BASEBAND COMPENSATION OF TRANSMITTER AND RECEIVER IQ IMBALANCE FOR OFDM IN FREQUENCY SELECTIVE FAADING CHANNELS

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ABSTRACT

In this paper, we present a baseband algorithm for estimation and compensation of transmitter and receiver IQ imbalance for OFDM in a frequency-selective fading channel. We show by simulations that the negative effect of IQ imbalance on system performance can be almost completely removed by the proposed algorithm. The algorithm is an iterative loop, consisting of consecutively channel estimation, Tx IQ imbalance estimation and data estimation.

1. INTRODUCTION

Because of the strong price erosion in the market for wireless local area network (WLAN) devices, it is crucial to develop low cost transceivers. One important way to do so is by ensuring a high level of integration, without requiring external IF components like SAW filters. Zero-IF front-ends have the potential to fulfill that promise. On the other hand, Zero-IF front-ends are suffering relatively strong imperfections, like for instance IQ imbalance.

Modern communication standards, like the 802.11 a, g and n systems, have been developed to achieve high spectral efficiency. As a consequence, OFDM has become the modulation technology of choice for many systems. In addition, each of the OFDM-subcarriers is modulated by a high-order QAM constellation, e.g. 64-QAM or 256-QAM. As a result, a high accuracy transmission is needed.

It would take expensive RF designs to achieve the necessary accuracy in the front-ends of transmitter and receiver. This would potentially remove all advantages of the zero-IF receiver. Therefore, in this paper, we study the estimation and compensation of IQ imbalance by additional baseband processing at the receiver.

The paradigm of estimating and compensating the IQ imbalance in baseband at the receiver is not new. A number of articles have been published that deal with this problem. For instance, Bourdoux et al. [1], Brötje et al. [2], Buchholz et al. [3], Lin and Tsui [4] and Tubbax [5] all have presented solutions for IQ estimation and compensation. None of these papers however, considers the combination of IQ imbalance in both transmitter and receiver with a frequency-selective fading channel. Our work is closest to the recent papers of Schenk et al. [6] Tarighat et al. [7].

In the paper of Schenk, an iterative algorithm for packet-based MIMO OFDM is presented, based on two separate estimators for TX IQ imbalance estimator and RX IQ imbalance, respectively. The estimators use the preamble for their estimation. The estimated values of the current packet are used as initial estimates for the next packet. This method requires many packets to converge to good estimates of transmitter and receiver imbalance. Note that this method requires the consecutive received packets in the iterations to originate from the same transmitter. Therefore, this solution is less suited for multi-user environments.

The main difference between the approach of Schenk and our approach is that we use a blind method which can operate both on the preamble and on the data. As a result, our algorithm iterates within a packet, rather than between packets. This allows for faster convergence, as well as easy application in a multi-user environment, in which different packets may originate from different transmitters.

Tarighat and Sayed present a compound channel model which contains the effects of the wireless channel as well as the IQ imbalance of transmitter and receiver. Their compensation is basically an equalizer which is based on estimates of the parameters of this compound channel.

The main difference with our approach is that we obtain separate estimates of the wireless channel parameters and the IQ imbalance parameters. This is especially useful because usually the channel will vary much faster than the IQ imbalance. Also, our approach allows for straightforward application of traditional channel smoothing techniques.
2. A MODEL OF IQ IMBALANCE FOR OFDM IN FREQUENCY SELECTIVE FADING CHANNELS

We model the IQ imbalance, by taking the in-phase branch as the reference and modelling the IQ imbalance to reside in the quadrature branch. Because IQ imbalance leads to crosstalk between the signals on mirror subcarriers (represented by subcarrier indices \( k \) and \(-k\)), the signals on both subcarriers are modelled jointly. It is convenient to split the model into three phases:

1. The transformation of the frequency-domain data symbols \((X_k, X_{-k})\) into transmitted signal with IQ imbalance, represented by a baseband equivalent signal in the frequency domain \((S_k, S_{-k})\).
2. The operation of the frequency selective fading channel, transforming the transmitted signal into the received signal before receiver IQ imbalance \((R_k, R_{-k})\), also represented as a baseband equivalent signal in the frequency domain.
3. The transformation of the frequency-domain received signal \((R_k, R_{-k})\) into the received signal with receiver IQ imbalance \((Y_k, Y_{-k})\).

The three steps above are described by the following three equations:

\[
\begin{bmatrix}
S^*_{-k} \\
S_k
\end{bmatrix} =
\begin{bmatrix}
\alpha^*_{Tx} & b^*_{Tx} \\
b_{Tx} & \alpha_{Tx}
\end{bmatrix}
\begin{bmatrix}
X^*_{-k} \\
X_k
\end{bmatrix},
\quad (1)
\]

\[
\begin{bmatrix}
R^*_{-k} \\
R_k
\end{bmatrix} =
\begin{bmatrix}
H^*_{-k} & 0 \\
0 & H_k
\end{bmatrix}
\begin{bmatrix}
S^*_{-k} \\
S_k
\end{bmatrix},
\quad (2)
\]

\[
\begin{bmatrix}
Y^*_{-k} \\
Y_k
\end{bmatrix} =
\begin{bmatrix}
a^*_{Rx} & b^*_{Rx} \\
b_{Rx} & a_{Rx}
\end{bmatrix}
\begin{bmatrix}
R^*_{-k} \\
R_k
\end{bmatrix}.
\quad (3)
\]

Or combining the 3 equations above,

\[
\begin{bmatrix}
Y^*_{-k} \\
Y_k
\end{bmatrix} = \Lambda_{Rx} \mathcal{H} \Lambda_{Tx}
\begin{bmatrix}
X^*_{-k} \\
X_k
\end{bmatrix},
\quad (4)
\]

where

\[
\Lambda_{Rx} =
\begin{bmatrix}
a^*_{Rx} & b^*_{Rx} \\
b_{Rx} & a_{Rx}
\end{bmatrix},
\quad (5)
\]

\[
\mathcal{H} =
\begin{bmatrix}
H^*_{-k} & 0 \\
0 & H_k
\end{bmatrix},
\quad (6)
\]

\[
\Lambda_{Tx} =
\begin{bmatrix}
a^*_{Tx} & b^*_{Tx} \\
b_{Tx} & a_{Tx}
\end{bmatrix}.
\quad (7)
\]

In the above equations, \(a_{TxE}, b_{TxE}\) are the transmitter IQ imbalance parameters and \(a_{Rx}, b_{Rx}\) are the receiver IQ imbalance parameters. Let \(\varepsilon_{Tx}, \varepsilon_{Rx}\) denote the gain imbalance and \(\phi_{Tx}, \phi_{Rx}\) the phase imbalance for transmitter and receiver, respectively. Then \(a_{TxE}, b_{TxE}, a_{Rx}, b_{Rx}\) are defined as

\[
\begin{align*}
a_{TxE} &= (1 + (1 + \varepsilon_{Tx}) e^{j\phi_{Tx}})/2, \\
b_{TxE} &= (1 - (1 + \varepsilon_{Tx}) e^{-j\phi_{Tx}})/2, \\
a_{Rx} &= (1 + (1 + \varepsilon_{Rx}) e^{j\phi_{Rx}})/2, \\
b_{Rx} &= (1 - (1 + \varepsilon_{Rx}) e^{-j\phi_{Rx}})/2.
\end{align*}
\]

Note that \(a_{TxE} + b_{TxE} = 1\) for all values of \(\varepsilon_{Tx}\) and \(\phi_{Tx}\). If there is no transmitter IQ imbalance, then \(a_{TxE} = 1\) and \(b_{TxE} = 0\). Similarly, \(a_{Rx} + b_{Rx} = 1\) for all values of \(\varepsilon_{Rx}\) and \(\phi_{Rx}\) and \(a_{Rx} = 1\) and \(b_{Rx} = 0\) in case there is no receiver IQ imbalance.

If we write out the combined equation for transmitter and receiver IQ imbalance for a single subcarrier, then we obtain

\[
Y_k = (a_{Rx} a_{TxE} H_k + b_{Rx} b_{TxE} \bar{H}^*_{-k}) X_k + (a_{Rx} b_{TxE} H_k + b_{Rx} a_{TxE} \bar{H}^*_{-k}) X^*_{-k} + \frac{\varepsilon_{Rx}}{a_{Rx}} N_k + \frac{\varepsilon_{Rx}}{b_{Rx}} N^*_{-k}.
\quad (8)
\]

Looking at the above equation (8), we see three effects of the IQ imbalance:

- The symbol at the mirror subcarrier \(-k\) acts as interference on the desired symbol \(X_k\);
- The desired symbol is scaled with a factor \(a_{Rx} a_{TxE} H_k + b_{Rx} b_{TxE} \bar{H}^*_{-k}\) instead of \(H_k\);
- Some correlation is introduced between the noise at subcarrier \(k\) and the mirror subcarrier \(-k\).

In the remainder of this paper we do not explicitly include the additive noise in the formulas. This does not significantly change the model, yet it considerably simplifies the formulas. Of course, in the simulations, we do take the noise into account.

2.1. Effect on channel estimation and equalization

Next we look at the effect of IQ imbalance on channel estimation and equalization. We assume the use of simple, per-tone channel estimation on the preamble. Later, we will consider compensation of the effect of IQ imbalance on the channel estimate. Therefore, we name the uncompensated estimate the raw channel estimate. Let \(P_k\) denote the pilot symbol transmitted on subcarrier \(k\) in the preamble and \(Y^P_k\) the received preamble signal, then the raw channel estimate becomes

\[
\begin{bmatrix}
\bar{H}^*_{-k} \\
\bar{H}^*_{k}
\end{bmatrix} =
\begin{bmatrix}
Y^P_k/P_{-k} \\
Y^P_{-k}/P_k
\end{bmatrix}.
\quad (9)
\]

\[
= \begin{bmatrix}
a^*_{Rx} & b^*_{Rx} Q_k & 0 \\
b_{Rx} Q_k & a_{Rx} & 0 \\
0 & a_{Tx} + b_{Tx} Q_k & \bar{H}_{-k}^* \\
0 & \bar{H}_k & 0
\end{bmatrix}.
\]
where $Q_k = P_{n,k}^* / P_k$ and we assume that $Q_k \in \{-1, 1\}$. This assumption is satisfied if the pilot symbols are BPSK coded.

Using this channel estimate in a zero-forcing equalizer, the data symbol estimate $\hat{X}_k$ coming out of the equalizer is

$$\hat{X}_k = \frac{Y_k}{\hat{H}_k} = \frac{a_{R_2} a_{T_2} H_k + b_{R_2} b_{T_2} H_k^* + (a_{R_2} b_{T_2}^* H_k + b_{R_2} a_{T_2}^* H_k^*) X_k}{a_{R_2} a_{T_2} H_k + b_{R_2} b_{T_2} H_k^* + (a_{R_2} b_{T_2}^* H_k + b_{R_2} a_{T_2}^* H_k^*) P_{n,k}^* / P_k} X_k. \quad (10)$$

### 3. IQ IMBALANCE COMPENSATION

In this section, we introduce our iterative IQ imbalance compensation algorithm. As mentioned before, the algorithm is based on equations (1)-(3). The idea is to sequentially estimate the entries of the matrices in the equations and to refine these estimates iteratively.

For the channel estimate, equation (9) models the effect of IQ imbalance on the raw per-tone channel estimate. Inverting (9) indicates how the channel estimate can be refined using the latest estimates of transmitter or receiver IQ imbalance parameters.

$$\left( \begin{array}{c} H_{-k}^* \\ H_k \end{array} \right) = \left[ \begin{array}{cc} a_{T_2}^* b_{R_2}^* Q_k & \frac{1}{a_{T_2}^*} \\ 0 & \frac{1}{a_{T_2}^*} \end{array} \right] \left[ \begin{array}{cc} a_{R_2} & b_{R_2} Q_k \\ b_{R_2} Q_k & a_{R_2} \end{array} \right]^{-1} \left( \begin{array}{c} \hat{H}_{-k}^* \\ \hat{H}_k \end{array} \right) \quad (11)$$

This correction of the channel estimate is applied after each new estimate of the receiver or transmitter IQ imbalance parameters.

To estimate the RX IQ imbalance parameters, we need a slightly rewritten version of equations (1)-(3):

$$\begin{pmatrix} S_{-k} \\ S_k \end{pmatrix} = \begin{pmatrix} X_{-k} & X_k^* \\ X_k & X_k^* \end{pmatrix} \begin{pmatrix} a_{T_2}^* \\ b_{T_2} \end{pmatrix} \quad (12)$$

$$\begin{pmatrix} R_{-k} \\ R_k \end{pmatrix} = \begin{pmatrix} H_{-k} & 0 \\ 0 & H_k \end{pmatrix} \begin{pmatrix} S_{-k} \\ S_k \end{pmatrix} \quad (13)$$

$$\begin{pmatrix} Y_{-k} \\ Y_k \end{pmatrix} = \begin{pmatrix} R_{-k} & R_k^* \\ R_k & R_k^* \end{pmatrix} \begin{pmatrix} a_{R_2} \\ b_{R_2} \end{pmatrix} \quad (14)$$

Unfortunately, it turns out that this straightforward and clean description of the system, though very appropriate for the estimation of the receiver imbalance parameters, does not provide a good transmit imbalance estimator. For the transmit imbalance estimator, we use a heuristic reformulation of equation (12) which considers $a_{T_2}, a_{T_2}^*, b_{T_2}$ and $b_{T_2}^*$ as four independent parameters:

$$\begin{pmatrix} S_{-k}^* \\ S_k \end{pmatrix} = \begin{pmatrix} 0 & X_{-k}^* \\ X_k & 0 \end{pmatrix} \begin{pmatrix} a_{T_2}^* \\ a_{T_2} \end{pmatrix} \begin{pmatrix} b_{T_2} \end{pmatrix} \begin{pmatrix} b_{T_2}^* \end{pmatrix} \quad (15)$$

For the received signal, we can now write:

$$\begin{pmatrix} Y_{-k}^* \\ Y_k \end{pmatrix} = M \begin{pmatrix} a_{T_2} \\ a_{T_2}^* \\ b_{T_2} \\ b_{T_2}^* \end{pmatrix} \quad (16)$$

with $M$ defined as

$$M = \Lambda_{R_2} \mathcal{H} \begin{pmatrix} 0 & X_{-k}^* & X_k & 0 \\ X_k & 0 & X_{-k}^* \end{pmatrix} \quad (17)$$

We can now present an outline of our algorithm. From the previous iteration $(n-1)$, we have estimates of transmitter imbalance $\hat{b}_{T_2,n-1}$, receiver imbalance $\hat{b}_{R_2,n-1}$ and channel frequency response $(\hat{H}_{-k,n-1}, \hat{H}_{k,n-1})$. These estimates and the estimate of the receiver imbalance parameters $\hat{\Lambda}_{R_2,n-1}$, $\hat{\Lambda}_{T_2,n-1}$ and $\hat{\mathcal{H}}_{n-1}$ can now be computed using the relations $\hat{a}_{R_2} = 1 - \hat{b}_{R_2}^*, \hat{a}_{T_2} = 1 - \hat{b}_{T_2}$.

1. Compute an estimate of the transmitted data symbols, $(\hat{X}_{-k,n}, \hat{X}_{k,n})$ by substituting the matrix estimates into equation (4):

$$\begin{pmatrix} \hat{X}_{-k,n} \\ \hat{X}_{k,n} \end{pmatrix} = \hat{\Lambda}_{T_2,n-1}^{-1} \hat{\mathcal{H}}_{n-1}^{-1} \hat{\Lambda}_{R_2,n-1}^{-1} \begin{pmatrix} Y_{-k} \\ Y_k \end{pmatrix}. \quad (18)$$

2. Compute a new estimate of the receiver imbalance parameter $\hat{b}_{R_2,n}$, as follows:

- Create an estimate of $(S_{-k}, S_k)$ using equation (12) and the estimates $\hat{b}_{T_2,n-1}, \hat{X}_{k,n}, \hat{X}_{-k,n}$.\n- Create an estimate of $(R_{-k}, R_k)$ using equation (13) and the estimates $(S_{-k}, S_k)$ and the just created estimates of $(S_{-k}, S_k)$.\n- Substitute the just created estimate of $(R_{-k}, R_k)$ into equation (14) and solve for the receiver imbalance parameter $\hat{b}_{R_2,n}$ to obtain the estimate $\hat{b}_{R_2,n}$.

3. Compute an intermediate update of the channel estimate $(\hat{H}_{-k,n}^{\text{int}}, \hat{H}_{k,n}^{\text{int}})$ as follows: substitute the new estimate of the receiver IQ imbalance parameter $\hat{b}_{R_2,n}$ and the old estimate of the transmitter IQ imbalance parameter $\hat{b}_{T_2,n-1}$ into equation (11) and optionally apply smoothing of the channel estimate. The result of this equation is the intermediate updated channel estimate.

4. Compute a new estimate of the transmitter imbalance parameters $\hat{b}_{T_2,n}, \hat{a}_{T_2,n}$, as follows:
use the latest receiver imbalance estimate \( \hat{b}_{Rx,n} \) and \( \hat{a}_{Rx,n} \), the intermediate updated channel estimate \( \hat{H}_{k,n}^{int}, \hat{H}_{k,n}^{int} \) and the latest data estimate \( \hat{\hat{X}}_{k,n}, \hat{X}_{k,n} \) in equation (17) to compute an estimate of matrix \( \hat{M} \).

- substitute the received signal \( (Y_k, Y_k) \) and the just created estimate of \( \hat{M} \) into equation (16) and solve for the transmit imbalance parameter \( b_{Tx,n}^* \).
- compute the estimates of the transmit imbalance parameters using \( \hat{a}_{Tx,n} = 1 - b_{Tx,n}^* \).

5. Compute a new estimate of the channel \( (\hat{\hat{H}}_{-k,n}, \hat{H}_{k,n}) \), by substituting the latest estimate of the transmitter and receiver imbalance parameters \( \hat{b}_{Tx,n} \) and \( \hat{b}_{Rx,n} \) into equation (11) and again optionally apply smoothing of the channel estimate.

6. Compute estimates \( \hat{\hat{H}}_{Rx,n}, \hat{\hat{H}}_{Tx,n} \) and \( \hat{\hat{H}}_n \) by substituting the estimates of transmitter imbalance \( \hat{b}_{Tx,n} \), receiver imbalance \( \hat{b}_{Rx,n} \) and channel frequency response \( \hat{H}_{-k,n}, \hat{H}_{k,n} \) into the equations (5)-(7).

3.1. initialization of the iterative algorithm

The iteration is initialized by assuming no IQ imbalance and computing the per-tone channel estimate, i.e.,

\[
\hat{b}_{Rx,0} = \hat{b}_{Tx,0} = 0, \\
\hat{H}_{0,k} = \hat{H}_{k}^{raw} = Y_k^P / P_k.
\]

3.2. extension to more subcarrier pairs

Above, we presented the iterative algorithm to estimate the transmitter and receiver IQ imbalance parameters for a subcarrier pair \((k, -k)\). Of course, in practice there are many such pairs. For 802.11a, for instance, there are 48 data subcarriers. For the case of frequency independent IQ imbalance, we propose to compute the estimates by creating a large joint system of equations for all subcarriers and using a pseudo inverse to calculate an estimate of the IQ imbalance parameters jointly for all subcarriers.

4. SIMULATION RESULTS

We have carried out simulations with the algorithm from the previous section. The 802.11a was implemented and simulated in its highest rate mode, using 64QAM and code rate 3/4. This data packet is preceded by the usual 802.11a training sequences and signal field. We used a Rayleigh fading channel with exponential power profile and an RMS delay spread of 50ns. The IQ imbalance in the simulations is fairly extreme: Both the transmitter and the receiver exhibit 10% gain imbalance and 5 degrees phase imbalance. It is frequency independent imbalance, so we combined all 48 data subcarrier pairs and used a pseudo inverse to create the estimates for the IQ imbalance parameters. The simulated packet size is 513 bytes. The receiver applies a zero-forcing equalizer. The estimator operates on the signal field, using 20 iterations.

Figure 1 shows the Bit Error Rate as a function of SNR and Figure 2 shows the Packet Error Rate as a function of SNR. In each of the figures 4 plots can be distinguished:

- The graph with 'x' markers corresponds to the ideal case of no IQ imbalance, and consequently the compensation algorithm is not used.
- The graph with '+' markers indicates the behavior of an 802.11a system with IQ imbalance in transmitter and receiver, but without IQ imbalance compensation.
- The graph with 'o' markers is the proposed approach: IQ imbalance in transmitter and receiver is compensated by the iterative algorithm, running in the receiver baseband processor.
- The graph with 'o' markers is added for completeness. It shows the system performance with the iterative algorithm in case there would be no IQ imbalance in transmitter nor receiver.

![Fig. 1. Bit Error Rate for 64 QAM rate 3/4 with 10% and 5° imbalance in transmitter and receiver](image-url)
In this paper we have introduced an iterative algorithm for IQ imbalance compensation in a packet-based OFDM system with a preamble in a frequency selective fading channel. The approach is based on iteratively estimating, in baseband at the receiver, the receiver IQ imbalance, channel frequency response, transmitter IQ imbalance and transmitted data. Simulations show that, even at extreme IQ imbalance level, the algorithm can completely annihilate the adverse effects of the receiver and transmitter IQ imbalance.

5. CONCLUSIONS

Finally, we show the convergence behavior of the estimators in absence of noise. In Figure 3 we plot the median squared error, averaged over all subcarriers, as a function of the number of iterations for the estimates for transmitter imbalance $b_{T,n}$, receiver imbalance $b_{R,n}$ and channel $H$. As can be observed, the squared error converges in 20 iterations to (squared) machine precision level. The number of iterations needed depends on the acceptable level of residual error, which in turn depends on the modulation order used.

6. REFERENCES


