### Four actuators at equi-spaced points of equal radii

The angle between the actuators is $90^\circ$, Figure 5.10.

![Figure 5.10: Four actuators located at equi-spaced points of equal radii.](image1)

![Figure 5.11: One actuator at the centre three actuators at equi-spaced points of equal radii.](image2)

The first three modes that should be prevented from excitation during positioning are mode two, three and four, see Table 4.2. Thus the forces are determined from Equation 4.10 and Equation 4.11, ($\Phi^T_2 \hat{F} = \Phi^T_3 \hat{F} = \Phi^T_4 \hat{F} = 0$). Independent on the radial actuator location the force vector is $\hat{F}^T = [1 \ 1 \ 1 \ 1]/4$. The
mode shapes are represented in Equation 4.9. As a result of the angular actuator location the mode shapes with zero, four, eight, twelve etc. nodal diameters are excited, \( n_p = 0, 4, 8, \ldots \) in Table 4.2. The excitation of the modes due to the angular location is determined in the same way as in Equation 4.25 with \( n_p = 0, 4, 8, \ldots \) in Equation 4.9, and \( \theta \) the angle of each actuator.

![Figure 5.12](image)

**Figure 5.12:** Frequency response at the side \((\theta = 0^\circ)\) when four actuators are used. The four actuators are located at equi-spaced points of equal radii, Figure 5.10. The radius is one case the outside \((r = 150 \text{ mm})\) and in the other case \(r = 102 \text{ mm}\). The actuators exert a force with a magnitude of 1 N. The angular orientation of one of the actuators is \(\theta = 0^\circ\).

The first mode that is excited is mode 6, as observed in Table 4.2. Mode 11-12 are the next modes that are excited during positioning. Mode 7-8 are for instance not excited due to this angular orientation [217]. In case modes are excited due to this angular orientation, the actual excitation depends furthermore on the radial location of the actuators. The excitation of mode 6 can be minimized when the radius of the actuator location is chosen equal to the nodal radius of mode 6. The nodal radius of mode 6 is at a radius of 102 mm as shown in Figure 4.22. The frequency response at different location is presented in Figure 5.12. The case with four actuators at the outer radius in Figure 5.12, is compared with the frequency response of a system with three actuators at the outer radius, see Figure 4.25. As result of the use of four actuators the lowest anti-resonance is shifted from 12.9 Hz in case of three actuators, towards 16.2 Hz. This can be explained with the amplitude of the exited modes in both case. In both cases mode 1 and 6 are the first modes that are excited. This is however equal in both cases. In Figure 4.25 the effect of mode 7-8 has to be taken into account as well, see also Figure 4.24. In case of four actuators, Figure 5.12, mode 7-8 is not excited but mode 11-12 are the first consecutive excited modes after mode 6 that need to be taken into account, see also Figure 5.13. The vibration amplitude of the first consecutive mode after mode 6, is reduced when four actuators are used instead of three, see Figure 4.24 and 5.13. As a result the first anti-resonance in Figure 5.12 is larger than the first anti-resonance in Fig-
The performance limiting frequency for consecutive positioning tasks is as a result of the increased lowest anti-resonance frequency shifted towards a higher frequency.

Due to the relative large modal density and complexity the frequency response is not discussed for any arbitrary location. All frequency response functions can be physically explained, however the explanation as provided in the beam and plate systems mentioned before is considered sufficient to obtain physical understanding. Instead of frequency response functions only the vibration amplitudes to step force profiles are mentioned. This gives a good approximation about the low frequent behaviour. Speed limitations can be indicated with the available system knowledge. The vibration amplitudes at the side and centre to a step force profile are represented in Figure 5.13.

![Figure 5.13](image_url)

**Figure 5.13:** Vibration amplitude at the centre and at the side when four actuators are used. The four actuators are located at equi-spaced points of equal radii, Figure 5.10. The radius is one case the outside ($r = 150$ mm) and in the other case $r = 102$ mm. The actuators exert a step force with a magnitude of 1 N.

The performance limiting frequency for consecutive positioning task of a system with four actuators located at the nodal radius of mode 6, still depends on the excitation of mode 4 or 5. In case three actuators are used which are located on the nodal radius of mode 6, the performance limiting frequency for consecutive positioning tasks resulted from the excitation of mode 4-5, see Chapter 4. Even though four actuators are used, errors in the actuator position will still result in excitation of either mode 4 or 5 during positioning. This results in an additional resonance at $19.5 \text{ Hz}$ in Figure 5.12. If the four actuators exert a force with a frequency equal to the natural frequency of mode 4 or 5 ($19.5 \text{ Hz}$) the amplitude will increase to infinite values when no material damping is present. In practice the amplitude is limited because damping is present. The frequency of mode 4 or 5 remains as a result a performance limiting frequency for consecutive positioning tasks when four actuators are located on the nodal radius of mode 6.

The guidance mass affects, as well as the feedback control, the dynamics of the system. Due to the actuator stiffness the mode shapes become a complex interplay between support stiffness and the stiffness of the elastic plate. The dynamic behaviour of a circular plate supported at four (and more) equally spaced points along the circumference of a concentric circle is also discussed by Irie and Yamada [112]. An upper limit of the error after a positioning task can
however be obtained when the actuator locations are assumed as nodes. The vibration amplitude after a positioning task with a ramped sine as reference path and the actuator locations as nodes can be calculated with Equation 4.29. Because all the actuators are located on the nodal lines of one of the mode pair 4-5 of the free vibration mode that mode is not affected due to the assumptions of nodes at the actuator location. (For ease of explanation it is assumed the actuators are located on the nodal lines of mode 5.) When the actuators are meanwhile also located on the nodal circle of the free vibration mode 6 this mode is also not affected by the assumption of nodes at the actuator location. The frequency of the lowest mode that is not used to calculate the force ratio forms a speed limitation of the open and closed loop system. In the open loop system mode 5 was the lowest frequency. When the actuators are located at a radius of 102 mm and the actuator locations form the nodes the first natural frequency is 20 Hz. A mode shape equivalent to the fifth free mode is obtained. Even when other control is applied the actuators are located in the nodes of mode 4 or 5. As a result one of these modes is not affected by the actuator mass or control, and a mode with a frequency of 20 Hz remains present. This also indicates the limited ability to damp modes with the actuators. An external disturbance that excites the fifth free mode with that frequency can simply not be damped with the actuators. Another actuator location can be applied, but then mode five will be excited during positioning and destroy the positioning performance. The actuators are in this case located for positioning purposes and thus in the nodes of the fifth mode. When mode five is disturbed the amplitude is only limited by material damping. The actuators can not be used to limit the vibration amplitude of that frequency. When the actuators are also located on the nodal diameter of mode 6 the natural frequency of that mode is not influenced by the actuator stiffness and mass and thus also not limited by the actuators.

**One actuator in the centre**

The situation where one actuator is located in the centre while the other three actuators are located at equi-spaced points of equal radii is described. The angle between the actuators at the radius is thus 120°, Figure 5.11. As result of the angular actuator location modes with zero, three, six, nine nodal diameters are excited, \( n_p = 0, 3, 6, \ldots \) in Table 4.2. The forces are determined from Equation 4.10 and Equation 4.11, \( \Phi_2^T \hat{F} = \Phi_3^T \hat{F} = \Phi_6^T \hat{F} = 0 \). The force ratio between the actuators with equal radius is \( \hat{F}^T = [1 \ 1 \ 1]/3 \). The ratio between the central actuator and the other actuator depends on the radial location. The force ratio in case the three actuators are located at the outer radius and one in the centre is determined. From Figure 4.22 the value of mode six at the side is \( \Phi_6(r = 150, \theta) = -2.8 \), in the centre \( \Phi_6(r = 0, \theta) = 3.8 \). The mode does not have an angular dependency. In order to prevent excitation of mode six during positioning the central force has to be a factor 2.8/3.8 of the total force exerted on the side. Furthermore Equation 4.10 has to be fulfilled as well. When the central actuator is the last element in the force vector the force vector is \( \hat{F}^T = [0.19 \ 0.19 \ 0.19 \ 0.43] \). Mode pair 7-8 and mode 17 are the first modes that are excited during posi-
tioning. The excitation of mode 17 can be minimized by the radial location and force ratio between the central actuator and the actuators on the radius. However due to the increased radius of the outer actuators also the excitation of mode 7-8 might change. The vibration amplitudes of the modes is determined at the side and at the centre in case a step force of $F = 1\text{ N}$ is applied, Figure 5.14. The three outer actuators are in one case located at the outside. In the other case the actuators are located at $r = 114\text{ mm}$, this radius is chosen because than mode 17 is not excited much during positioning. At that radius the mode shape value of mode six is $(\Phi_6(r = 114, \theta) = -0.7$ This results in a force vector $F^T = [0.28\ 0.28\ 0.28\ 0.16]$. 

\[
\begin{array}{c}
\text{actuators at } r=114; \theta=n \times 2/3 \pi \ (n=1,2,3) \\
\text{actuators at } r=150; \theta=n \times 2/3 \pi \ (n=1,2,3)
\end{array}
\]

![Figure 5.14: Vibration amplitude at the centre and at the side when four actuators are used. One actuator is located at the centre while three actuators are located at equi-spaced points of equal radii, see Figure 5.11. The radius is one case the outside (r = 150 mm) and in the other case r = 114 mm. The actuators exert a step force with a magnitude of 1 N.](image)

Errors in actuator location results in another force ratio to prevent the excitation of the lowest numbered modes during positioning. The force ratio can be such that mode 2, 3 and 6 are not excited during positioning. As a result of actuator location errors, the force ratio is changed thus mode 4-5 is excited during positioning. Due to the excitation of this mode the system is sensitive for internal disturbances with a frequency equal to the natural frequency of mode 4-5. The vibration amplitude will not change much when the actuator location is not ideal, however the frequency of mode 4-5 limits the performance. If the actuators exert a force with a frequency of 20 Hz, the amplitude of mode 4-5 will increase to infinite values when no material damping is present. In practice the amplitude is limited because damping is present. The possible instability at the frequency of mode 4-5 should however be taken into account. An instability at a frequency equal to that of mode 6 will not appear due to this actuator location.

The system is not sensitive for disturbances forces with a frequency equal to the natural frequency of mode 6. One of the modes of mode pair 7-8 is not affected by the actuator stiffness or damping. The system is thus sensitive for a disturbance with that frequency.

The focus of this thesis is on minimal mass. The excitation of the lowest modes during positioning should be as less as possible. When Figure 5.13 and 5.14 are compared the vibration amplitude is in case of four actuators at an equal radius of $r = 102\text{ mm}$, the smallest. This actuator location is thus preferred. In
both cases mode 4-5 is not excited as result of the actuator location. When the actuators are however not exactly located at the specified location mode 4-5 is excited. The frequency of these modes forms a speed limitation for consecutive positioning tasks in both cases.

### 5.2.3 Plate positioning system using five actuators

Two symmetric configurations are considered. The situation where one actuator is located in the centre while the other four actuators are located at equi-spaced points of equal radii will be discussed further on.

**actuators at five equi-spaced points of equal radii**

The angle between the actuators is \(\frac{360}{5}\)°, Figure 5.15.

![Figure 5.15: Five actuators located at equi-spaced points of equal radii.](image1)

![Figure 5.16: One actuator at the centre four actuators at equi-spaced points of equal radii.](image2)

As result of the angular actuator location modes with zero, five, ten, fifteen etc. nodal diameters are excited, \(n_p = 0, 5, 10, 15, \ldots\) in Table 4.2. Mode 6 and mode 13-14 are thus the first modes that are excited during positioning. The mode shape value depends furthermore on the radial location of the actuators. The actuator force is determined from Equation 4.10 and 4.11 what results in: \(\Phi_T^{T} \hat{F} = \Phi_2^{T} \hat{F} = \Phi_3^{T} \hat{F} = \Phi_4^{T} \hat{F} = \Phi_5^{T} \hat{F} = 0\). This results in the force vector \(\hat{F}^T = [1 \ 1 \ 1 \ 1 \ 1]/5\). The excitation of mode 6 can be minimized when the radius of the actuator location is chosen equal to the nodal radius of mode 6. The magnitude of the vibration amplitude at the side and centre are represented in Figure 5.17. The actuators are in one case located at the outside \(r = 150\) mm. In the other case the actuators are located at \(r = 102\) mm what is near the nodal radius of mode 6.

Errors in actuator location results in another force ratio to prevent the excitation of the lowest numbered modes during positioning. Due to errors in actuator location, and thus a changed force ratio \((\hat{F})\), mode 6 and mode pair 7-8 are the first modes that are excited during positioning. Due to the excitation of these modes, the system is sensitive for internal disturbances with a frequency of mode 6 and mode 7-8. The vibration amplitude will not change much when the actuator location is not ideal, however the frequency of mode 6 and mode 7-8 limits the performance. If the five actuators exert a force with a frequency equal to the natural frequency of mode 6 (35 Hz) the amplitude of
Figure 5.17: Vibration amplitude at the centre and at the side when five actuators are used. The five actuators are located at equi-spaced points of equal radii, Figure 5.15. The radius is one case the outside \(r=150\) mm and in the other case \(r=102\) mm. The actuators exert a step force with a magnitude of 1 N.

mode 6 will increase to infinite values when no material damping is present. In the same way the vibration amplitude of mode 7-8 becomes infinite when the actuators exert a force with a frequency equal to the natural frequency of mode 7-8 (46 Hz). In practice the amplitude is limited because damping is present. The possible instability at the frequency of mode 6 and mode 7-8 should however be taken into account.

The first mode that is not affected by actuator stiffness and mass is mode 6 in case the actuators are on the nodal radius of that mode. The actuators are furthermore also located on the nodal lines of one of the free modes of mode pair 13-14. The control of the actuator does as well as its mass not affect the free mode when all actuators are located on the nodal line. An external disturbance that excites the system with a frequency equal to one of the free modes 13-14 is only limited by material damping. The actuators can not be used to limit the vibration amplitude of that frequency. The vibration amplitudes of other modes depends on the support stiffness and damping of the actuators.

One actuator in the centre

The situation where one actuator is located in the centre while the other four actuators are located at equi-spaced points of equal radii is considered. The angle between the actuators at the radius is thus 90°, Figure 5.16. \(\Phi_2^T \hat{F} = \Phi_3^T \hat{F} = \Phi_4^T \hat{F} = \Phi_6^T \hat{F} = 0\). As result of the angular actuator location modes with one, four, eight, twelve etc. nodal diameters are excited, \(n_p = 0, 4, 8, \ldots\) in Table 4.2. Due to the prevention of excitation of certain modes, mode pair 11-12 and mode 17 are the first modes that are excited during positioning. The ratio between the outer actuators is such that \(\hat{F}^T = [1 1 1 1]/4\) the ratio with the central actuator is such that mode six is not excited due to the radial location of the outer actuators.

The force ratio can be determined similar to the earlier discussed situation with three actuators at the outer diameter and one in the centre. The mode shape values from Figure 4.22 remain the same. The mode shape value of mode six at the side is \((\Phi_6(r=150, \theta) = -2.8,\) in the centre \((\Phi_6(r=0, \theta) = 3.8). The mode does not have an angular dependency. In order to prevent excitation
of mode six during positioning the central force has to be a factor 2.8/3.8 of the total force exerted on the side. However in this case the total force of the actuators at the side is spread over four actuators instead of three. When the central actuator is the last element in the force vector the force vector is 
\[
\hat{F}^T = \begin{bmatrix} 0.144 & 0.144 & 0.144 & 0.144 & 0.424 \end{bmatrix}.
\]

The vibration amplitudes of the modes is determined at the side and at the centre in case a step force of \( F = 1 \text{ N} \) is applied, Figure 5.18. The four outer actuators are in one case located at the outside (\( r = 150 \text{ mm} \)). In the other case the actuators are located at \( r = 114 \text{ mm} \), this radius is chosen because then mode 17 is also not excited much during positioning. At that radius the mode shape value of mode six is \( \Phi_6(r = 114, \theta) = -0.7 \) This results in a force vector \( \hat{F}^T = [0.21 \ 0.21 \ 0.21 \ 0.21 \ 0.16] \). Compared to the situation with three actuators at the radius the total force of the actuators at the radius remains constant. The only difference is that the force of the actuators at the radius is spread over four actuators instead of three actuators.

![Figure 5.18: Vibration amplitude at the centre and at the side when five actuators are used. One actuator is located in the centre while four actuators are located at equi-spaced points of equal radii, Figure 5.11. The radius is one case the outside (\( r = 150 \text{ mm} \)) and in the other case \( r = 114 \text{ mm} \). The actuators exert a step force with a magnitude of 1 N.](image)

Errors in actuator location results in another force ratio to prevent the excitation of the lowest numbered modes during positioning. As result of errors mode 5 is excited during positioning. Due to the excitation of this mode the system is sensitive for internal disturbances with a frequency equal to the natural frequency of mode 5. The vibration amplitude will not change much when the actuator location is not ideal, however the frequency of mode 5 limits the performance. If the actuators exert a force with a frequency equal to the natural frequency of mode 5 (20 Hz) the amplitude of mode 5 will increase to infinite values when no material damping is present. In practice the amplitude is limited because damping is present. The possible instability at the frequency of mode 5 should however be taken into account. An instability at a frequency equal to that of mode 6 will not appear due to this actuator location.

The system is not sensitive for disturbances forces with a frequency equal to the natural frequency of mode 6. Mode 5 is not affected by the actuator stiffness or damping. The system is thus sensitive for an external disturbance with that frequency. The focus of this thesis is on minimal mass. The excitation of the lowest modes during positioning should therefore be as small as possible.
The vibration amplitude is smallest in case the five actuators are located at an equal radius of \( r = 102 \, \text{mm} \), Figure 5.17 and 5.18. This actuator location is thus preferred.

### 5.2.4 Plate positioning system using six actuators

In order to maintain (rotation) symmetry when six actuators are used, two actuator configurations are possible. The situation where one actuator is located in the centre is first considered, while the situation with five actuators located at equi-spaced points of equal radii will be discussed further on.

#### Six actuators at equi-spaced points of equal radii

The angle between the actuators is \( 360/6^\circ \), Figure 5.19.

![Figure 5.19: Six actuators located at equi-spaced points of equal radii.](image)

![Figure 5.20: One actuator at the centre; five actuators at equi-spaced points of equal radii.](image)

The actuator force is determined such that \( \Phi^T_2 \hat{F} = \Phi^T_3 \hat{F} = \Phi^T_4 \hat{F} = \Phi^T_5 \hat{F} = \Phi^T_7 \hat{F} = 0 \). This results in force vector \( \hat{F}^T = [1 \ 1 \ 1 \ 1 \ 1 \ 1]/6 \). As result of the angular actuator location modes with zero, six, twelve, etc nodal diameters are excited during positioning, \( n_p = 0, 6, 12, \ldots \) in Table 4.2. This means mode 6 and mode 18-19 are the first modes that are excited during positioning. The mode shape value depends furthermore on the radial location of the actuators. The excitation of mode 6 can be minimized when the radius of the actuator location is chosen equal to the nodal radius of mode 6. The magnitude of the vibration amplitude at the side and centre are represented in Figure 5.21 when a step force of \( F(t) = 1 \, N \) is applied. The actuators are in one case located at the outside \( (r = 150 \, \text{mm}) \). In the other case the actuators are located at \( (r = 102 \, \text{mm}) \) what is near the nodal radius of mode 6.

Errors in actuator location results in another force ratio to prevent the excitation of the lowest numbered modes during positioning. Mode 6 and mode 8 are the first modes that are excited during positioning. Due to the excitation of these modes the system is sensitive for internal disturbances with a frequency of mode 6 and mode 8. The vibration amplitude will not change much when the actuator location is not ideal, however the frequency of mode 6 and mode 8 limits the performance. If the five actuators exert a force with a frequency equal to the natural frequency of mode 6 (35 Hz) the amplitude of mode 6 will increase to infinite values when no material damping is present. In the same
5.2. Overactuated beam and plate positioning system

way the vibration amplitude of mode 8 becomes infinite when the actuators exert a force with a frequency equal to the natural frequency of mode 8 (46 Hz). In practice the amplitude is limited because damping is present. The possible instability at the frequency of mode 6 and mode 8 should however be taken into account.

The first mode that is not affected by actuator stiffness and mass is mode 6 in case the actuators are on the nodal radius of that mode. One of the modes of mode pair 18-19 is also not affected by actuator stiffness. When an external disturbance is present with a frequency equal to mode 19 the amplitude is only limited by material damping. The actuators can not be used to limit the vibration amplitude of that frequency. The vibration amplitudes of other modes depends on the support stiffness and damping of the actuators.

One actuator in the centre

The situation where one actuator is located in the centre while the other five actuators are located at equi-spaced points of equal radii. The angle between the actuators at the radius is thus $72^\circ$, Figure 5.20. $\Phi_2^T \hat{F} = \Phi_3^T \hat{F} = \Phi_4^T \hat{F} = \Phi_5^T \hat{F} = \Phi_6^T \hat{F} = 0$. As result of the angular actuator location modes with zero, five, ten, etc. nodal diameters are excited during positioning, $n_p = 0, 5, 10, \ldots$ in Table 4.2. The first modes that are excited during positioning are thus mode 13-14 and mode 17.

In case modes are excited due to this angular orientation, the excitation depends furthermore on the radial location of the actuators. The force ratio of the actuators on the radius is $\hat{F}^T = [1 \ 1 \ 1 \ 1 \ 1]/5$. The ratio between the central actuator and the outer actuators is such that mode 6 is not excited during positioning. The mode shape values of mode six at the actuator locations are used to calculate the force ratio.

The force ratio can be determined similar to the earlier discussed situation with three and four actuators at the outer diameter and one in the centre. The total force of the actuators at the side is spread over five actuators. When the central actuator is the last element in the force vector the force vector is $\hat{F}^T = [0.115 \ 0.115 \ 0.115 \ 0.115 \ 0.115 \ 0.425]$. 

Figure 5.21: Vibration amplitude at the centre and at the side when six actuators are used. The six actuators are located at equi-spaced points of equal radii, Figure 5.15. The radius is one case the outside ($r = 150 \text{ mm}$) and in the other case $r = 102 \text{ mm}$. The actuators exert a step force with a magnitude of 1 N.
The vibration amplitudes of the modes is determined at the side and at the centre in case a step force of $F(t) = 1\ \text{N}$ is applied, Figure 5.18. The five outer actuators are in one case located at the outside ($r = 150\ \text{mm}$). In the other case the actuators are located at ($r = 114\ \text{mm}$), this radius is chosen because then the excitation of mode 17 during positioning is also relatively small. This results in a force vector $\hat{F}^T = [0.168\ 0.168\ 0.168\ 0.168\ 0.168\ 0.160]$. The force of the actuators at the radius is spread over five actuators instead of three or four actuators in the previously mentioned similar cases with three and four actuators at a radius and one actuator in the centre.

![Figure 5.22: Vibration amplitude at the centre and at the side when six actuators are used. One actuator is located at the centre while five actuators are located at equi-spaced points of equal radii, Figure 5.20. The radius is one case the outside ($r = 150\ \text{mm}$) and in the other case $r = 114\ \text{mm}$. The actuators exert a step force with a magnitude of 1 N.](image)

Errors in actuator location results in another force ratio to prevent the excitation of the lowest numbered modes during positioning. As result of errors mode 7-8 is excited during positioning. Due to the excitation of this mode the system is sensitive for internal disturbances with a frequency of mode 7-8. The vibration amplitude will not change much when the actuator location is not ideal, however the frequency of mode 7-8 limits the performance. If the actuators exert a force with a frequency equal to the natural frequency of mode 7-8 (46 Hz) the amplitude of mode 7-8 will increase to infinite values when no material damping is present. In practice the amplitude is limited because damping is present. The possible instability at the frequency of mode 7-8 should however be taken into account.

The system is not sensitive for disturbances forces with a frequency equal to the natural frequency of mode 6. One of the modes of mode pair 13-14 is not affected by the actuator stiffness or damping. The system is thus sensitive for a disturbance with that frequency.

The focus of this thesis is on minimal mass. The excitation of the lowest modes during positioning should be as less as possible. The vibration amplitude is smallest in case the six actuators are located at an equal radius of ($r = 102\ \text{mm}$), Figure 5.21 and 5.22. This actuator location is thus preferred.

For positioning purposes six in a hexagram is preferred. However when one actuator is not located exactly at the nodal circle of mode six that mode is excited. Therefore the location with one actuator at the centre and five in a pentagram is preferred from stability point of view.
5.2.5 More actuators for the circular plate system

The previous mentioned plate systems can be expressed with the plate in Figure 5.23. The actuators are located on different radii. The actuators located on an equal radius are located at equally-spaced points. The plate system in Figure 5.23 is sufficient to indicate the position to a number of 16 actuators in case mode 16 and preceding free modes are used to calculate the force ratio $\hat{F}$. With that system the excitation of all 16 modes can be prevented while positioned. Mode 17 of the free system is in that case the lowest frequency that is limiting the speed of successive positioning tasks. In Table 5.1 the position of the actuators is indicated. The nodal circle of mode 6 has special meaning and the radius is indicated with $B_6$. The radii of the actuators are indicated with respect to the nodal radius of mode 6. A radius of zero is used to express the centre location.

![Figure 5.23: The actuators are located as indicated in Table 5.1.](image)

Figure 5.23 and the values in Table 5.1 are explained using the case where four actuators are located at equi-spaced points of equal radii as presented in Figure 5.10. All four actuators are located at the nodal radius thus $R_2 = B_6$ in Table 5.1. The angle between the equi-spaced actuators is $\angle_{2\{o\}} = 90^\circ$. Mode 2, 3, and 4 are minimal excited as result of the applied force ratio, see Equation 4.11. Positioning of the rigid body should be possible 4.10. Mode four and the preceding ones of the free system are used to calculate the force $\hat{F}$. When a force profile is applied mode 11-12 is excited. Even though not excited mode 5 is limiting the frequency of successive positioning tasks.

The force vector $\hat{F}$ is determined such that consecutive modes are not excited. The excitation of the higher numbered modes affects the accuracy of the positioning system. When a force profile is applied that consists of only a single harmonic the vibration amplitude of the vibration modes is calculated with Equation 4.23. Due to location errors of the actuators modes might be excited. The force ratio $\hat{F}$ is however determined such that in open loop operation consecutive modes are not excited despite small errors in the required actuator location. The excitation of the higher numbered modes affects the accuracy of the positioning system. When a force profile is applied that consists of only a single harmonic the vibration amplitude of the vibration modes is calculated with Equation 4.23.

Different aspects of actuator locations were discussed. As was explained with
Table 5.1: Performance of a circular plate positioning system dependent on the number of actuators and their location. The total number of actuators is indicated with the number on the left. The actuators are located as drawn in Figure 5.23, where the values in this table have to be used to determine each particular situation. The radius of $B_6 = 0$ corresponds to a radius at the nodal circle of mode 6, see Figure 4.22. In the last four columns mode numbers, natural numbers, of the free, unconstrained, system without actuator mass are indicated. A force ratio is determined while the excitation of some modes is set to zero $(\Phi^*_i \cdot \hat{F})$. With this force vector some modes are still excited during positioning. Despite not excited the frequency of the lowest mode to which the force $\hat{F}$ is not tuned remains a performance limiting frequency for consecutive positioning tasks.

<table>
<thead>
<tr>
<th>#</th>
<th>#</th>
<th>$R_1$</th>
<th>$\angle_1[^\circ]$</th>
<th>#</th>
<th>$R_2$</th>
<th>$\angle_2[^\circ]$</th>
<th>force determ. from</th>
<th>excited $\hat{F}$ during positioning</th>
<th>limiting speed mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$B_6$</td>
<td>180</td>
<td>$\leq 1$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>$B_6$</td>
<td>180</td>
<td>2</td>
<td>$B_6$</td>
<td>90</td>
<td>$\leq 2$</td>
<td>4-5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>$B_6$</td>
<td>120</td>
<td>3</td>
<td>$B_6$</td>
<td>90</td>
<td>$\leq 3$</td>
<td>7-8</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>4</td>
<td>$B_6$</td>
<td>90</td>
<td>4</td>
<td>$B_6$</td>
<td>45</td>
<td>$\leq 4$</td>
<td>11-12</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
<td>$R_2 &gt; B_6$</td>
<td>120</td>
<td>5</td>
<td>$R_2 &gt; B_6$</td>
<td>72</td>
<td>$\leq 6$</td>
<td>13-14</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>6</td>
<td>$R_2 &gt; B_6$</td>
<td>72</td>
<td>6</td>
<td>$B_6$</td>
<td>60</td>
<td>$\leq 7 \times 6$</td>
<td>17, 18-19</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>7</td>
<td>$R_2 &gt; B_6$</td>
<td>60</td>
<td>7</td>
<td>$R_2 &gt; B_6$</td>
<td>72</td>
<td>$\leq 7$</td>
<td>17, 18-19</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>9</td>
<td>$R_1 &lt; B_6$</td>
<td>120</td>
<td>9</td>
<td>$R_2 &gt; B_6$</td>
<td>$\approx 51$</td>
<td>$\leq 8$</td>
<td>17, 24-25</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>10</td>
<td>$R_1 &lt; B_6$</td>
<td>120</td>
<td>10</td>
<td>$R_2 &gt; B_6$</td>
<td>$\approx 51$</td>
<td>$\leq 9$</td>
<td>17, 24-25</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>11</td>
<td>$R_1 &lt; B_6$</td>
<td>120</td>
<td>11</td>
<td>$R_2 &gt; B_6$</td>
<td>45</td>
<td>$\leq 10$</td>
<td>17, 24-25</td>
</tr>
</tbody>
</table>
six actuators, a choice has to be made between minimum excitation and stability. For minimum excitation the six actuators should be located on the nodal radius of the sixth mode. However this mode might result in instability during positioning and therefore limits the performance. With five actuators at an equal radius and one central actuator the error increases slightly, however mode six does not result in instability. In the experiments the actuators are located for minimal excitation.

5.2.6 Annotation to the applied sensor configuration

The previously applied control method in case of closed loop operation was: collocated decentralized control. The sensor configuration should, as a result, be such that at every actuator location also a sensor is present. As mentioned, an important advantage of this type of control is that a proportional and differential controlled actuator forces can be replaced with the mechanical analogs: linear springs and dampers, see page 127. Another sensor configuration, e.g. changed sensor location or different number of sensors used, might also require another closed loop control structure than the hitherto used collocated decentralized control structure. The dynamic response is, for instance, changed when mechanical flexibility is present between a sensor and an actuator. In case a system with a different number of sensors than actuators is considered, a possible solution might be to combine, or to subdivide signals. Even though the uncertainty management might be changed, e.g. as a result of non-collocated actuator and sensor locations or other control structures, the deterministic approach can in the design phase, be used to motivate the choice for the applied uncertainty management method in operation.

A collocated decentralized control structure is in this thesis used as a start in the initial design phase of parallel positioning systems. Systems using other sensor configurations, and control structures, might evolve from this initial system. Even though other sensor configurations and control structures might be easily implemented in the detailed design of specific systems, it is a specialty on its own to indicate the opportunities and limitations, that result from the application of other sensor configurations and control structures, in the initial design phase of parallel positioning systems in general. The application and description of control structures other than collocated proportional and differential decentralized control, is in this thesis not considered.

5.3 Validation of simulated results

In order to verify the results obtained with simulation, experiments are performed. The simulation results depend on the used model and the values used for the required physical parameters. A beam system as well as a plate system were created. The beam was mainly used to increase the knowledge level about parallel positioning systems. The plate system is considered a more realistic product carrier for application in a parallel positioning system. In this thesis only the experiments performed on the plate system are incorporated. This selection is based on the strategy of experiments, as mentioned on page 14.
Experiments to verify the reliability of the simulations

Simulations are in this thesis used to increase the insight and understanding of the positioning system. The reliability of the results obtained from simulations is determined with a comparison to the results obtained from experiments. As explained in Chapter 1 the in this thesis presented simulations and experiments are selected according to the guidelines of Montgomery [166]. Knowledge about the system was the major selection criterium to determine relevant experiments. Despite experiments might be used to increase model accuracy or parameters [149], the aim of the experiments is to determine the reliability of the results obtained from simulations.

The results of simulations are compared with experiments, while the model parameters in the simulations are assumed correct. Despite the model parameters are assumed to be known, the efficiency and potential of calibration, described by Soons [242], is applied for the selection of relevant experiments. Efficiency of an experiment concerns the estimation of the value and accuracy of the parameters. Potential of an experiment concerns the ability to verify the proposed model itself. The efficiency and potential of an experiment performed can be assessed by two criteria:

- the assembly of parameters that can be determined from the experiment.
- the accuracy of these parameter values, given a certain repeatability of an experiment.

According to [242]: The number of experiments is a compromise between an extreme with a large number of experiments, known as "brute force" validation, and the other extreme with a small number of experiments. In the "brute force" method little intelligence is used in selecting the experiments. The validation efficiency is rather low, i.e. many experiments are required. The lack of assumptions in the experimental validation results in a maximum ability to detect (unexpected) differences from the experiments that are not implemented in the proposed model. On the other extreme the number and choice of experiments is optimized according to the proposed effect. This method is valid within limitations.

The number and choice of experiments as presented in this thesis is optimized such that the reliability of the simulations can be determined with a limited number of experiments. Because knowledge about the system is used to select the simulations and experiments a "brute force" validation is not required. In this research the experiments are considered not to be a purpose on its own. Therefore also the limited available time and resources are taken in account to select relevant experiments.

Created overactuated plate system

Overactuation and the excitation of vibration modes is a major subject in this thesis. For this reason experiments are conducted on an overactuated plate. In the previous and current chapter the dynamic behaviour of a circular brass plate is discussed. A plate with the same properties as mentioned in section 4.3.2 is used for the experiments. Forces are obtained with voice coil
actuators. An advantage of a brass plate is that no reluctance force is generated due to the permanent magnet of the actuators. The experimental setup is designed by de Ruijter [217].

![Experimental plate setup](image)

**Figure 5.24:** Experimental plate setup. Three wire flexures are used to create the constraint lines as presented in Figure 5.24. Two constraints are connected in vertical direction to support the weight. A third wire flexure is used to constrain motion in sideways direction. The voice coil actuators are attached at the back side of the plate at a nodal radius of 102 mm. Two accelerometers are attached to the plate to measure the response. Different measurement setups, with a different number of actuators, were used. Besides the shown setup with four actuators attached to the plate, also plates with three and six actuators were used for the experiments.

The performance of individual positioning tasks can be compared. The time domain response is then dependent on the frequency content of individual positioning tasks. Instead of single positioning tasks the frequency response behaviour of the simulations is compared with experiments. From this frequency response the excitation of individual modes can be determined easier, because the response peaks at the resonance frequencies. The frequency response to an applied force is determined. The advantage of this open loop system is that feedback control does not affect the experiments. When feedback control is applied the frequency response to base accelerations can be determined. Despite the requirement to use tight control the dynamic behaviour at certain frequencies is the result of the complex interaction between control and structural properties of the system. The open loop frequency response is independent of this interaction.

In the experimental setup the plate is constrained by three wire flexures as schematically shown in Figure 3.2 and 3.25. The system is orientated such that
the wire flexures carry the weight as result of gravity. These wire flexures are connected to the plate at the outer radius, see Figure 5.24. Other constraint patterns are also possible. For instance the connection of the two parallel constraint can be moved upwards and directed such that they create a rotation pole. Without the third constraint connected, an equilibrium position in the gravity field is present. If this is a stable equilibrium 'stiffness' is experienced when the plate is moved in sideways (in-plane) direction. This 'stiffness' is obtained as result of an increased amount of potential energy stored in the system that is located in a gravity field. The change in potential energy of a stable system, and as a result 'stiffness', depends among others on the length of the constraints and the location of the pole. To enhance this 'stiffness' in sideways (in-plane) direction, the third constraint can be applied. Despite the existence of such alternative constrained patterns, the constraint pattern as shown in Figure 5.24 is among others used for ease of creation and adjustment. The forces are obtained from voice coils actuators that are connected to the plate without own guidance. The actuators are located at the nodal radius of the sixth free mode at equi-spaced points, schematically shown in Figures 4.23, 5.10, and 5.19. In the measurement setup different circular brass plates are used, i.e. with three, four, and six actuators equispaced at the nodal radius of mode 6. For the experiments with three and six actuators a plate with six $2\text{ mm}$ holes has been used. For the experiment with four actuators another plate is used with four holes of $2\text{ mm}$. These holes are required to connect the actuators. The holes have a minor effect on the dynamic behaviour of the lower numbered modes due to their number, location, and dimension. However, to minimize the number of holes two different plates are used.

The response is measured at two different locations. One sensor measures the behaviour at the actuator location. Because not all modes are observable at that location, another sensor measures the behaviour at the centre. Two accelerometers are used to measure the response. These sensors might not be sufficient for positioning purposes, they provide sufficient information to measure the frequency response. Despite not used as positioning system in the described experiments, the system is designed for that purpose. For contactless operation as positioning system optical displacement sensors can be used. The reaction force of the actuators was however able to enter the measurement frame. The measurement frame forms a reference and should not be subjected to disturbances that vary over time. In order to exclude the additional dynamics of the measurement frame in the considered dynamic experiments, the optical sensors are replaced with accelerometers. The accelerometers are connected to the plate and detect changes in inertia forces originating from changed accelerations.

Despite the models used for simulation do not incorporate all aspects, it is validated if the models represent the most important system behaviour. The stiffness and mass of the guidance and the cables attached to the actuators and sensors is for instance not modelled. The actuators are modelled as point masses of 8 $g$, without rotational inertia. Plate damages, such as scratches and other plastic deformations, turned out to affect the dynamic behaviour of the relative thin plate. To minimize the effect of such damages in the conducted
and described experiments, the plates are carefully selected. The frequency response of the model is determined with ANSYS® and compared with the experimental results. Despite the positioning system with six actuators is not able to operate at frequencies above the natural frequency of the sixth free mode \( f_6 = 35 \, \text{Hz} \) the frequency response to 200 Hz is determined. When the actuators are located in the nodes of mode 6, this mode is theoretically not excited. In practice it is complicated to realize systems with actuators exactly in the nodes of free vibration modes. When mode six is excited, the vibration amplitude is dependent on the amount of excitation, and on the internal damping of that mode. As result of the unknown amplitude at resonance, the frequency of mode 6 is regarded a speed limitation for consecutive positioning tasks in case six actuators are used. In case of four or five actuators in the nodes of mode six at equi-spaced locations, a small misalignment results in excitation of the mode with a frequency of 20 Hz. The increase of the number of parallel actuators result in an increase of the speed limiting frequency.

In case one positioning task is considered, only a single harmonic is performed. The experiments are conducted up to a frequency of 200 Hz, despite the indicated speed limitation. In this way the effect of the additional actuators on the excitation of the modes can be determined with comparisons of individual resonance frequencies. When the frequency range of the experiments is limited to the quasi-static frequencies, the magnitude of the frequency response can mainly be used to compare the systems. The increased number of actuators affects also the magnitude of the low frequent amplitude, however this is hard to observe. In the frequency response it is more easy to observe if modes are excited, or not excited as result of the number of actuators.

**Determination of the frequency response and coherence**

The force ratio between the actuators \( \hat{F} \) is fixed. The amplitude of the force of each actuator is equal due to the location and the requirements of minimum excitation of the lowest modes, as explained in this chapter. The time signal of the force is present in scalar \( F(t) \). For the time input signal \((F(t))\) white noise within a specified frequency range is used. The mutual actuator force remains equal, due to the constant ratio \( \hat{F} \). Via Fourier analysis of the time signals the frequency response is determined that relates the measured output acceleration to the applied input force at any frequency \((F(\Omega))\). The displacement response is obtained via integration. The accelerometers are unable to detect motions below a certain frequency, such as 'static' motions. In the measurement frequency range of the sensors the cyclic displacement response can be obtained via integration of the acceleration signal, even though the initial speed and initial position are not taken into account.

Besides the frequency response also the coherence between the measured output and the input force \((F(\Omega))\) has been determined. The coherence function indicates the degree of causality in a frequency response function [95]. It is obtained from the square of the cross power spectrum between input and response, divided by the auto power spectrum of the input and the response [15]. A unity coherence value at a certain frequency indicates perfect causality. The
measured response power is caused totally by the input power. A coherence value less than unity indicates that the measured output response power is not perfectly related to the input power. Disturbances might be a cause of a coherence value other than one. The output power near an anti-resonance frequency is another cause of low coherence at that frequency. Low coherence does not necessarily imply poor estimates of the frequency response, it only indicates the reliability at that frequency.

System knowledge obtained from simulations is used to link the peaks found in the measured frequency response to the eigenmodes. The vibration modes might be determined from the experiments as well. This requires however, much more measurement positions. In order to minimize the effect of the sensors on the dynamics contactless measurement is then preferred, for instance optically scanning the plate [109].

**Measurement equipment**

The measurement setup to determine the frequency response is schematically shown in Figure 5.25. A personal computer is connected to a HP VXI-system to operate the HP-VXI system [97] and perform the experiments. The signal generator in the VXI system generates a noise signal with the frequency range of interest. This signal is send to the amplifiers (TUE/dacs amplifiers) of the Lorentz actuators. The response is measured with miniature DeltaTron® accelerometers type 4507. The signal of each sensor is conditioned with a NEXUS™ condition amplifier, not shown in Figure 5.25. The measured data from the sensors is, as well as the measured noise signal send to the amplifiers, collected in the digital signal processor (type E1432A) of the VXI system. The measured output and input data are after the experiments is finished retrieved by he personal computer. The frequency response of the system is then determined with MATLAB®.

To verify the validity of the simulations experiments were conducted. However, to verify the results obtained with the experimental setup, i.e. the measurement equipment and the accompanying data analysis, the frequency response of other systems has additionally been determined as well. These systems were specifically selected for their simplicity, in order to minimize the contribution of potential error sources between theory and practice. After it was assured the measurement equipment and the data analysis were correct, the dynamic behaviour of the brass plate with only a single voice coil attached to it, was determined. From these verification experiments it turned out the initial used brass plate was dented, which had a large effect on the dynamic behaviour. The brass plates used for the experiments, as presented in this thesis, are therefore carefully selected such that the brass plate can be described with the theory as referred to in this thesis.

### 5.3.1 Experiments

**Effect of the suspension of the plate**

The constrained plate is subjected to gravity. As a result of the orientation of the constraints with respect to the gravity field the potential energy is changed
when the plate is positioned over a certain stroke. The required rigid body mode is thus subjected to restoring forces, also known as quasi elastic forces. The angular natural frequency of the created pendulum is $\omega_1 = \sqrt{g/L}$. Where $L$ is the length of the pendulum and $g$ the gravitational constant. The rigid body mode frequency introduces an additional resonance in the frequency response. Below the natural frequency the stiffness affects the vibration amplitude, above the natural frequency the rigid body mass determines the amplitude. The length of the constraints in the created experimental setup results in a rigid body mode frequency much smaller than the lowest elastic frequency of the plate. A changed excitation of the elastic modes is of interest in the conducted experiments. The considered frequency range starts at a frequency larger than the rigid body mode frequency. Even though the frequency response is in this case not affected by the rigid body mode frequency, this frequency should be taken into account when the positioning system is operated. In that case a need for position measurement and feedback control might be identified as result of the rigid body mode frequency.

Besides the length of the constraints, the stiffness of the constraints might affect the dynamic behaviour of the plate. In order to determine the effect of the wire flexures the dynamic behaviour is compared to a plate where the suspension is created with ropes. The ropes do not possess bending stiffness, while the wire flexures do. Alignment of the moving part of the system with respect to the stators is difficult, despite the relative large clearance between individual mover and stator. During operation the mover should not touch the stator because that affects the dynamic behaviour and the description becomes much more complicated. Alignment of the system is easier obtained with wire flexures and therefore applied. To determine the effect of the wire flexures the plate with three actuators connected to it is used. Only one actuator is actu-

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**Figure 5.25:** Experimental setup used for open loop experiments. This figure represents the particular case with three actuators and sensors. In practice the number of actuators and sensors can be varied.
ated. Alignment of two stators is hard to obtain with the ropes. The effect of the guidance on the dynamics is shown in Figure 5.26.

![Graph](image)

**Figure 5.26:** Three actuators are connected to the plate but only one actuator exerts a force. The suspension of the plate is varied, ropes and rods were used. The acceleration is measured at the actuated actuator.

In Figure 5.26 can be seen that the rods affect the dynamics. At a frequency of 8 Hz a ‘resonance’ appears present. This is the result of the stiffness of the guidance of the plate that is added to the rigid body modes. The rigid body modes as result of gravity field is approximately 1 Hz. As result of guidance stiffness the rigid body modes frequency is increased. The cables connected to the actuators and sensors also might add stiffness. The resonance frequency near 20 Hz differs from the rope and the wire flexures. The wire flexures add mass and stiffness. Above 50 Hz more natural frequencies appear present when rods are used. If the number of peaks is considered in the frequency band larger than 60 Hz, more peaks are found in case rods are used. Due to the addition of rods the mode pairs with equal resonance frequencies are split into two modes with different frequency. The dynamic behaviour of the slender rods might also introduce disturbances in the measurement results.

As observed in Figure 5.26 the suspension affects the frequency response and thus the dynamics of the system. Despite ropes results in a more smooth graph, and thus less effect on the dynamic behaviour, wire flexures are used to overcome some practical problems, e.g. for alignment.

**Plate with three actuators**

Three actuators are connected and excite the plate simultaneous. Compared to Figure 5.26 the stiffness at low frequencies is increased in Figure 5.27. Rigid body mode rotations are not present what might result in an apparent higher stiffness at low frequencies. The resonance near 20 Hz is not excited. From 60 Hz more frequencies appear present, this might however be the result of the wire flexures.
5.3. Validation of simulated results

Figure 5.27: Dynamic behaviour at one actuator location. Three actuators are located at 102 mm radius of the plate at equally spaced points as shown in Figure 4.23.

Figure 5.28: Dynamic behaviour at the centre. Three actuators are located at 102 mm radius of the plate at equally spaced points as shown in Figure 4.23.

In Figure 5.28 the sensor is located at the centre. In theory only the modes with non-zero modal values at the centre are observed. In theory only mode 6, (35 Hz) and mode 17 (147 Hz), see Figure 4.22 are present. However, in the simulation actuator mass is added and even in the simulation other modes are excited as well in Figure 5.28. Besides the modes that are excited in the simulation other modes are also present. When the system is not perfectly symmetric such dynamic behaviour might be obtained. Also modal interaction might result in excitation of modes, that are in theory not present [85]. The natural frequency of mode 6 is in theory 35 Hz, in Figure 5.28 it appears at 27 Hz. This
can be explained with the guidance and central sensor. The guidance is connected at a position where mode 6 is affected much. The mass and stiffness of the guidance affects, as well as the mass of the central sensor, this mode. In Figure 5.28 more anti-resonance and resonance frequencies are present in practice than in the simulations. As explained in the previous chapter, this effect appears when a mode is present but only slightly excited or the sensor is located in the node of that mode. In theory modes are not excited, in practice they are. The excitation of the most dominant modes affects the global shape of the frequency response. From Figures 5.27 and 5.28 it is initially concluded that the global shape of the measured frequency response matches with the simulated frequency response. The local differences can be assigned to aspects, such as the effect of the guidance, no perfect symmetry in practice, and the addition of sensors and their cables that are not modelled in the simulations. To obtain more insight other experiments with a different number of actuators are performed.

**Plate with four actuators**
The frequency response when four actuators are connected and apply an equal force is determined. As mentioned, a different plate is used compared to the experiments with three and six actuators. Compared to Figures 5.27 the frequency response in Figure 5.29 is changed. Still local differences are present between simulations and experiments. The local effects are not discussed in detail. The global differences between a different number of actuators is discussed later.

**Plate with six actuators**
The frequency response when six actuators are connected and apply an equal
5.3. Validation of simulated results

Figure 5.30: Dynamic behaviour at the centre. Four actuators are located at 102 mm radius of the plate at equally spaced points as shown in Figure 5.10.

force is determined.

Figure 5.31: Dynamic behaviour at one actuator location. Six actuators are located at 102 mm radius of the plate at equally spaced points as shown in Figure 5.19.

Comparison between the systems

The frequency response at the actuator location is compared when four actuators are used, Figure 5.29, and when three actuators are used, Figure 5.27. As observed the mode at 45 Hz is excited in Figure 5.27, while it is not excited when four actuators are used, Figure 5.29. The mode at 80 Hz is much more excited when four actuators are used, compared to the situation when three actuators
In case the situation with six actuators, Figure 5.31 is compared with the use of four actuators, Figure 5.29 the mode, at 80 Hz, is not excited in case six actuators are used. Only the mode at 135 Hz is excited. This mode is however also excited with three and four actuators. It is therefore concluded that this mode is the one with two nodal circles.

**Validity of the results obtained with simulations**

The excitation of the most dominant modes affects the global shape of the frequency response. From the comparison between the experiments and the simulations presented above it is concluded that the global shape of the measured frequency response matches with the simulated frequency response. The local differences between theory and practice can be explained and assigned to different practical aspects.

Despite local differences between the results from simulations and experiments are present, it is concluded that the models used in the simulations provide sufficient accurate information. The global changes predicted with the simulations are also obtained with the experiments. Local resonances might be present in the presented frequency response functions, they will however not appear in case the positioning system is operated. The operation frequency of the positioning system is namely limited to frequencies smaller than the lowest resonance or anti-resonance frequency, as motivated in the previous chapter. Even in case six actuators are used the frequency of mode six is limiting the performance. Sufficient time should be present between successive positioning tasks, as explained in the previous chapter.

The results obtained with the simulations are assumed accurate enough for the conceptual design. However, robustness has to be considered in the design...
phase as well. For instance in theory the frequency of the mode that is limiting the performance is $35 \, Hz$, while in practice the frequency of that mode appears at $27 \, Hz$.

5.4 Conclusion

Overactuation is an extension of the parallel actuation discussed in the previous chapter. The damping capabilities of the actuators used for positioning, such as applied in serial positioning system, remains limited. Initially the actuator forces are such that consecutive modes are not excited. In the design phase the actuator locations can be considered nodes of the vibration modes. The vibration amplitude after a positioning task can then be determined with Equation 4.29. In case of open loop operation the error after a positioning task for open loop operation can be determined with the same equation.

When the number of parallel actuators is increased, the location and the force ratio during open loop operation is such that the excitation of consecutive numbered free vibration modes is minimized. As result of the decreased deformation at low frequencies the lowest anti-resonance frequency of open loop operation shifts to higher values. The resonance frequencies remain the same in case of mass less actuators. In practice the system dynamics are the results of the complex interaction between the control forces, the actuator mass, and the structure. To obtain an upper limit of the performance the case is considered where the actuator locations form the nodes. The highest possible resonance and anti-resonance frequency is obtained when the actuators are located in the nodes of consecutive free vibration modes. In order to also minimize the excitation of some higher numbered modes, the actuator location can slightly be shifted when due to symmetry the first mode of the controlled system with nodes at the actuator location is not excited. This results however only in small improvements.

Thanks to the use of an increased number of actuators the deformation during accelerations is decreased and thus the accuracy of a single positioning task within the same motion time can be increased. When the tolerance specification is maintained the motion time can be decreased. Another option is to reduce the structural mass because less structural stiffness is required. This will be treated in the next chapter.

The performed experiments indicate that trends are well predicted, but local differences are present between simulation and experiment. These local differences can however be explained with physical understanding about the dynamic behaviour of the system. It is therefore concluded that the simulations can be used to obtain information and knowledge about parallel positioning systems in the design phase. Despite, the conceptual parallel positioning systems can be modelled in the design phase, the design should be robust for limited model accuracy in order to guarantee an upper limit of the performance.
Chapter 6

Mass reduction of positioning systems by overactuation

6.1 Introduction

Overactuation is researched in order to reduce mass without losing precision and speed of the positioning system. The tradeoff between mass that limits the operating speed, and mass required for precision might be shifted as a result of overactuation. The effect on the moving mass as result of additional parallel actuators is determined while the speed and precision requirements remain constant. Reduction of the moving mass, is often accompanied with smaller actuator forces, and accompanied actuator mass, see [239]. In that case mass reduction at the start of the mass spiral, thus the moving mass closest by the product, is most effective, because that results in a waterfall of mass reduction. Prior to considering the whole mass of positioning systems, it is in this thesis studied if the moving mass can be reduced with the use of additional parallel actuators.

The application of an increased number of parallel actuators results primarily in:

1. Reduction of the mechanical stiffness requirements of the product carrier. The structural mass used to create the required stiffness, can as a consequence be reduced.

2. Reduction of the required actuator force of the individual actuators.

3. Changed control structure [224].

Reduction of the required structural mass, as result of the reduced structural stiffness requirements, is accompanied with a smaller required total actuator force to obtain the required acceleration. A smaller required total actuator force might be accompanied with smaller actuator systems. In smaller actuator systems the mass required to generate the actual force is reduced as well. A reduc-
tion of the actuator force also results in reduced power dissipation, what might be beneficial to reduce the error as result of the thermomechanic behaviour. From this point of view it might be concluded that the mass of the positioning system always profits from the application of an increased number of parallel actuators. Proper operation of the system might require additional mass besides the mass required to generate actuator force at an instant time, and mass required to provide structural stiffness. E.g. mass required to create a guidance, or mass required to provide force over the total stroke. The mass of an individual actuator systems is therefore expressed as:

1. Material used to generate the required force [148].
2. Structural parts required for proper operation of the individual actuator. The requirements for these individual parts is also affected by the requirements and configuration of the total system.

Even though the mass of the individual actuator systems is reduced as result of overactuation, the total actuator mass at a certain moment even increases as result of the increased number of parallel actuators. The possible mass reduction with an increased number of parallel actuators is thus limited. In this chapter the effect on the moving mass as result of an increased number of actuators is shown for the two benchmark systems. The aspects discussed in the previous chapters are used to relate the mass of the individual elements to the number of parallel actuators used.

### 6.2 Minimum mass positioning system

The mutual dependency of many parameters in parallel positioning systems complicates the design for minimum mass [256]. The requirements for the different parts depend on the total configuration, while the positioning performance of the complete configuration depends on the individual parts. Despite the thermo-mechanic, static, dynamic and control aspects of a positioning system should be considered as well, in this thesis is only focussed on dynamical changes as result of additional parallel actuators. The dynamical properties are initially assumed to change most significant as result of mass and stiffness changes.

In order to estimate the mass and its dependency on the number of actuators first order approximations are in this chapter performed instead of tailored specific design solutions. The focus is on obtaining knowledge and insight in the effects of mass as result of overactuation. More accurate results might be obtained when more details are incorporated in the design, and the models [234]. The first order approximation should however reveal all relevant aspects at system level. The obtained knowledge can be used to specify tolerances, and to cope with scarce, time, money and capacity, when designed in more detail.

**Actuator configuration of the positioning system**

The number of parallel actuators used in a positioning systems affects the con-
figuration. In order to determine the total mass, the mass of the different elements needs to be related to the number of actuators. Therefore it should be clear how the different elements, and thus parameters, scale with the number of actuators. Similar to the scaling of the voice coils, as mentioned in section 3.5, the independent variables are assumed to be constant or vary in proportion to a known power of scale. This variation with scale can either be due to a physical law and geometry or due to man made design requirements (such as design rules or tolerances). In this chapter the mass is related to the number of actuators, for different configurations and thus different scaling.

The benchmark systems are used to show the effect of overactuation on the moving mass of the system. The requirements of the positioning system are specified in Chapter 1 and remain unchanged. Despite scaling, a certain constant mass is assumed required, for instance for the guidance. The ratio between constant mass and scaleable mass changes as well when the number of actuators is varied. As result an optimum might be present. The presence of constant mass and scaleable mass can be motivated with understanding of the complete system. The required configuration is affected by the assumptions and requirements of the system and this might also affect the possible mass reduction as result of overactuation.

The effects of the assumptions on the mass is for instance obtained in Figure 3.14 and 3.15 can for instance be compared. The system in Figure 3.14 allows relative small airgaps of the actuators. The airgap depends on the guidance of the actuators itself, what might be related to the size of the actuators. In Figure 3.15 the actuators are attached direct to the ‘product carrier’ without own guidance. The airgap between stator and mover depends on the guidance of the ‘product carrier’ and is thus independent on the size of the actuators itself. Thus even though the required actuator force might be reduced as result of more actuators, the airgap is not reduced and this affects the attainable mass reduction of the actuator.

The mass of the product carrier is determined dependent on the number of actuators. The deformation in a static acceleration field is initially used for the design of the product carrier. After evaluation of the dynamic behaviour the product carrier is adapted. When the product carrier mass is known other mass is assigned to the product carrier, that does not scale with the number of actuators. For instance part of the guidance or elements used to connect the product to the product carrier. Initially only the conductive mass of the drives is assumed to scale. Besides the conductive mass required to provide the actuator force, the other requirements for the drive system are assumed to be independent on the actuator force. The clearance and guidance mass are thus assumed independent on the required force.

The additional actuators need to be controlled as well. In case of open loop operation it might be sufficient to change the force ratio, see Chapter 4. The system is however assumed to be operated in closed loop, because it is uncertain if all aspects are incorporated in the deterministic description used for open loop control. In order to reduce the complexity of the control structure, decentralized co-located control is in this thesis applied. The stability of the
individual control loops is ensured as result of the lack of mechanical flexibility between the sensor and the actuator.

### 6.3 Method to determine the mass of overactuated positioning systems

In a positioning system stiffness is required to resist the effect of loading, including inertia forces. Even though parallel positioning systems are considered, the stiffness in these systems is the result of a serial arrangement of elements required to create stiffness, e.g. the serial mechanical- and control stiffness. The stiffness of the mechanical part is in these systems often dominated by bending stiffness. With bending stiffness is in this case referred to stiffness that is obtained as result of stress and deformation in a direction perpendicular to the loading. The structural material can not be assigned to individual stiffness requirements, e.g. bending, shear, or torsion. Nevertheless, the material mainly used to create the required mechanical bending stiffness is in this thesis referred to as structural material. The amount of structural material required in a parallel positioning system depends, among others, on the number of parallel actuators.

Besides the structural mass, other elements, that are required for proper operation of the system, also contribute to the total moving mass. The moving mass of these additional elements is referred to as support mass. A part of the support mass is directly related to the structural mass, e.g. the mass used to generate force and accelerate the system. The remainder of the support mass is not related to the structural mass, however still required for proper operation of the system. Application of an increased number of parallel actuators affects both the structural mass and the support mass.

**Positioning system split up in elements**

The product requires positioning with respect to some other system, e.g. other product or tool. Both sides of the positioning system might be moved in order to obtain the required relative position. However, in this thesis only one side of the positioning system is assumed movable. The terms product and product carrier can be replaced with other terms such as tool and tool holder when for instance the chisel of a fast tool servo has to be positioned [283].

The product is considered as given fact, with dictated parameters that can not be changed. A positioning system can be designed for each particular product. The connection between product and product carrier depends also on the specific application and should be taken in account as well. Furthermore the interaction between product and the product carrier, e.g. the dynamics, can be considered in detail for each specific product and positioning system. In this thesis the effects on the moving mass due to overactuation in general are described. In the first order approximation it is assumed the product only possesses mass, while the stiffness is neglected. The stiffness of the product can be neglected when the stiffness of the product carrier is much larger. However when the mass, and thus stiffness of the product carrier is reduced, the product
stiffness might then start to affect the total stiffness. This complicated interaction depends as well on the specific application, and is not considered in this first order approximation. The product is thus only assumed to possess mass, does not possess any stiffness and is always and everywhere in contact with the product carrier. Despite these assumptions are valid over a limited range, they can for specific applications be taken in account later.

In order to determine the mass of the parallel actuated positioning system dependent on the number of actuators, and thus the configuration, the system is split in several subsystems:

- product that requires positioning; the mass is $m_{prod}$
- dynamic mass of the system $m_{dyn}$ that is subdivided in:
  - product carrier mass $m_{car}$; further divided in:
    * mass dependent on the number of actuators: $m_{struct}(n_{act})$
    * mass independent on the number of actuators: $m_{ind}$. For instance the parts of the guidance mass, or elements required to attach the product, are considered part of the product carrier and require acceleration as well.
  - a number of actuator systems to provide the required forces. The dynamic mass of individual drive systems, $m_{mov}$, see Equation 3.8, is determined and consists of:
    * mass dependent on the maximum required force of the actuator $m_{con}(F_{L\ max})$.
    * mass assumed independent on the required force $m_{fix}$ for instance the connection between mover and product carrier, cup mass, or the guidance mass that is connected to the product mover. The guidance and mover mass of the actuator, might for instance scale with the required stroke. The mass contained in this parameter might in practice be related to the required force. However, practical aspects limit scaling.
- static part of the system; the mass is $m_{stat}$. This mass is all mass not assigned to dynamic mass. For instance the static part of the actuators and parts of the guidance systems.

For serial or hybrid systems this division has to be extended, because the total mass of one system may form the payload mass of the other system. In this thesis only parallel (redundant) actuated systems are discussed. Focussed is on the effects on the dynamic mass ($m_{dyn}$) of the system. The static mass might be important in case of a serial machine. The dynamic part of the system requires accurate positioning. The accuracy of positioning systems might be improved thanks to overactuation. In literature about specific positioning systems, other names are sometimes used to refer to the dynamic mass, e.g. translator mass in case of a planar actuator [115]. In literature also other names are used to refer to the product carrier, e.g. working platform [65].
To seize the different elements the forces acting on the positioning systems have to be known. In the high speed operated high precision positioning system the inertia forces are assumed to contribute most to the total loading. The calculation of the forces starts with the inertia force as result of acceleration of the actual product. The specified stroke and positioning time require a certain acceleration. As mentioned in Chapter 4 in the design phase the required maximum acceleration is determined in case the acceleration force profile of the reference path contains only the fundamental harmonic. To obtain the required acceleration a force ($F_{eff}$) is exerted on the product:

$$F_{eff} = a \cdot m_{prod}$$

(6.1)

The total mass that requires acceleration in order to position the system consists of the product with mass ($m_{prod}$) but also the dynamic mass of the system ($m_{dyn}$). The dynamic mass ($m_{dyn}$) consists of the product carrier mass and the summation of the individual dynamic mass of the actuators ($m_{mov}$). In this first order approximation the product mass and the product carrier mass are homogeneous. The inertia force is in this first order approximation assumed equally divided over the actuators. Due to this assumption equal drives can be used. The dynamic mass of the system consists of carrier mass and $n_{act}$ times the mover mass:

$$m_{dyn}(n_{act}) = m_{car}(n_{act}) + n_{act} \cdot m_{mov}(n_{act})$$

(6.2)

The mass of the mover is related to the maximum required force $F_{L\ max}$ expressed in Equation 3.8.

The stiffness of the product carrier should be sufficient to fulfil the accuracy specification. Stiffness is obtained at the cost of mass. The main loading is the inertia force, for this reason the mass of the product carrier is kept as low as possible. In order to limit the required actuator force, or to obtain high operation speed, the mover mass is also as low as possible. The stiffness, and thus structural mass, of the product carrier can be reduced when more parallel actuators are used. Besides the structural mass required to provide stiffness, also the elements that are required for proper operation contribute to the carrier mass, e.g. carrier guidance or product attachment. The mass of these elements is assumed not to change when more actuators are used. Furthermore is the guidance in this first order approximation assumed not to add stiffness in motion direction, as mentioned in Chapter 3.

The effective required force of each actuator ($F_{act}$), see Equation 3.13, is in this first order approximation determined with:

$$F_{act} = \frac{a(m_{prod} + m_{car})}{n_{act}} = \frac{F_{eff} + a \cdot m_{car}}{n_{act}}$$

(6.3)

Assumed is thus that each actuator should exert the same force on the carrier. In Equation 6.3 the force $F_{act}$ of each actuator is calculated. As shown in Chapter 3 the static force of the actuator $F_{L}$ should be larger to obtain the required actuator force $F_{act}$. How much larger depends on the mass of the mover and
the required acceleration.

The number of actuators that result in the lowest dynamic mass is that number of actuators that correspond with the minimum mass is determined with:

\[ \min \left( m_{\text{dyn}}(n_{\text{act}}) \right) \]  

(6.4)

**Example; the parallel beam system split in elements**

In Figure 6.1 the positioning system is split in elements. Several elements are present only once, such as the product carrier and its guidance. These elements are part of the subsystem carrier. Other elements are present twice, such as the drive and its guidance, or the connection to the carrier. The properties of these elements is assigned to two equal subsystems (movers). Even though a mover guidance is drawn in Figure 6.1 it is in some cases not necessarily needed. The requirements for the mover guidance, the product carrier guidance, and the connection between mover and product carrier depends on the configuration, e.g. Figure 3.14 and 3.15. If the connection between mover and product carrier constraints all the mutual degrees of freedom no mover guidance is required. This might however result in relatively large airgaps when compared with the situation where each mover has its own guidance.

![Figure 6.1: The positioning system of Figures 3.14 and 3.15 where two actuators are used is split in elements. One product carrier including its guidance and two equal drives. Even though an individual guidance system is drawn for the drives, the guidance is not necessarily present. In that case the connection between carrier and drive constrains the 6 dof.](image)
Because a product with homogeneous mass distribution is used the acceleration of the product results in a continuous distributed load on the product carrier. Even though it is a distributed load on the product carrier only the summation of this load is drawn $F_{\text{eff}}$ in the Figures 6.1, 6.2 and 6.3. The summation equals the inertia force of the product.

In the same way as in Figure 6.1 also the application of more actuators can be modelled. See Figure 6.2 and 6.3 for the application of three and four actuators.

![Figure 6.2: Three actuators are used for two rigid body degrees of freedom, this is over-actuation](image1)

![Figure 6.3: Four actuators are used for two rigid body degrees of freedom, this is over-actuation](image2)

As determined in Chapters 2, 4 and 5 the mass of a homogeneous carrier is almost equally divided over the actuators when they are located in their optimal locations. The required force of the actuators, to accelerate the product and the carrier, is thus almost equal. The guidance of the carrier is however not equally distributed over the carrier. Dependent on the location of the guidance and the drives, a force difference between the actuators is created. This is however in this first order approximation not taken into account.

### 6.4 Mass of the overactuated benchmark systems

The design is started with an initial proposed product carrier that is regarded to be able to meet the stiffness requirements. Thereafter the total dynamic mass, e.g. product carrier mass and the moving mass of the actuators, is determined.
Chapter 6. Mass reduction of positioning systems by overactuation

The effect of the initial design constraints, and decisions on the mass is indicated in the next paragraph. Via iteration of analysis and design adaptations a minimum mass product carrier is determined for the applied constraints and assumptions.

Thereafter the drives are added. The scaling of the voice coil actuators is based on heat reduction as result of convection. The mass is determined with the drives connected to the product carrier. In case the the stroke $h_m$ and motion time $t_m$ are known the required actuator forces can be determined from the uncontrolled system. The force vector is such that consecutive vibration modes are not excited, while the rigid body is positioned. With the proper actuators added, the open loop performance can be determined with Equation 4.23. For minimum mass positioning system the location of the actuators is such that the lowest free vibration modes are not excited during positioning.

To ensure the required position is obtained, closed loop operation is applied. A reference profile is created of which the acceleration content contains only one harmonic with a period that is equal to the positioning time. The magnitude of the acceleration amplitude depends on the stroke and the positioning time. The performance of the positioning system can then be determined with Equation 4.29. The actuators are dimensioned using the forces that are calculated from open loop operation.

Despite the opportunity arises to optimize the results of the interaction between structural system and control system, a limited amount of time is assumed present to create designs [88, 161]. Therefore the product carrier, beam and plate, are dimensioned such that the performance specifications are met independent on the complex interaction of control and structural properties.

6.4.1 Overactuated beam positioning system

The properties of the product and the requirements of positioning system are specified in Chapter 1. The product specifications are: length $L = 500 \text{ mm}$, width $b = 20 \text{ mm}$, height $h = 0.5 \text{ mm}$, density $\rho = 3000 \text{ kg/m}^3$ thus weighing 15 g. The required stroke is $h_m = 1 \text{ mm}$, with a tolerance specified as $\pm 25 \text{ µm}$ has to be obtained within a motion time $t_m = 20 \text{ ms}$. A reference profile is applied where the acceleration consists of only the fundamental harmonic. The amplitude of the acceleration is calculated with Equation 4.26 and is $\ddot{x}_1 = h_m \frac{2\pi}{t_m^2} = 15.7 \text{ m/s}^2$. The nett effective force required to accelerate the product is $F_{\text{eff}} = m_{\text{prod}} \cdot \ddot{x}_1 = 0.236 \text{ N}$, Equation 6.1.

Simplifying assumptions are made to obtain insight in the effect of the mass as result of an increased number of actuators. For instance the product stiffness is assumed negligible and the product sticks to the product carrier. The product carrier is made of steel and only the height can be varied. The product carrier is designed with the assumption that the actuator locations form the nodes. This initial assumption is used because information about guidance and actuator mass is unknown as well as the control. The calculation method is only briefly mentioned.
6.4.1.1 Product carrier mass

The effect of actuator and guidance mass is neglected by assuming the guidance elements are located at the actuator location, and the actuator locations form the nodes. With Equation 4.27 the vibration amplitude as result of a sinusoidal acceleration profile is related to the steady state deformation in a constant acceleration field of the same amplitude. The initial proposed design is therefore such that the deformation as result of the product mass, product carrier mass and product carrier stiffness is \(25\mu m\) when placed in a constant acceleration field of \(a = 15.7 \, m/s^2\).

Because a solid steel beam is used the dependency of the mass and the stiffness on the height can be used to calculate the required height, without tailoring the design to this specific problem. In a static acceleration field the distributed load due to the product is \(w_p = 15.7 \cdot 2 \times 10^{-3} \cdot 3000 \cdot 0.5 \times 10^{-3} = 0.471 \, N/m\). The total distributed load also depends on the inertia load of the beam. The inertia force of the beam is dependent on the height \(w_c = 15.7 \cdot 7850 \cdot h \cdot 20 \times 10^{-3} \, N/m\). The total distributed load is then \(w = w_p + w_c = 15.7 \cdot 7850 \cdot h \cdot 20 \times 10^{-3} + 0.471 \, N/m\).

Actuators located at both sides of the beam

The dynamic behaviour is dependent on the location of the actuators. The stiffness of the beam is dependent on the height as \(I = bh^3/12 \, [m^4]\). An initial beam height is determined from the static deformation in acceleration field, which is \(\delta = 5 \cdot w \cdot 0.5^4/384/E/I = 25 \, \mu m\). From an iterative calculation the required beam height is \(h = 15.2 \, mm\). A beam with this height results in a deflection of \(25 \, \mu m\) as result of the loading in an acceleration field of \(15.7 \, m/s^2\). In case this beam is used in the positioning system the envelope of the vibration amplitude is determined with Equation 4.29. ANSYS © and the Equations 4.21, 4.22 and 4.15 are used to calculate the different parameters in Equation 4.29. The error envelope is presented in Table 6.1. In Table 6.1 only the contribution of the lower numbered modes is indicated because of their relative large amplitude compared to higher numbered modes. When a beam with a height of \(15.7 \, mm\) is used the summation of the vibration amplitudes is smaller than the specified tolerance of \(25 \, \mu m\). The height is therefore reduced. After a few iteration steps of the mechanical system a steel beam with a height of \(14.4 \, mm\) is proposed. The error envelopes are calculated and presented in Table 6.1. From the results in Table 6.1 it is concluded the total error after positioning remains within the specified tolerance of \(25 \, \mu m\) and a beam with a height of \(14.4 \, mm\) is thus able to fulfil the requirements.

Two actuators in the nodes of the first elastic mode

An initial beam height is calculated using the the static deflection in a constant acceleration field of \(15.7 \, m/s^2\). The deflection of a cantilever of length \(L = 112 \, mm\) has to remain within the specified tolerance of \(25 \, \mu m\). The minimum required beam height from the static deflection is \(2.2 \, mm\). The envelope of the vibration error is calculated and presented in Table 6.1. In Table 6.1 the total error after positioning is smaller than \(25 \, \mu m\) and thus fulfils the specifications. The envelope of the vibration amplitude is also calculated in case the height of
Chapter 6. Mass reduction of positioning systems by overactuation

The location of the actuators w.r.t. the side

<table>
<thead>
<tr>
<th>h [mm]</th>
<th>( f_1 ) [Hz]</th>
<th>( f_2 ) [Hz]</th>
<th>( f_3 ) [Hz]</th>
<th>( \Phi_1^T M_L \hat{a}/m_1 ) [-]</th>
<th>( \Phi_2^T M_L \hat{a}/m_2 ) [-]</th>
<th>( \Phi_3^T M_L \hat{a}/m_3 ) [-]</th>
<th>( y ) [mm]</th>
<th>( \Phi_1 ) [-]</th>
<th>( \Phi_2 ) [-]</th>
<th>( \Phi_3 ) [-]</th>
<th>( u_{01} ) [( \mu )m]</th>
<th>( u_{02} ) [( \mu )m]</th>
<th>( u_{03} ) [( \mu )m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.2</td>
<td>14.4</td>
<td>2.2</td>
<td>2.1</td>
<td>0.99</td>
<td>0.95</td>
<td>0</td>
<td>250</td>
<td>1.29</td>
<td>1.32</td>
<td>4.61</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>112</td>
<td>142</td>
<td>134</td>
<td>45</td>
<td>45</td>
<td>0.99</td>
<td>0.95</td>
<td>0</td>
<td>250</td>
<td>1.29</td>
<td>1.32</td>
<td>4.61</td>
<td>1.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 6.1: The envelope of the error of the first three modes is shown in the last three rows for different heights of the product carrier. These errors are calculated with Equation 4.29. The results from ANSYS® are required to determine the error envelope, and are shown to be able to calculate the required values of Equations 4.21, 4.22 and 4.15. The parameter \( y \) represents the location of interest; \( y = 250 \text{ mm} \) corresponds to the centre, \( y = 0 \text{ mm} \) to the side.

The beam is reduced. This beam is however not able to fulfil the requirements and therefore a beam with a height of 2.2 mm has to be used.

Increased number of actuators

The height of the product carrier is calculated dependent on the location and the number of actuators. In Table 6.2 the required product carrier height and mass is presented. The number of actuators is indicated in the first column. Their location is indicated in the consecutive columns with the amount of millimetres from the left side of the beam in Figure 4.10.

<table>
<thead>
<tr>
<th>( n_{act} )</th>
<th>act1 [mm]</th>
<th>act2 [mm]</th>
<th>act3 [mm]</th>
<th>act4 [mm]</th>
<th>act5 [mm]</th>
<th>beam properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>height [mm]</td>
<td>mass [g]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>500.0</td>
<td>14.4</td>
<td>1130.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>112.0</td>
<td>388.0</td>
<td>2.2</td>
<td>172.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>66.1</td>
<td>250.0</td>
<td>1.3</td>
<td>102.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>72.0</td>
<td>250.0</td>
<td>0.95</td>
<td>74.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>47.2</td>
<td>177.9</td>
<td>1.65</td>
<td>51.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>53.0</td>
<td>184.0</td>
<td>0.55</td>
<td>43.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>36.7</td>
<td>138.4</td>
<td>0.40</td>
<td>31.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2: The height and mass of a product carrier, a steel beam with a length of 500 mm and width \( b = 20 \text{ mm} \), is determined as function of the number of actuators and the location of the actuators. The number of actuators is expressed in the first column. Their location is indicated in millimetres from the left side of the beam in Figure 4.10. When no value is indicated no actuator is present.
In Table 6.2 can be observed that as result of an increased number of actuators the beam height and mass is reduced. In Table 6.3 another actuator location is indicated. A symmetric system is created, where two actuators are located at the outside. The remaining actuators divide the beam in parts of equal length.

<table>
<thead>
<tr>
<th>n_{act}</th>
<th>height [mm]</th>
<th>mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14.4</td>
<td>1130.4</td>
</tr>
<tr>
<td>3</td>
<td>2.40</td>
<td>188.4</td>
</tr>
<tr>
<td>4</td>
<td>1.25</td>
<td>98.1</td>
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<td>5</td>
<td>0.73</td>
<td>57.3</td>
</tr>
<tr>
<td>6</td>
<td>0.48</td>
<td>37.7</td>
</tr>
<tr>
<td>7</td>
<td>0.36</td>
<td>28.3</td>
</tr>
<tr>
<td>8</td>
<td>0.28</td>
<td>22.0</td>
</tr>
<tr>
<td>9</td>
<td>0.22</td>
<td>17.3</td>
</tr>
<tr>
<td>10</td>
<td>0.19</td>
<td>14.9</td>
</tr>
</tbody>
</table>

Table 6.3: Beam product carrier mass as function of the number of actuators. The number of actuators is indicated in the first column. A symmetric system is created where two actuators are located at the outside of the beam. The remaining actuators are located such that the distance between the two outer actuators is divided in parts of equal length. The dimensions of the steel beam are: length 500 mm, width 20 mm.

From Table 6.2 and 6.3 it is concluded that the mass of the product carrier depends on the number of actuators and their location. The minimum required mass of the beam product carrier as function of the number of actuators is plotted in Figure 6.4. The actuators are assumed to be located such that the vibration amplitude of the dynamic error is minimal.

Figure 6.4: Carrier mass as function of the number of actuators. A support area of 500 × 20 mm² is created with a solid steel beam. The minimum product carrier mass from Table 6.2 is used, in case of 3 actuators the product carrier weights 74.6 g. The guidance of the product carrier is assumed to weight 5 g.

A positioning system not only consists of a product carrier but also of guidances and other elements. As guidance for the product carrier a lightweight elastic guidance is used, built with only rods. The guidance of the product carrier is assumed to weight 5 g. This mass is independent on the number of actuators and does not change when the number of actuators is changed. The mass of
the product carrier guidance is as well as the mass of the product also plot in Figure 6.4.

6.4.1.2 Mass of the actuators

The mass summation in Figure 6.4 multiplied with the required acceleration of \( a = 15.7 \, m/s^2 \) results in the total required actuator force \( \Sigma F_{\text{act}} \). To determine the mass of the actuators the scaling of the actuators has to be known. For every number of actuators a different actuator might be selected. However, the stroke and clearance have to be equal in every case. Actuators that fulfil this requirements are not available. Therefore an existing actuator that meets the specification of the initial system is selected, and thereafter scaled according to the scaling laws presented in Chapter 3. The conductive mass is assumed to scale with the static actuator force as \( m_{\text{con}} \propto F^{6/5} \). The scaling law is assumed to be valid only over a limited range around the original actuator specifications. The actuator used for scaling is an actuator of the company Bei-Kimco [14]; type: LA08-10-000A with a stroke of 2 mm. According to the specifications the actuator is able to exert a static force of 2 N with a mover mass of 8 g in total. This 8 g is assumed to consists of 3 g conductive material \( (m_{\text{con}}) \) and 5 g other mass that does not scale \( (m_{\text{fix}}) \). The weight of the guidance and the connection between the mover and the carrier is assumed negligible. The mover mass of the actuator as function of the static actuator force that can be generated is plotted in Figure 6.5.

In Figure 6.5 it can be observed that in case the actuator should be able to generate 2 N the mover mass is 8 g. This consists of 5 g static guidance mass and 3 g conductive material. This matches exactly with the specifications of the original actuator.

The generated actuator force can be used to accelerate a load. Not only the
load is accelerated but also the mover mass of the actuator. The effective force of the actuator to accelerate the actual load is determined. This is obtained by subtraction of the inertia force of the mover from the actuator force. When an acceleration of $15.7 \, m/s^2$ is required and the mover weights $8 \, g$, the inertia is $0.13 \, N$. The actuator generates $2 \, N$, what results in an effective actuator force of $F_{\text{act}} = 1.87 \, N$. The effective actuator force for an acceleration of $15.7 \, m/s^2$ is plotted in Figure 6.6 as function of the mover mass of an actuator.

### 6.4.1.3 Total dynamic mass of the positioning system

As mentioned the total required actuator force $\Sigma F_{\text{act}}$ can be determined from the mass summation in Figure 6.4 multiplied with the required acceleration of $a = 15.7 \, m/s^2$. In this example is assumed each actuator should exert an equal force on the system. The total required actuator force is divided by the number of actuators in order to determine the required effective actuator force per actuator. With Figure 6.6 the required mass of each mover can be determined. The total mover mass is obtained by multiplication of each mover mass with the total number of actuators used.

The mass of the positioning system can now be determined with Equation 6.2. The dynamic mass as function of the number of actuators used, is plotted in Figure 6.7.

![Figure 6.7: Moving mass of the positioning system as function of the number of actuators](image)

The number of actuators that result in the lightest construction is that number of actuators that correspond with the minimum mass in Figure 6.7, see Equation 6.4. In this case 6 actuators result in the least dynamic mass ($57 \, g$) of the positioning system. When less actuators are used the mass of the carrier increases what result in an increased total actuator force to obtain the required acceleration. When more than 6 actuators are used the mass is not reduced anymore. In case of 7 actuators the mass is $58 \, g$. The total mass of the movers increases when more actuators are used. This is mainly because a mover always has a certain mass, even when almost no static force is required. The mass of the actuators increases with approximately $5 \, g$ per additional actuator when more than six actuators are used while the product carrier mass reduces only a small amount.
6.4.1.4 Increased amount of drive guidance mass

In this case the effect of a changed guidance is examined. In the previous example the mass of the guidance and the cup resulted in a fixed mass of each drive of $m_{fix} = 5g$. When another guidance is used for the actuators the mass of each mover changes. Now is assumed that the constant mass of each mover is $m_{fix} = 15g$. The same actuator as used before is applied and scaled. Due to the additional mass of the mover guidance the force as function of the mover mass presented in Figure 6.8 changes. The mover mass as function of the generated actuator force is plotted in Figure 6.9. As result of the increased mover mass also the effective actuator force $F_{act}$ for an acceleration of 15.7 $m/s^2$ is changed. When 3 g conductive material is used, still 2 N is generated. However the mover mass is 18 g, what results in an inertia force of 0.28 N when accelerated with 15.7 $m/s^2$. The effective actuator force is therefore $F_{act} = 1.72$ N when the mover weights 18 g. The effective actuator force for an acceleration of 15.7 $m/s^2$ is plotted in Figure 6.9 as function of the mover mass of an actuator.

The mass of the positioning system as function of the number of actuators is plotted in Figure 6.10. Due to the relatively large mover mass of each actuator the minimum mass is shifted to 4 actuators. When more actuators are added the mass increases with approximately 15 g per extra actuator.

6.4.1.5 Further increased amount of drive guidance mass

In this case another guidance of the mover is used. The ratio between mass that generates force and mass required for proper operation is as a result also changed. The mass of the guidance of each mover is assumed to be 150 g. The same actuator as used before is applied and scaled. Due to the additional mass of the mover guidance the force as function of the mover mass presented in Figure 6.8 changes. The mover mass as function of the generated actuator force is
plotted in Figure 6.11. As result of the increased mover mass also the effective actuator force $F_{\text{act}}$ for an acceleration of $15.7 \, \text{m/s}^2$ is changed. When 3 g conductive material is used, still 2 N is generated. However the mover mass is 153 g, what results in an inertia force of 2.4 N when accelerated with $15.7 \, \text{m/s}^2$. The effective actuator force is therefore $F_{\text{act}} = -0.4 \, \text{N}$ when the mover weights 153 g. This negative effective force means that the generated force is not sufficient to accelerate the drive itself to a value of $15.7 \, \text{m/s}^2$. To obtain the required acceleration more conductive mass has to be used. The effective actuator force for an acceleration of $15.7 \, \text{m/s}^2$ is plotted in Figure 6.11 as function of the mover mass of an actuator.

The mass of the positioning system as function of the number of actuators is plotted in Figure 6.13. Due to the relatively large mover mass of each actuator the minimum mass is shifted to a not overactuated system with 2 actuators. When more actuators are added the mass increases with approximately 150 g.
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2

Figure 6.13: Positioning system mass as function of the number of actuators

as result of an additional actuator.

6.4.1.6 Sandwich material product carrier

Instead of a solid beam, sandwich material is now applied. The optimization of the product carrier is then much more difficult. In the previous example only the height was adapted and then the dynamic behaviour was analyzed. In case of the sandwich beam the number of variables that need to be optimized is larger. Initially the beam is dimensioned in a static acceleration field, as mentioned in Chapter 2. Thereafter the performance is evaluated using the methods presented in Chapter 4 and 5. Then an iteration step is made to adapt the dimensions of the sandwich beam. The dimensions of the sandwich beam remain determined in a static acceleration field. Initially 25 µm was allowed as deflection, during the iteration the allowed deformation is changed. When the evaluation indicates the beam does not fulfil the requirements, the deformation in a static acceleration field is reduced, and a new beam is designed.

Compared with the results in Chapter 2, the dimension of the beam in Table 6.4 are changed. This is the result of the increased acceleration, and due to the dynamic system in interaction with the applied force profile.

In the Figures 6.4 and 6.14 the same amount of additional mass \( m_{\text{ind}} \) is used for the product carrier. This mass is for instance used for the guidance of the product carrier or to connect the product. In Figure 6.4 the amount of constant mass \( m_{\text{ind}} \) is initially small compared to the carrier mass \( m_{\text{struct}} \). Because the mass of the product carrier is in Figure 6.4 dominated by the structural mass \( m_{\text{struct}} \gg m_{\text{ind}} \) the amount of mass reduction is initially large. Due to the mass reduction this ratio changes and therefore the amount of mass reduction decreases when more actuators are used. In Figure 6.14 the ratio between structural mass \( m_{\text{struct}} \) and other mass \( m_{\text{ind}} \) is initially different than in Figure 6.4. The mass reduction as result of additional additional actuators is with the sandwich beam therefore much smaller than in case of a solid beam.

The dynamic mass of the system is determined for the case the same drive system as discussed in Figure 6.5 and 6.6 is used. The results of the dynamic mass are represented in Figure 6.15.
Table 6.4: The dimensions of an optimized sandwich panel with a length of 500 mm and width \( b = 20 \text{ mm} \), are determined as function of the number of actuators and the location of the actuators. The number of actuators is expressed in the first column. Their location is indicated in millimetres from the left side of the beam in Figure 4.10. When no value is indicated no actuator is present.

![Graph 6.14: Product carrier mass in case a sandwich beam is used. The results from Table 6.4 are used.](image)

![Graph 6.15: Dynamic mass of the positioning system in case a sandwich beam is used. A minimum mass is obtained when three actuators are used.](image)

The dynamic mass of the sandwich system in Figure 6.15 is compared with the dynamic mass of the steel system, Figure 6.7. The optimum number of actuator is shifted toward a much lower number of actuators in case a sandwich beam is used, see Figure 6.15. This is mainly the result of the changed ratio between
structural mass required for stiffness and other mass. The absolute amount of mass reduction is less in case overactuation is applied to a sandwich system, compared with a steel system. The changed ratio between mass required for structural stiffness and other mass results in another number of actuators that have to be used to obtain a minimum mass system.

6.4.2 Plate positioning system

A circular product with a radius of \( r = 150 \, mm \), and a height \( h = 0.5 \, mm \) has to be positioned, see Chapter 1. The density of the product is \( \rho = 3000 \, kg/m^3 \) what results in a mass of 106 g. In this thesis the product stiffness is neglected as well as the connection to the product carrier. The assumptions are only used to indicate the effects on the mass as result of an increased number of actuators. The required stroke is \( h_{m} = 1 \, mm \), with a tolerance specified as \( \pm 25 \, \mu m \) has to be obtained within a motion time \( t_{m} = 20 \, ms \).

6.4.2.1 Mass of the product carrier

The product carrier is assumed to be constructed of brass plate with constant thickness. The required thickness of the plate is calculated for different actuator locations and different number of actuators.

In first consideration the actuators are located at the outer radius as for instance shown in Figure 4.23, 5.10, 5.15 and 5.19. The required height and thus mass of the product carrier is presented in Table 6.5. The amplitude of the remaining vibration depends on the location. In Table 6.5 the centre location is considered, indicated with \( u_{cen} \). Furthermore the error at the outer radius is calculated at an angle between two actuators. When for instance three actuators are used, the mutual angle between the actuators is \( 120^\circ \). The error is determined at \( 60^\circ \). This error is indicated with parameter \( u_{rad} \).

<table>
<thead>
<tr>
<th>( n_{act} )</th>
<th>thick [mm]</th>
<th>( u_{stat} ) [( \mu m )]</th>
<th>( u_{rad} ) [( \mu m )]</th>
<th>( u_{cen} ) [( \mu m )]</th>
<th>( f_1 ) [Hz]</th>
<th>mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5.2</td>
<td>28</td>
<td>( u_3 = 21 )</td>
<td>( u_3 = 25 )</td>
<td>131</td>
<td>3.099</td>
</tr>
<tr>
<td>4</td>
<td>4.3</td>
<td>29</td>
<td>( u_1 = 10 )</td>
<td>( u_1 = 25 )</td>
<td>140</td>
<td>2.562</td>
</tr>
<tr>
<td>5</td>
<td>4.0</td>
<td>29</td>
<td>( u_1 = 5 )</td>
<td>( u_1 = 25 )</td>
<td>143</td>
<td>2.384</td>
</tr>
<tr>
<td>6</td>
<td>3.8</td>
<td>30</td>
<td>( u_1 = 3 )</td>
<td>( u_1 = 25 )</td>
<td>142</td>
<td>2.264</td>
</tr>
<tr>
<td>8</td>
<td>3.7</td>
<td>30</td>
<td>( u_1 = 1 )</td>
<td>( u_1 = 25 )</td>
<td>142</td>
<td>2.205</td>
</tr>
<tr>
<td>15</td>
<td>3.7</td>
<td>30</td>
<td>( u_1 = 0 )</td>
<td>( u_1 = 23 )</td>
<td>144</td>
<td>2.205</td>
</tr>
<tr>
<td>15</td>
<td>3.6</td>
<td>31</td>
<td>( u_1 = 0 )</td>
<td>( u_1 = 26 )</td>
<td>141</td>
<td>2.145</td>
</tr>
</tbody>
</table>

Table 6.5: Product carrier mass when a brass plate is used as product carrier. The actuators are located at the outer diameter, e.g. Figure 4.23, 5.10, 5.15 and 5.19. The number of actuators is indicated with parameter \( n_{act} \).

Because the central deflection becomes the limiting factor, the case is considered with an additional central actuator. The other actuators remain at the outside. The number of actuators at the outside is indicated with parameter \( n_{act \, rad} \) in Table 6.6. The actuator location for some cases is shown in Figures,
5.11, 5.16 and 5.20. The required height and thus mass of the product carrier is presented in Table 6.6.

<table>
<thead>
<tr>
<th>$n_{\text{act rad}}$</th>
<th>thick [mm]</th>
<th>$u_{\text{stat}}$ [µm]</th>
<th>$u_{\text{rad}}$ [µm]</th>
<th>$u_{\text{cen}}$ [µm]</th>
<th>$f_1$ [Hz]</th>
<th>mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.1</td>
<td>30</td>
<td>$u_3 = 23$</td>
<td>$u_3 =$&lt; 25</td>
<td>77</td>
<td>1.847</td>
</tr>
<tr>
<td>4</td>
<td>2.1</td>
<td>31</td>
<td>$u_4 = 26$</td>
<td>$u_4 =$&lt; 25</td>
<td>82</td>
<td>1.251</td>
</tr>
<tr>
<td>5</td>
<td>1.6</td>
<td>28</td>
<td>$u_5 = 25$</td>
<td>$u_5 =$&lt; 25</td>
<td>105</td>
<td>0.953</td>
</tr>
<tr>
<td>6</td>
<td>1.4</td>
<td>27</td>
<td>$u_4 = 18$</td>
<td>$u_4 = 24$</td>
<td>120</td>
<td>0.834</td>
</tr>
<tr>
<td>10</td>
<td>1.2</td>
<td>30</td>
<td>$u_3 = 3$</td>
<td>$u_3 = 26$</td>
<td>121</td>
<td>0.715</td>
</tr>
</tbody>
</table>

Table 6.6: Product carrier mass when a brass plate is used as product carrier. One actuator is located at the centre the other actuators are located at the outer diameter. The number of actuators at the outer diameter is indicated with parameter $n_{\text{act rad}}$.

In Table 6.7 the actuators are located in a circular pattern as discussed in Table 6.5. However, in this case the actuators are located at a radius of 102 mm, which is optimal for minimum excitation of the sixth free mode as motivated in Chapter 5.

<table>
<thead>
<tr>
<th>$n_{\text{act}}$</th>
<th>thick [mm]</th>
<th>$u_{\text{stat}}$ [µm]</th>
<th>$u_{\text{rad}}$ [µm]</th>
<th>$u_{\text{cen}}$ [µm]</th>
<th>$f_1$ [Hz]</th>
<th>mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.5</td>
<td>30</td>
<td>$u_3 = 24$</td>
<td>$u_3 = 10$</td>
<td>73</td>
<td>1.490</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>30</td>
<td>$u_5 = 24$</td>
<td>$u_5 = 11$</td>
<td>58</td>
<td>0.894</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
<td>28</td>
<td>$u_6 = 23$</td>
<td>$u_6 = 17$</td>
<td>74</td>
<td>0.656</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>34</td>
<td>$u_7 = 25$</td>
<td>$u_7 = 26$</td>
<td>60</td>
<td>0.536</td>
</tr>
<tr>
<td>10</td>
<td>0.8</td>
<td>35</td>
<td>$u_6 = 27$</td>
<td>$u_6 = 26$</td>
<td>53</td>
<td>0.477</td>
</tr>
</tbody>
</table>

Table 6.7: Product carrier mass when a brass plate is used as product carrier. The actuators are located at a radius of 102 mm. The number of is indicated with parameter $n_{\text{act}}$.

For the product carrier the masses from Table 6.7 are used.

### 6.4.2.2 Mass of the actuators

For the actuators another voice coil actuator, designed and described by de Ruijter [217], is selected and scaled. The actuator consists of $m_{\text{con}} = 9$ g conductive mass and $m_{\text{fix}} = 8$ g other mass. With this mass the actuator is able to generate a force of 3.9 N.

Now the mass of the carrier is known as well as the mass of each mover the mass of the positioning system can be determined with Equation 6.2. The dynamic mass as function of the number of actuators used is plotted in Figure 6.19. The number of actuators that result in the lightest construction is that number of actuators that correspond with the minimum in Figure 6.19, see Equation 6.4. In this case 6 actuators result in the least dynamic mass of the positioning system. When less actuators are used the mass of the carrier increases what result in an increased total actuator force to obtain the required acceleration. When more than 6 actuators are used the mass reduction as result of an
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Figure 6.16: Carrier mass as function of the number of actuators. The carrier mass \( m_{\text{car}} \) consists mainly of structural mass \( m_{\text{struct}} \). A circular support with a radius of \( r = 150 \text{ mm} \) is created with a solid brass plate. The actuators are located for minimal deformation. This is at a radius of \( 102 \text{ mm} \). With three actuators at their optimal location the height is \( 2.5 \text{ mm} \), what results in a brass plate mass of 1490 g.

Figure 6.17: Conductive mass as function of the actuator force. The fixed mass is \( m_{\text{fix}} = 8 \text{ g} \).

Figure 6.18: Effective actuator force \( F_{\text{act}} \) as function of the mover mass when an acceleration of \( 15.7 \text{ m/s}^2 \) is required.

additional actuator decreases. On the other hand the total mass of the movers increases when more actuators are used. This is mainly because a mover always has a certain mass needed for proper operation, even when almost no static force is required.

6.4.2.3 Increased amount of drive guidance mass

In this case the effect of a changed guidance is examined. In the previous example the contribution in the mass of the mover was only a few grams. Now another mover guidance is assumed to be used. The product; the product carrier; the actuator; and the required acceleration; are not changed. The only
thing that is changed is the mass of each mover. The mass of the guidance is assumed to be \( m_{\text{fix}} = 100 \, \text{g} \). The actuator acceleration of a load mass of each mover as function of the static actuator force is plotted in Figure 6.20. The corresponding effective actuator force \( F_{\text{act}} \) as function of the mover mass is plotted in Figure 6.21.

The effective actuator force in Figure 6.21 is shifted downward compared to the force in Figure 3.13. This is the result of the increased mover mass. The mass of the positioning system as function of the number of actuators is plotted in Figure 6.22. The minimum mass is in this case obtained with four actuators. When more actuators are added the mass increases with approximately 100 g per extra actuator.
Optimization of a sandwich panel is relatively easy for a beam material, because analytical formulas are available. Due to the lack of analytical formulas of the deflection of point supported sandwich plates, optimization of these plates is much more complicated. Due to the application of sandwich material the ratio between structural mass that depends on the number of actuators $m_{\text{struct}}$ and the constant mass $m_{\text{ind}}$ is changed. The amount of mass reduction thanks to additional parallel actuators is, as a result, changed. The optimum is shifted towards a smaller number of actuators compared to the case with solid plates.

6.5 Annotation to the examples and assumptions

The minimum mass was determined under a number of simplifying assumptions. The research is directed less towards details of the individual components but more towards aspects of the integration. The mass of particular positioning systems might be reduced thanks to tailored optimization techniques that require more accurate calculations. In this thesis is however focussed on the possible mass reduction as result of overactuation. The benchmarks were only used to apply the theory and to obtain information and insight in the aspects that affect the mass of overactuated positioning systems. First order approximations are in this chapter used to reveal trends in the mass as result of overactuation. The qualitative information obtained from these trends might be used in the conceptual design phase. The quantitative accuracy might for instance profit from optimization and more accurate calculations. The presence of a global minimum and the trends are however not expected to change as result of improved calculation accuracy. A beam and plate with homogeneous height were used, because they reveal trends without obscuring the calculation with specific local optimization techniques.

Via iteration of synthesis and analysis with the methods mentioned in this thesis the optimum number of actuators might be determined for particular systems. In this chapter the actuator location is assumed to form nodes in order to
obtain an initial indication of the opportunities and limitations of the system. This information can thereafter be used to adapt and improve the design. The forces are known, thus the actuators can be dimensioned. The control might be optimized etc.. The initial assumption of nodes at the actuator locations is used as start for the initial design phase. According to [74]:“Good designs evolve rather than appear in their best form after the first pass.”

Scaling of the individual elements
The number of actuators that should be used for the least dynamic mass depends on the requirements and the configuration. The examples discussed in this chapters are mainly used to indicate the effect of the configuration on the mass scaling of the different element. The structural stiffness of the product carrier is direct related to the number of actuators. The mass of the product carrier depends among others on the required structural stiffness. A reduction of the required stiffness can thus be used to reduce the structural mass that is used for the stiffness. Besides mass used to obtain the required stiffness, also carrier mass is present that is used for other purposes and does not depend on the number of actuators. The relative amount of mass reduction of the product carrier depends on the ratio between the mass that is related to the number of actuators and the mass that is independent on the number of actuators.

The configuration of the system affects also the scaling of the individual drive systems. A reduced mass of the product carrier results in a reduction of the total actuator force to obtain the required acceleration. The required force of the individual actuators is furthermore also reduced as result of the load that is spread over the increased number of actuators. The mass reduction of the individual actuators depend on the configuration. The mass of the drive is related to the required force. Besides mass required to generate the force also other mass is present in the system that does not depend on the required force. The mass reduction of an individual drive as result of decreased forces is limited. The summation of the mass reduction of the individual drives should be less than the mass increase as result of the additional drive. Otherwise the total mass of the drive even increases as result of an increased number of actuators. Due to the application of sandwich materials or other lightweight structural material of elements the ratio between mass to provide stiffness and mass required for other purposes, for instance guidance, changes. In that case result the application of an additional actuator in a relative smaller mass reduction. The optimum number of actuators appears as result even at a lower number of actuators.

Effect of the configuration on the mass
The mass of the product carrier and the mass reduction as result of additional actuators depends on the ratio between structural mass that depends on the number of actuators $m_{\text{struct}}$, and the assumed other mass $(m_{\text{ind}})$. This additional mass $(m_{\text{ind}})$ is assumed independent on the number of actuators, and comprises for instance the guidance of the carrier, or elements to hold the product. In the tables and Figures 6.4, 6.14 and 6.16 it is shown the structural
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Mass that is required for the structural stiffness $m_{\text{struct}}$ is related to the number of actuators. However the additional mass $m_{\text{ind}}$ affects the dynamics as well and limit the mass reduction of the product carrier. The ratio between the mass $m_{\text{struct}}$ and $m_{\text{ind}}$ affects the possible mass reduction as result of additional actuators. An initial large amount of structural mass with respect to the constant mass ($m_{\text{struct}} >> m_{\text{ind}}$) results in a relative large amount of mass reduction of the product carrier as result of additional actuators. This ratio changes as result of additional actuators and therefore the amount of mass reduction decreases as result of additional actuators.

The mass of individual actuators is determined as function of the force that is required. Besides mass that is required to generate force $m_{\text{con}}$, also a constant mass is assumed to be present. This additional mass is for instance used to connect the conductive mass to the product carrier, or to guide the conductive mass. The amount of mass independent on the force, is indicated with $m_{\text{fix}}$. In case the amount of conductive mass is initially large compared to the other mass $m_{\text{con}} >> m_{\text{fix}}$ the mass of the actuator can be reduced much as result of decreased force requirements. The mass ratio changes when the amount of conductive mass is reduced. When the fixed mass is relatively large compared to the conductive mass ($m_{\text{fix}} >> m_{\text{con}}$) the mass reduction as result of decreased force requirements is small. The total dynamic mass of the drive system is formed by the summation of the individual mover mass. A relative small amount of conductive mass with respect to the fixed mass ($m_{\text{con}} << m_{\text{fix}}$) affects the combined dynamic mass of the actuators. This mass limits the possible mass reduction of the system. The dynamic mass and the effect of a relative large amount of fixed mass is shown in Figures 6.11 and 6.22.

6.6 Conclusion

An increased number of parallel actuators results in reduction of the structural material. The total dynamic mass is however also dependent on other elements, e.g. actuators, guidance but also the elements used to ensure the positioning system remains in contact with the payload. In the considered benchmark systems a minimum dynamic mass is obtained when the number of parallel actuators is equal or just above, the number of rigid body modes that needs to be affected by the parallel actuators, see Figures 6.7, 6.10, 6.13, 6.15, 6.19 & 6.22. In case additional parallel actuators result in mass reduction, the mass reduction as result of an additional parallel actuator decreases with the increased number of additional parallel actuators. I.e. the first additional parallel actuator results in the largest mass reduction.

Application of an increased number of parallel actuators affects both the structural mass and the support mass. The effect on the moving mass as result of an additional parallel actuator depends on the initial structural mass with respect to the support mass, and on the changed support mass of the (new) system with the additional parallel actuator.

Mass reduction as result of the application of an increased number of parallel actuators is most likely to appear when the initial structural mass is relatively
large compared to the support mass. Furthermore, proper operation of the system with additional parallel actuators, should not result in an increase of the support mass that is larger than the reduced structural mass. Dependent on support mass change, as result of an increased number of parallel actuators, the ratio between structural mass and support mass is changed as well. As a result a minimum moving mass system is obtained with a limited number of parallel actuators. The initial moving mass of the benchmark systems is reduced by a factor 2 to 4 in case of an initial large structural mass compared to the support mass, see Figures 6.7, 6.10 & 6.19. The moving mass of the benchmark systems with a relative small amount of structural mass compared to the support mass, is not or only slightly reduced when additional parallel actuators are used, see Figures 6.13, 6.15 & 6.22.

The ratio between support mass and structural mass is among others affected by the requirements and the selected configuration of the positioning system. Independent on the number of parallel actuators the moving mass can often be reduced via optimization, for instance due to the application of metal foam cores in sandwich panels. The opportunity to attain more accuracy at higher speeds with additional parallel actuators is difficult to recognize when aimed for mass reduction of current available precision manufacturing machines with preservation of speed and accuracy. The application of additional parallel actuators provides however opportunities to create new systems with new functionality, or to improve the performance of existing systems. Despite these systems might even be accompanied with additional mass, the improved performance or new functionality as result of additional parallel actuators would be hard to realize without conceptual changes of existing design solutions. The additional parallel actuators can for instance be used to adapt the quasi static shape of the structure. In case shape adaptation is an important requirement, e.g. in adaptive optics, the bending stiffness of the structure is relatively small in order to reduce the required actuator force. In precision manufacturing machines the acceleration introduced inertia force and the limited stiffness might result in dynamic errors, that limit the attainable speed; also known as dynamic performance barrier [222]. Hardware improvements in a conventional way, that aims for an increased structural stiffness with a reduced structural mass, might become difficult to realize when structures are further optimized. Additional parallel actuators can be used to reduce the dynamic error, or to obtain higher speeds for the same dynamic error.
Chapter 7

Conclusions & recommendations

7.1 Conclusions

The operation speed of a positioning system used in precision manufacturing machines, is often limited as result of the inertia forces and the flexibility of the structural elements. In this thesis the effect of the number of actuators and their location on the required structural material distribution is determined. Stiffness is required to resist the effect of loading, including inertia forces in the positioning system. Even though parallel positioning systems are considered, the stiffness in these systems is the result of a serial arrangement of elements required to create stiffness. The stiffness of the mechanical part is in these systems is often dominated by bending stiffness. With bending stiffness is in this case referred to stiffness that is obtained as result of stress and deformation in a direction perpendicular to the loading. The structural material can not be assigned to individual stiffness requirements, e.g. bending, shear, or torsion, nevertheless the material mainly used to create the required mechanical bending stiffness, is in this thesis referred to as structural material. The amount of structural material required in a parallel positioning system depends, among others, on the number of parallel actuators.

The structural stiffness requirements of the product carrier can be reduced as result of an increased number of parallel actuators. Besides the structural mass other elements, that are required for proper operation of the system, also contribute to the total dynamic mass. The moving mass of these additional elements is referred to as support mass. A part of the support mass is directly related to the structural mass, e.g. the mass used to generate force and accelerate the system. The remainder of the support mass is not related to the structural mass, and is required for proper operation of the system. Application of an increased number of parallel actuators affects both the structural mass and the support mass. The effect on the dynamic mass as result of an additional parallel actuator depends on the initial structural mass with respect to the support mass, and on the changed support mass of the (new) system with the additional
parallel actuators.

The major conclusions from the individual chapters are:

- The effect of the stiffness and mass of the used material of the product carrier is determined in a static acceleration field with a constraint on the maximum deflection. The lightest structure is obtained when material is used that provides the highest stiffness for the least weight, see Chapter 2. The ratio $E/\rho^3$ can be used as material selection criterium to obtain a minimum mass product carrier. Optimized structures can be used to decrease the structural mass even further, for instance due to the application of metal foam cores in sandwich panels. The absolute amount of mass reduction as result of additional supports decreases when more supports are used.

- Scaling of the actuator systems is dependent on the configuration. The dynamics mass contains besides structural mass to obtain stiffness also other mass that is required for proper operation. The configuration of the positioning system affects the requirements of the additional elements and their mass, e.g. the systems shown in Chapter 3 (page 74). Besides on the configuration, depends the mass of an individual actuator also on the required force. The configuration and the requirements limit the apparent initial benefits of an increase number of parallel actuators, for instance as result of the requirements for the airgap size of the actuator used.

- The dynamics of parallel positioning systems depends on the complex interaction between the controlled actuator forces and the mechanical system. The location of the actuators affect, as well as the spatial stiffness and mass distribution, the dynamic behaviour of the positioning system. Methods to analyze the dynamic performance of parallel actuated systems in the design phase are obtained and presented in Chapters 4 and 5. The knowledge and information obtained with these methods can be used to adapt the design prior to construction while an upper limit of the dynamic performance is guaranteed. The residual vibration amplitude after a single transient positioning task is used as initial performance measure. The natural frequency limits the speed of consecutive positioning tasks.

- Overactuation is an increase of the number of parallel actuators, above the minimum number of parallel actuators required to actuate the degrees of freedom of rigid bodies. The aspects are similar to the well known aspects of parallel actuated systems. The dynamic performance of parallel positioning systems is dependent on the number of actuators and their location. The experiments on an overactuated beam and plate system show that the results from simulations match with practice.

- The mass of the product carrier and the mass of individual elements depends, besides on the number of actuators, on the requirements, assumptions, and configuration of the system. In Chapter 6 the effect
on the mass of different configurations is indicated. The benchmarks systems are used to show that minimum mass systems exists, and that the assumptions and constraints affect the location of the minimum to a large extend. The location of the optimum number of actuators depends on the positioning task, which implies: stroke, positioning time, tolerance specification, but also the product that needs to be positioned. The optimum is furthermore affected by the initial ratio between mass that provides structural stiffness and mass required for proper operation. When the number of parallel actuators is increased, the ratio between the mass that is dependent on the number of actuators and the mass that is independent on the number of actuators, changes. The mass reduction as result of an additional actuator is therefore larger in case of a small number of actuators compared to an actuator added to a system with a large number of actuators.

A beam and a plate system are used as benchmark system in order to evaluate the effect of additional actuators on the mass of parallel positioning systems. Whether additional parallel actuators result in mass reduction depends on the requirements, constraints and assumptions that are applied in the design of the electromechanical positioning system. Mass reduction is most likely to appear in systems where a relative large amount of mass is required to generate the required structural bending stiffness. The exchange of structural stiffness and mass with additional actuator systems might seem beneficial for mass reduction, other aspects, such as control, requires at least more attention. The mass reduction is however dependent on the configuration of the system and thus the mass ratio between different elements. In case overactuation results in mass reduction the absolute amount of mass reduction decreases with an increased number of parallel actuators. The number of additional parallel actuators, above the minimum required for rigid body motion, that should be used to get a minimum mass system, is therefore in general small.

7.2 Recommendations

The application of overactuation affects the configuration of the system. To motivate conceptual design decisions, such as the configuration, knowledge and understanding about the synergy of the concept is important. Also knowledge about the limitations and opportunities of the individual elements might be more important than increased accuracy of these individual elements. Even though recommendations for scientific research to increase accuracy of the individual elements are easily made, such recommendations are left to the specialists in these particular fields. Prior to identification of a need for increased accuracy of individual aspects, the research on overactuation might focus on obtaining even more understanding of the involved physics and the synergy. Besides, for conceptual design decisions such increased knowledge can also be used to specify better motivated tolerances, and to better cope with scarce money, time, or other resources, in the design phase.
Overactuation is a special type of a parallel-axis configuration in a machine. For useful application of overactuation design rules should be developed. The applicability of such design rules, that affects the configuration of the machine axes, increases when they are in the same trend as currently available for the selection of parallel-, serial- or hybrid axis configurations. 

Apart from the apparent benefits of minimum mass systems, e.g. reduction of power dissipation, obtaining minimum mass systems should not be considered as a purpose on its own. Most of all, productivity and cost effectiveness are required in manufacturing industry. A conventional design where the minimum number of actuators necessary for rigid body motion is applied, might for instance require more mass but might be much more cost effective. Overactuation can open new opportunities for the accuracy and speed of precision machines, and shift the dynamic performance barrier, however it also has its costs. In this thesis overactuation is applied in a single direction positioning system. When opting for a more realistic practical application, e.g. for application in flat panel display-, or semiconductor industry, further research can be conducted towards overactuated positioning systems that are able to position in more directions.
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Curriculum vitae

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