Generation of high-field, single-cycle terahertz pulses using relativistic electron bunches

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Generation of high-field, single-cycle terahertz pulses using relativistic electron bunches

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus, prof.dr.ir. C.J. van Duijn, voor een commissie aangewezen door het College voor Promoties in het openbaar te verdedigen op dinsdag 24 november 2009 om 16.00 uur

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Introduction

1.1 Terahertz radiation

The terahertz (1 THz = $10^{12}$ Hz) spectral range roughly extends from 100 GHz to 10 THz, which positions it in between electronics and photonics, see Fig. 1.1. Because both electronic and photonic techniques are difficult to implement at THz frequencies, sources and detectors of electromagnetic radiation have always been scarce in this spectral range. This has limited the amount of research conducted at THz frequencies. However, since the advent of ultrafast lasers in the late 1980’s the THz spectral range has opened up to researchers, leading to an enormous increase in studies performed at THz frequencies.

One of the first applications of ultrafast THz pulses was time-domain spectroscopy (TDS). Single-cycle pulses containing frequency components over the entire THz spectral range can be used to extract spectroscopic information from gases, liquids [1] and solids [2]. Because the electric field of a THz pulse can be measured directly, both the amplitude and the phase modulation of the electric field can be obtained simultaneously over a wide range of frequencies. In this way both the real and the imaginary part of the dielectric function can be determined over the entire THz spectral range, making it a very effective spectroscopic method.

Besides spectroscopy, THz radiation is also used to image materials. Terahertz radiation can penetrate through clothing, packaging materials or building materials. Metallic objects and substances with high water content, however, are opaque to THz radiation. Because of this contrast THz radiation has often been suggested for security and packaging control applications. Moreover, since THz radiation is non-ionizing, it is expected to have applications in medical imaging as well [3]. Terahertz imaging was first demonstrated by Hu et al. [4], by scanning an object through the THz focus. Since this is a time consuming process, soon methods were developed to create two-dimensional images of THz pulses as a function of time [5]. Later the time-of-flight information present in the reflection of a THz pulse on a sample was used to create a tomographic image [6]. A very recent example of the latter is the imaging of hidden paint layers in a painting on canvas [7].

To give an impression of the fascinating opportunities in fundamental science at THz
frequencies we give a few examples [8]: Electrons in highly-excited atomic Rydberg states orbit at THz frequencies; small molecules rotate at THz frequencies; biologically important collective modes of proteins vibrate at THz frequencies; electrons in semiconductor nanostructures resonate at THz frequencies.

Powerful THz sources are very desirable, simply from signal-to-noise point of view. In addition, they could prove valuable tools to investigate matter in the non-linear regime. The influence of a time-dependent electric field on a quantum mechanic system can not be described as a small perturbation, once the electric field strength becomes too strong. Studies in this regime have largely been confined to the optical regime using atoms and high-power lasers. In the same non-linear regime it is expected that semiconductors show fascinating new phenomena [8]. However, accessing this regime in semiconductors at optical frequencies may require electric fields above the laser damage threshold of the material.

At THz frequencies, on the other hand, this regime is accessible at electric field strengths ranging from 0.1 MV/m - 100 MV/m, well below the dielectric breakdown threshold of most materials [8]. Another application of high field strength electric, and magnetic pulses, is the study of thin film metallic ferromagnets [9, 10]. In [10], e.g., the Coulomb field of a short, ultra-relativistic electron bunch was used to create electric-field-induced magnetic anisotropy in a thin film ferromagnet. Similar results should be possible using free-space THz pulses having equally strong magnetic fields of a few Tesla.

Another topic relevant for the content of this thesis is the research into near-field techniques at THz frequencies. Because the wavelength at 1 THz is relatively large, $\lambda \simeq 0.3$ mm.
mm, it is difficult to apply free-space THz techniques at the nanometer, or even micron-scale. To overcome this limitation apertures [11, 12], waveguides [13, 14], or probes [15–18] have been used to localize the electric field. Very recently researchers have optically pumped an array of aligned germanium nanowires, and probed the nanowires using single-cycle THz pulses [19]. Probing a single nanowire at THz-frequencies using near-field techniques, for example, would offer very interesting research opportunities.

More recently a tapered metal tip has been used to obtain subwavelength resolution at THz frequencies [20]. These so-called Sommerfeld waveguides were originally investigated in the 60s [21] for THz signal transmission. They recently received renewed interest because of there application in THz-TDS [22–25]. The wire geometry acts as an efficient waveguide for Sommerfeld waves, allowing them to propagate over long distances along the wire with low attenuation and dispersion. This enables endoscopic delivery of THz radiation to samples in applications where line-of-sight access is not available [26]. Sommerfeld waves are weakly bound electromagnetic surface waves, also known as surface plasmon polaritons (SPPs). In the optical regime it has been shown that tapering the metal tip can lead to superfocusing of SPPs into a volume much smaller than the wavelength [27]. If the metal wire is periodically corrugated [28, 29] the same effect is possible for SPPs in the THz frequency range. Finding a method to generate THz SPPs of appreciable amplitude on a metal wire could lead to new, exciting THz plasmonic applications [30].

In conclusion, there is a need for a broadband THz source, capable of delivering powerful pulses which are either propagating in free space, or coupled onto a metal wire. In this thesis we present novel concepts to generate such THz pulses. In this introduction we therefore limit ourselves to an overview of the relevant pulsed THz sources.

1.2 Sources of intense, few-cycle THz pulses

1.2.1 Accelerator-based sources

Currently the most powerful sources of free-space single-cycle THz radiation are large accelerator-based sources. These sources generate THz radiation using ultra-relativistic ultra-short electron bunches via various schemes. Examples are coherent undulator radiation (CUR) [31] and coherent synchrotron radiation (CSR) [32]. These sources are capable of delivering pulses with energies ranging up to $\sim 10 \, \mu J$. Another example of the large accelerator-based source is the free-electron laser (FEL) [33], which delivers powerful quasi-continuous-wave (CW) radiation, tunable between 0.1 THz and 10 THz.

Among the large accelerator-based sources the most powerful ones are those based on coherent transition radiation (CTR) as emission process [34, 35]. They are capable of delivering single-cycle pulses with energies up to $\sim 100 \, \mu J$, and a frequency bandwidth of 0.1 - 10 THz. Note that both examples [34, 35] are accelerators that are not specifically intended as THz sources. In both accelerators the electrons are ultra-relativistic, but only because this is necessary for their intended application, not because it is necessary for the CTR process. For the CTR process the electron bunches have to be ultra-short and very
dense, but a few MeV of kinetic energy is already sufficient. Such electron electron bunches can readily be created in a single RF photogun [36].

Because the accelerator-based sources are all huge and expensive their practical use for the THz community is limited.

1.2.2 Laser based sources

At present the favorite method for generating few-cycle intense THz radiation is optical rectification of femtosecond laser pulses. Already in 1994 it was shown that it is possible to create single-cycle pulses containing $\sim 0.4 \mu J$ using a large aperture photoconductive switch, but only in a limited bandwidth [37]. Since then different approaches of optical rectification, with several different non-linear materials, have been investigated [38–41]. This has led to single-cycle THz pulses having energies ranging up to $\sim 10 \mu J$, with roughly a frequency bandwidth of 0.1 - 3 THz. In [42] a bandwidth of 0.1 - 7 THz was reported, using four-wave mixing in a ionized air-plasma. Due to the large bandwidth they were able to generate electric field strengths of $\sim 40$ MV/m with only $\sim 30$ nJ of energy per pulse.

In a laser-wake-field-accelerator (LWFA) sub-ps electron bunches are created and accelerated by shooting a femtosecond TW laser into a plasma. Electron bunches passing the plasma-vacuum boundary create single-cycle THz radiation via the CTR process [43]. It can be argued that the LWFA-THz source is an accelerator based-source. However, because it is a compact setup we placed the LWFA-THz source in the same category as the laser-based sources. Although it is compact, the LWFA-THz source still require a femtosecond TW laser. This in contrast to the laser based sources mentioned so far, which typically require femtosecond pulses having peak powers between $\sim 10$ GW and $\sim 100$ GW.

A last method to generate free-space THz radiation we want to mention is difference frequency generation (DFG) using two femtosecond optical pulses, which have a offset in the carrier frequency [44, 45]. Pulses have been generated with electric fields of $\sim 10$ GV/m with a locked carrier envelope phase (CEP). However, the center frequency of the generated THz pulses ranges from 10 to 72 THz [45]. So far the method has not been demonstrated to work in the 0.1 - 10 THz range.

In recent years laser-based source have closed most of the THz gap. They are, therefore, the favorite method for generating intense single-cycle free-space THz pulses. Most of them, however, still have to balance between energy per pulse and bandwidth.

Although laser-based sources are capable of delivering intense free-space THz radiation, up to now efficient coupling of these free-space THz pulses into the guided mode on a wire has proven difficult. Currently, THz SPPs are generated by scattering the linearly polarized free-space waves into a radially polarized wave, which is then coupled onto the wire [26]. However, due to the poor spatial overlap between the free-space radiation waveform and the SPP waveform, the coupling efficiency is very low (typically less than 1% [46]). A proposed method to overcome this poor coupling efficiency is to create radially polarized THz radiation using a radially symmetric photoconductive antenna [46].

So far no method to generate intense half-cycle THz SPPs on a metal wire has been demonstrated experimentally.
1.2.3 A hybrid approach: RF Photogun as a compact source of intense, few-cycle THz pulses

We present a novel method that combines a small accelerator, a so-called RF photogun, with a footprint comparable to a regular laser-based system, and a femtosecond laser to create THz radiation. The recently developed combination of femtosecond photoemission with radiofrequency (RF) accelerator technology, makes it possible to create MeV, sub-ps electron bunches containing more than 100 pC of charge using a single RF photogun [36]. Such bunches can be used to generate intense single-cycle broadband THz radiation in a compact manner, via CTR as emission process. A relativistic charged particle sheds part of its Coulomb field in the form of a pulse of radiation when passing through the interface between two dielectrics. This so-called transition radiation (TR) usually has a negligible effect on the beam and is often used for diagnostic purposes. However, when the charged particles are concentrated in a short bunch, the electric fields in the radiated pulse at wavelengths larger than the bunch length add up coherently. The energy collectively released at long wavelengths is therefore proportional to the number of particles squared [47].

Besides free-space THz radiation, the CTR process can be used to generate THz SPPs coupled onto a metal wire. Similar to the method proposed in ref. [46], in our method the guided mode on the wire is excited by a radially polarized field, thereby avoiding the poor coupling efficiency described above. We generate THz SPPs by launching electron bunches onto a metal wire which is tapered into a conical tip. When passing the conical vacuum-metal boundary, the bunch will generate a radially polarized CTR field, of which THz SPPs along the boundary are part. These excited SPPs will propagate onto the wire subsequently.

Generating THz SPPs by launching electron bunches onto a tapered wire tip, instead of coupling free-space CTR emitted at a planar interface onto a metal wire, has two benefits; first, electrons are capable of exciting SPPs directly, in contrast to photons where an additional coupling medium is necessary to match the wave vectors of the photons and SPPs. Second, for sharp tips the electrons pass the vacuum-metal boundary at grazing incidence, which enhances the transition radiation due to an increased radiation formation length [48].

The central theme of this thesis is the use of CTR to produce intense single-cycle THz pulses, either propagating in free-space or coupled onto a metal wire. In the next section we address the general characteristics of CTR, and make an estimate of the performance of an RF photogun as source of CTR THz pulses, either propagating in free-space or coupled onto a metal wire.

1.3 Coherent transition radiation

In general, an electron traveling through a spatially inhomogeneous dielectric, induces a time-varying polarization current that leads to radiation. When the spatially inhomoge-
Figure 1.2: Schematic picture of TR-pulse emitted when an electron travels from a metal into vacuum. The Coulomb field of the electron is built up by the TR-pulse. Although in most experiments the electron will be impinging on a metal, rather than exiting from it, we took the latter situation for clarity.
neous dielectric is formed by an interface between two dielectrics it is called transition radiation (TR) [49]. When an electron travels from a perfectly conducting metal into vacuum, the Coulomb field of the electron has to be built up in the form of a pulse of TR. This is because the Coulomb field of the electron is screened while it is traveling inside the metal. The screening ends abruptly the moment the electron passes the metal-vacuum interface. However, the Coulomb field can not fill the entire vacuum instantaneously. Hence the TR-pulse, which builds up the Coulomb field. The polarization of the TR-pulse is therefore determined by the Coulomb field of the electron.

1.3.1 CTR generated at a flat metal surface

For an electron incident at a normal angle with respect to the metal-vacuum interface the TR-pulse is shown schematically in Fig. 1.2. The TR pulse is polarized in the radial direction. The electric field lines curve towards the metal surface because of the induced image charge. The compression of electric field lines above and beneath the electron is because the electron is traveling with a relativistic speed. The radiated energy $W$, emitted per unit frequency and unit solid angle, is given by [49]

$$\frac{\partial^2 W}{\partial \omega \partial \Omega} = \frac{e^2 \beta^2 \sin^2 \theta}{4 \pi \varepsilon_0 c \pi^2 (1 - \beta^2 \cos^2 \theta)^2},$$

(1.1)

with $\varepsilon_0$ the permittivity of free space, $e$ the electronic charge, $c$ the speed of light, $\beta = \frac{v}{c}$, and $\theta$ the polar angle with the $z$-direction, see Fig. 1.2 and Fig. 1.3. Figure 1.3 shows a schematic polar plot of the intensity of the emitted radiation. The TR is emitted into a hollow cone of light which is brightest at $\theta \simeq 1/\gamma$, with $\gamma^{-1} = \sqrt{1 - \beta^2}$ the relativistic Lorentz factor. The hollow cone of light can be interpreted as a dipole pattern, created by the electron and its image charge, relativistically stretched in the forward direction. If we integrate Eq. (1.1) over the solid angle we obtain the energy $W$ emitted per unit frequency,

$$\frac{\partial W}{\partial \omega} = \frac{e^2}{4 \pi^2 \varepsilon_0 c} \left( \frac{1 + \beta^2}{2 \beta} \ln \frac{1 + \beta}{1 - \beta} - 2 \right),$$

(1.2)

which can be approximated by

$$\frac{\partial W}{\partial \omega} = \frac{e^2}{4 \pi^2 \varepsilon_0 c} \left( 2 \ln 4 \gamma - 2 \right),$$

(1.3)

when $\gamma \gtrsim 2$. From Eq. (1.3) it is clear that a TR source does not require ultra-relativistic electron energies. The radiated energy scales logarithmically with $\gamma$, so a mildly relativistic electron is sufficient. According to Eq. (1.1) the spectral bandwidth is unlimited, which is a consequence of the assumption of a perfectly conducting metal and a point-like elementary charge. In reality the dielectric inhomogeneity at the interface will disappear when the frequency reaches the plasma frequency $\omega_p$ of the metal. For a mildly relativistic electron (3 MeV, $\gamma = 7$), passing a vacuum-metal interface ($\omega_p/2\pi \simeq 10^{16}$ Hz), the total radiated energy $W \approx 0.5$ eV.
When an electron bunch travels through a metal-vacuum interface the TR fields emitted by the individual electrons add up coherently at wavelengths larger than the bunch size. This coherently emitted radiation is generally called coherent transition radiation (CTR). We calculated that the TR pulse emitted by a single electron typically contains only $\sim 0.5$ eV. However, since the energy radiated in the coherent spectrum scales quadratically with the number $N$ of electrons in the bunch the radiated energy in the case of CTR can be considerable. Especially if one realizes that an electron bunch produced in a RF photogun typically contains $\sim 100$ pC = $6 \times 10^8$ electrons. Moreover, since the electron bunch length is $\sim 1$ ps, the CTR spectrum will extend into the THz range. We can now make a straightforward estimate of the expected energy, $W_{\text{THz}}$, contained in the coherent spectrum,

$$W_{\text{THz}} = 0.5 \text{ eV} \times N^2 \times \frac{\omega_{\text{THz}}}{\omega_p} \approx 2 \mu\text{J},$$

where $\omega_{\text{THz}}/2\pi = 10^{12}$ Hz. We therefore conclude that by using an RF photogun it is in principle possible to create short pulses of THz radiation containing a few $\mu\text{J}$ of energy. Focussing such pulses into a small spot of $\sim \text{mm}$, should enable electric fields of $\sim 10$ MV/m.

### 1.3.2 THz SPPs generated by an electron bunch impinging on a thin metal wire

Besides free-space radiation, the CTR process can also be used to generate intense THz SPPs propagating on a metal wire. Using a-priori arguments we can make an estimate of the expected electric field strength of the THz SPP propagating on the wire. To show this we will consider an electron impinging onto a metal wire having a very small opening angle $2\delta$, and a radius $\rho_w$. Figure 1.4 shows a sketch of the situation. Consider an electron,
traveling with a speed $v$ in the $z$-direction, which reaches the metal wire at $t = 0$, see Fig. 1.4. When the electron is inside the metal wire the fields are screened for an observer outside the metal. However, the screening is not instantaneous, instead it occurs on a sphere traveling outwards with the speed of light. Within the sphere there is no electric field, outside the sphere the electric field has to be consistent with the presence of the electron and the induced surface current and charge densities at the metal wire. Because the metal cone is collapsed into a very small line, the electric field outside the expanding sphere is that of an undisturbed charge traveling with a speed $v$ in the $z$-direction. We can now easily calculate the electric field on the metal wire at the location of the expanding sphere. Imagine a ‘Gauss box’ with a surface located outside the expanding sphere of light, which closes over the metal wire, see the dotted line in Fig. 1.4. Because the ‘Gauss box’ does not enclose any charge the net flux over its surface has to be zero. However, the flux going through the part of the ‘Gauss box’ located outside the expanding sphere equals that of a single electron $-e/\varepsilon_0$. So the electric flux on the metal wire is $e/\varepsilon_0$. The electric field on the metal wire $E(z, t)$ is then given by

$$E(z, t) = \frac{-e}{2\pi \rho_w \varepsilon_0} \delta(z - ct) e_{\rho},$$

(1.5)

with $e_{\rho} = \cos \varphi e_x + \sin \varphi e_y$, with $\varphi$ the azimuthal angle in the x-y plane. The Fourier transform of $E_{\rho}(z, t)$ is given by

$$E_{\rho}(z, \omega) = \frac{-e}{(2\pi)^2 \rho_w \varepsilon_0 c} e^{i\omega z}.$$  

(1.6)

Here again we find that an electron excites a white, unmodulated spectrum. For an electron bunch only wavelengths larger than the bunch size add up coherently.

We can now show the potential of generating THz SPPs using electron bunches. Take, e.g, electron bunches containing 100 pC of charge and a bunch length of $\tau_g = 1$ ps, impinging on a wire having a radius of $\rho_w = 0.5$ mm. The generated SPPs will have a estimated peak electric field $E_{\rho}^{\text{peak}}$ of

$$|E_{\rho}^{\text{peak}}| = \frac{eN}{(2\pi)^2 \rho_w \varepsilon_0 c} \times \frac{2\pi}{\tau_g} \simeq 5 \text{ MV/m},$$  

(1.7)

and the spectrum will be coherent up to 1 THz. If such THz SPPs would be focused by tapering the wire back into a metal tip [27], fields strengths in excess of 100 MV/m are obtainable in spot sizes far below the wavelength.

### 1.4 This thesis

This thesis deals with an experimental study of a CTR-based THz source employing mildly relativistic electron bunches from a compact RF photogun. Novel concepts are presented for generating both free-space THz CTR and THz SPPs coupled onto a metal wire.
The moment the electron reaches the metal line, which begins in the origin $O$, a sphere of light is emitted into vacuum. On the surface of the sphere the electric field lines change to the new situation. Within the sphere there is no electric field because the charge is totally screened by the metal. Outside the sphere the electric field lines are described by that of an electron traveling in the z-direction. The field lines end on the metal wire where they induce a surface charge density. The combination of the electric field on the metal wire, the surface currents and the charge densities constitutes the propagating SPPs.
In Ch. 2 we extensively discuss the theory of free-space CTR. We will show that a realistic metal-vacuum interface can be regarded as a perfectly conducting metal-vacuum interface in the THz frequency range. We derive the CTR electric fields for 45° incidence, because this is the used configuration in the experiments described in Ch. 5. In the experiments we used electro-optic detection to measure the CTR pulses. Therefore we calculate the temporal electric field of the generated CTR pulses.

In Ch. 3 we present the theory of CTR generated by ellipsoidal electron bunches. We calculate analytical expressions for the electric field spectrum, the power spectrum, and the temporal electric field of CTR, generated by cylindrically symmetric ellipsoidal electron bunches with hard and “soft” edges. This theory is relevant for diagnostics of ellipsoidal electron bunches. Realization of such bunches would solve the problem of space-charge induced emittance degradation, which is an important issue in accelerator physics.

The production of few-MeV, sub-ps, 100 pC electron bunches, which can be focussed to a sufficiently small spot size, requires state-of-the-art RF photogun technology. A major part of the experimental effort has therefore gone into the development of such a RF photogun. Chapter 4 treats design considerations, technical details, and the commissioning of the 1.5 cell RF photogun used to generate high-brightness electron bunches. The design is characterized by several innovative features, in particular the clamped, cylindrical symmetric structure. Another innovative aspect of our method is that we employ \( \sim 100 \) fs photoemission laser pulses generating \( \sim 100 \) pC bunches (instead of \( \sim 1 \) nC bunches generated with \( \sim 10 \) ps pulses). The achieved normalized transverse emittance and brightness of the electron bunches are comparable to the present state-of-the-art [50].

In Ch. 5 we present the measurements of single-cycle free-space THz pulses, generated using the CTR process. To enable the emitted CTR to be measured, we need to focus it into a small spot in order to increase the signal-to-noise ratio. This complicates matters because the behavior of focused broadband pulses is non-trivial. Especially since we are dealing with a radially polarized, half-cycle broadband THz pulse. Application of the physical theory of diffraction allows us to analyze the electric field profile in the focal spot in quite some detail, both the qualitative behavior and the quantitative field strengths. This enables us to make a realistic estimation of the potential of the 1.5 cell RF photogun as a source of free-space THz CTR. We show that is is possible to create THz pulses with peak-electric fields of 100 MV/m, having energies per pulse up to 10 \( \mu \)J, and bandwidths of 0.1 - 10 THz, using the CTR process in a compact manner. However, besides a single RF photogun this requires additional electron optics because the electron bunches need to be compressed in the longitudinal dimension to \( \sim 100 \) fs. We also find that by focusing a broadband THz pulse one inevitably throws away a substantial part of the pulse energy, because the low frequency components can not be focused as well as the high frequency components. This problem can be resolved by coupling a THz CTR pulse onto a metal wire.

In Ch. 6 we show that intense THz SPPs can be generated on a metal wire by launching electron bunches onto a tapered end of the wire. Half-cycle THz SPPs were generated on a metal wire of 1.5 mm thickness, characterized by a peak electric field strength of 0.5 MV/m and a full-width-half-maximum (FWHM) pulse length of \( \sim 6 \) ps. We have compared the
measured properties of the SPPs with a newly developed theory\textsuperscript{1}. The theory predicts the bandwidth correctly, however, the measured spectral amplitude was typically a factor of 5 less than the calculated amplitude. Probable causes for the discrepancy are the tip-to-wire transition and electron scattering in the metal tip, which are both not modeled in the theory. By optimizing the electron beamline and focusing the SPPs, by tapering the metal wire back into a tip, electric field strengths in excess of $\sim 100$ MV/m localized to a subwavelength spot become possible.

Bibliography

Chapter 1.


Coherent Transition Radiation

**Abstract.** Starting from the general theory of transition radiation (TR), we derive the TR fields generated by an electron traveling through a metal-vacuum interface. We show that the equations for a perfect metal-vacuum interface are applicable in the terahertz frequency range. The TR fields in the case of a metal-vacuum interface are also derived in an heuristic manner. This method gives physical insight in the TR generation process. In addition it allows straightforward calculation of TR created when an electron travels from vacuum into metal under $45^\circ$ incidence. We calculate the coherent transition radiation (CTR) power spectrum, generated when a bunch of electrons travels through a metal-vacuum interface, by adding the contributions of all individual electrons. Finally the temporal electric field of CTR is calculated, which is the physical observable measured by electro-optic detection methods in the experiments described in Ch. 5 and 6. To make this calculation physically relevant we have to take the finite transverse extent of the metal-vacuum interface into account.
2.1 Introduction

In general, an electron traveling through a spatially inhomogeneous dielectric, induces a time-varying polarization current that leads to radiation. When the spatially inhomogeneous dielectric is formed by an interface between two dielectrics it is called transition radiation (TR) [1]. In this chapter we discuss the theory of coherent transition radiation (CTR), emitted when a bunch of electrons travels through a metal-vacuum interface, which is applicable in the terahertz (THz) frequency range. We start with the general TR fields, as described by Ginzburg and Tsytovich in Ref. [2], and derive the conditions for which a dielectric can be regarded as a perfectly conducting metal. We will show that a typical metal fulfills these conditions in THz frequency range, especially when the electron is traveling with relativistic speed. The TR fields in the case of a perfect metal-vacuum interface can be derived in an heuristic approach [3]. We briefly discuss this approach in section 2.2.1 and use it to derive the TR fields of an electron incident under 45° at a metal-vacuum interface, because this geometry was used in the experiments.

When an electron bunch travels through a metal-vacuum interface the TR fields emitted by the individual electrons add up coherently at wavelengths larger than the bunch size. This coherently emitted radiation is generally called coherent transition radiation (CTR). The TR pulse emitted by a single electron typically contains only \( \sim 1 \) eV. However, since the energy radiated in the coherent spectrum scales quadratically with the number of electrons in the bunch, the radiated energy in the case of CTR can be considerable. Especially if one realizes that an electron bunch produced in a RF photogun typically contains \( \sim 10^8 \) electrons. Moreover, since the electron bunch length is \( \sim 1 \) ps, the CTR spectrum will extend into the THz range. We calculate the CTR power spectrum assuming a 3D Gaussian electron distribution.

In the experiments we used electro-optic detection to measure the CTR pulses. Therefore we calculate the temporal electric field of CTR in section 2.4. To make this calculation physically relevant the frequency spectrum has to be cut off at high and low frequencies. The high frequency components of the coherent radiation are either cut off by the dimensions of the electron bunch, or by the dielectric response of the medium. The low frequency components are cut off due to the finite extent of the metal, resulting in diffraction radiation [4]. To take this into account we will use Babinet’s [5] principle in section 2.4 to calculate the coherent diffraction radiation of a dielectric disc much larger than the transverse bunch size [6, 7].

2.2 Metal-vacuum interface

We discuss the TR fields for the situation sketched in Fig. 2.1, where \( \mathbf{r}_j \) is the position of the electron at \( t = 0 \) and \( \mathbf{v} = v\mathbf{e}_z \) is the velocity of the electron. The interface between the two dielectrics is located at \( z = 0 \), for \( z < 0 \) the dielectric constant equals \( \varepsilon^- \), for \( z > 0 \) the dielectric constant equals \( \varepsilon^+ \). For the moment we will assume that \( \mathbf{r}_j = \mathbf{0} \). In Appendix A we have calculated the TR fields of an electron traveling through an interface between
two dielectrics. The electric field spectrum $E_\theta(r, \omega)$ of TR for $\varepsilon^+ = 1$ and $z > 0$ is given by, see appendix Eqs. (A-14) and (A-30),

$$
E_\theta(r, \omega) = \frac{e \beta \sin \theta \cos \theta}{(2\pi)^2 \varepsilon_0 c (1 - \beta^2 \cos^2 \theta)} \frac{e^{i \pi r}}{r} \times \left( \frac{\sin^2 \theta + \beta^{-2} - 1}{\sin^2 \theta + \beta^{-2} - \varepsilon^-} - \left( \beta \sqrt{\varepsilon^- \sin^2 \theta + 1} \right) \right)
$$

$$
- (\varepsilon^- \cos \theta + \sqrt{\varepsilon^- - \sin^2 \theta})
$$

(2.1)

with $\varepsilon_0$ the permittivity of free space, $e$ is the electronic charge, $c$ is the speed of light, $\beta = \frac{v}{c}$, and $r$ is the distance to an observer. The observation vector $r$ makes an angle $\theta$ with the $z$ axis. For a metal $|\varepsilon^-| \gg 1$, therefore we expand the second part of the right hand side of Eq. (2.1) in negative powers of $\varepsilon^-$

$$
E_\theta(r, \omega) = \frac{e \beta \sin \theta}{(2\pi)^2 \varepsilon_0 c (1 - \beta^2 \cos^2 \theta)} \frac{e^{i \pi r}}{r} \times
$$

$$
\left( 1 - \frac{1 - \beta \cos \theta}{\cos \theta} \frac{1}{\beta \cos \theta} + \frac{1}{\cos^2 \theta} \right) - \frac{1}{\varepsilon^-} + O\left( \frac{1}{\varepsilon^-} \right)^{3/2}.
$$

(2.2)

We can now immediately see that

$$
\lim_{|\varepsilon^-| \to \infty} E_\theta(r, \omega) = \frac{-e \beta \sin \theta}{(2\pi)^2 \varepsilon_0 c (1 - \beta^2 \cos^2 \theta)} \frac{e^{i \pi r}}{r}.
$$

(2.3)
In a similar fashion it can be shown that for \( z < 0 \)

\[
\lim_{|\varepsilon| \to \infty} E_\theta(r, \omega) = 0,
\]

as it should in a perfectly conducting metal. If we calculate the power spectrum using Eq. (2.3) we obtain (see Appendix A)

\[
\frac{\partial^2 W_{\text{metal}}}{\partial \omega \partial \Omega} = \frac{e^2 \beta^2 \sin^2 \theta}{4\pi \varepsilon_0 c \pi^2 (1 - \beta^2 \cos^2 \theta)^2}.
\]

Equation (2.5) is the well-known expression for the radiated energy in the case of an electron traveling through the interface between a perfect mirror and vacuum [2, 8]. So, as far as transition radiation is concerned, we can regard a dielectric as a perfectly conducting metal if the second term in the power series (2.2) is negligible:

\[
\frac{1 - \beta \cos \theta}{\sqrt{|\varepsilon - | \cos \theta}} \ll 1.
\]

This condition shows that for all values of \( \beta \) and \( \varepsilon^- \), there always exists an angle \( \theta \) at which the dielectric can not be regarded as a metal, because of the \( \cos \theta \) term in the denominator. In Fig. 2.2 the radiated energy \( \frac{\partial^2 W_{\text{metal}}}{\partial \omega \partial \Omega} \) radiated by a single electron, is plotted as a function of \( \theta \), for a perfect metal-vacuum interface (\( \varepsilon^- = \infty, \varepsilon^+ = 1 \)), and two different dielectric-vacuum interfaces (\( \varepsilon^- = 10^2, 10^5, \varepsilon^+ = 1 \)). The calculations have been done for four different values of \( \gamma^{-1} = \sqrt{1 - \beta^2} \). If we examine the plot for \( \gamma = 1.2 \), and focus on the curve for \( \varepsilon^- = 10^2 \), we see that the perfect metal-vacuum expression is inapplicable almost regardless of \( \theta \). However, when the dielectric constant increases, see the curve for \( \varepsilon^- = 10^5 \), the perfect metal-vacuum expressions closely follows the exact curve. It only deviates at large angles. For increasing \( \gamma \) the restriction on \( \varepsilon^- \) becomes even less, see the plots for \( \gamma = 4, \gamma = 8 \) and \( \gamma = 12 \), because most of the radiation is emitted at small angles. For larger angles, i.e. \( \theta > 1 \), there is a significant deviation, but only a small fraction of the total radiation is emitted at those angles. Since the typical dielectric constant of a real metal is of the order of \( 10^5 \) for frequencies in the THz regime [9], we can use the perfect metal-vacuum expression without seriously limiting the applicability of the theory.

### 2.2.1 Heuristic derivation of coherent transition radiation for a metal-vacuum interface

The TR emitted when an electron passes a metal-vacuum interface (see Eq. (2.3)) can be derived using charged particles which suddenly start or stop moving. Consider, for example, an electron and a positively charged image particle traveling towards each other both stopping when they meet. The image particle takes care of the boundary conditions at the interface, before the particles meet. When the electron and image particle suddenly
Figure 2.2: The energy emitted when an electron passes a perfect metal-vacuum interface, or a dielectric-vacuum interface ($\varepsilon^+ = 1$) as a function of $\theta$. The plots are made for two different values of $\varepsilon^-$ and four different values of $\gamma = (1 - \beta^2)^{-1/2}$. 
Figure 2.3: Electrical field of the charge in the case of an instant change of velocity. Initially the charge was at rest in the point $z=0$. At the moment of time $t=0$ the velocity instantly changed to some value $v$. Taken from [10].
stop moving as they meet, they screen each others fields from then on, which is precisely what happens when an electron travels into a metal.

To calculate TR using starting charges, we first consider an electron initially at rest, which suddenly starts moving (see Fig. 2.3). The electric field lines have to adapt to the new situation, but this can not happen instantaneously. The reordering of field lines will happen on a sphere traveling outwards with the speed of light. Within the sphere the electric field is given by that of a charge traveling with a speed \( v \), outside the sphere the electric field is that of an electron at rest. This means that the electric field lines make a sudden jump going through the sphere. But since electric field lines can only break at the location of charges, they have to close over the surface of the sphere. The field lines at the sphere are thus tangent to the surface, which is perpendicular to the direction of propagating. They therefore form the radiation pulse which is emitted due to the sudden start of the electron. For an electron located at \( z = 0 \) which started moving at \( t = 0 \) with a velocity \( v \) in the +\( z \)-direction this radiation pulse is given by [3],

\[
E(r, \omega) = \frac{-e\beta \sin \theta}{8\pi^2 \epsilon_0 c (1 - \beta \cos \theta)} \frac{e^{i \omega r}}{r} e^\theta. \tag{2.7}
\]

When an electron passes the interface between a metal and vacuum the situation for \( z > 0 \) resembles that of a starting electron. Once again the electric field lines are reordered on a sphere traveling outwards with the speed of light, originating from the position where the electron passed the interface. Outside the sphere no electric field is present. Inside the sphere the electric field is that of a electron traveling with a speed \( v \), plus that of a positively charged particle imaged in the metal-vacuum interface. This second particle is necessary for the boundary conditions at the metal surface. The field lines close on the sphere ending on the metal surface, which is allowed due to the surface charge density at the metal surface.

Using Eq. (2.7) we can therefore construct the TR fields of an electron traveling from a perfect metal into vacuum. If we assume that at \( t = 0 \) the electron suddenly started moving in the +\( z \)-direction, and that at the same moment a particle of opposite charge started moving in the −\( z \)-direction, we can write

\[
E_\theta = \frac{-e\beta \sin \theta}{8\pi^2 \epsilon_0 c} \frac{e^{i \omega r}}{r} \left( \frac{1}{1 - \beta \cos \theta} + \frac{1}{1 + \beta \cos \theta} \right) \tag{2.8}
\]

\[
E_\theta = \frac{-e\beta \sin \theta}{(2\pi)^2 \epsilon_0 c (1 - \beta^2 \cos^2 \theta)} \frac{e^{i \omega r}}{r}, \tag{2.9}
\]

giving the transition radiation pulse of a single electron, see (Eq.2.3).

### 2.2.2 Transition Radiation at 45° incidence

Using image charges we can also find the TR fields created when an electron travels from vacuum into metal under 45° incidence. The situation is drawn schematically in Fig. 2.4. Because the metal acts simultaneously as TR interface and mirror, the CTR pulse is emitted
Figure 2.4: Schematic picture of transition radiation in the case of 45° incidence. The moment the electron passes the vacuum-metal interface a sphere of light is emitted into vacuum. On the surface of the sphere the electric field lines change to the new situation. Within the sphere there is no electric field because the charge is totally screened by the metal. Outside the sphere the electric field lines are described by that of an electron and its corresponding image charge.
into vacuum at right angles with respect to the incoming electron bunch. We can describe the TR pulse with an electron traveling in the +x-direction, and a positively charged particle traveling in the +z-direction, which both suddenly stop in the origin at \( t = 0 \).

The radiation emitted when an electron suddenly stops in the origin is equal in magnitude but opposite in sign to Eq. (2.7). We can therefore use Eq. (2.7) to construct the radiation emitted by an electron traveling in the +x-direction stopping in the origin at \( t = 0 \), by changing sign, rotating the coordinates 90° clockwise around the y-axes, and expressing the result into spherical coordinates of the rotated frame

\[
E(\mathbf{r}, \omega) = \frac{e\beta}{8\pi^2\epsilon_0c} \frac{e^{i\nonumber{2}r}}{r} \left( \frac{\sin \varphi}{1 - \beta \cos \varphi \sin \theta} e_{\varphi} - \frac{\cos \varphi \cos \theta}{1 - \beta \cos \varphi \sin \theta} e_{\theta} \right).
\]  

(2.10)

The TR pulse at 45° incidence, \( E_{45^\circ}(\mathbf{r}, \omega) \), is the sum of Eq. (2.7) and Eq. (2.10),

\[
E_{45^\circ}(\mathbf{r}, \omega) = \frac{e\beta}{8\pi^2\epsilon_0c} \frac{e^{i\nonumber{2}r}}{r} \left( \frac{\sin \varphi}{1 - \beta \cos \varphi \sin \theta} e_{\varphi} + \frac{(\beta - \cos \theta) \cos \varphi - \sin \theta}{(1 - \beta \cos \varphi \sin \theta)} e_{\theta} \right).
\]  

(2.11)

Figure 2.5 shows the radiated energy for an electron traveling from vacuum into metal under 45° incidence, for \( \varphi = 0 \). The energy is radiated in an asymmetric hollow cone and is brightest at an angle of \( \theta \simeq \gamma^{-1} \). Figure 2.6 shows a schematic polar plot of the TR energy in case of 45° incidence, to illustrate that the TR is emitted at 90° with respect to the incoming electron.

### 2.3 Coherent transition radiation

If a bunch of \( N \) electrons passes an interface between two dielectrics, we can calculate the CTR by adding the contributions of all individual electrons. We will assume that all electrons travel in the \( z \)-direction with the same velocity.

#### 2.3.1 Form factor at normal incidence

If the electron is located at \( \mathbf{r}_j \neq 0 \) at \( t = 0 \), the field (2.3) acquires and additional phase factor, which can be can written for \( \mathbf{r} >> \mathbf{r}_j \) as

\[
E_\theta(\mathbf{r}, \omega) = \frac{-e\beta \sin \theta}{(2\pi)^2\epsilon_0c(1 - \beta^2 \cos^2 \theta)} \frac{e^{i\nonumber{2}r}}{r} e^{-i\frac{\mathbf{n} \cdot \mathbf{r}_j}{c}} e^{-i\frac{\mathbf{v}_j t}{c}},
\]  

(2.12)

with \( \mathbf{n} = \hat{\xi} \) the unit vector in the direction of observation and \( \mathbf{r}_j \) the projection of \( \mathbf{r}_j \) on the x-y plane.

Eq. (2.12) can be used to calculate the CTR radiated by a bunch of \( N \) electrons traveling through a perfect-metal vacuum interface. We define \( \rho(\mathbf{r}) \) as the electron density
Figure 2.5: The energy emitted when an electron travels from vacuum into metal under $45^\circ$ incidence. Plot is made for $\varphi = 0$ and $\gamma = 6.9$. 
distribution of the bunch. We assume that all electrons travel in the \( z \) direction with the same velocity, and that each electron is located at a different position \( r_j \) at \( t = 0 \). The electric field spectrum of the total CTR can then be written as

\[
E_\theta(r, \omega) = \frac{-eN\beta \sin \theta}{(2\pi)^2 \varepsilon_0 c(1 - \beta^2 \cos^2 \theta)} f(\omega) \frac{e^{i\frac{\omega}{c} r}}{r}.
\] (2.13)

The term \( f(\omega) \) is called the form factor and is given by

\[
f(\omega) = \frac{1}{N} \sum_{j=1}^{N} e^{-i\frac{\omega}{c} n \cdot r_j - i\frac{\omega}{c} z_j},
\] (2.14)

Because \( N \) is typically very large, we can replace the summation in Eq. (2.14) with an integral over the electron distribution function \( h(r) \equiv -\rho(r)/eN \)

\[
f(\omega) = \int h(r)e^{-i\frac{\omega}{c} n \cdot r - i\frac{\omega}{c} z} d^3r.
\] (2.15)

Note that \( h(r) \) is subject to the normalization condition

\[
\int h(r) d^3r = 1.
\] (2.16)

Since we replaced the summation over the spatial distribution of the electron bunch with an integral over a well-behaved continuous function, we can not describe the incoherent

---

**Figure 2.6:** Schematic polar plot of TR energy in case of 45° incidence. The plot is made for \( \gamma = 6.9 \). The TR is emitted at 90° with respect to the incoming electron. The radiated energy is an asymmetric hollow cone, which peaks at \( \theta \approx \gamma^{-1} \).
part of the transition radiation with Eq. (2.15). Calculation of the incoherent part would require taking into account the exact position of each electron. However, the electron density distribution scales with \(N\), while in general, the local density fluctuations scale with \(\sqrt{N}\) [11]. Due to the large number of electrons in a typical bunch (\(N \sim 10^8\)), the fluctuations and thus the incoherent radiation, can safely be neglected. We will therefore proceed with the coherent radiation only.

### 2.3.2 Form factor of a Gaussian electron distribution for normal incidence

If we assume a Gaussian electron distribution we can calculate the form factor. In this case the electron distribution is given by,

\[
h_g(r) = \frac{1}{(2\pi)^{3/2} \sigma_t^2 \sigma_l^2} \exp[-\frac{1}{2}(\frac{\rho^2}{\sigma_t^2} + \frac{z^2}{\sigma_l^2})],
\]

where \(\sigma_t\) is the root-mean-square (RMS) transverse size and \(\sigma_l\) the RMS longitudinal size of the electron bunch. Substitution of Eq. (2.17) into Eq. (2.15) leads to

\[
f_g(\omega) = \exp[-\frac{1}{2} \omega^2 (\frac{1}{\beta c^2} (\sigma_t^2 \sin^2 \theta + \sigma_l^2 \beta^{-2})),
\]

where \(f_g(\omega)\) is defined as the form factor of a electron bunch with a Gaussian electron distribution.

### 2.3.3 Form factor of a Gaussian electron distribution at 45°

The form factor at 45° incidence given by

\[
f_{45°}(\omega) = \int h(r) \exp[-i \frac{\omega}{\beta c} (x + \beta \sin \varphi \sin \theta y + (\beta \cos \varphi \sin \theta + \beta \cos \theta - 1) z)]d^3 r.
\]

For a Gaussian electron distribution this becomes,

\[
f_{45°}(\omega) = \exp[-\frac{1}{2} \frac{\omega^2}{c^2 \beta^2} (\sigma_t^2 + \sigma_l^2 (\beta^2 \sin^2 \varphi \sin^2 \theta + (\beta \cos \varphi \sin \theta + \beta \cos \theta - 1)^2))].
\]

If \(\gamma >> 1\) most of the radiation is emitted at small angels \(\theta \simeq \gamma^{-1}\) which means we can neglect the transverse dimensions of the electron bunch as long as \(\frac{\sigma_t}{\gamma} << \sigma_l\), resulting in a particularly simple form factor

\[
f_{45°}(\omega) = \exp[-\frac{1}{2} \frac{\omega^2}{c^2 \beta^2} \sigma_t^2].
\]
2.3.4 Power Spectrum of CTR

Fig. 2.7 shows a plot of the power spectrum emitted at an angle \( \theta \) by an electron bunch with a Gaussian electron density, normally incident on a metal-vacuum interface. We observe a flat spectrum, up to the point where the frequency reaches \( \omega_{\text{incoh}} \equiv c/\sqrt{\sigma_t^2 \sin^2 \theta + \sigma_z^2 \beta^{-2}} \), with \( \omega_{\text{incoh}} \) the frequency at which the power spectrum drops to zero. To shed some light on this condition for the frequency, we will assume for the moment that we can neglect the transverse dimension of the electron bunch. It is customary to express the length of an electron bunch in a time scale, \( \sigma_l = \frac{1}{2} c \beta \tau \), with \( \tau \) the time it takes for a bunch to pass by. Combining all this we can write \( \omega_{\text{incoh}} = \frac{\omega_0}{2} \). This is not a surprise, because when \( \omega \tau \ll 1 \) the electrons emit the radiation with almost no phase difference, resulting in constructive interference. On the other hand when \( \omega \tau \gg 1 \), the radiation has collected a considerable amount of phase difference during the emission process, resulting in destructive interference. This argument remains the same when the transverse dimension of the electron bunch is not negligible. In that case the phase difference is the result of both the spatial and temporal extension of the electron bunch: The spatial phase difference leads to the \( \sin \theta \) term in \( \omega_{\text{incoh}} \).
2.4 Temporal electric field of CTR

The last step in reconstructing the CTR pulse in space and time domain, is the inverse Fourier transformation over the frequency

\[ E_\theta(r, t) = \int E_\theta(r, \omega)e^{-i\omega t}d\omega. \]  
(2.22)

If we substitute Eq. (2.3) into Eq. (2.22) we can write

\[ E_\theta(r, t) = \frac{-eN\beta \sin \theta}{(2\pi)^2\varepsilon_0 c(1 - \beta^2 \cos^2 \theta)} \frac{1}{r} \int f(\omega)e^{-i\omega t_0}d\omega, \]  
(2.23)

with \( t_0 = t - \frac{r}{c} \). At this point we need to address a problem with the form factor. If we examine Eq. (2.18) we see that the spectrum is coherent down to \( \omega = 0 \). According to Eq. (2.23) this implies a DC offset in the radiation pulse, which is physically impossible. This is a consequence of the assumption of an infinitely large metal-vacuum interface. In reality the polarization currents, responsible for the CTR, are spatially limited by the transverse size of the metal-vacuum interface. They therefore create coherent diffraction radiation at the edges of the interface [4]. Coherent diffraction radiation (CDR) will be generated only at wavelengths larger than the spatial dimensions of the interface, i.e. small \( \omega \), since smaller wavelengths will add up incoherently. As we will show, CDR leads to the cancelation of CTR at \( \omega \to 0 \).

2.4.1 Coherent diffraction radiation

To show the influence of CDR on the radiated CTR spectrum, we will discuss it here briefly. Diffraction radiation can be calculated by applying Kirchhoff’s diffraction theory to the fields incident on the interface [5], in our case the fields of the electron bunch. For an electron traveling perpendicular through a circular aperture the expressions for diffraction radiation are well known [6], and can be used to model the finite extent of the perfect metal-vacuum interface, using Babinet’s principle [5]. The necessary assumption of a circular interface is not a problem, we are only interested in the general influence of the CDR, independent of a particular geometry. Since a circular interface maintains the cylindrical symmetry of the problem, it is a natural choice. We choose the radius of the interface \( a \gg \sigma_t \). This allows us to model the bunch as a point particle with charge \(-eN\), passing through the center of the circular interface. Under these assumptions we can write

\[ E^{cr}_\theta(r, \omega) = \frac{-eN\beta \sin \theta}{(2\pi)^2\varepsilon_0 c(1 - \beta^2 \cos^2 \theta)} \frac{e^{i\frac{2\pi}{r}r}}{r} f(\omega)(1 - d(\omega)), \]  
(2.24)

for the spectrum of the electric field of the total coherent radiation \( E^{cr}_\theta(r, \omega) \), which describes both CTR and CDR. The function \( d(\omega) \) describes the diffraction radiation and is
Figure 2.8: Frequency dependence of the electric field spectrum of the total coherent radiation pulse, including CTR and CDR. The parameters used to make the plot are listed in the upper right corner.
given by [6, 7]

\[
d(\omega) = \frac{a \sin \theta \omega}{c} \left[ J_0\left( \frac{a \omega}{\beta \gamma c} \right) K_1\left( \frac{a \omega}{\beta \gamma c} \right) + \frac{1}{2} \left( \frac{a \omega}{\beta \gamma c} \right)^2 K_0\left( \frac{a \omega}{\beta \gamma c} \right) \right] + \frac{1}{2} \left( \frac{a \omega}{\beta \gamma c} \right)^2 J_2\left( \frac{a \omega}{\beta \gamma c} \right) K_0\left( \frac{a \omega}{\beta \gamma c} \right),
\]

(2.25)

with \( J_m \) and \( K_m \) the \( m \)-th order regular and modified Bessel functions, respectively. Fig. 2.8 shows a plot of the frequency dependence of \( E_{\theta}^{cr}(r, \omega) \). Note the break in the frequency scale, which serves to illustrate the spectrum at both low and high frequencies. The values in the upper right corner indicate the parameters used in the plot. The important feature of the plot, however, is the fact that \( E_{\theta}^{cr}(r, \omega) \to 0 \), as \( \omega \to 0 \). The DC offset in the radiation is thus indeed removed by taking CDR into account.

The expression in Eq. (2.24) can be simplified to

\[
E_{\theta}^{cr}(r, \omega) = \frac{-eN\beta \sin \theta}{(2\pi)^2 \varepsilon_0 c (1 - \beta^2 \cos^2 \theta)} \frac{e^{i\pi r}}{r} \left( f(\omega) - d(\omega) \right).
\]

(2.26)

This is allowed because we can write \( f(\omega)d(\omega) = d(\omega) \), since \( f(\omega) \approx 1 \) when \( d(\omega) \neq 0 \). This is a direct consequence of the assumption \( \sigma_t \ll a \), and is the mathematical consequence of treating the electron bunch as a point like particle with charge \(-eN\), passing through the center of the circular interface. If we look at Eq. (2.26) we see that the total coherent radiation can be described by subtracting the CDR from the CTR, independent of the CTR. We can thus continue with calculating the temporal electric field of CTR, by evaluating the inverse Fourier transform of \( f(\omega) \), as long as we keep in mind that the CDR has to be subtracted. This means that we have to evaluate the inverse Fourier transform of \( f(\omega) \) and \( d(\omega) \). Both will be done in the next two subsections.

### 2.4.2 Inverse Fourier transform of \( d(\omega) \)

Evaluating the inverse Fourier transform of \( d(\omega) \) analytically is not straightforward and outside the scope of this thesis. Instead we will use the fast fourier transform (FFT) algorithm to evaluate Eq. (2.24) numerically. This will illustrate the influence of CDR on CTR quite generally, as long as the assumption \( R_t \ll a \) holds. The result of the FFT is shown in Fig. 2.9. The relevant parameters are listed in the upper left corner. We observe two half cycle pulses superposed on each other. The long negative pulse is CDR, the short positive pulse is CTR. The CTR pulse is shown in more detail in the upper right corner. The important feature of Fig. 2.9 is that the CDR pulse is almost constant on the time scale of the CTR pulse, and the electric field strength is much lower. This conclusion holds generally as long as \( \frac{a}{\beta \gamma} \gg \sigma_t \) and \( \frac{a}{\beta \gamma} \gg \sigma_l \). We will therefore proceed with the temporal CTR pulse only and neglect the CDR.
\[ a/\beta \gamma = 10^2 \left( \sigma_\gamma^2 \gamma^{-2} + \sigma_\beta^2 \gamma^{-2} \right)^{1/2} \]

\[ \gamma = 6.5 \]

\[ \sin \theta = \gamma^{-1} \]

\[ \sigma_t/c = \sigma_t/c = 0.2 \text{ ps} \]

\[ eN = 100 \text{ pC} \]

Figure 2.9: Plot of temporal electric field, with coherent diffraction radiation taken into account. The long negative pulse is the CDR. The CTR pulse is shown in the inset in the upper right corner. The parameters used to make the plot are listed in the upper left corner.
2.4.3 Inverse Fourier transform of $f_g(\omega)$

The inverse Fourier transform of $f_g(\omega)$ is,

$$f_g(t_0) = \int f_g(\omega)e^{-i\omega t_0}d\omega = \frac{\sqrt{2\pi}}{\tau_g}e^{-\frac{1}{2} \left( \frac{\omega}{\tau_g} \right)^2},$$

(2.27)

with $\tau_g = c^{-1}\sqrt{\sigma_t^2 \sin^2 \theta + \sigma_t^2 \beta^{-2}}$. The CRT pulse in the case of a Gaussian electron bunch $E_{\theta g}(r, t)$ is thus

$$E_{\theta g}(r, t) = \frac{-eN\beta \sin \theta}{(2\pi)^2 \epsilon_0 c(1 - \beta^2 \cos^2 \theta)} \frac{\sqrt{2\pi}}{\tau_g r} e^{-\frac{1}{2} \left( \frac{\omega}{\tau_g} \right)^2}.$$  

(2.28)

The inset of Fig. 2.9 shows a plot of Eq. 2.28. It is a half-cycle pulse with a Gaussian shape. The peak electric field strength decreases with distance. For example, at 10 cm distance the peak electric field strength would be $\sim 400$ kV/m, for an electron bunch with the specifications listed in the upper left corner of Fig. 2.9. Since the electron bunches used in the experiments described in Ch. 5 typically have bunch lengths of $\sim 1$ ps, we expect peak electric fields of $\sim 80$ kV/m. To increases the electric field we used a mirror to focus the CTR onto an electro-optic detection crystal. The mirror will be discussed in Ch. 5.

2.5 Summary and Conclusions

We have analytically calculated the electric field spectrum, power spectrum and temporal electric field of CTR, created by electron bunches with a Gaussian electron density distribution. We have shown that TR in the case of a perfect metal-vacuum interface can also be derived using instantaneously starting and stopping charges and their respective images. Using this method we have derived the fields emitted when an electron bunch travels into a perfect metal making an angle of $45^\circ$ with the interface. To make the calculation of the temporal electric field of CTR physically relevant, the finite transverse size of the perfect metal-vacuum interface has to be taken into account. This is done by calculating CDR, assuming that the radius of the interface is much larger than the transverse dimensions of the electron bunch. It is shown that the temporal electric field of CDR can be neglected with respect to the CTR electric field pulse, as long the radius of the interface is much larger than the transverse dimension of the electron bunch.
Bibliography

Coherent Transition Radiation as diagnostic tool of ellipsoidal electron bunches

Abstract. We present the theory of coherent transition radiation (CTR) generated by ellipsoidal electron bunches. We calculate analytical expressions for the electric field spectrum, the power spectrum, and the temporal electric field of CTR, generated by cylindrically symmetric ellipsoidal electron bunches with hard and "soft" edges. This theory is relevant for diagnostics of ellipsoidal electron bunches. Realization of such bunches would solve the problem of space-charge induced emittance degradation.

3.1 Introduction

For a long time it has been realized that uniformly filled ellipsoidal electron bunches, also known as "waterbag" bunches, are the ideal particle distributions for controlled, high-brightness charged particle acceleration. Due to their linear internal fields they do not suffer from brightness degradation caused by space-charge forces [1, 2]. In spite of this they were never considered as a realistic option, because there was no method to create them. In 1997 Serafini was the first to suggest that waterbag bunches may be created by using high-gradient radio-frequency (RF) photoguns, operated in the space-charge blowout regime [3]. In 2004 Luiten et al. presented a practical recipe which results in an almost ideal hard-edged ellipsoidal electron bunch [4–6]. The deviation from the ideal situation is caused by the finite duration of the photoemission process and results in "soft" edges; the density does not fall off to zero abruptly, but over a distance comparable with the laser pulse length [4].

Coherent transition radiation (CTR), created by an electron bunch experiencing a sudden change in dielectric constant [7–9], is a well-known diagnostic tool to characterize the spatial distribution of electron bunches [10–13]. In this chapter we extend the analytical theory of CTR to ellipsoidal electron bunches with soft-edges, which include all bunch shapes from perfectly smooth Gaussians, to hard-edges ellipsoids. This makes the detection possible of realistic waterbags bunches using CTR.

In section 3.2 we will calculate the form factor of an ellipsoidal spatial distribution, both with soft and hard edges. The power spectrum of CTR generated by ellipsoidal electron bunches traveling through a perfect metal-vacuum interface is discussed in 2.3.4. In section 3.3 the temporal electric field of CTR will be calculated.

3.2 Form factor of ellipsoidal electron bunch

We evaluate the form factor for three different electron distributions with ellipsoidal symmetry. First, we evaluate the form factor of a uniformly filled ellipsoidal electron bunch with hard edges; second, for the case of an electron bunch with a Gaussian distribution; third, we use these two distributions to construct an electron distribution with ellipsoidal symmetry, that falls of smoothly from a uniform value to zero. This will serve as a model for a uniform filled ellipsoidal electron bunch with soft edges. To this purpose we write Eq. (2.15) as a one dimensional integral, by assuming that $h(r)$ is a function only of the radial coordinate of a spheroid; $R \equiv \sqrt{x^2 + y^2 + \alpha^2 z^2}$, where $\alpha$ is defined as the aspect ratio of the electron bunch. To illustrate the radial coordinate of a spheroid, the contour line defined by $\sqrt{x^2 + \alpha^2 z^2} = R_t$ is plotted in Fig. 3.1. A spheroid is an ellipsoid having two equal semi-axes, in our case $x$ and $y$. We will refer to it as an ellipsoid in the remainder of this chapter. The three different electron distribution; $h_e$ the uniform filled ellipsoidal electron bunch with hard edges, $h_g$ the Gaussian electron distribution, and $h_{e \otimes g}$ the electron distribution that falls of smoothly from a uniform value to zero, are drawn in Fig. 3.2.
Coherent Transition Radiation as diagnostic tool of ellipsoidal electron bunches

\[(x^2 + \alpha^2 z^2)^{1/2} = R_t\]

\[\alpha_o > 1\]
\[\alpha_s = 1\]
\[0 < \alpha_p < 1\]

\[R_t\]

\[0 - R_{\alpha_o} \quad R_{\alpha_s} \quad R_{\alpha_p}\]

\[z\]

**Figure 3.1**: Illustration of radial coordinate of a spheroid, which is an ellipsoid with two equal semi-axes. Shown is the contour line defined by \(\sqrt{x^2 + \alpha^2 z^2} = R_t\). The electron density is, by definition, constant on this contour line. The electron bunch can be oblate (\(\alpha > 1\)), spherical (\(\alpha = 1\)) and prolate (\(0 < \alpha < 1\)).

The form factor of an electron distribution with ellipsoidal symmetry can be written as

\[
f(\omega) = \frac{4\pi}{\alpha} \int_0^\infty h(R) \sin\left(\frac{R\Theta}{c}\right) \frac{R}{\Theta} dR,
\]

where \(R = \sqrt{x^2 + y^2 + \alpha^2 z^2}\) and

\[
\Theta = (\alpha \beta)^{-1} \sqrt{\alpha^2 \beta^2 \sin^2 \theta + 1}.
\]

The derivation from Eq. (2.15) to Eq. (3.1) is given in Appendix B-1.

We start with the electron density of a hard edged uniformly filled ellipsoidal electron bunch, which is given by

\[
h_e(R) = \begin{cases} 
\frac{3\alpha}{4\pi R_t^3} & \text{if } |R| \leq R_t; \\
0 & \text{if } |R| > R_t,
\end{cases}
\]

(3.3)
Figure 3.2: Plot of the three different electron distributions. The upper picture shows the electron distribution $h_e$, the uniform filled ellipsoidal electron bunch with hard edges. In the middle the Gaussian electron distribution $h_g$ is shown. The lower picture shows $h_{e\otimes g}$, the electron distribution that falls of smoothly from a uniform value to zero. The choices $\sigma_t = \frac{1}{4} R_t$ and $\sigma_R = \frac{1}{10} R_t$ have been made for illustrative purposes.
where \( R_t \) is the transverse radius of the ellipsoidal electron bunch, see Fig. 3.1. Substitution of Eq. (3.3) into Eq. (3.1) leads to

\[
 f_e(\omega) = 3 \frac{\sin(R_t \Theta \omega c) - (R_t \Theta \omega c) \cos(R_t \Theta \omega c)}{(R_t \Theta \omega c)^3}, \tag{3.4}
\]

where we defined \( f_e(\omega) \) as the form factor of a hard edged uniformly filled ellipsoidal electron bunch.

The next is a Gaussian electron distribution,

\[
 h_g(R) = \frac{\alpha}{(2\pi)^{3/2} \sigma_t^3} e^{-\frac{R^2}{2\sigma_t^2}}, \tag{3.5}
\]

where \( \sigma_t \) is the root-mean-square (RMS) transverse dimension of the electron bunch. Substitution of Eq. (3.5) into Eq. (3.1) leads to

\[
 f_g(\omega) = e^{-\frac{1}{2}(\sigma_t \Theta \omega c)^2}, \tag{3.6}
\]

where \( f_g(\omega) \) is defined as the form factor of a electron bunch with a Gaussian electron distribution.

We can model a uniformly filled ellipsoidal electron bunch, that falls of smoothly from the uniform value to zero, by writing the electron distribution function as the convolution of a hard-edged distribution of size \( R_t \) and a Gaussian distribution with \( \sigma_t = \sigma_R \)

\[
 h_{e\otimes g}(R) = A h_e \otimes h_g = A \int_{-R_t}^{R_t} e^{-\frac{(R-\xi)^2}{2\sigma_R^2}} d\xi =

 A \sqrt{\frac{\pi}{2}} \sigma_R \left( \text{erf}(\frac{R + R_t}{\sqrt{2} \sigma_R}) - \text{erf}(\frac{R - R_t}{\sqrt{2} \sigma_R}) \right), \tag{3.7}
\]

where \( \text{erf} \) is the error function \[14\]. The normalization constant is given by

\[
 A = \frac{3\alpha}{4\sqrt{2\pi}^3 R_t^3 \sigma_R (1 + 3\frac{\sigma_t^2}{R_t^2})}. \tag{3.8}
\]

If we calculate the form factor by substituting Eq. (3.7) into Eq. (3.1) we obtain

\[
 f_{e\otimes g}(\omega) = \frac{3}{(1 + 3\frac{\sigma_t^2}{R_t^2})} e^{-\frac{1}{2}(\sigma_R \Theta \omega c)^2} \times

 \frac{(1 + (\sigma_R \Theta \omega c)^2) \sin(R_t \Theta \omega c) - R_t \Theta \omega c \cos(R_t \Theta \omega c)}{(R_t \Theta \omega c)^3}. \tag{3.9}
\]

Note that Eq. (3.9) can be used to generate the form factor of the uniformly filled ellipsoidal, and the Gaussian electron bunch. If we e.g. take the limit of vanishing \( \sigma_R \)

\[
 \lim_{\sigma_R \rightarrow 0} f_{e\otimes g}(\omega) = f_e(\omega), \tag{3.10}
\]
we obtain the form factor of the uniformly filled ellipsoidal electron bunch with hard edges. In the limit of vanishing $R_t$

$$\lim_{R_t \to 0} f_{e\otimes g}(\omega) = f_g(\omega),$$

we obtain the form factor of the electron bunch with a Gaussian electron density.

The three different form factors are plotted in Fig. 3.3. If we compare $f_e$ and $f_g$, we observe that the CTR emitted by an ellipsoidal electron bunch is coherent up to higher frequencies than a Gaussian electron bunch with comparable dimensions. Partly this is because $\sigma_t$ is defined as the RMS value, while $R_t$ characterizes the entire width of the electron bunch. However, taking this difference in definition into account, we still see that $f_e$ is coherent up to higher frequencies than $f_g$. This is due to the hard edges of the ellipsoidal electron bunch, which introduce infinitely high frequencies in the spectrum of the CTR. If we look at the form factor of a uniformly filled ellipsoid with soft edges, $f_{e\otimes g}$, we see that these high frequencies are suppressed. However, the form factor is only slightly affected by the soft edges for frequencies $\omega \leq \sqrt{2\sigma_t}/\sigma_R \Theta$. 

![Figure 3.3: Plot of the form factors belonging to the three different electron distributions discussed in this paper; the ellipsoidal electron bunch with hard edges $f_e (X_t = R_t)$, the Gaussian electron bunch $f_g (X_t = \sigma_t)$, and the ellipsoidal electron bunch with soft edges $f_{e\otimes g} (X_t = R_t)$. The choice $\sigma_R = 0.2R_t$ has been made for illustrative purposes.](image)
3.3 Temporal electric field of CTR

3.3.1 Inverse Fourier transform of $f(\omega)$

The form factor of the ellipsoidal electron bunch with soft edges can be written as

$$f_{e \otimes g}(\omega) = f_{e}^{\sigma R}(\omega) f_{g}(\omega),$$  \hspace{1cm} (3.12)

where we defined $f_{e}^{\sigma R}(\omega)$ as

$$f_{e}^{\sigma R}(\omega) = \frac{3}{(1 + 3 \frac{\sigma_{R}^{2} R}{R_{c}^{2}})} \times$$

$$\left(1 + (\sigma_{R}^{2} T_{c}^{2}) \sin(R_{c} T_{c}^{\omega}) - R_{c} T_{c}^{\omega} \cos(R_{c} T_{c}^{\omega})\right)$$

$$(R_{c} T_{c}^{\omega})^{3}. \tag{3.13}$$

Since a product of Fourier components leads to a convolution in time domain, we can use Eqs. (3.12) and (3.13) to find the temporal electric field of the ellipsoidal electron bunch with soft edges.

However, we first calculate the temporal electric field of the ellipsoidal electron bunch with hard edges. We do this by calculating the inverse Fourier transform of $f_{e}^{\sigma R}(\omega)$ and substituting $\sigma_{R} = 0$ afterwards. The inverse Fourier transform of $f_{e}^{\sigma R}$ is

$$f_{e}^{\sigma R}(t_{0}) = \int f_{e}(\omega) e^{-i\omega t_{0}} d\omega = \frac{\pi}{2} \frac{3}{1 + 3 \frac{\sigma_{R}^{2} R}{R_{c}^{2}}} \times$$

$$\left(1 + 2 \frac{\sigma_{R}^{2}}{R_{c}^{2}} - \frac{t_{0}^{2}}{\tau_{c}^{2}}\right) \times \begin{cases} 0 & \text{if } |t_{0}| > \tau_{c}; \\ 1 & \text{if } |t_{0}| \leq \tau_{c}, \end{cases} \tag{3.14}$$

with $\tau_{c} = \frac{R_{c} T_{c}}{c}$ the duration of the CTR pulse, and $f_{e}^{\sigma R}(t_{0})$ the inverse Fourier transform of the form factor. If we assume $\sigma_{R} = 0$ and substitute Eq. (3.14) into Eq. (2.23), we obtain $E_{h e}(r, t)$, the CTR pulse of a hard edged ellipsoidal electron bunch

$$E_{h e}(r, t) = -e N \beta \sin \theta \frac{2 \pi}{(2\pi)^{2} \varepsilon_{0} c(1 - \beta^{2} \cos^{2} \theta)} \times$$

$$\left(1 - \frac{t_{0}^{2}}{\tau_{c}^{2}}\right) \times \begin{cases} 0 & \text{if } |t_{0}| > \tau_{c}; \\ 1 & \text{if } |t_{0}| \leq \tau_{c}. \end{cases} \tag{3.15}$$

Next we calculate the inverse Fourier transform of $f_{g}(\omega)$,

$$f_{g}(t_{0}) = \int f_{g}(\omega) e^{-i\omega t_{0}} d\omega = \frac{\sqrt{2\pi}}{\tau_{g}} e^{-\frac{1}{2} \left(\frac{t_{0}}{\tau_{g}}\right)^{2}}. \tag{3.17}$$

with $\tau_{g} = \frac{\sigma_{T} T_{c}}{c}$. The CRT pulse in the case of a Gaussian electron bunch $E_{h g}(r, t)$ is thus

$$E_{h g}(r, t) = -e N \beta \sin \theta \frac{2 \pi}{(2\pi)^{2} \varepsilon_{0} c(1 - \beta^{2} \cos^{2} \theta)} \frac{1}{\tau_{g} c} e^{-\frac{1}{2} \left(\frac{t_{0}}{\tau_{g}}\right)^{2}}. \tag{3.18}$$
We can now find \( E^h_{\theta e \otimes g}(r, t) \), the temporal electric field of the electron bunch with soft edges. To do so we write
\[
E^h_{\theta e \otimes g}(r, t) = \frac{1}{2\pi} f_e^{\sigma^h}(t_0) \otimes f_g(t_0),
\]
which leads to
\[
E^h_{\theta e \otimes g}(r, t) = \frac{-eN\beta \sin \theta}{(2\pi)^2\varepsilon_0 c(1 - \beta^2 \cos^2 \theta)} \frac{1}{r} \frac{1}{2\pi} f_e(t_0) \otimes f_g(t_0).
\]
(3.19)

The result of the convolution in Eq. (3.19) is
\[
\frac{1}{2\pi} f_e(t_0) \otimes f_g(t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_e(\tau) f_g(t_0 - \tau) d\tau = \int_{-\tau_e}^{\tau_e} \frac{3 \tau^2}{8 \tau_e^2 + 3 \tau_g^2 \tau_e \tau_s} \frac{1}{\tau_e \tau_s} \int_{-\tau_e}^{\tau_e} (1 + 2 \tau_s^2) e^{-\frac{1}{2} \left(1 - \tau_s^2\right)} d\tau = \end{align*}
\[
\int_{-\tau_e}^{\tau_e} \frac{3 \tau^2}{8 \tau_e^2 + 3 \tau_g^2 \tau_e \tau_s} \left(1 - \frac{t_0}{\tau_e} + \frac{t_0}{\tau_e} e^{-\frac{t_0}{\tau_e}} \right) + \int_{-\tau_e}^{\tau_e} \frac{\tau^2}{2} \left(\tau_s^2 + 1 - \frac{t_0^2}{\tau_e^2}\right) \left(\text{erf}\left(\frac{t_0 + \tau_e}{\sqrt{2} \tau_s}\right) - \text{erf}\left(\frac{t_0 - \tau_e}{\sqrt{2} \tau_s}\right)\right). \end{align*}
\]
(3.20)

The time scale corresponding to the soft edges of the electron bunch is defined as \( \tau_s = \frac{2\pi \sigma \Theta}{c} \).

Fig. 3.4 shows a plot of three CTR pulses, corresponding to the three different electron densities discussed in this article. The contribution of CDR is not taken into account in the figure. We observe three half-cycle pulses. The pulse \( E^h_{\theta e}(r, t) \) has a parabolic shape. We furthermore observe that \( E^h_{\theta e \otimes g}(r, t) \) has the highest peak electric field, if we assume a similar time scale for all three CTR pulses (\( \tau_g = \tau_e \)). The CTR pulse \( E^h_{\theta g}(r, t) \) has the lowest peak electric field. This is because the Gaussian electron density has a much larger spatial extent, due to its long tails. If we look at \( E^h_{\theta e \otimes \Theta}(r, t) \), we see that the soft edges in the electron bunch lead to soft edges in the temporal profile of the CTR pulse. The peak electric field is less with respect to \( E^h_{\theta e}(r, t) \), which is a consequence of the larger spatial dimension of the electron bunch due to the soft edges. The shape of all three CTR pulses appears to be a direct fingerprint of the corresponding electron density. However, one should be careful with this argument because of the \( \theta \) dependence of \( \tau_e \) and \( \tau_g \). So, if we observe the electron bunch from a different angle, we observe a different CTR pulse length.

### 3.4 Conclusions

We have analytically calculated the electric field spectrum and temporal electric field of CTR, created by ellipsoidal electron bunches, both with hard and soft edges.
Coherent Transition Radiation as diagnostic tool of ellipsoidal electron bunches

Figure 3.4: Plot of the temporal electric field of the CTR pulse, for the three different electron bunches discussed in this article; the ellipsoidal electron bunch with hard edges \( E_{\theta e}^h (T = \tau_e) \), the Gaussian electron bunch \( E_{\theta g}^h (T = \tau_g) \), and the ellipsoidal electron bunch with soft edges \( E_{\theta e \otimes g}^h (T = \tau_e) \). The choice \( \tau_s = 0.2 \tau_e \) has been made for illustrative purposes.
Chapter 3.

The theory described in this paper can be used for diagnosing ellipsoidal electron bunches. These bunches are of major interest because of their linear self-fields, which solves the problem of space-charge induced emittance degradation.
Coherent Transition Radiation as diagnostic tool of ellipsoidal electron bunches

Bibliography


Second Generation TU/e RF-Photogun

Abstract. At Eindhoven University of Technology (TU/e) radio-frequency photo-injectors are being developed which are characterized by full cylindrical symmetry and photoemission in the bunch blow-out regime. A new feature of the 2nd generation TU/e RF photogun is the fact that the different parts are clamped instead of brazed, preventing deformation during the brazing process. We show that the TU/e approach results in picosecond electron bunches with state-of-the-art brightness. At low charges the normalized transverse emittance is limited by the thermal emittance of copper. At 70 pC bunch charge we can place an upper bound of 1 mm-mrad on the normalized transverse emittance. The electron bunch length is less than 2 ps (FWHM) 1.4 m from the cathode, corresponding to a brightness of $0.4 \times 10^{14}$ Am$^{-2}$. The peak current is on the order of 100 A directly behind the RF photogun. The electron bunch arrival time jitter has been estimated to be 20 fs at 1.4 m from the cathode. This shows that the TU/e 1.5 cell RF photogun is ideally suited for pump-probe experiments in a small-scale setup.

\footnote{An article based on this chapter has been submitted to Phys. Rev. ST Accel. Beams.}
4.1 Introduction

Presently, the brightest pulsed electron sources are so-called radiofrequency (RF) photoguns [1, 2]. These are compact RF accelerators, consisting of a few pillbox cavities (‘cells’) in TM_{010} mode, connected in series with a common symmetry axis. High density 0.1-1 nC electron bunches are generated by (sub-)picosecond pulsed laser photoemission from the back plate of the first pillbox and are accelerated to, typically, a few MeV, in fields strengths as high as 120 MV/m. The resulting relativistic bunches are characterized by peak currents as high as 100 A, and root-mean-squared (RMS) normalized transverse emittances as low as 1 mm-mrad [2], making RF photoguns the brightest pulsed sources around.

RF photoguns were originally developed as injectors for Free Electron Lasers (FELs), and are considered as the only option for injection of an X-ray FEL [2, 3]. In recent years other applications have emerged as well: pulsed radiolysis [4–9], femtosecond electron diffraction [10–12], generation of intense THz pulses [13, 14], generation of X-rays for medical diagnostics [15, 16], and injection of bunches into Laser Wakefield Accelerators (LWA) [17, 18]. These small-scale applications make them interesting tools for university labs as well. At present, nearly all RF photoguns are based on the successful BNL/SLAC/UCLA 1.6 cell S-band RF gun design [19], most prominently the LCLS gun, which holds the record in brightness [2].

At Eindhoven University of Technology (TU/e) we have adopted a different design strategy with respect to the BNL/SLAC/UCLA approach. The combination of advanced simulation tools and precise machining techniques makes it possible to design and fabricate a RF photogun with micrometer precision. Tuning plungers are therefore not necessary making it possible to create fully cylindrically symmetric RF cavities. The RF power is fed into the cavity using a coaxial RF feed, originally suggested by the DESY lab, and implemented in the first TU/e RF photogun. The cylinder symmetry minimizes emittance growth caused by field asymmetries associated with higher transverse modes. This is an intrinsic property of the design and does not require balancing transverse RF feeds or making a slightly astigmatic rf-cavity, as in the LCLS injector [2]. In addition, due to the absence of transverse RF feeds a solenoid can be placed around the rf-cavity. This allows the electrons to be focused during the early stages of acceleration, thus reducing emittance growth due to off-axis non-linearities in the RF fields.

The first TU/e RF photogun [20] already demonstrated that it is possible to design and build RF cavities with micrometer precision. At the time it was chosen to braze the different cells of the RF cavity together. Heating the high-purity copper during brazing, however, can easily cause small deformations, that in turn can affect the resonant frequencies and field-balance. Furthermore, traces of solder can easily reach the inner surface causing breakdown problems later on. The 2nd generation TU/e RF photoguns therefore use a different approach: The individual cells and the cathode plate are all mechanically clamped together, avoiding brazing altogether. An additional advantage is that the individual cells and the cathode plate can be replaced in case this is needed.

The TU/e approach also differs from mainstream RF photogun development in that it employs ∼ 100 fs photoemission laser pulses generating ∼ 100 pC bunches (instead of
In this paper we show that the TU/e approach results in electron bunches with state-of-the-art brightness. This paper focuses particularly on the technical aspects: design considerations, construction details, and commissioning results of the 1.5 cell 2nd generation TU/e RF photogun. The most important results of the commissioning are: a measured emittance of 1 mm-mrad at 70 pC and 0.4 mm-mrad at 5 pC; an arrival time jitter of 20 fs and a full-width-half-maximum (FWHM) electron bunch length of 1.9 ps, corresponding to 35 A peak current, measured 1.4 m from the cathode.

4.2 Cavity Design

The design process of the 2nd generation 1.5 cell RF photogun described here started from an earlier 2.6 cell version [20]. Pioneered in the 1st generation 2.6 cell rf-photogun was micrometer-precise design of the inner dimensions, combined with single-point diamond turning manufacture. This combination fully eliminated the need for any adjustments or corrections.

The 1.5-cell RF photogun is tailor-made for the production of ‘pancake’ electron bunches with a charge in the order of 100 pC, to be used for the generation of THz radiation via various schemes of CTR [13, 14]. As such, the design criteria for the inner dimensions differ slightly from the more usual design aiming at extracting more charge during several picoseconds. In the ‘pancake’ regime the accelerating field must be much stronger than the image charge fields during photoemission. This is particularly relevant for the extraction of ‘ellipsoidal’ bunches [22]. To get the highest possible fields at the cathode during extraction, two changes were incorporated: The cross sectional shape of the irises was made elliptical, and the length of the first half cell was shortened.

Inspired by the Alpha-X RF photogun [17, 24], having inner dimensions designed by the same ‘Pulsar’ team, the irises with circular cross-section have been replaced by irises with an elliptical shape. This results in a relatively higher field at the cathode than at the iris. Visual inspection of the 1st generation 2.6 cell RF photogun revealed significant surface roughness caused by breakdown exactly at the iris locations with highest field-strength. Elliptical irises have the advantage that potential breakdown damage, most likely to occur at the location of highest field-strength, is now at the easily replaceable cathode and not on the irises.

The length of the first RF cell of an RF photogun is typically optimized for maximum output energy. However, for ‘pancake’ emission it is best to emit the bunches at maximum field at the cathode, defined as 0° phase. Maximum energy and 0° launching phase are conflicting requirements. A compromise was found making the first half-cell 0.5 times the
nominal length, resulting in a $-30^\circ$ launching phase for maximum output energy, thereby reducing the field only by 13%. A summary of the design specifications of the 1.5 cell RF photogun is provided in Table 4.1.

| Design specification following from field calculations of the 2nd generation 1.5 cell cavity. |
|---------------------------------|-----------------|
| RF input power                  | 8.5 MW          |
| Maximum field at cathode        | 110 MV/m        |
| Maximum field at irises         | 100 MV/m        |
| Maximum output energy           | 4.4 MeV         |
| Corresponding launch phase      | $30^\circ$      |
| Cathode field at emission       | 95 MV/m         |
| Frequency $\pi$-mode (vacuum)   | 2998.9 MHz      |
| Unloaded quality factor         | 11300           |

### 4.3 Clamped Cavity Construction

The cavity consists of three parts, a cathode, a middle part containing the half and full cell, and a front part containing the exit of the cavity and the connection to the coaxial RF feed (see Fig. 4.1). The middle part has been equipped with an alignment rim which fits into a groove made in the front part of the RF cavity. All the copper parts are clamped together using screws. The clamped RF cavity is placed inside a stainless steel vacuum can (see Fig. 4.2). The copper RF cavity is suspended within the can in such a way that it can slightly be moved in the axial direction. The last part of the RF cavity is attached to the vacuum can using spring loaded screws, in such a manner that the RF cavity protrudes slightly from it. This allows us to make a vacuum seal between the can and the coaxial RF feed using a viton O-ring, while simultaneously ensuring good electrical conductivity between the RF cavity and the RF feed.

The RF cavity can is mainly pumped through the RF feed using a turbopump in a similar fashion as described in Ref. [20]. In addition the space in between the stainless steel can and the copper cavity is pumped at the backside of the cathode plate.

### 4.4 Temperature control

The temperature of the cavity needs to be controlled. This is important because a change in temperature leads to shift in resonance frequency of approximately $-49$ kHz/K [20]. This results in a reduction of the RF power fed into the cavity and, more importantly, a change in phase $\phi$ between the RF wave going towards the cavity and the standing wave within the cavity. Due to the large heat capacity of the RF cavity the temperature drifts
on a timescales of minutes. A drifting phase leads to a change in electron bunch arrival time, which is undesired in e.g. pump-probe experiments. To keep the drift in arrival time of electrons injected at crest at 1.5 m from the cathode less than a picosecond, a phase stability of $\Delta \phi \approx 0.02$ rad is required. The measurements from which this is deduced will be discussed later (section 4.8.3).

For an ideal pillbox it can be shown that [20]

$$\frac{d\phi}{dT} = -\frac{p_{01}cQ_uk_T}{\pi rf_0} = -0.33 \frac{rad}{K}, \quad (4.1)$$

where $p_{01}$ is the 1$^{st}$ root of the 0$^{th}$ order bessel function, $c$ is the speed of light, $k_T$ the thermal expansion coefficient of copper, $f_0$ is the resonance frequency of the used mode, and $r$ the radius of the cavity. A phase stability of $\Delta \phi \approx 0.02$ rad during experiments therefore leads to a required temperature stability of $\Delta T \approx 0.06$ K. In order to reach this temperature stability the setup schematically depicted in Fig. 4.3 has been built.

In the vacuum a water cooled copper plate is attached to the backside of the cathode plate (see Fig. 4.2). A gear pump outside the vacuum pumps water with 3 l/min through the channel, which can enter the vacuum via two feedthroughs in the vacuum can (see Fig. 4.2). The water reservoir serves as buffer. To ensure a good heat conductance between the water and the copper channel the water flow was made turbulent by choosing a 6 mm diameter water channel (Reynolds number of $10^5$).

The RF cavity is operated at a repetition rate of 3 Hz, and each shot approximately 25 J is dissipated. This leads to a required cooling power of 75 W. The temperature of the

---

**Figure 4.1:** Picture of RF cavity to illustrate the three parts of which it has been constructed.
a) Clamping of cathode on middle part using screws.

b) Clamping of middle part on last part using screws.

c) Spring loaded attachment of RF cavity to the stainless steel can.

Figure 4.2: Three different cross-sections of the RF cavity to illustrate the clamping of the different parts.
RF cavity is controlled by a heat exchanger consisting of 5 peltier elements of 70 W each. Because each peltier element both pumps and dissipates 70 W, the total cooling power is approximately 175 W. The peltier elements exchange energy from the water going to the cavity with water which is kept at 20 ± 2 °C and flowing with a few liters per minute. The diameter of the water channel within the heat exchanger is also 6 mm to ensure good heat conductance.

The thermocouple in the first cell is used to control the temperature. It is linked to a PI-controller which determines the amount and direction of current flowing through the peltier elements. In Fig. 4.4 the temperature of the RF cavity is shown as a function of time after switching on the RF power. After a settling time of 800 seconds the temperature is within 0.05 K from its setpoint. From the measurement shown in Fig. 4.4 we conclude that the temperature stability is better than the required 0.06 K.

4.5 RF Commissioning

4.5.1 Low Power Measurements of RF-Photogun

The rf cavity was first tested at low power using a network analyzer. The measurements were performed in ambient conditions. In Fig. 4.5 the measured absorption is shown, together with two fitted lorentz curves. From the fit it follows that the resonance frequency $f_0$ of the $\pi$-mode is at 2997.9 MHz, and that the loaded quality factor $Q_l = 5100$, implying
Figure 4.4: Temperature of the RF cavity versus time when RF power is applied. After 800 seconds the temperature is within 0.05 °C of its final value. From this measurement we conclude that the temperature stability is better than the required stability of 0.06 °C.

Figure 4.5: Measured absorption spectrum. The dotted lines are fitted lorentz curves.
an unloaded quality factor of $Q_u = 10200$. The measured frequency should be corrected for the difference in dielectric constant between dry atmospheric air and vacuum (0.95 MHz) [20] before it can be compared with the design value. If we do so we see an excellent agreement between the design frequency and the actual frequency of the cavity. The measured quality factor is 10% less than the design value.

The electric field distribution of the $\pi$-mode has been measured using the bead-ball method [25]. A nylon bead having a 4 mm diameter was glued on a thin nylon fishing line ($\sim 150\mu$m diameter) which entered the cavity through a small hole in the cathode ($\sim 200\mu$m diameter). The frequency shift at each position of the bead was measured using a network analyzer. Because the nylon line was slightly elastic the bead has been moved in both directions through the cavity. In Fig. 4.6 the measurements are shown together with the designed cavity field profile and the field profile if the radius of either the first or second cell is increased by 5 $\mu$m.

The measured electric field distribution is in good agreement with the calculated design distribution. From the calculated field profiles of the cells with increased radius it can be inferred that the dimensions of the actual cavity are within 5 $\mu$m of the design values. Note that the measured field differs from the calculated field at the cathode. This is due to image charges which are induced when the bead is close to the cathode surface.

Figure 4.6: Measured electric field distribution together with designed field. Also shown are calculated field distributions of a cavity in which the radius of the first (half) cell is 5 $\mu$m larger (dashed curve); and the field distribution in case the radius of the second (full) cell is 5 $\mu$m larger (dotted curve).
4.5.2 RF Conditioning

The RF conditioning of the RF photogun is fully automated. The incoming RF power increases linearly with $\sim 0.5$ MW/h until a breakdown occurs, which is detected by a sudden change in the reflected RF power. This is done by using a high pass filter, which effectively differentiates the reflected RF power, in combination with a level detector. Whenever a breakdown is detected the incoming RF power is immediately removed. After 10 seconds the RF power is reapplied. The RF power is increased from zero to the level at which the breakdown occurred via an analog RC-filter with a time constant of 20 s. After 200 seconds the linear increase of the RF power is resumed.

Using the above mentioned method the RF-photogun was conditioned to an electric field of 76 MV/m at the cathode in $\sim 10^6$ shots at 3 Hz repetition rate. The clamping construction did not result in problems at high RF power. With the obtained field strength a kinetic energy of 3 MeV is achieved when electrons are injected 40° before crest. During the conditioning the current through the solenoid was set at 120 A, corresponding to an on-axis field of 0.2 T. The bucking coil was used to null the field at the cathode [20]. The pressure in the cavity was typically $10^{-8}$ mbar during conditioning. Occasionally the pressure increased to $10^{-7}$ mbar after a breakdown.
4.6 Electron Beamline and Diagnostics

Fig. 4.7 shows a schematic overview of the RF cavity in combination with the electron beam line and the optical setup. The solenoid around the RF cavity is used to focus the electrons in the early stages of acceleration reducing emittance growth due to off-axis nonlinearities in the RF field. The bucking coil is used to null the magnetic field at the center of the cathode. Both the solenoid and the bucking coil are described in detail in Ref. [20].

Directly behind the valve, which separates the RF cavity from the electron beamline, two 45° aluminum (Al) mirrors with a magnesium fluoride coating are placed off-axis inside a 4-way vacuum cross. The mirrors leave an opening of 15 mm for the electron bunches. Behind the 4-way cross containing the Al mirrors a phosphor screen can be moved into the beamline. The phosphor screen is imaged onto a ccd-camera outside the vacuum. The next item in the electron beamline is a triplet lens, consisting of three magnetic quadrupole lenses. A dipole magnet is used as spectrometer with on the off-axis port a Faraday cup allowing bunch charge measurements. When the dipole magnet is switched off the electron bunches enter a vacuum cube containing either another phosphor screen (not shown in Fig. 4.7), or a 8 μm thick Al foil having a radius of 18 mm placed at an angle of 45° with respect to the incoming electron bunches. The CTR emitted when the electrons hit the foil leaves the vacuum through a 3 mm thick z-cut crystalline quartz window. The CTR is focused using an ellipsoidal Al mirror, allowing characterization of the electron bunch length. The second phosphor screen in combination with a single magnetic quadrupole lens is used to measure the transverse normalized emittance.

The quadrupole lenses, dipole magnet, Faraday cup, and construction and imaging of the phosphor screens are described in detail in Ref. [20].

4.7 Optical Setup

Our Ti:Sa femtosecond laser system consists of a Femtosource oscillator (Femtolasers Produktions GmbH) and an Omega Pro multipass amplifier (Femtolasers Produktions GmbH). The laser is designed to operate at 1 kHz delivering pulses of 30 fs containing 0.8 mJ per pulse with a central wavelength of 800 nm. A Pockels cell pulse picker within the amplifier selects pulses at 3 Hz. The setup described in [26] has been used to synchronize the RF with the Ti:Sa oscillator.

4.7.1 Third Harmonic Generation

The 800nm pulses need to be frequency tripled to 267nm for photoemission from the copper photocathode. Third harmonic generation (THG) is done in a collinear setup using four crystals, which are depicted schematically in Fig. 4.8. The collinear setup results in a better transverse beam quality, making beam shaping easier with smaller losses. This is important for the creation of ellipsoidal electron bunches. The frequency tripling is achieved by two sum frequency processes, first second harmonic pulses (400nm) are generated, which are
The first crystal is a type 1 $\beta$-barium borate (BBO) crystal of 0.15 mm thick grown at the optimal orientation for second harmonic generation (SHG) [28]. The thickness of the BBO has been chosen such that the second harmonic pulses generated at the beginning of the crystal lag 30 fs in time with the second harmonic pulses generated at the end of the crystal.

The third harmonic waves are generated via sum frequency generation by passing the remaining fundamental and second harmonic waves through another BBO crystal (the fourth crystal in Fig. 4.8). This BBO crystal is a 0.16 mm thick type 1 phase matched crystal optimized for THG of 800nm pulses [28]. The fundamental pulses travel through the THG crystal the fastest, the second harmonic pulse take 50 fs longer to traverse it. Increasing the thickness of the crystal does therefore not lead to a higher THG efficiency because the fundamental and second harmonic pulses do not overlap anymore. In order to align the polarization directions of the fundamental and second harmonic waves a retarder (the third crystal in Fig. 4.8) is used, which acts as a half-wave plate for the fundamental waves and as a full-wave plate for the second harmonic waves [28].

Because of group velocity mismatch, the fundamental waves need to be delayed with respect to the second harmonic waves. This is achieved using a BBO crystal (the second in Fig. 4.8) [28], which is grown in such a manner that the fundamental waves polarized in the ordinary direction have a lower group velocity than the second harmonic waves polarized in the extra-ordinary direction.

Using 0.7 mJ, 30 fs 800 nm pulses with an RMS radius of approximately 1.5 mm, 35 $\mu$J, 267 nm pulses are generated. This is sufficient for photoemission of $\sim$ 0.2 nC charge.

The third harmonic pulses generated at the beginning of the crystal lag 120 fs with respect to the fundamental pulse at the end of the crystal. Based on this we estimate that the third harmonic pulses are stretched in time to 120 fs.

The 267 nm pulses reflected from the cathode were imaged on a ccd camera, see Fig. 60.
4.9. The grooves visible in the cathode were used to align the laser with respect to the center of the cathode.

4.7.2 Electro-optic detection

In order to measure the CTR created when the electron bunches hit the Al foil, a 0.5 mm thick ZnTe crystal cut in the $<110>$ direction is placed in the focus of the ellipsoidal mirror (see Fig. 4.7). The birefringence induced by the CTR is probed with the 800nm light remaining after THG. Since the 800 nm pulses have a slight elliptical polarization after THG a linear polarizer in combination with a $\lambda/4$ waveplate is used to make the 800nm probe pulses circularly polarized. The phase rotation $\phi_{EO}$ induced in the ZnTe crystal is measured using a Wollaston prism and two balanced photodiodes. The delay $\tau$ between the CTR and the 800 nm probe pulses is varied using two mirrors placed on a motorized stage.

4.8 Generation of high-brightness electron Bunches

The energy and charge of the electron bunches were measured using the dipole magnet in combination with the Faraday cup. Using the dipole magnet the injection phase of the laser pulses on the RF was optimized for highest kinetic energy $E_k$. The maximum kinetic energy $E_k = 3.3$ MeV, corresponds to an electric field strength of 82 MV/m at the cathode.
However, at this level occasionally a breakdown occurred which required new conditioning of the cavity. Therefore the cavity was typically operated to produce electrons with $E_k = 3.0$ MeV, corresponding to an electric field strength of 76 MV/m at the cathode. The measured quantum efficiency $QE = 3 \cdot 10^{-5}$ is in agreement with [2, 11, 29].

### 4.8.1 Normalized Transverse Emittance of low charge bunches

A scan with the second quadrupole in the triplet was made in order to measure the normalized transverse emittance $\epsilon_x^n$. In a waist $\epsilon_x^n = \frac{1}{mc} \sigma_x \sigma_{px}$, with $\sigma_x$ the RMS size in the $x$-direction in phase-space and $\sigma_{px}$ the RMS size in the $px$-direction in phase-space. The electrons were focused in the $x$-direction (plane of optical table) and measured at the second phosphor screen. A typical picture taken during the scan is shown in Fig. 4.10. In order to determine $\sigma_x$, a Gaussian distribution was fitted over the pixel row containing the most energy (see the red line in Fig. 4.10). Using this method $\sigma_x$ was measured at several values of the quadrupole focal length $f_x$.

In order to determine $\epsilon_x^n$ from the measured $\sigma_x$ versus $f_x$ data points they were fitted with the following equation

$$\sigma_x = \sqrt{(l_1 l_2 - (l_1 + l_2) f_x)^2 \epsilon_x^2 + \sigma_{v f_x}^2}$$

(4.2)
Figure 4.11: Three quadrupole scans performed at different bucking coil currents, respectively 5.1 A, 5.6 A and 6.1 A. The current through the solenoid was 110 A, the kinetic energy of the electron bunches was 2.8 MeV and they contained 5 pC of charge. The solid lines are fits using Eq. 4.2.
Table 4.2: Results derived from fitted data points. The current through the solenoid was 110 A.

<table>
<thead>
<tr>
<th>$I_{bc}$ (A)</th>
<th>5.1</th>
<th>5.6</th>
<th>6.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_x^n$ (mm mrad)</td>
<td>$0.46 \pm 0.05$</td>
<td>$0.40 \pm 0.05$</td>
<td>$0.45 \pm 0.05$</td>
</tr>
<tr>
<td>$\sigma_v$ (mm)</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$l_1$ (m)</td>
<td>0.66</td>
<td>0.64</td>
<td>0.73</td>
</tr>
</tbody>
</table>

with $\epsilon_x = \frac{1}{\gamma \beta} \epsilon_x^n$ the emittance in trace-space, and $l_2$ the distance from the second quadrupole to the phosphor screen. Here $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$ is the relativistic lorentz factor with $\beta = \frac{v}{c}$. The parameter $\sigma_v$ can be interpreted as the transverse RMS size of a virtual source point located at a distance $l_1$ before the quadrupole lens. We call this a virtual source point because the phase-space is non-skewed at this position, assuming free space propagation over the distance $l_1$. Eq. (4.2) can be derived with the Twiss parameters and assuming a Gaussian phase-space distribution neglecting space-charge forces [30].

Three different quadrupole scans were made at a solenoid current of $I_{s0l} = 110$ A and bucking coil currents of respectively $I_{bc} = 5.1$ A, $I_{bc} = 5.6$ A, and $I_{bc} = 6.1$ A. The electron bunches contained 5 pC of charge and had a kinetic energy of 2.8 MeV. The RMS size of the intensity of the 267 nm laser pulses $\sigma_I = 0.43$ mm. The measured data points together with the fitted curves are shown in Fig. 4.11. The values of $\epsilon_x$, $\sigma_v$, and $l_1$ are derived from the fitted curves and shown in Table 4.2. The distance $l_2$ can be measured $l_2 = 0.63$ m.

The $\epsilon_x^n$ is lowest at a bucking coil current of 5.6 A. If we assume that the magnetic field at the cathode is zero for this setting we can deduce from data in Ref. [20] that the magnetic field is $\pm 4$ mT for the two other settings. According to [31] this would lead to an emittance growth of approximately

$$\Delta \epsilon_x^n \approx \frac{1}{8} \frac{q |B|}{2mc} \sigma_x^2 = 0.03 \text{ mm} \cdot \text{mrad},$$

for a uniform distribution. Although the measurement error is larger than the estimated contribution from the non-zero magnetic field, the increase in emittance is of the same order of magnitude as calculated with Eq. (4.3).

The values of $\sigma_v$ and $l_1$ derived from the three fits are consistent with each other and can be used to calculate the RMS beam size of $\sigma_x = 0.25$ mm at the first phosphor screen for $I_{bc} = 5.1$ A and $I_{bc} = 5.6$ A. For $I_{bc} = 6.1$ A we calculate $\sigma_x = 0.32$ mm. This was confirmed by measurements.

### 4.8.2 Normalized Transverse Emittance of high charge bunches

The emittance was also measured at a charge of 70 pC. The kinetic energy of the electron bunches was 2.8 MeV and the injection phase was optimized for maximum kinetic energy. The cathode electric field strength at injection was 55 MV/m. The used laser spot is shown in Fig. 4.13 together with a Gaussian fit. In Fig. 4.12 the measured quadrupole scan is

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Figure 4.12: Measured quadrupole scan at 70 pC of charge and 2.8 MeV.

shown together with a fit using Eq. 4.2. From the fit it follows that $\epsilon_n^x = 1 \pm 0.1 \text{ mm-mrad}$.

A major contribution to the error of the measured emittance is due to the fact that the actual trace-space distribution deviates from a Gaussian distribution. We estimate an error of approximately 10% in the measured emittances both at high and low charge (see appendix C-1).

Anderson et. al. [32] show that emittance measurements using a quadrupole scan can be incorrect due to the influence of space-charge forces on the beam envelope. In that case a quadrupole scan always results in an higher emittance than the actual phase-space surface $\sigma_x \sigma_{p_x}$. The measured emittance of 1 mm-mrad is therefore an upper bound.

4.8.3 Arrival time jitter & Electron bunch length

The RF photogun has been built as a CTR THz source. Although this specific topic will not be discussed here we have performed measurements during the CTR optimization process which give an upper bound for the bunch length and an estimate of the arrival time jitter, which is relevant for pump-probe experiments. These measurements will be discussed here.

Arrival Time Jitter

The arrival time of the electron bunches has been measured as function of the injection phase $\phi$. To find a reference the injection phase was optimized to maximum kinetic energy,
Chapter 4.

**Figure 4.13:** Image of laser pulse measured at virtual cathode. The transverse dimensions are in mm. The red line is a fit using a Gaussian distribution giving $\sigma_{t_x} = 0.56$ mm.

**Figure 4.14:** Plot of measured EO signals as function of delay time for several values of the injection phase.
Figure 4.15: Measured and simulated arrival time as function of injection phase.

denoted as $\phi_{E_k}$. According to GPT simulations this corresponds to $41^\circ$ before crest. The electron bunches contained 10 pC of charge at an energy of 3 MeV. The distance between the Al foil and the cathode was 1.4 m. In Fig. 4.14 the measured EO signals are shown as a function of delay time for several values of the injection phase. The arrival time of the radiation pulses changes with $\phi$. We define the arrival time $\tau_a$ as the moment when $\phi_{EO}$ changes sign (black dots in Fig. 4.15), taking the average value of $\phi_{EO}$ before the arrival of the pulse as zero. In Fig. 4.15 the measured $\tau_a$ is plotted as function of the injection phase $\phi$, using the arrival time at maximal kinetic energy $\tau_{E_k}^a$ as reference. Besides the measured values Fig. 4.15 also shows arrival times simulated with GPT. Since the simulated and measured values correspond to a high degree we conclude that the measured delay is indeed determined by the injection phase.

From Fig. 4.15 it is apparent that the arrival time is quadratically dependent on $\phi$ around $\phi_{E_k}$. To illustrate this a $2^{nd}$ order polynomial has been fitted through the measured data points. Based on this measurement we can calculate the arrival time jitter of the electrons for each injection phase, by calculating the derivative $d\tau_a/d\phi$ and multiplying this with the phase jitter $\Delta\phi_{jitter}$ of our setup. In our setup $\Delta\phi_{jitter} = 0.5^\circ$, which is mainly caused by the klystron [20]. In Fig. 4.16 a plot is shown of $d\tau_a/d\phi$ with on the righthand scale $(d\tau_a/d\phi)\Delta\phi_{jitter}$ to give the arrival time jitter. From this we can conclude that if the injection phase $-35^\circ$ the arrival time jitter is 20 fs at 1.4 m from the cathode. This is much less than the electron bunch length, which is on the order of picoseconds.
Bunch Length

From the measured radiation pulses shown in Fig. 4.14 it is difficult to resolve the temporal shape and length of the electron bunches. This is because it is determined by a combination of coherent diffraction radiation (CDR) and CTR. The CDR is coherent up to wavelengths of \( \lambda_{\text{CDR}} \approx \frac{\rho}{\sqrt{2}} = 0.9 \text{ mm} \) [13] at 3 MeV, with \( \rho \) the radius of the foil and the factor \( \sqrt{2} \) due to the 45° orientation. The bunch length determines the coherence of the CTR, and since we expect bunches with FWHM of 2 ps, this is approximately 0.6 mm.

In order to decrease the influence of CDR we removed the foil from the beamline and replaced it with 1 mm thick Al plate of 68 mm in transverse directions. The plate was also placed at an angle of 45° with respect to the incoming electron bunches, the rest of the setup remained the same. In Fig. 4.17 the measured \( \phi_{EO} \) is shown for electron bunches containing 70 pC of charge, having a kinetic energy of 3 MeV. The bunches where focused to an rms transverse spot size of \( \sigma = 0.25 \) mm. As can be seen the measured electro-optic signal contains a half cycle peak having a FWHM of 1.9 ps. It remains difficult to resolve the exact bunch length from this measurement. However, since the electron bunches have the shortest time scale within the experiment we can place on upper bound on their FWHM bunch length of 1.9 ps.
4.8.4 Brightness

The brightness of the electron bunches can be calculated using

\[
B = \frac{\hat{I}}{e_x^n e_y^n} = \frac{Q}{\tau_B e_x^n e_y^n}, \quad (4.4)
\]

with \( \tau_B \) the FWHM electron bunch length and \( Q \) the charge. Assuming that \( e_y^n = e_x^n \) the electron bunches have a brightness of 0.4\( \cdot 10^{14} \) Am\(^{-2} \) at 1.4 m from the cathode. The electron bunch length increases with the distance from the cathode. From GPT simulations it is known that the electron bunch length is roughly 2-3 times smaller at the position of the first phosphor screen at 0.6 m from the cathode (see Fig. 4.7), corresponding to a brightness of approximately 10\( ^{14} \) Am\(^{-2} \).

This brightness is comparable with the brightness of the LCLS injector [2]. However, because we use femtosecond photoemission at lower charge the electron bunch length is on the order of picoseconds instead of ten picosecond. It is therefore possible to create high-brightness picoseconds electron bunches without using bunch compression schemes. Using 'pancake' electron bunches it is possible to increase the brightness further. As predicted by Luiten et al. [22] and shown by P. Musumci et. al. [23] it is possible to obtain the thermal emittance of the used cathode material if the laser pulse is properly shaped and the electric field at the cathode is substantially larger than the image charge fields. In this way an additional factor of \( \sim 4 \) can be gained in brightness.
4.9 Conclusions

We have shown that the TU/e approach results in electron bunches with state-of-the-art brightness: Using fully cylindrically symmetric cavities and operating in the bunch blow-out regime we produce (sub)-ps electron bunches with \( \sim 100 \) A peak current and 1 mm-mrad normalized emittance. The arrival time jitter of the electron bunches has been estimated to be well below 100 fs (\( \sim 20 \) fs at 1.4 m from the cathode) making this compact setup ideally suited for small-scale pump-probe experiments.

We show that it is possible to clamp the different parts of a RF cavity, instead of brazing them, thus preventing deformation of the RF cavity during the brazing process. The clamping construction does not result in RF problems. An important advantage is that the clamping allows great flexibility in replacing damaged parts of the cavity, or even adding cells.
Bibliography


In this chapter\textsuperscript{1} we present experimental results on the generation of free-space terahertz (THz) pulses by means of the coherent transition radiation (CTR) process. The experiments were done to test the performance of the 1.5 cell RF photogun, discussed in chapter 4, as a source of THz radiation. To enable the CTR measurements we have focused the CTR pulses using a custom made ellipsoidal aluminum (Al) mirror onto a ZnTe crystal used for electro-optic (EO) detection. In the focus of the ellipsoidal mirror the field distribution of the THz pulses resembles a radially polarized ‘donut’ mode [1]. We have performed detailed measurements of the full spatiotemporal structure of the focussed CTR THz pulses, which reveals a rich transverse and temporal structure. We used ccd-cameras for single shot measurements of the two-dimensional electric field distribution as a function of the delay between the EO-probe pulse and the CTR pulse. In order to compare the measurements with the theory of CTR discussed in Ch. 2, we calculate the focused electric field distribution using the combination of the theory of Ch. 2 and the physical theory of diffraction. We end the chapter with a realistic estimation of the potential of the 1.5 cell RF photogun as a free-space CTR THz source.

5.1 Introduction

What kind of free-space CTR pulses can we expect in the case of our experimental situation? The electron bunches, which are created and accelerated in the RF photogun described in Ch. 4, are sent through a 1 mm thick, 68 mm x 68 mm square Al plate. The Al plate makes an angle of 45\textdegree with respect to the incoming electron beam, as is schematically shown in Fig. 5.1. We want to measure the CTR pulses outside the vacuum chamber, which places a lower bound on distance between the Al plate and a detector of typically 10 cm. The electron bunches typically contain 60 pC of charge, with a kinetic

\textsuperscript{1}Manuscript in preparation.
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Variable Delay Line
ZnTe 800 nm remaining after THG polarizer
λ/4 waveplate
electrons
CCD A
CCD B
polarizing beamsplitter cube
scintillator foil
foil Al plate
z-cut crystalline quartz window
eellipsoidal Al mirror

\textbf{Figure 5.1:} Schematic picture of setup used to measure free space terahertz radiation, generated with CTR.
Figure 5.2: Plot of energy flow, $S$, of CTR pulses traveling away from Al plate. The plot has been made for $r = 10$ cm and $t = r/c$. The electron bunches containing 60 pC of charge, with a kinetic energy of 2.8 MeV and a RMS electron bunch length of 1 ps.
unfocused CTR for $r = 10$ cm and $t = r/c$

**Figure 5.3:** Plot of unfocused transverse CTR electric fields. The plot has been made for $z = 10$ cm and $t = r/c$. The electron bunches contain 60 pC of charge, with a kinetic energy of 2.8 MeV and a RMS electron bunch length of 1 ps.

energy of 2.8 MeV, and a root-mean-square (RMS) electron bunch length $\sigma_l/(c\beta) = 1$ ps. Here $\beta = v/c$, with $v$ the speed of the electrons, and $c$ the speed of light. The transverse RMS bunch size $\sigma_t = 0.3$ mm. In Fig. 5.2 we have plotted the absolute value of $S$, which is the Poynting vector of the CTR pulses. We have calculated $S$ at $r = 10$ cm and $t = r/c$, with $r = \sqrt{x^2 + y^2 + z^2}$, using the theory discussed in Ch. 2. See Fig. 5.4 for the definition of the coordinate system $(x,y,z)$. The energy is emitted into a asymmetric 'donut' shaped profile. The asymmetry is due to the fact that the TR interface is oriented at 45$^\circ$ with respect to the incoming electron bunches. The energy is spread-out into a spot of a few cm$^2$. If we calculated the total energy we find that the CTR pulse contains roughly $\sim 150$ nJ. In Fig. 5.3 we have plotted the $x$- and $y$-component of $E^{\text{THz}}$, the electric field vector of the CTR pulse. Because $E^{\text{THz}}$ is polarized (mainly) in the radial direction, we see the 'donut' shape projected onto the $x$- or $y$ axis, resulting in two oppositely polarized lobes. Due to the large spot size the electric field strength is only on the order of $\sim 40$ kV/m. Because the coherent diffraction radiation (CDR) is negligible, the temporal pulse shape will closely resemble a half-cycle pulse of ps duration, as is shown in Fig. 2.9.

We have decided not to measure the THz CTR profile directly. Instead, it is first focused to a small spot for two reasons:

- We want to measure $E^{\text{THz}}$ using two-dimensional electro-optic detection (EO). For this purpose we use two ccd-cameras in a balanced detection scheme. This resulted in a phase rotation noise of roughly $\sim 5$ mrad during real-time measurements. This corresponds with electric fields strengths of 25 kV/m, i.e. of the order of magnitude of the maximum expected signal. So, we need to focus the CTR THz pulses to increase...
the signal-to-noise ratio.

- Our goal is to show that electron bunches created and accelerated in an RF photogun, can be used to create intense half-cycle THz radiation using the CTR process. In applications such half-cycle THz pulses will always be focussed. We, therefore, feel it is appropriate to characterize the CTR in its focus.

We have chosen to focus the THz CTR pulses by means of a custom made ellipsoidal mirror, which is shown in Fig. 5.4. The ellipsoidal mirror focusses the CTR onto a ZnTe crystal used for EO detection. We have measured the transverse components of $E^{\text{THz}}$ in its focus, as function of transverse position and time. Sect. 5.2 treats the experimental details of the ellipsoidal mirror and the two-dimensional EO-detection using ccd-cameras.

In Sect. 5.3 the CTR THz measurements are presented. First, in Sect. 5.3.1, optimization of the CTR THz signal as a function of the injection phase is discussed. Then, in Sect. 5.3.2, the main result is presented: EO measurements of the full transverse electric field distribution (polarization and amplitude) of a focussed CTR THz pulse, as function of transverse position and time.

To make a realistic estimation of the potential of the 1.5 cell RF photogun as source of THz radiation using the CTR process, we have to verify that we understand all relevant physical details. To do so, we will compare the measured THz CTR pulses with the theory described in Ch. 2. There is a complication, however, because we focus the CTR pulses using the ellipsoidal mirror. To make the comparison possible we have to calculate the electric fields in the focal spot. For this purpose we use the physical theory of diffraction (PTD), which is discussed in Sect. 5.4. Both the measured and the theoretically calculated fields in the CTR focus are quite complicated, and we will observe some surprising, subtle features in both:

- Apparent superluminal pulse propagation in the focus of the THz CTR pulses.
- Because the low frequency components can not be focused as well as the high frequency components, the focusing acts as a ‘high-pass filter’ on the CTR spectrum. This leads to major temporal pulse distortions in the focus of the CTR.

Note that other groups [2–5] observed similar behavior in focused single-cycle pulses. From the comparison between experiments and theory we can show that we understand in quite some detail, both the qualitative behavior and the quantitative field strengths, of the focused THz CTR pulses.

Since we understand all relevant physical details we can make a realistic estimation of the potential of the 1.5 cell RF photogun as a source of free-space CTR THz radiation. We show that, in principle, we have produced CTR pulses containing $\sim 70$ nJ, which is roughly in agreement with expectations. However, due to the limited bandwidth of $\sim 1$ THz the radiation can not be focussed very well. The energy flowing through an area of $4\pi \, \text{mm}^2$ located in the focal spot is only a few nJ. The peak electric field of the focused CTR is $\sim 0.4 \, \text{MV/m}$. So, in order to create THz pulses of $\sim 100 \, \text{MV/m}$ the bandwidth needs to be increased, by compressing the electron bunches in the longitudinal dimension to $\sim 100$ fs.
5.2 Experimental Setup

Fig. 5.1 shows the setup used for the generation and detection of CTR. The electron bunches, which are created and accelerated in the setup described in chapter 4 (see Fig. 4.7), are sent through a 1 mm thick, 68 mm x 68 mm square Al plate. The Al plate makes an angle of 45° with respect to the incoming electron bunches. The CTR is emitted at right angles with respect to the incoming electron beam, and leaves the vacuum through a 3 mm thick z-cut crystalline quartz window. The CTR is focused using an ellipsoidal Al mirror onto a ZnTe crystal used for electro-optic (EO) detection.

5.2.1 Terahertz Focussing Mirror

Fig. 5.4 shows a schematic picture of the Al ellipsoidal mirror used to focus the THz pulses. The surface of the mirror is a part of the prolate spheroid obtained by rotating an ellipse about its major axis. In this case an ellipse with a semimajor axis $a = 80$ mm, and a semiminor axis $b = 20$ mm. One of the foci of the ellipse coincides with the ZnTe crystal used for EO detection. If the mirror is properly aligned, the other focus is located where the electron bunch travels into the Al plate.

In order to align the electron bunches with respect to the ellipsoidal mirror, the Al plate can be replaced by a scintillator foil (NE-102A). By imaging the focus of the scintillation light onto a ccd-camera it is possible to align the ellipsoidal mirror with respect to the electron bunches and the Al plate. The Al plate and the scintillator foil are placed 2.5 mm off-axis (as defined by the incoming electron bunches) on a rotatable vacuum feedthrough. This allows the electron bunches to be imaged on a phosphor screen placed behind the Al plate and scintillator foil (not shown in Fig 5.1).

5.2.2 Electro-optic detection

The crystal used for EO detection is a 0.5 mm thick ZnTe crystal cut in the <110> direction. The birefringence induced by the CTR is probed using 800 nm pulses remaining after THG (see Ch. 4). Since the 800 nm pulses have a slight elliptical polarization after THG, a linear polarizer in combination with a λ/4 waveplate is used to make the 800 nm probe pulses circularly polarized. The delay $\tau$ between the CTR and the 800 nm probe pulses is varied using two mirrors placed on a motorized stage. The probe pulse illuminates a spot of approximately 2 mm radius on the ZnTe crystal. A polarizing beamsplitter cube splits the 800nm beam into an s-polarized and a p-polarized beam, which are each imaged on a different ccd-camera, see Fig. 5.1. In this way the phase rotation $\phi_{EO}$ induced in the ZnTe crystal is measured in a single shot for each transverse position in the probe beam as a function of delay time $\tau$.

Various authors have described EO-detection for numerous situations [6–8]. We will discuss the theory here briefly to find the relation between $\phi_{EO}$ and the THz electric field $E_{THz}$. Fig. 5.5 defines the orientation of the THz electric field vector $E_{THz}$, the circularly polarized probe pulse $E_{probe}$, and the ZnTe crystal with respect to a lab coordinate system.
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Figure 5.4: Schematic picture of Al ellipsoidal mirror used to focus CTR on ZnTe crystal.
Figure 5.5: Geometry of circularly polarized probe pulse, the THz electric field $E_{\text{THz}}$, and the ZnTe crystal. All with respect to a lab coordinate system $(x, y, z)$. The plane of the page is the $<110>$ plane of the crystal.

The angle between $E_{\text{THz}}$ and the $x$-axis is $\varphi$. The angle between the [001] crystallographic axis of the ZnTe crystal and the $x$-axis is $\Gamma$. The semimajor axis of the index ellipsoid, characterizing the induced birefringence $[9]$, makes an angle $\Gamma_{ie}$ with the $x$-axis direction and is given by $[6]$

$$\Gamma_{ie} - \Gamma = \tan^{-1} \frac{2\sin \Omega}{\sqrt{1 + 3\sin^2 \Omega - \cos \Omega}},$$  

(5.1)

with $\Omega = \varphi - \Gamma$, the angle between $E_{\text{THz}}$ and the [001] direction of the ZnTe crystal. The difference in index of refraction between the two principle axes $\Delta n$ is given by $[6]$

$$\Delta n = (n_{y'} - n_{x'}) = \frac{1}{2} n_0^3 r_{41} E_{\text{THz}} \sqrt{1 + 3\sin^2 \Omega},$$

(5.2)

with $n_0$ the index of refraction of ZnTe at 800 nm, and $r_{41}$ the EO coefficient of ZnTe involved in the Pockel effect.

We define the intensity of the s-polarized probe pulse on the crystal as $I_s(x, y)$, and the intensity of the p-polarized probe pulse on the crystal as $I_p(x, y)$. We can now find $\phi_{\text{EO}}(x, y)$ for each transverse position in the probe beam as follows

$$\phi_{\text{EO}}(x, y) = \frac{I_s(x, y) - I_p(x, y)}{I_p(x, y) + I_s(x, y)},$$

(5.3)

If the probe pulses incident on the ZnTe crystal are polarized circularly this becomes

$$\phi_{\text{EO}}(x, y) = -\sin(\Delta n(x, y) kL) \sin(2\Gamma_{ie}) \equiv -\Delta n(x, y) kL \sin(2\Gamma_{ie}),$$

(5.4)

with $k$ the wavenumber at 800 nm, and $L$ the crystal thickness. The last step is allowed if $\Delta n(x, y) kL \ll 1$. The expression for $\phi_{\text{EO}}(x, y)$ becomes particulary simple for $\Gamma = 0$. If we assume that $\Gamma = 0$ and substitute Eqs. (5.1)-(5.2) into Eq. (5.4) we obtain

$$\phi_{\text{EO}}^0(x, y) = \eta E_{\text{THz}} y(x, y) \sin \varphi \equiv \eta E_{\text{THz}}^y(x, y),$$

(5.5)
where we defined $\eta \equiv n_0^3 r_{\text{tr}} k L$. Following the same reasoning as above we can find

$$\phi_{90^\circ}^{\text{EO}}(x,y) = \eta E_{\text{THz}}^x(x,y) \cos \varphi \equiv \eta E_{\text{THz}}^x(x,y),$$

when $\Gamma = \frac{\pi}{2}$. So depending on the orientation of the ZnTe crystal we can measure either the x or y-component of the THz electric field.

### 5.3 Measurements Terahertz Radiation

Before discussing detailed measurements of the THz electric field distribution in the focal spot as function of $x$, $y$ and $\tau$, we first discuss CTR measurements made at various values of the injection phase, $\phi$ (see Ch. 4 on pag. 52), which have been done to optimize the CTR yield.

#### 5.3.1 Optimization of CTR as a function of injection phase

The ZnTe orientation was such that $E_{\text{THz}}^x(x,y)$ was measured. Because we want to optimize the CTR we are not yet interested in the measured $E_{\text{THz}}^x(x,y)$ as function of $x$ and $y$, for a certain $\tau$. Instead we have calculated the corresponding energy flux $P_x$ using

$$P_x = \varepsilon_0 c n_{\text{THz}} \int E_{\text{THz}}^x(x,y)^2 \, dx \, dy,$$

where $\varepsilon_0$ is the permittivity of free space, $c$ the speed of light, and $n_{\text{THz}} = 3.2$ the index of refraction of ZnTe between $\sim 0.01$ THz and $\sim 2$ THz. The total energy $E_n$ of the THz pulse is calculated by integrating $P_x$ over time. Fig. 5.6 shows the measured $P_x$ (black line) and $E_n$ (red line) versus $\tau$, for five different values of $\phi$. In the lower right corner the energy versus $\phi$ is plotted. The delay was varied in steps of 40 fs. The measured energy flux shown has been filtered by averaging each data-point with 15 adjacent data-points.

The electron bunches contained 95 pC of charge, had a kinetic energy of 3 MeV, and a transverse RMS spot size $\sigma_x$ of 0.3 mm, all measured at $\phi = -40^\circ$.

In Fig. 5.6, in the plot for $\phi = -40^\circ$ we observe three peaks. As the electrons are injected further before crest ($\phi$ decreases), the first peak remains approximately the same while the other two grow, eventually forming a single structure. As $\phi$ goes from $-40^\circ$ to $-75^\circ$ the electron bunch length becomes shorter. This is because the electrons at the back of the bunch are accelerated more than those at the front, which counteracts the bunch blow-out caused by space-charge forces. A shorter electron bunch length leads to a wider coherent spectrum and therefore to more radiated energy (see Ch. 2). That is why we identify the part of the measurements which grows with decreasing $\phi$ as CTR. The first peak can be identified as a wakefield of diffraction radiation for two reasons; first, it is created at a moment in time corresponding to a position where the electron bunch sees a sudden change in the inner diameter of the vacuum tube. Second, the radiated energy remains the same as the bunch shortens, indicating that the coherence is not determined
Figure 5.6: Measured energy flux (black line) and energy (red line) in ZnTe crystal versus delay, for five different injection phases. In the lower right figure the energy versus the injection phase is plotted. The delay was varied in steps of 40 fs. The measured energy flux shown has been filtered by averaging each data-point with 15 adjacent data-points.
by the dimensions of the electron bunch, but instead by the dimensions of the vacuum tube.

So, if we want to optimize our setup on THz radiation we should inject the electrons at $\sim 75^\circ$ before crest.

5.3.2 Measured transverse electric field components of focused CTR

In figures 5.7 and 5.8 the measured electric field components $E_{x}^{\text{THz}}(x,y)$ and $E_{y}^{\text{THz}}(x,y)$ are shown, respectively, measured at different values of the delay $\tau$. The 267 nm laser intensity profile has a RMS size in the $x$-direction of $\sigma_x = 0.4$ mm, and in the $y$-direction $\sigma_y = 0.2$ mm. The electron bunches contained 100 pC of charge, had a kinetic energy of 3 MeV, and a RMS transverse size of $\sigma_x = 0.3$ mm, all measured at an injection phase $\phi = -40^\circ$. The CTR itself was measured at the optimized phase $\phi = -72^\circ$, corresponding to 60 pC of charge, and a kinetic energy of 2.8 MeV.

In Fig. 5.7 a dashed circle is shown in the first image. During the digital dataprocessing, which is necessary to convert the measured intensity profiles into a phase rotation angle $\phi_{\text{EO}}$, we truncated the measured $\phi_{\text{EO}}$ outside the circle. The digital dataprocessing was performed on-the-fly during measurements. Afterwards noise was removed from the measured $\phi_{\text{EO}}$ profile using a spatial low-pass-filter.\(^2\) Any structure lying outside the dashed circle in the measured profiles are therefore artifacts resulting from the truncation and filtering of the measured profiles.

Qualitative analysis of measured electric field profile

The general structure of Figs. 5.7 and 5.8 shows two lobes of opposite sign, and zero signal in the center. The two lobes approach each other and eventually switch side when the radiation goes through its focus. The CTR focus is always $\sim 1 \times 1$ mm$^2$, roughly corresponding to the diffraction limit. Combination of Fig. 5.7 and 5.8 suggest a single-cycle radially polarized pulse: $E_{\text{THz}}$ is pointing outwards in the $1^{\text{st}}$ half-cycle, and pointing inwards in the $2^{\text{nd}}$ half-cycle. At first sight this observation agrees with expectations, see Fig. 5.3. The CTR field is radially polarized and is focussed to a $\sim$ mm spot by the ellipsoidal mirror.

However, closer inspection of Figs. 5.7 and 5.8 reveals deviations from a full cylindrically symmetric radially polarized ‘donut’ mode. The deviations are due to the fact that the TR interface is oriented at $45^\circ$ with respect to the incoming electron bunches:

- The radial component, $E_{\rho}^{\text{THz}}$, of the CTR is not cylindrically symmetric, see Fig. 5.2 and Fig. 2.6.
- The CTR electric field has a non-zero azimuthal component, $E_{\phi}^{\text{THz}}$, see Eq. (2.11).

\(^2\)Spatial frequencies lying outside a surface defined by an ellipse were removed. The ellipse had a semi-axis of $f_x = 4$ mm$^{-1}$, and a semi-axis of $f_y = 1.5$ mm$^{-1}$.
Figure 5.7: Measured $E_{x}^{THz}(x, y)$ for different values of the delay.
Figure 5.8: Measured $E_{y}^{\text{THz}}(x,y)$ for different values of the delay.
Figure 5.9: Density plot of and $E_{\rho}^{\text{THz}}$ at a delay of 113.2 ps. The dashed arrows indicate the directions of $E_{\rho}^{\text{THz}}$ and $E_{\varphi}^{\text{THz}}$ for four values of $\varphi$. The solid arrows indicate the direction of the transverse THz electric field $E^{\text{THz}}$. As can be seen the projection of $E^{\text{THz}}$ onto the y-axis tends to be larger for $x > 0$ than for $x < 0$.

The detailed profiles of $E_{x}^{\text{THz}}$ and $E_{y}^{\text{THz}}$ are difficult to decipher from Eq. (2.11), since we are focusing the CTR using an ellipsoidal mirror. Later in this chapter (Sect. 5.4) we will make a detailed calculation of the fields in the focus of the radiation. However, we can already make some general remarks: From Eq. (2.11) it follows that $E_{\rho}^{\text{THz}}$ is always $\sim 5 \times$ larger than $E_{\varphi}^{\text{THz}}$. Therefore we expect a mode roughly resembling a radially polarized 'donut' mode, slightly distorted by a smaller azimuthal component.

The radial component can be recognized easily. Because we are measuring the x- or y-component of $E^{\text{THz}}$, we see the 'donut' shape projected onto the x- or y-axis, resulting in two oppositely polarized lobes. We also observe a surprising feature: If one interprets the positive (red) and negative (blue) lobes of $E^{\text{THz}}(x, y)$ as two separate pulses, they appear to be traveling with superluminal speeds. For example, the 'red pulse' in Fig. 5.7 appears to have traveled roughly 0.5 mm between $\tau = 117.9$ ps and $\tau = 118.4$ ps, corresponding to a speed of $10^9$ ms$^{-1}$. This can be explained if one realizes that $E_{x}^{\text{THz}}(x, y)$ is formed by
a superposition of pulses, all traveling with the speed of light in the forward and radial direction. Their superposition, however, can have features which appear to be traveling with speeds faster than that of light. The origin of this effect will be discussed in more detail in Sect. 5.4.

The influence of the azimuthal electric field component can be seen most clearly in Fig. 5.8 at $\tau = 113.2$ ps. To illustrate this we have plotted the measured $E_{\text{THz}}^y$ for $\tau = 113.2$ ps in Fig. 5.9 using a different scale and a more stringent spatial filter.\(^3\) According to Eq. (2.11) the CTR pulse moving away from the Al plate is characterized by $E_{\rho}^{\text{THz}} < 0$ and $E_{\varphi}^{\text{THz}} \propto \sin \varphi$, with $\varphi$ the azimuthal angle in the x-y plane from the x-axis. Subsequently the pulse is focussed by the ellipsoidal mirror. The radial component $E_{\rho}^{\text{THz}}$ acquires no phase shift when reflected by the ellipsoidal mirror. However, $E_{\varphi}^{\text{THz}}$ does change sign when reflected because it is polarized perpendicular to the plane of incidence. Therefore we expect $E_{\rho}^{\text{THz}}$ and $E_{\varphi}^{\text{THz}}$ to have the same sign for $0 < \varphi < \pi$, and opposite sign for $\pi < \varphi < 2\pi$. This is depicted schematically in Fig. 5.9. The dashed arrows indicate the directions of $E_{\rho}^{\text{THz}}$ and $E_{\varphi}^{\text{THz}}$ for four values of $\varphi$. We have drawn $E_{\rho}^{\text{THz}}$ in the positive direction because this corresponds with the measured profile. Since the absolute sign of the measured $E_{\rho}^{\text{THz}}$ has not been resolved, only the relative sign, we are only interested in the relative orientation of $E_{\rho}^{\text{THz}}$ with respect to $E_{\varphi}^{\text{THz}}$. The solid arrows indicate the direction of the transverse THz electric field $E^{\text{THz}}$. As can be seen the projection of $E^{\text{THz}}$ onto the y-axis tends to be larger for $x > 0$ than for $x < 0$. So we expect the measured $E_{y}^{\text{THz}}$ to be larger for $x > 0$ than for $x < 0$. This behavior is clearly reflected in the measurements.

If we calculate the discrete Fourier transforms (DFTs) of the peak electric fields of $E_{x}^{\text{THz}}$ and $E_{y}^{\text{THz}}$ versus $\tau$, both contain frequency components up to 1 THz.

5.3.3 Energy flux and total radiated energy

The energy flux, $P$, has been calculated as follows:

$$P = \varepsilon_0 c n_{\text{THz}} \int (E_{x}^{\text{THz}}(x,y)^2 + E_{y}^{\text{THz}}(x,y)^2) dx dy.$$

(5.8)

The total energy, $E_n$ of the THz pulse is calculated by integrating $P$ over time. In Fig. 5.10 the result is shown. We shifted the delay of $E_{y}^{\text{THz}}$ to compensate for the difference in arrival time. We again observe a small peak caused by a wakefield, and a large pulse corresponding with the CTR. From Fig. 5.10 we conclude that the part of the CTR pulse transmitted into the ZnTe crystal contained approximately 1 nJ.

5.4 Comparison with theory

In Ch. 2 we calculated the CTR fields emitted when the electron bunch travels into the Al plate. If we want to compare those expressions with the measured CTR we have to take

\(^3\)Spatial frequencies lying outside a surface defined by an ellipse were removed. The ellipse had a semi-axis of $f_x = 1 \text{ mm}^{-1}$, and a semi-axis of $f_y = 0.75 \text{ mm}^{-1}$
Figure 5.10: Measured energy flux versus delay (black curve), and total energy (red curve).
the ellipsoidal mirror into account. The CTR induces surface charges and currents in the metal surface of the ellipsoidal mirror. Those surface charges and currents give rise to a reflected radiation pulse, which will have a focus at the position of the ZnTe crystal, see Fig. 5.4. We can calculate the electric fields of the focused CTR, using the physical theory of diffraction (PTD) [10], which assumes that the surface current density $\mathbf{K}$ on the metal surface is given by,

$$\mathbf{K} = 2\mu_0^{-1} \mathbf{n} \times \mathbf{B}_{\text{in}}, \quad (5.9)$$

where $\mathbf{B}_{\text{in}}$ is the incident magnetic field, $\mu_0$ is the permeability of free space, and $\mathbf{n}$ the outward normal vector at the metal surface. We supplement this by assuming that the surface charge density $\sigma$ is given by,

$$\sigma = 2\varepsilon_0 \mathbf{n} \cdot \mathbf{E}_{\text{in}}, \quad (5.10)$$

with $\mathbf{E}_{\text{in}}$ the incident electric field, and $\varepsilon_0$ the permittivity of free space. Using $\sigma$ and $\mathbf{K}$ we calculate the scattered fields via the vector potential $\mathbf{A}$,

$$\mathbf{A}(\mathbf{r}, t) = \int_S \frac{\mathbf{K}(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})}{4\pi \mu_0 |\mathbf{r} - \mathbf{r}'|} d^2 r', \quad (5.11)$$

and the scalar potential $\phi$,

$$\phi(\mathbf{r}, t) = \int_S \frac{\sigma(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})}{4\pi \varepsilon_0 |\mathbf{r} - \mathbf{r}'|} d^2 r', \quad (5.12)$$

where $S$ is the surface of the ellipsoidal mirror. The scattered electric field $\mathbf{E}$ is given by

$$\mathbf{E}(\mathbf{r}, t) = -\nabla \phi(\mathbf{r}, t) - \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t}, \quad (5.13)$$

and the scattered electric field $\mathbf{B}$ by

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t). \quad (5.14)$$

We will evaluate the integrals for $\mathbf{E}(\mathbf{r}, t)$ without any additional assumptions other than the assumed surface current and charge densities shown in (5.9) and (5.10). This raises the question how good of an approximations Eqs. (5.9) and (5.10) are. They are correct for a perfectly conducting infinite metal plane. So, considering a Fourier component of the radiation field with a wavelength $\lambda$, we can expect Eqs. (5.9) and (5.10) to be reasonable approximations if the surface is flat on a wavelength scale. Or to quantify this statement: $\kappa \lambda \ll 1$, with $\kappa$ the local curvature of the surface. The curvature is smallest at the entrance of the ellipsoidal mirror, as seen by the CTR, and is given by $\kappa = b/a^2$. The curvature increases towards the location of the ZnTe crystal, where it approaches $\kappa = a/b^2$. For THz frequencies $\kappa \lambda < 1$, so we have a reasonable approximation for the high frequency components in our CTR pulse. Since the high frequency components dominate the focus of the CTR we have a reasonable approximation of the radiation in the focus. However,
if we want to evaluate the CTR pulse at times later than the moment of focus, we have
to take the lower frequency components into account for which \( \kappa \lambda \geq 1 \). In that case the
approximations used for the surface current and charge densities become invalid. We will
therefore only compare the measured and calculated pulses in a time window of a few
picoseconds.

5.4.1 Behavior of CTR in and near focus

In Appendix D-1 we discuss how the integrals for \( \mathbf{E}(\mathbf{r}, t) \) can be found from Eqs (5.9)-(5.13). We will evaluate them without any additional assumption. However, the integrals
for \( \mathbf{E}(\mathbf{r}, t) \) can be simplified if they are evaluated in or near the vicinity of the focus of the
radiation. We will discuss the simplification here because it gives physical insight into the
electric fields in and near the focus. For those readers who are not particulary interested
in the derivation, the result is listed in Eq. (5.21).

We assume that the CTR is emitted at one of the foci of the ellipsoidal mirror, which
we will call \( F_1 \), see Fig. 5.11. The focus coinciding with the ZnTe crystal is called \( F_2 \). We
define a coordinate system \((x, y, z)\) with the origin located in \( F_1 \). As incident fields we use
the expression for CTR for an electron bunch traveling from vacuum into metal under 45°
incidence,

\[
\mathbf{E}_{in}(\mathbf{r}, t) = \frac{eN\beta}{8\pi^2\varepsilon_0c} \frac{f(t - \frac{\mathbf{r}}{c})}{r} \mathbf{E}(\varphi, \theta),
\]

(5.15)

with \( N \) the number of electrons in a bunch, \( f(t - \frac{\mathbf{r}}{c}) \) the inverse Fourier transform of the
form factor, and \( \mathbf{E}(\varphi, \theta) \) the vector components as defined in Appendix D-3. We defined \( \varphi \) as the azimuthal angle in the x-y plane from the x-axis, \( \theta \) as the polar angle from the
z-axis, and \( r \) the distance from a point to the origin. The incident magnetic field is given by

\[
\mathbf{B}_{in}(\mathbf{r}, t) = \frac{eN\beta}{8\pi^2\varepsilon_0c^2} \frac{f(t - \frac{\mathbf{r}}{c})}{r} \mathbf{B}(\varphi, \theta),
\]

(5.16)
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with \( \mathbf{B}(\varphi, \theta) = \mathbf{e}_r \times \mathbf{E}(\varphi, \theta) \), and \( \mathbf{e}_r \) the unit vector in radial direction. If we substitute Eq. (5.16) into Eqs. (5.9) and (5.11) we can write

\[
A(r, t) = \frac{eN\beta}{4\pi^2\varepsilon_0 c^2} \int_S (\mathbf{n} \times \mathbf{B}) \frac{f(t - \frac{|r-r'|+|r'|}{c})}{4\pi|r-r'||r'|} d^2r',
\]

(5.17)

for the vector potential. If we substitute Eq. (5.15) into Eqs. (5.10) and (5.12) we obtain

\[
\phi(r, t) = \frac{eN\beta}{4\pi^2\varepsilon_0 c^2} \int_S (\mathbf{n} \cdot \mathbf{E}) f(t - \frac{||r-r'||}{c}) \frac{d^2r'}{4\pi|r-r'||r'|},
\]

(5.18)

for the scalar potential.

The term \( |r-r'| + |r'|/c \) within the argument of \( f \) is the transit time it takes the CTR to travel from \( F_1 \) to a point of observation, via the surface of the ellipsoidal mirror. For an observer located in \( F_2 \), \( r = (0, 0, 2f) \) with \( f = \sqrt{a^2 - b^2} \), the transit time is equal to \( 2a/c \). To simplify the mathematics we can expand \( |r-r'| + |r'| \) around \( F_2 \),

\[
|r-r'| + |r'| \approx 2a + s \cdot r_f,
\]

(5.19)

with \( s \) a unit vector pointing from a surface element on the ellipsoidal mirror towards \( F_2 \), and \( r_f = r - (0, 0, 2f) \). So, we can replace the argument of \( f \) in Eqs. (5.17) and (5.18) with \( t - 2a/c - s \cdot r_f/c \), and we can replace \( |r-r'| \) in the denominators with \( 2a - |r'| \), because \( r_f \cdot s \ll 2a \). Since the only remaining \( r \) dependence in the integrals (5.17) and (5.18) is in the argument of \( f \), we can use Eq. (5.13) to write

\[
E = \frac{eN\beta}{4\pi^2\varepsilon_0 c^2} \int_S ((\mathbf{n} \cdot \mathbf{E})s - \mathbf{n} \times \mathbf{B}) \frac{1}{4\pi|r'|(|2a - |r'|)} \frac{\partial f}{\partial t} d^2r'.
\]

(5.20)

If use \( \mathbf{B} = \mathbf{e}_r \times \mathbf{E} \) and \( s = \mathbf{e}_r - 2(\mathbf{e}_r \cdot \mathbf{n})\mathbf{n} \) we can write Eq. (5.20) as

\[
E = \frac{eN\beta}{4\pi^2\varepsilon_0 c^2} \int_S \mathbf{E}^{\text{reflected}} \frac{\mathbf{n} \cdot s}{4\pi|r'|(|2a - |r'|)} \frac{\partial f}{\partial t} d^2r',
\]

(5.21)

with \( \mathbf{E}^{\text{reflected}} = 2(\mathbf{n} \cdot \mathbf{E})\mathbf{n} - \mathbf{E} \), the vector components of the reflected electric field on a surface element on the ellipsoidal mirror.

Most terms in Eq. (5.21) allow a straightforward interpretation: We have to propagate an electric field having the components of CTR, which is reflected by the surface of the ellipsoidal mirror, towards the focus of the mirror. The terms in the denominator take into account, respectively, the distance from the location where the CTR is emitted to a surface element on the ellipsoidal mirror, and the distance from the surface element to an observer. The term in the numerator often appears in scattering and diffraction problems, and is known as the inclination factor [9] or the obliquity factor [11].

Surprisingly however, the secondary droplets on the surface of the ellipsoidal mirror do not create pulses proportional to \( f \), which is the temporal pulse shape of the emitted CTR. Instead the pulses are proportional to \( \partial f/\partial t \) in the focus of the radiation. So, even
if we neglect coherent diffraction radiation (CDR) and assume that the CTR is almost a half-cycle pulse, the pulses traveling towards the focus of the radiation are full-cycle pulses. The radiation in the focus, which is the superposition of the full-cycle pulses, is therefore certainly not a half-cycle pulse. This is less strange than it seems, since we are dealing with a very broadband pulse. The low frequency components can not be focused as well as the high frequency components. This causes a dispersion of the initial pulse leading to a significantly altered pulse in the focus. Similar behavior has been observed in focused full- and half-cycle pulses [2–5].

5.4.2 Calculated transverse electric field components of focused CTR

Figures 5.12 and 5.13 show, respectively, the calculated $E_{x}^{\text{THz}}$ and $E_{y}^{\text{THz}}$, at the position of the ZnTe crystal. We used a Gaussian electron distribution for the calculations and assumed that the transverse dimension of the electron bunch is negligible compared to the longitudinal dimension $\sigma_l$. The calculations have been done for electron bunches containing 60 pC of charge, with a kinetic energy of 2.8 MeV, and $\sigma_l(c\beta)^{-1} = 1$ ps, corresponding to the conditions of the optimized injection phase (Sect. 5.3.1).

In the calculated spatiotemporal profiles we clearly recognize the radial component. We see two lobes which are the projection of the ‘donut’ shape onto the respective $x$- or $y$ axis. We also observe the the influence of the azimuthal electric field component. In the calculated $E_{y}^{\text{THz}}$, see Fig. 5.13 at $\tau = -1.34$ ps, we observe an asymmetry with respect to the x-axis. This clearly resembles the measured $E_{y}^{\text{THz}}$ profile shown in Fig. 5.9. To compare the calculated and measured electric field strengths we have to take the influence of the crystalline quartz window and the ZnTe crystal into account. The transmission coefficient for the crystalline quartz window $t_{w} = 0.8$ at 1 THz [12, 13]. For the ZnTe crystal the transmission coefficient $t_{\text{ZnTe}} = 0.5$. If we correct the calculated peak electric field of 0.4 MV/m with $t_{w}$ and $t_{\text{ZnTe}}$ we obtain 0.16 MV/m, which is in good agreement with the measured peak electric field.

There are, however, some marked differences between the calculated and measured spatiotemporal profiles.

- The calculated spot sizes are larger than the measured spot sizes.

- The evolution of the lobes is different: In both the calculated $E_{x}^{\text{THz}}$ and $E_{y}^{\text{THz}}$ the main lobes appear to enter and leave from the same side. For example, the red lobe in the focus of the calculated $E_{x}^{\text{THz}}$ first appears from the right side around -1.1 ps, then travels towards the focus where it reverses direction, and finally leaves on the right side again at approximately 1.1 ps. This in contrast with the measurements where each lobe keeps on propagating in the same direction.

- Another feature worth mentioning is the sign of the calculated profiles around zero delay: For example, $E_{x}^{\text{THz}}$ is positive for $x > 0$ around $\tau = 0$, which appears to be in contrast with $E_{\rho}^{\text{THz}} < 0$. 

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**Figure 5.12:** Calculated $E_{THz}^{x}(x, y)$ at the position of the ZnTe crystal for different times. We evaluated $E_{THz}^{x}(x, y)$ at 24 positions.
Figure 5.13: Calculated $E_{y}^{\text{THz}}(x,y)$ at the position of the ZnTe crystal for different times. We evaluated $E_{y}^{\text{THz}}(x,y)$ at 24 positions.
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\[ \frac{\partial^2 E_x}{\partial \phi \partial \theta} \]

Figure 5.14: Several plots of the x-component of two CTR pulses created by secondary droplets on the surface of the ellipsoidal mirror. The pulses are evaluated at \( y = 0 \) and \( z = 2f_e \), as function of \( x \). Each plot is made for a different moment in time. The droplets are located at \( \theta = 0, 1 \), and respectively, \( \varphi = 0 \) (dashed curves) and \( \varphi = \pi \) (dotted curves). The right figures show the sum of two pulses (solid curves).
The sign of the fields around zero delay is caused by the fact that the secondary droplets at the surface of the ellipsoidal mirror do not radiate pulses with the temporal profile of the initial pulse, but instead with the time derivative of the initial pulse profile, see Eq. (5.21). To show the influence of this on the x-component of the electric field in the focus, we have plotted the contribution of two secondary droplets located at $\theta = 0.1$ and, respectively, $\varphi = 0$ (dashed curves) and $\varphi = \pi$ (dotted curves), on the left side in Fig. 5.14. The right side of Fig. 5.14 shows the superposition of the two pulses (solid curves). If we look at the two separate pulses we observe two counter propagating pulses, both with a temporal profile corresponding to the time derivative of a Gaussian pulse. The pulse coming in from the right (dashed curve) is the time derivative of a negative Gaussian pulse. At $\tau = 0$ the center of the negative Gaussian pulse is located at $x = 0$. We therefore observe a positive time derivative for $x > 0$, leading to a positive $E_x^{\text{THz}}$ for $x > 0$ at $\tau = 0$. From Fig. 5.14 we can also understand why the lobes in the calculated $E_x^{\text{THz}}$ and $E_y^{\text{THz}}$ appear to reverse direction when reaching the focus. If we look at the solid curve at $\tau = -1.2$ ps, we see that the positive part of the total pulse (solid curve) is dominated by the pulse traveling from right to left (dashed curve). However, when $\tau = 1.2$ ps, the positive side of the total pulse (solid curve) is dominated by the pulse traveling in the opposite direction (dotted curve). A last note on Fig. 5.14 is that we again observe superluminal pulses, since both the dashed and dotted curves travel with a speed $v_{\text{pulse}} = c/s_x > c$, see Sect. 5.4.1.

As already mentioned, the evolution of lobes in the measured spatiotemporal profiles are different than in the calculated ones. The lobes in the measured $E_x^{\text{THz}}$ and $E_y^{\text{THz}}$ do not appear to reverse direction in the focus, and keep on propagating in the same direction. An explanation could be that the electron bunches have a 'comet' like shape: Most of the electrons are in the front part of the bunch, but, a substantial part of the electrons is trailing in a tail. The measured $P$, see Fig. 5.10, also hints in that direction. The time derivative of a 'comet' shape has a short intense peak at the front, followed by a much longer and lower peak with opposite sign. Therefore we can easily recognize the two separate counter propagating pulses, by which the total field is formed. It could also explain the fact that the measured spot sizes are smaller than the calculated spot sizes. The short peak in front of the pulse can be focussed to a smaller spot than the long tail which is following. We therefore mainly observe the first short peak. In the calculations we use a symmetric electron bunch distribution, so the first peak is always followed by an equal peak with opposite sign. This typically creates a spot size which is twice as large as a single peak.

### 5.4.3 Calculated energy flux and total energy

We calculate the energy flux $P^{\text{calc}}$ using

$$P^{\text{calc}} = \varepsilon_0 c \int \left( E_x^{\text{THz}}(x, y) B_y^{\text{THz}}(x, y) - E_y^{\text{THz}}(x, y) B_x^{\text{THz}}(x, y) \right) dx dy,$$

(5.22)

where the integration area is a circular disc with a radius of 2 mm. The energy $E_n^{\text{calc}}$ is calculated by integrating $P^{\text{calc}}$ over time. Figure 5.15 shows the theoretically calculated
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$P_{\text{calc}}$ (black lines) and $E_n^{\text{calc}}$ (red lines) as function of time. The presence of the two 'pedestals' next to the main peak raises the question whether the first peak in Fig. 5.6 really is a wakefield. The answer is affirmative for two reasons: the wakefield is separated further in time from the main peak than the pedestals, and the wakefield does not scale with the electron bunch length. To compare the calculated and measured $P$ we have to take the influence of the crystalline quartz window and the ZnTe crystal into account. The combined transmittance of the window and the ZnTe crystal is $\sim 0.5$. If we correct the theoretically calculated $E_n$ for this, we obtain 3 nJ, which is larger than the measured value of 1 nJ. The pulse form of $P_{\text{calc}}$ is also different from the measured $P$. $P_{\text{calc}}$ is a sharp peak with two pedestals, the measured $P$ has the 'comet' shape already discussed in Sect. 5.4.2. If we want to find a better match between theory and measurements, we have to find a better approximation for the temporal electron distribution. However, the match between theory and measurements is good enough to conclude that we have captured and described the relevant physical details of the measurements.

The amount of measured energy in the THz pulse is somewhat disappointing. If we calculate the total energy radiated towards the surface of the ellipsoidal mirror, assuming an electron bunch containing 60 pC, a kinetic energy of 2.8 MeV, $\sigma_l(c\beta)^{-1} = 1$ ps, and neglecting the finite size of the Al plate, we obtain 34 nJ. But even for the theoretically calculated electric fields we only end up with 5 nJ in a spot size of $4\pi$ mm$^2$. This is because the 34 nJ is emitted at all wavelengths, and only wavelengths smaller than $\sim 2$ mm can be focussed in a spot of $4\pi$ mm$^2$ in a time window of 10 ps. This is not a specific problem of our setup, it is a general problem when focusing broadband THz pulses.

### 5.5 RF photogun as a source of free-space THz radiation

Now that we have shown that we can describe and understand the relevant physical details, we can make a realistic estimation of the potential of the 1.5 cell RF photogun as a source of free-space CTR THz radiation. A very simple and straightforward improvement would be to place the Al plate directly behind the RF photogun. The Al plate can be placed at 0.6 m from the cathode without any additional changes to the RF photogun itself, see Fig. 4.7 in Ch. 4. Because the Al plate is closer to the RF photogun the electron bunches are shorter, and hardly affected by the injection phase. In this way we can create electron bunches having 100 pC of charge, with a kinetic energy of 3 MeV, with an electron bunch length $\sigma_l(c\beta)^{-1} = 0.5$ ps at the Al plate. Since we can inject the electrons at $-40^\circ$ the arrival time jitter is less than $\sim 20$ fs, see Fig. 4.15 in Ch. 4. Using the solenoid around the cavity we can focus the electron bunches to a transverse RMS spot size $\sigma_t$, such that $\sigma_t/\gamma << \sigma_l$.

For these conditions we calculate focused THz pulses with peak transverse electric fields of $\sim 2$ MV/m, and frequency components up to 2 THz, and $P_{\text{calc}} \sim 35$ kW, using a circular disc of 2 mm radius as integration surface. The total energy passing through the circular disc $E_n^{\text{calc}} = 55$ nJ. So far we have not discussed the z-component of the electric field in the focus of the CTR. The z-component of the THz pulses all add up in phase in the focus of
Figure 5.15: Calculated energy flux as function of time (black curve), and total energy (red curve).
the radiation, leading to a longitudinal peak electric field strength of $\sim 2$ MV/m.

The estimated performance of the RF photogun is not as good as has been outlined in Ch. 1, i.e., field strengths of $\sim 100$ MV/m and energy per pulse of $\sim 1$ $\mu$J. In the above estimated performance these numbers are clearly not obtained.

However, it could be argued that the THz collecting optics is poorly designed. In Ch. 1 we explained that $E_n \propto \ln \gamma$, making it unnecessary to accelerate the electrons to very high $\gamma$. However, because we are using electron bunches with $\gamma = 7$ the radiation is emitted into a large solid angle. Looking at Fig. 2.5 we see that we might gain by increasing the acceptance angle of the THz collecting optics. To check this we redid the calculation for an ellipsoidal mirror, having a semimajor axis $a = 320$ mm, and a semiminor axis $b = 80$ mm.

We used the specifications of the last mentioned electron bunches, and we integrated over $0.05 < \theta < \pi/4$ and $0 < \varphi \leq 2\pi$. The calculated THz pulses have transverse peak electric fields of $\sim 3$ MV/m, and contain frequency components up to 2 THz. The $P_{\text{calc}} \sim 160$ kW, if we use a circular disc of 2 mm radius as integration surface. The longitudinal peak electric field strength is $\sim 3$ MV/m. The total energy passing through the circular disc $E_{n,\text{calc}} = 160$ nJ. So, the THz electric field strength and energy indeed increase by improving the THz collecting optics. But the performance is still not as good as the numbers discussed in Ch. 1. Note that if we calculate the total energy radiated towards the surface of the ellipsoidal mirror, we obtain $\sim 1$ $\mu$J. So, we can create THz pulses containing $\sim 1$ $\mu$J, but they can not be focussed into a small enough space-time volume to reach electric fields of 100 MV/m.

Finally, we would like to point out that the most efficient way to boost the focussed CTR THz yield is decreasing the electron bunch length, because the peak electric fields of $E_{\text{THz}} \propto \tau_g^{-2}$. This scaling clearly shows that there is a huge gain to be made if the electron bunches are compressed. Van der Geer et al. [14] have shown that is possible to compress 3.7 MeV electron bunches of 100 pC to 100 fs RMS bunch length, using an RF photogun and an additional RF cavity to compress the electron bunches in the longitudinal dimension. Such electron bunches would lead to THz pulses with peak electric fields of $\sim 75$ MV/m.
Chapter 5.

Bibliography


Direct generation of terahertz surface plasmon polaritons on a wire using electron bunches

Abstract. We have performed a proof-of-principle experiment for generating intense terahertz (THz) surface plasmon polaritons (SPPs) on a metal wire by launching electron bunches onto a tapered end of the wire. An SPP field strength of 0.5 MV/m and a full-width-half-maximum (FWHM) SPP pulse length of \( \sim 6 \) ps are attained on a metal wire of 1.5 mm thickness. We have compared the measured properties of the SPPs with a newly developed theory. The theory predicts the bandwidth correctly, however, the measured spectral amplitude was typically a factor of 5 less than the calculated amplitude. Probable causes for the discrepancy are the tip-to-wire transition and electron scattering in the metal tip, which are both not modeled in the theory. By optimizing the electron beamline and focusing the SPPs, by tapering the metal wire into a tip, electric field strengths in excess of \( \sim 100 \) MV/m localized in a subwavelength spot become possible.

\(^{a}\)Manuscript in preparation.

6.1 Introduction

Terahertz Surface Plasmon Polaritons (THz SPPs) on a metal wire recently received a lot of attention [1–10]. It has been shown that these SPPs can be focused efficiently below the diffraction limit by periodically corrugating the wire [7, 8], or tapering the wire into a tip [9]. This leads to electromagnetic THz pulses that are both very strong and highly localized, making it possible to study materials at THz frequencies with sub-wavelength spatial resolution [11, 12]. Applications include near-field microscopy [13, 14], imaging of semiconductor structures [15, 16] or biological tissues [17, 18], single particle sensing [19, 20] and THz spectroscopy [21, 22].

Another important benefit of the wire geometry is that it acts as an efficient waveguide for THz SPPs. Recently it has been shown that THz SPPs can propagate long distances along a wire with low attenuation and dispersion [1–5]. This enables endoscopic delivery of THz radiation to samples in applications where line-of-sight access is not available [1]. Several other structures have been proposed as waveguides for THz SPPs, including coaxial lines [23], metal tubes [24] and non-metallic guides [25, 26]. However, the feasibility of these guides is limited by either high attenuation or high dispersion. An exception is the parallel-plate waveguide [27], but in this case the large cross-sectional area may be a problem for many THz applications.

Despite the promising properties of THz SPPs guided by a metal wire, it has proven difficult to efficiently generate SPPs of appreciable amplitude. In contrast, over the last years several sources have become available that generate intense free-space THz radiation pulses, with broad bandwidth and peak electric fields that approach the 100 MV/m regime. Technologies of the latter include accelerator-based sources generating coherent synchrotron, undulator, and transition radiation [28–30] and table-top systems producing radiation by optical rectification of femtosecond laser pulses [31]. However, up to now efficient coupling of these free-space THz pulses into the guided mode on a wire has proven difficult. Currently, THz SPPs are generated by scattering the linearly polarized free-space waves into a radially polarized wave, which is then coupled onto the wire [1]. However, due to the poor spatial overlap between the free-space radiation waveform and the SPP waveform, the coupling efficiency is very low (typically less than 1% [32]). A proposed method to overcome this low coupling efficiency is to create radially polarized THz radiation using a radially symmetric photoconductive antenna [32].

We have adopted a novel method to generate THz SPPs on a wire directly, that is, without the creation of free-space THz radiation as an intermediate step. Similar to the method proposed in ref. [32], in our method the guided mode on the wire is excited by a radially polarized field, thereby avoiding the poor coupling efficiency described above. We generate THz SPPs by launching electron bunches onto a metal wire which is tapered into a conical tip, as is illustrated in Fig. 6.1. When passing the conical vacuum-metal boundary, the bunch will generate a radially polarized coherent transition radiation (CTR) field, of which THz SPPs along the boundary are part. These excited SPPs will propagate onto the wire subsequently.

Generating THz SPPs by launching electron bunches onto a tapered wire tip, instead of
Direct generation of terahertz surface plasmon polaritons on a wire using electron bunches

Figure 6.1: Principle of THz SPP (blue pulse) generation on a wire by launching electron bunches (red) onto a conical tip.
coupling free-space CTR emitted at a planar interface onto a metal wire, has two benefits; first, electrons are capable of exciting SPPs directly, in contrast to photons where an additional coupling medium is necessary to match the wave vectors of the photons and SPPs. Second, for sharp tips the electrons pass the vacuum-metal boundary at grazing incidence, which enhances the transition radiation due to an increased radiation formation length [33].

In this chapter we present the measurements of SPPs on a metal wire, which were generated directly using electron bunches. We will first briefly discuss the properties of THz SPPs propagating on a metal wire in Sec. 6.2.1. To illustrate how we can generate THz SPPs on a wire, we discuss in Sec. 6.2.2 a simple model describing the SPPs generated by a line charge impinging onto a 1D line. This provides an insight into the physics of the SPP generation process. Subsequently, in Sec. 6.2, we briefly discuss the recently developed detailed theory of the generation of SPPs on a metal cone using electron bunches [34]. The experimental setup is treated in Sec. 6.3. In Sec. 6.4 we discuss the SPPs measured at 8 cm distance from the tip, and we will compare the measurements with theory in Sec. 6.4.3. In Sec. 6.4.4 we discuss possible reasons for the observed difference between the theoretically calculated and measured SPP spectra. In Sec. 6.5 we present measurements of SPPs which have propagated around a bend in the wire. The bend in the wire is to check if we indeed excited SPPs on a metal wire and not free-space radiation. We end this chapter with a short discussion of the potential of using a 1.5 cell RF photogun to generate THz SPPs on a metal wire.

6.2 Theory

In order to generate SPPs propagating on a metal wire, we will launch electron bunches onto a conical tip of the wire. The tip has an opening angle of $2\delta$ and a radius $\rho_w$, see Fig. 6.2. We briefly discuss the general properties of a THz SPP propagating on a metal wire. Next we discuss a simple model describing the SPP generated by a line charge impinging onto a 1D line. We end this section with a discussion of the recently developed detailed theory of the generation of SPPs on a metal cone using electron bunches [34]. This detailed theory will be compared with the measured SPPs.

6.2.1 THz SPP propagation on a metal wire

In Ref. [35] the dispersion and absorption of a THz SPP propagating on a cylindrical metal wire are discussed. In vacuum the electric field, $\mathbf{E}$, of the SPP on a cylindrical wire can be written as

$$\mathbf{E}(r, \omega) = \frac{E^0_\rho(\omega)}{H^1_1(k_\rho \rho_w)} \left( \frac{k_z}{k} H^1_1(k_\rho \rho) e_\rho - i \frac{k_\rho}{k} H^0_1(k_\rho \rho) e_z \right) e^{ik_z z}, \rho \geq \rho_w \quad (6.1)$$

with $E^0_\rho(\omega)$ the spectral amplitude, $H^1_n$ a $n$th-order Hankel function of the first kind, $k = \sqrt{k^2_\rho + k^2_z} = \omega/c$, and $e_z$ a unit vector in $z$-direction. The values of $k_\rho$ and $k_z$ follow...
from the boundary conditions for $E$ on the surface of the metal wire, and the dielectric function of the metal $\varepsilon_w(\omega)$. In Ref. [35] it is shown that $k_\rho \ll k_z \simeq k$ for a copper wire having a radius $\rho_w = 0.5$ mm in the THz frequency range. The THz SPP, therefore, propagates in the $z$-direction with almost the speed of light and little attenuation. The $1/e$ absorption length at 1 THz is 3 m. Close to the metal wire, if $k_\rho \rho < 1$, the expression for $E$ can be simplified by writing $H_1^{(1)}(k_\rho \rho)$ as

$$H_1^{(1)}(k_\rho \rho) \simeq \frac{-2i}{\pi k_\rho \rho}, \hspace{1cm} (6.2)$$

which is a good approximation for $\rho \leq 3$ mm [35]. If we furthermore assume that $k_z = k$ the expression for $E$ near the wire becomes

$$E(r, \omega) = E_0^0(\omega) \frac{\rho_w}{\rho} e^{i\vec{\rho} \cdot \vec{e}_\rho}, \hspace{1cm} (6.3)$$

where we neglected the $z$-component of $E$. So, close to the wire the SPP electric field is, to a good approximation, radially polarized with an electric field amplitude proportional to $1/\rho$.

### 6.2.2 Simple model for line charge launched onto semi-infinite line

To see how we can generate a SPP propagating on a metal wire we will consider a line charge impinging onto a thin wire. To find the electric field of the SPP we will use a description of radiating fields originally developed by Purcell, already sketched in Ch. 1, and also used in Ch. 2 to calculate $45^\circ$ transition radiation (TR). We assume that $\delta \rightarrow 0$, and that $\rho_w$ is small but non-zero. We furthermore assume that an electron, traveling with a speed $v$ in the $z$-direction, reaches the metal wire at $t = 0$, see Fig. 6.3. When the electron is inside the wire, the fields are screened for an observer outside the metal. The screening is not instantaneous, but instead it takes place on a sphere traveling outwards with the speed of light. Within the sphere there is no electric field, outside the sphere the
electric field has to be consistent with the presence of the electron and the induced surface
current and charge densities at the metal wire. Because the metal cone is collapsed into a very small line, the electric field outside the expanding sphere is that of a undisturbed charge traveling with a speed \( v \) in the \( z \)-direction. We can now easily calculate the electric field on the metal wire at the location of the expanding sphere. All electric field lines have to close over the expanding sphere towards the metal wire, where they eventually have to end at the location of the surface charge density. The electric flux on the metal wire, therefore, has to be equal and opposite in sign to the flux of the electron, which is \( -e/\varepsilon_0 \) with \( \varepsilon_0 \) the permittivity of free space and \( e \) the electronic charge. So the electric field on the metal wire \( E(z,t) \) is given by,

\[
E(z,t) = -\frac{e}{2\pi \rho_w \varepsilon_0} \delta(z - ct) e_\rho,
\]

with \( \rho = \sqrt{x^2 + y^2} \), \( e_\rho = \cos \varphi e_x + \sin \varphi e_y \) with \( \varphi \) the azimuthal angle in the x-y plane from the x-axis. The Fourier transform of \( E_\rho(z,t) \) is given by

\[
E_\rho(z,\omega) = -\frac{eN}{(2\pi)^2 \rho_w \varepsilon_0 c} e^{i\omega \tau g} e^{i \frac{\omega}{c} z},
\]

with \( c \) the speed of light. For a line charge impinging on a metal wire this becomes

\[
E_\rho(z,\omega) = -\frac{eN}{(2\pi)^2 \rho_w \varepsilon_0 c} f_l(\omega) e^{i \frac{\omega}{c} z},
\]

where \( f_l(\omega) \) is the form factor of a line charge, defined in Eq. (2.15) of Ch. 2. If we assume a Gaussian electron distribution with a RMS transverse size \( \sigma_t = 0 \) and a RMS longitudinal size \( \sigma_l \), \( f_l(\omega) \) is given by Eq. (2.18).

We find that the spectrum of the SPP excited by a line charge is solely determined by the form factor. In Ch. 2 we explained that the form factor describes the degree of coherence of a certain frequency component. When the electrons emit the radiation with almost no phase difference then \( \omega \tau_g \ll 1 \), with \( \tau_g \) the electron bunch length, so that \( f(\omega) \approx 1 \). On the other hand when \( \omega \tau_g \gg 1 \), the radiation has collected a considerable amount of phase difference during the emission process, resulting in \( f(\omega) \approx 0 \).

### 6.2.3 SPP electric field spectrum for \( \delta \neq 0 \) and \( \sigma_t \neq 0 \)

For cones with an opening \( \delta \neq 0 \) and electron bunches with \( \sigma_t \neq 0 \) the theory is much more complicated, although the principle remains the same. The propagating surface charge density on the metal cone has to compensate for the ‘disappearing’ electric flux, and the coherence is determined by the dimensions of the electron bunch. However, the amount of ‘disappearing’ flux is influenced by the metal cone and the fact that an electron can hit the cone off-axis. Moreover, electrons at different transverse positions enter the metal cone at different moments in time influencing the addition of electric fields.
Figure 6.3: Schematic picture of transition radiation in the case a semi-infinite metal wire. The moment the electron reaches the metal line, which begins in the origin $O$, a sphere of light is emitted into vacuum. On the surface of the sphere the electric field lines change to the new situation. Within the sphere there is no electric field because the charge is totally screened by the metal. Outside the sphere the electric field lines are described by that of an electron traveling in the $z$-direction. The field lines end on the metal wire where they induce a surface charge density. The combination of the electric field on the metal wire, the surface current and surface charge densities constitutes the propagating SPPs.
Smorenburg et al. [34] have developed an analytic theory predicting what THz SPP electric far fields can be obtained by launching electron bunches onto a tapered end of a metal wire. These calculations amount to finding a solution of Maxwell’s equations for the electric field, consistent with the presence of the electron bunch and the appropriate boundary conditions at the metal surface. It is assumed that the metal is an ideal conductor, so that the electric field is perpendicular to the metal surface outside the tip and is zero inside the tip \(^1\). The calculated electric fields on the metal tip have to be related to the electric fields propagating on a metal wire, having a radius \(\rho_w\). This is done by evaluating the electric fields on the tip at \(r = \rho_w / \sin \delta\), with \(r\) the distance from the sharp end of the tip to the tip-to-wire transition, see Fig. 6.2. The theory therefore predicts the SPP electric field amplitude \(E_\rho^0(\omega)\) on the surface of the wire, see Eq. (6.3).

In the detailed theory described in [34] two important assumptions are made; first, electron scattering is neglected. It is assumed that all electrons travel into the metal tip, and remain in the metal indefinitely; second, it is assumed that the SPPs excited on the metal cone adiabatically transform into SPPs which are propagating on the wire.

In [34] it is shown that in the limit of \(\delta \to 0\) the radiated electric field can be divided into two contributions. A part of the field resembles the free-space transition radiation created by a point charge impinging on a metal cone. The other part of the field is dominant near the surface of the tip, and grows to infinity proportional to \(1 / \sin \delta\) on the metal surface. This is because it describes a surface wave collapsed into a 1D line. The electric field of the surface wave can still be recovered, however, because the fields are evaluated at \(r = \rho_w / \sin \delta\) removing the singularity and resulting in Eq. (6.5).

Since we want to compare the theory and measurement later in this chapter we have calculated the electric field spectra corresponding with the experimental situation using the theory in [34]. Note that the theory in [34] assumes a uniformly filled ellipsoidal electron distribution [36], with \(a\) the transverse half axis and \(b\) the longitudinal half axis. If we need to compare \(a\) or \(b\) with RMS bunch dimensions we will take twice the corresponding RMS value. In Fig. 6.4 the spectra are shown for \(\delta = 4.3^\circ\) and \(\gamma = 7\), with \(\gamma^{-1} = \sqrt{1 - \beta^2}\), for several values of \(b/(a + b)\).

We will now briefly discuss the difference between the detailed theory described in [34] and the simple model described in Sec. 6.2.2. The SPP spectral amplitude created by a line charge impinging on a cone, \(b/(a + b) = 1\), is approximately twice that of a SPP being generated by the same line charge impinging onto a thin 1D wire. If the transverse dimensions of the electron bunches are taken into account, we see that the coherence starts to break down at \(\lambda < 2b\). Take, e.g, the curve for \(b/(b + a) = 0.5\). The spectral amplitude is already slightly decreased at wavelengths as low as \(\lambda = 200b\). This in contrast with the line charge, which remains fully coherent until \(\lambda \simeq 2b\). Although all the SPP spectra for which \(a \neq 0\) show this behavior, they all extend up to \(\lambda \simeq 2b\). The sharp dents present in the spectra near \(\lambda = 2b\) are artifacts of a uniformly filled ellipsoidal electron distribution. In reality the spectra will fall of more smoothly.

---

\(^1\)In making this assumption one inherently neglects the existence of SPPs. Nevertheless, for good conductors, the SPPs can be recovered from the idealized field by well-known perturbation techniques [34]
Figure 6.4: Electric field spectra of SPP for $\delta = 4.3^\circ$ and $\gamma = 7$ and different values of $b/(a+b)$. 

Artifact of hard-edged ellipsoids. Falls off more smoothly in reality.
6.3 Experimental setup

Fig. 6.5 shows the setup used for the generation of SPPs using electron bunches, which are created and accelerated in the setup described in chapter 4. The setup shown is positioned behind the dipole magnet, see Fig. 4.7. The aluminum (Al) wire has a radius $\rho_w = 0.75$ mm, and has been tapered into a tip with an opening angle $2\delta = 8^\circ$.

Starting from the tip, the Al wire consist of (1) a straight section 8 cm in length; (2) a 90° circular bend with 33 mm radius of curvature; and (3) a second straight section, connected to the inner conductor of a coaxial feedthrough (see Fig. 6.5). The electron beam can be aimed at the wire by maximizing the charge deposited on the wire, measured using an oscilloscope. Two ZnTe crystals cut in the $<110>$ direction are positioned next to the wire for EO-detection of the THz SPPs. Both crystals make contact with the wire. The 1st crystal is placed at the end of the first straight section, at $\sim$8 cm from the tip. The mirror reflecting the EO-probe beam towards the 1st ZnTe crystal can be replaced with a phosphor screen (not shown in Fig 6.5), allowing imaging of the electron bunch. The setup with the ccd-cameras was used to measure the EO-signal in the 1st ZnTe crystal, see Ch. 5 in Sec. 5.2.2. The delay $\tau$ between the SPPs and the 800 nm probe pulses is varied using two mirrors placed on a motorized stage. Figure 6.6 shows a picture of the 1st ZnTe crystal illuminated by the 800 nm laser light. We briefly discuss the labeled characteristic features in the picture because they leave fingerprints in the measured EO-profile. The edge labeled with (a) is due to blocking of the 800 nm beam by the wire. The edge labeled with
The 1\textsuperscript{st} ZnTe crystal is mounted on a rotatable feedthrough, so that it can be removed from the wire, allowing free passage of the THz SPPs. The 2\textsuperscript{nd} ZnTe crystal can then be used to measure THz SPPs which have propagated around the bend. This allows us to ascertain whether we are really dealing with SPPs. Because we expected less EO-signal behind the bend, we used the setup with the balanced diodes to measure the EO-signal in the second ZnTe crystal, see Ch. 4 in Sec. 4.7.2.
Fig. 6.7 shows measurements of $E_y$ (the y-component of the electric field vector $\mathbf{E}$), as function of $x$ and $y$. The 267 nm laser intensity profile, used for photoemission, had a RMS size in the x-direction of $\sigma^I_x = 0.1$ mm and a RMS size in the y-direction of $\sigma^I_y = 0.15$ mm. The electron bunches contained 160 pC of charge, had a kinetic energy of 3.1 MeV, and a RMS transverse size of $\sigma_x = 0.2$ mm, all measured at $\phi = -40^\circ$.

The wire has its center at $x_w = 0.2$ mm and $y_w = -2.2$ mm. The measured $E_y$ decreases with distance from the wire. Close to the wire the measured $E_y$ is enhanced in two 'ear-like' features. These 'ear-like' features indicate that the incoming SPP profile, see Eq. (6.3), is distorted by the presence of the ZnTe crystal. Since we only have a theory for the undistorted vacuum SPP profile (Eq. (6.3)), we have to find a method to relate the measured distorted SPP profile to the undistorted SPP amplitude $E_0^\rho(\tau)$ on the wire surface.
6.4.1 Microwave studio simulations of SPP distortion by ZnTe crystal

To find out how we can extract $E_\rho^0(\tau)$ from the measurements we have done simulations of the experimental setup using Microwave Studio (MWS). Figure 6.8 shows the geometry used in the simulations. We modeled the ZnTe crystal with a rectangular cuboid having a height and width of 5 mm, and a thickness of 0.5 mm. We used a dielectric constant of $\varepsilon = 10$ for the ZnTe crystal. The wire and the surrounding vacuum chamber are modeled as a uniform coaxial waveguide, having an inner conductor radius of 0.75 mm, and a outer conductor radius of 7 mm. The crystal is placed against the inner conductor. The incident pulse on the ZnTe crystal is the lowest TEM-mode appropriate with the uniform coaxial waveguide, and has a full-width-half-maximum (FWHM) pulse length of 4 ps. The temporal profile of the incident pulse is Gaussian, and in the $x - y$ plane the transverse electric field strength is proportional to $1/\rho$.

Figure 6.9 shows two frames of the simulations at different moments in time. Each frame shows $E_y$ as function of $x$ and $y$ evaluated at $z = 0$, which corresponds to the center.
of the ZnTe crystal. The left-hand frame corresponds with the time when the propagating pulse would reach \( z = 0 \) if the ZnTe crystal were absent. As can be seen the part of \( E_y \) propagating under the wire, \( y < 0 \), is not influenced by the ZnTe crystal. We will use this profile as reference. The SPP electric field inside the crystal is lagging behind due to the lower velocity in the ZnTe crystal. The right-hand frame corresponds with the moment in time when the pulse propagating in the ZnTe crystal reaches \( z = 0 \). We clearly see a distortion of the incoming SPP profile which is caused by the ZnTe crystal. Close to the wire the simulated \( E_y \) is also enhanced in two ‘ear-like’ features, which corresponds very well with the measured profile, shown in Fig. 6.7.

Figure 6.10 illustrates how we can retrieve the undistorted value of the electric field at the wire. Figure 6.10 shows \(-t_{ZnTe}E_y(-y)\) simulated at 0 ps, with \( t_{ZnTe} \) a transmission coefficient, \( E_y(y) \) simulated at 2 ps, and a measured \( E_y(y) \) profile taken from Fig. 6.7. All profiles are evaluated at \( x = x_w \). Here \( t_{ZnTe} = 0.48 \), and is calculated for a frequency component of 1 THz normal incident onto a ZnTe crystal, using the Fresnel equations. From Fig. 6.10 it is clear that close to the wire both the simulated and measured profiles in the ZnTe crystal are not proportional to \( 1/y \). The part of the SPP traveling under the wire (\( E_y(y) \) at 0 ps), however, is proportional to \( 1/y \) since this is the appropriate behavior of the lowest TEM-mode in a uniform coaxial waveguide. We therefore see that at large enough distance from the wire, \( y - y_w \geq 0.5 \) mm, the electric field in the crystal resembles the incoming profile, corrected by the transmission coefficient \( t_{ZnTe} \). So, to retrieve \( E_0^\rho(\tau) \) from the measurement we evaluate the measured \( E_y \) directly above the wire at a distance of \( 2\rho_w \), and multiply the value by \( 2/t_{ZnTe} \).
Figure 6.10: Plot of $E_y$ versus $y$ simulated inside and outside the ZnTe crystal, together with a measured $E_y$ versus $y$. 

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6.4.2 Undistorted SPP amplitude on the wire surface

Fig. 6.11 shows the measured undistorted SPP amplitude on the wire surface $E^0_\rho(\tau)$. We varied the electron bunch length by measuring at two different values of the injection phase of the RF photogun, being $\phi = -40^\circ$ (black line) and $\phi = -60^\circ$ (red line). The solid lines are generated by removing frequency components above 0.25 THz from the measurement. The difference in arrival time is caused by the difference in the injection phase, see Fig. 4.15. In Ch. 5 we explained that decreasing $\phi$ leads to shorter electron bunches and thus shorter CTR pulses. For the measured SPPs this is not the case because its spectrum is also strongly dependent on the transverse dimensions of the electron bunches, which slightly increases when $\phi$ is decreased. Note that the main peak in the measured pulse at $\phi = -60^\circ$ has larger gradients than the main peak measured at $\phi = -40^\circ$, indicating the presence of higher frequency components.

Figure 6.12 shows the spectrum of $E^0_\rho(\omega)$ for $\phi = -40^\circ$ (black line) and $\phi = -60^\circ$ (red line). We used a discrete Fourier transformation (DFT) to calculate $E^0_\rho(\omega)$ from $E^0_\rho(\tau)$. The two measured spectra are almost identical. The differences between them are smaller than the estimated error in the measurements (see error bars). However, the trend between the two measured spectra agrees with the theoretically calculated spectra shown in Fig. 6.4. Because the electron bunch length $2b/\beta c$ is shorter for $\phi = -60^\circ$ (red line) than for
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\[ \text{measured } E_0^\rho \text{ for } \phi = -40^\circ \]
\[ \text{measured } E_0^\rho \text{ for } \phi = -60^\circ \]
\[ \text{calculated } E_0^\rho /5 \]

Figure 6.12: SPP spectra measured for \( \phi = -40^\circ \) (black line) and \( \phi = -60^\circ \) (red line). Also shown is the calculated SPP spectrum for the bunch specifications measured at \( \phi = -40^\circ \) (dashed line).

For \( \phi = -40^\circ \) (black line), we expect the spectrum for \( \phi = -60^\circ \) (red line) to extend up to higher frequencies. But, since both the number of electrons \( N \), and \( b/(a + b) \) decrease with decreasing \( \phi \), the spectral amplitude for \( \phi = -60^\circ \) (red line) drops below that for \( \phi = -40^\circ \) (black line) for low frequency components. To make any solid conclusions on this, however, we should check whether this trend continues at lower frequencies by measuring the SPPs in a larger time window.

### 6.4.3 Comparison with and theory

Besides the measured spectra, Fig. 6.12 also shows a spectrum calculated using the theory described in [34] (dashed line). As input for the theory we used the electron bunch specifications measured at \( \phi = -40^\circ \), which are mentioned at the beginning of Sect. 6.4. For the electron bunch length we use \( b = 2\sigma_l = 1.6 \) mm, which is based on GPT simulations. We have divided the calculated spectrum by a factor of 5 to make it overlap with the measured spectra. We see that the measured and calculated spectra both become incoherent at \( \sim 0.1 \) THz. We therefore conclude that the highest frequency component present in the SPP spectrum is indeed determined by the electron bunch length. However, the ampli-
chapter

6.4.4 Discussion

We will now briefly discuss some possible causes for the discrepancy between the measured and calculated spectral amplitudes.

A possible cause for the discrepancy between the theory and the measurements is the precise shape of the conical tip and the transition of the tip into a straight wire. The SPPs excited on the tip will not adiabatically transform into SPPs propagating on the wire, which is assumed in the theory. In reality a part of the SPP will be radiated into vacuum, although it is doubtful that this can account for a factor of 5. The radiative losses, however, will depend on the wavelength, since small wavelengths will radiate relatively more energy than large wavelengths. So it will certainly influence the shape of the spectrum. The amount of losses is difficult to estimate from a priori arguments. However, it could be modeled with MWS.

The theory also neglects electron scattering. It is assumed that all electrons travel into the metal tip, and remain in the metal indefinitely. In reality the electrons will scatter on the atomic nuclei and electrons in the Al tip. The mean free path of a relativistic electron in a metal is on the order of a few microns. If the electron bunches were launched into a large piece of Al, they would penetrate \( \sim 1 \) cm deep, and scatter into a transverse spot size of \( \sim 1 \) cm. Since the opening angle of the used tip is only \( 2\delta = 8^\circ \), it is very likely that a considerable number of electrons will scatter out of the metal tip. This is confirmed by the charge measured via the coaxial feedthrough, see Fig. 6.5. For an electron bunch containing \(-160\) pC of charge launched onto the wire, we typically measure a charge of \(+20\) pC. So, we knock more electrons off the wire than we actually launch onto it.

However, in the measurements we observed that the largest frequency component present in the SPP spectrum is determined by the longitudinal electron bunch. So apparently electron scattering influences the SPP generation only during the initial impact of the electron bunch into the tip. Any electrons scattering out of the tip at later times will give rise to an incoherent tail. But, since the electrons hardly loose any energy in the first part of the metal tip, they can easily scatter out of the metal before the entire electron bunch has traveled into the tip. Each electron leaving the metal tip during the initial impact will emit a radiation pulse, which is oppositely polarized to the pulse it emitted on entrance, and approximately in phase with the generated SPP. The net amount of charge creating the SPP can therefore be much less than the total charge of the electron bunch. A detailed study of the electron scattering is outside the scope of this thesis. It is possible, however, to model the multiple scattering of relativistic electrons in a solid using the theory described in [37–39].
6.5 SPPs after the bend

Figure 6.13 shows the SPP electric field measured inside the ZnTe crystal placed after the bend in the wire, see Fig. 6.5. The specifications of the electron bunches used to generate the SPPs are listed in the beginning of Sect. 6.4. The measurements were performed using an injection phase of $\phi = -40^\circ$. We used two balanced diodes to measure the EO-signal, and thus the electric field strength of the SPPs at a single position. The RMS intensity spot size of the probe beam $\sigma_{\text{probe}} \sim 50 \, \mu\text{m}$. The distance between the probe spot and the center of the wire, $\Delta \rho$, is estimated to be $\Delta \rho = 1.75 \pm 0.75 \, \text{mm}$.

Because the excitation generated by the electrons propagates around the bend, the measurement confirms that we indeed generated and measured SPPs instead of free space radiation.

We see that the SPP is attenuated and dispersed by traveling around the bend. Before the bend, see black line in Fig. 6.11, the main peak has a FWHM length of $\sim 6 \, \text{ps}$ and a peak electric field of $\sim 0.5 \, \text{MV/m}$. The peak electric field of the SPP after the bend is estimated to be $\sim 0.02 \Delta \rho/(\rho_\text{w} t_{\text{ZnTe}}) = 0.1 \pm 0.05 \, \text{MV/m}$, and the FWHM of the negative peak is $\sim 22 \, \text{ps}$. So by traveling around the bend the SPP electric field strength is...
attenuated by a factor $6 \pm 3$, and dispersed to a 4 times longer pulse length. The dispersion also causes a small overshoot of the electric field.

### 6.5.1 Microwave studio simulations of SPP propagation around bend

To see if we can model the attenuation and dispersion of the SPP caused by the bend in the wire, we performed simulations with MWS. The simulation geometry is shown in Fig. 6.14. The SPP enters from port 1, which is assumed to be connected with a uniform coaxial waveguide having the same cross-section. The incident pulse is the lowest TEM-mode appropriate for the uniform coaxial waveguide, and has a FHWM pulse length of 4 ps. Port 2 is also assumed to be connected with a uniform coaxial waveguide of equal cross-section. Port 3 is assumed to be open. It can be used to evaluate the fields that have radiated from the wire. Except the ports, all boundaries are assumed to be perfect conductors. The two probes indicate the position where we can evaluate the simulated electric field as function of simulated time. Both probes are located beneath the wire at the same distance from the wire.

In reality, the (metal) walls of the surrounding vacuum chamber are at a larger distance from the wire. However, increasing the volume of the geometry would lead to computational problems which makes it impossible to perform the simulations on a single PC. We,
Figure 6.15: Electric field evaluated at $z = 0$ for several simulated moments. The coordinate system used in the simulations is shown in the lower right-hand corner. In the first two frames, 20 ps and 110 ps, we have evaluated the absolute value of the simulated electric field. All other frames show the y-component of the electric field. The dashed circle indicates the influence of the sharp edge in the metal surrounding the wire on the simulated electric field.
therefore, have to account for the influence of the metal walls on the simulated results. In Fig. 6.15 the simulated electric field is shown, evaluated at \( z = 0 \), for several moments in time. The coordinate system used in the simulations is shown in the lower right-hand corner of Fig. 6.15. In the first two frames, 20 ps and 110 ps, we have evaluated the absolute value of the simulated electric field. All other frames show the y-component of the electric field. The initial pulse enters the structure at 20 ps. At 110 ps the pulse has traveled partly around the bend. The part of the pulse traveling around the outside of the bend is attenuated much less than the the part traveling along the inside of the bend. So far the ‘outer’ pulse is hardly affected by the metal surrounding. It is, however, about to reach a sharp feature in the metal surrounding. As can be seen in the next frames, the sharp feature introduces a significant disturbance, which we have marked with a dashed circle in Fig. 6.15. The part of the initial pulse remaining after the bend in the wire reaches the position of probe 2 at \( \sim 160 \) ps. The disturbance caused by the sharp feature reaches probe 2 at \( \sim 176 \) ps.

Figure 6.16 shows the simulated electric fields evaluated in the two probes. The left-hand side shows the electric field evaluated at the position of probe 1, which corresponds with the initial pulse. The right-hand side shows the pulse evaluated at probe 2, which corresponds with the SPP after it has propagated around the bend in the wire. The dashed
line indicates the moment in time when the disturbance, created by the sharp feature in the metal surrounding, reaches the position of probe 2. If we neglect the electric field at probe 2 after the dashed line, we see that the incoming pulse is attenuated by a factor of $\sim 5$, and dispersed to a FWHM pulse length of $\sim 20$ ps, by traveling around the bend in the wire. This agrees with the experimental observations.

### 6.6 Generation of THz SPP using a 1.5 Cell RF photogun

The measurements shown in this chapter clearly demonstrate that strong THz SPPs can be generated on a metal wire using electron bunches. We can make an estimate of the potential of the 1.5 cell RF photogun as a source of THz SPPs on a metal wire, just as we did in Ch. 2. We could repeat the experiment described in this chapter, but instead place the wire directly behind the RF photogun. Because the transverse spot size $\sigma_t$ is important for the coherence of the SPPs, we would require an additional focusing element directly behind the RF-photogun. If one would place a additional solenoid directly behind the RF-photogun to focus the electrons transversely, it should be possible to obtain electron bunches containing 100 pC of charge, with a kinetic energy of 3 MeV, a RMS bunch length $\sigma_t \simeq 1$ ps, and a RMS transverse size $\sigma_t \simeq \sigma_l/4$. It is not directly clear how the radiative losses at the tip-to-wire transition and the influence of the electron scattering in the metal scale with electron bunch length. Extrapolating from the measurements, however, it is safe to conclude that such electron bunches launched onto a metal tip lead to the generation of picosecond SPPs of $\sim 1$ MV/m on a metal wire having a thickness of $\sim 1$ mm. If such SPPs would be focussed below the wavelength, by tapering the metal wire into a tip, electric field strengths in excess of $\sim 100$ MV/m become possible.
Chapter 6.

Bibliography

Direct generation of terahertz surface plasmon polaritons on a wire using electron bunches


Appendix A

A-1 Ginzburg-Frank equations

We derive the TR fields, created by an electron moving with a uniform velocity through the interface between two dielectrics. The problem is to find a homogeneous solution of Maxwell’s equations, such that the particular and homogeneous solution together fulfill the boundary conditions at the interface. We can identify the particular solution with the field created by the moving electron, also known as the coulomb field, or the velocity field. The homogeneous solution is the emitted transition radiation. We start by solving the inhomogeneous wave equation for a non-magnetic medium

\[ \nabla^2 \mathbf{E}^p - \varepsilon \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}^p}{\partial t^2} = \frac{1}{\varepsilon \varepsilon_0} \nabla \varrho + \mu_0 \frac{\partial \mathbf{J}}{\partial t}, \]  

(A-1)

where \( \mathbf{E}^p \) is the particular solution, \( \mu_0 \) and \( \varepsilon_0 \) are the permeability and permittivity of free space respectively, and \( \varepsilon \) is the dielectric constant. The charge density \( \varrho \) and current density \( \mathbf{J} \) of the electron are

\[ \varrho = -e \delta(\mathbf{r} - \mathbf{r}_j - \mathbf{v}t), \]  

(A-2)

\[ \mathbf{J} = \mathbf{v} \varrho, \]  

(A-3)

where \( e \) is the electronic charge, \( \mathbf{r}_j \) is the position of the electron at \( t = 0 \) and \( \mathbf{v} = ve \) is the velocity of the electron. The interface between the two dielectrics is located at \( z = 0 \), for \( z < 0 \) the dielectric constant equals \( \varepsilon^- \), for \( z > 0 \) the dielectric constant equals \( \varepsilon^+ \). A sketch of the situation is shown in Fig. 2.1 for illustration.

In order to find the solution of Eq. (A-1), we will expand all quantities in Fourier series of time \( t \) and transverse position \( (x \text{ and } y) \). Because the boundary conditions have to be applied at \( z = 0 \), we will not use a Fourier expansion of longitudinal position \( z \). For a scalar \( X(\mathbf{r}, t) \) we thus write

\[ X(\mathbf{r}, t) = \int X(k_\rho, z, \omega) e^{ik_\rho \cdot \mathbf{r} - i\omega t} dk_\rho dk_\rho dk_\rho d\omega, \]  

(A-4)

(A-5)
where \( \mathbf{k}_p \equiv k_x \mathbf{e}_x + k_y \mathbf{e}_y \). The Fourier transforms of \( \varrho \) and \( \mathbf{J} \) can be found by applying the inverse Fourier transform to Eqs. (A-2) and (A-3), resulting in

\[
\varrho(\mathbf{k}_p, z, \omega) = \frac{e^{-i \mathbf{k}_p \cdot \mathbf{r}_j - i \frac{\pi}{2} z_j + i \frac{\pi}{2} z}}{(2\pi)^3 \nu},
\]

(A-6)

\[
J_z(\mathbf{k}_p, z, \omega) = \nu \varrho(\mathbf{k}_p, z, \omega).
\]

(A-7)

If we decompose Eq. (A-1) in its Fourier components and substitute Eqs. (A-6) and (A-7), in combination with the ansatz

\[
E^p(\mathbf{k}_p, z, \omega) = E^p(\mathbf{k}_p, \omega) e^{i \frac{\pi}{2} z},
\]

(A-8)

into it we obtain the following particular solution

\[
E^p(\mathbf{k}_p, \omega) = \frac{-ie(\mathbf{k}_p + (\beta^{-2} - \epsilon) \beta \mathbf{e}_z)}{(2\pi)^3 \varepsilon_0 \varepsilon \beta (k_p^2 + (\beta^{-2} - \epsilon) \beta^2)} e^{-i \mathbf{k}_p \cdot \mathbf{r}_j - i \frac{\pi}{2} z_j},
\]

(A-9)

where \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \) and \( \beta = \frac{v}{c} \). The homogeneous solution, \( \mathbf{E}^h \), can be found by taking \( \varrho = 0 \) and \( \mathbf{J} = 0 \) resulting in

\[
\mathbf{E}^h(\mathbf{k}_p, z, \omega) = \mathbf{E}^h(\mathbf{k}_p, \omega) e^{i \frac{\pi}{2} z} \quad \text{if} \quad z \leq 0.
\]

(A-10)

where \( k_p^\pm = \sqrt{\varepsilon \mu - k_p^2} \). The sign of \( k_p^\pm \) is chosen such that the transition radiation travels away from the interface in each half-space. The homogeneous solution needs to be divergence free in order to fulfill Maxwell’s equations. Thus an extra requirement for the homogeneous solution is

\[
k_p^\pm \mathbf{E}^h(\mathbf{k}_p, \omega) = \pm k_p^\pm \mathbf{E}^h(\mathbf{k}_p, \omega) \quad \text{if} \quad z \leq 0.
\]

(A-11)

The boundary conditions for the Fourier components at the interface can be written as

\[
E^p_0^+ + E^h_0^+ = E^p_0^- + E^h_0^-,
\]

(A-12)

\[
\varepsilon^+(E^p_0^+ + E^h_0^+) = \varepsilon^-(E^p_0^- + E^h_0^-),
\]

(A-13)

where the minus sign indicates the relevant Fourier component for \( z < 0 \), and the plus sign for \( z > 0 \). If Eq. (A-9) is substituted into Eqs. (A-11)-(A-13), the homogeneous solution can be found:

\[
E^h_0^+(\mathbf{k}_p, \omega) = \frac{-ie \mathbf{k}_p k_p^+ e^{-i \mathbf{k}_p \cdot \mathbf{r}_j - i \frac{\pi}{2} z_j}}{(2\pi)^3 \varepsilon_0 \varepsilon \beta (k_p^2 + (\beta^{-2} - \epsilon)^2)} \left( \frac{\varepsilon k_p^+ \beta + 1}{k_p^2 + (\beta^{-2} - \epsilon)^2 \beta^2} - \frac{\varepsilon k_p^+ \beta + \varepsilon^+}{k_p^2 + (\beta^{-2} - \epsilon)^2 \beta^2} \right),
\]

(A-14)

\[
E^h_0^-(\mathbf{k}_p, \omega) = \frac{-i e \mathbf{k}_p k_p^- e^{-i \mathbf{k}_p \cdot \mathbf{r}_j - i \frac{\pi}{2} z_j}}{(2\pi)^3 \varepsilon_0 \varepsilon \beta (k_p^2 + (\beta^{-2} - \epsilon)^2)} \left( \frac{\varepsilon k_p^- \beta - 1}{k_p^2 + (\beta^{-2} - \epsilon^+)^2 \beta^2} + \frac{\varepsilon^+ - \varepsilon k_p^- \beta}{k_p^2 + (\beta^{-2} - \epsilon)^2 \beta^2} \right),
\]

(A-16)

\[
E^h_0^- = \frac{k_p}{k_p^0} E^h_0^-(\mathbf{k}_p, \omega).
\]

(A-17)
A-2 Spectrum of electric field

To obtain the spectrum of the electric field we have to evaluate the inverse Fourier transformation over the transverse wave numbers \((k_x, k_y)\). If we assume that the electron passes the interface in the origin \(r_j = 0\), we can use the cylindrical symmetry allowing the following substitution:

\[
\begin{pmatrix}
E_x^h(k_\rho, z, \omega) \\
E_y^h(k_\rho, z, \omega)
\end{pmatrix} = \begin{pmatrix}
\cos k_\phi \\
\sin k_\phi
\end{pmatrix} E_{k_\rho}^h(k_\rho, z, \omega),
\]

(A-18)

for the transverse radiation fields. To investigate the consequences of this on the Fourier transform, we will substitute a Fourier component that is separable in polar coordinates

\[
F(k_\rho) = F_{k_\rho}(k_\rho) e^{ik_\phi r}.
\]

Equation (A-19) can then be written as

\[
F(\rho) = \sum_{n=-\infty}^{\infty} i^n e^{-in\phi} \int_0^\infty k_\rho F_{k_\rho}(k_\rho) J_n(k_\rho \rho) dk_\rho \int_0^{2\pi} F_{k_\phi}(k_\phi) e^{in\phi} dk_\phi,
\]

(A-20)

which is an infinite sum of weighted Hankel transforms. To see what the consequence of this is on our situation we have to calculate

\[
\int_0^{2\pi} \cos k_\phi e^{in\phi} dk_\phi = \pi \delta(n + 1) + \pi \delta(n - 1),
\]

(A-21)

and

\[
\int_0^{2\pi} \sin k_\phi e^{in\phi} dk_\phi = \pi i \delta(n + 1) - \pi i \delta(n - 1),
\]

(A-22)

in which \(\delta(n)\) is Dirac’s Delta function. We can now use Eqs. (A-18)-(A-22) to calculate

\[
\begin{pmatrix}
E_x^h(r, \omega) \\
E_y^h(r, \omega)
\end{pmatrix} = \int \int \begin{pmatrix}
\cos k_\phi \\
\sin k_\phi
\end{pmatrix} E_{k_\rho}^h(k_\rho, z, \omega) e^{ik_\phi r} dk_\rho dk_y = 2\pi i \int_0^\infty k_\rho E_{k_\rho}^h(k_\rho, z, \omega) J_1(k_\rho \rho) dk_\rho,
\]

(A-24)

which immediately leads to

\[
E_x^h(r, \omega) = 2\pi i \int_0^\infty k_\rho E_{k_\rho}^h(k_\rho, z, \omega) J_1(k_\rho \rho) dk_\rho.
\]

(A-25)
A-2.1 Evaluation of transverse Fourier transformation

We will evaluate the integral in Eq. (A-25) under the assumption \( r \to \infty \), with \( r = \sqrt{\rho^2 + z^2} \), or more specifically \( \mathbf{r} \cdot \mathbf{k} \gg 1 \). Note that we require both \( k_{\rho} \rho \gg 1 \) and \( k_{z} z \gg 1 \) independently, which means we have to be careful at \( \rho = 0 \) or \( z = 0 \). However, no radiation is emitted when \( \rho = 0 \) or \( z = 0 \) in the case of a dielectric-vacuum interface. The resulting expression will scale with \( 1/r \), since this will be the largest term under the assumption \( r \to \infty \). So, besides making the evaluation of the integral possible, it also ensures that the result is the radiative part of the homogeneous solution.

We start with replacing \( J_1 \) with its asymptotic expansion [2]

\[
J_1(\rho k_\rho) = \frac{1}{\sqrt{2\pi \rho k_\rho}} \left( e^{i\rho k_\rho - i\frac{3}{4}\pi} + e^{-i\rho k_\rho + i\frac{3}{4}\pi} \right) + O(|\rho k_\rho|^{-1}),
\]

(A-26)

which is valid because we assume \( \mathbf{r} \cdot \mathbf{k} \gg 1 \) (or in this case \( \rho k_\rho \gg 1 \)). If we neglect the higher orders in Eq. (A-26) and substitute it, together with \( k_z = \sqrt{\omega^2 - k_{\rho}^2} \), into Eq. (A-25) we obtain

\[
E_{\rho}^h(\mathbf{r}, \omega) = \sqrt{2\pi i} \int_0^\infty \frac{\mathbb{E}_p^h(k_\rho, \omega)}{\rho k_\rho} e^{i\pi (\frac{k_\rho + z}{\sqrt{\rho^2 + z^2}} - k_{\rho}^2)} d\rho + \frac{\mathbb{E}_p^h(k_\rho, \omega)}{\rho k_\rho} e^{-i\pi (\frac{k_\rho + z}{\sqrt{\rho^2 + z^2}} + k_{\rho}^2)} d\rho.
\]

(A-27)

Because we assume \( r \to \infty \), we are able to evaluate the two integrals in Eq. (A-27) with the method of stationary phase, see Ref [3]. The method allows us to evaluate the integrands at their critical points. The critical points of the second kind of both integrals, the begin and end points of the interval of integration, give no contribution. This is because both integrands are zero at those points. The critical point of the first kind, of the second integral \( (k_\rho = -\frac{\omega}{c} \frac{\rho}{r}) \), gives no contribution either. This is because it is not included in the integration interval. The only contribution comes from the critical point of the first kind, of the first integral \( (k_\rho = \frac{\omega}{c} \frac{z}{r} = \frac{\omega}{c} \sin \theta) \), and is given by

\[
E_{\rho}^h(\mathbf{r}, \omega) = -2\pi i \frac{\omega}{c} \frac{z}{r^2} \mathbb{E}_p^h(\frac{\omega}{c} \frac{\rho}{r}, \omega) e^{i\pi z r}.
\]

(A-28)

So far we have only calculated \( E_{\rho}^h(\mathbf{r}, \omega) \), but the electric field of the transition radiation also contains a component in the \( z \)-direction. We furthermore know that the total electric field is directed in the \( \mathbf{e}_\theta \) direction, which is by definition the direction perpendicular to the wave vector \( \mathbf{k} \). For a illustration of this, see Fig. A-1. We can thus write \( E_{\theta} \mathbf{e}_\theta = E_{\theta} \cos \theta \mathbf{e}_\rho - E_{\theta} \sin \theta \mathbf{e}_z = E_{\rho} \mathbf{e}_\rho + E_{z} \mathbf{e}_z \), which means

\[
E_{\theta}^h(\mathbf{r}, \omega) = \frac{E_{\rho}^h(\mathbf{r}, \omega)}{\cos \theta} = -\frac{E_{z}^h(\mathbf{r}, \omega)}{\sin \theta},
\]

(A-29)

and since \( \cos \theta = \frac{z}{r} \),

\[
E_{\theta}^h(\mathbf{r}, \omega) = -\frac{2\pi i}{r} \frac{\omega}{c} \mathbb{E}_p^h(\frac{\omega}{c} \frac{\rho}{r}, \omega) e^{i\pi z r}.
\]

(A-30)
A-2.2 Power spectrum

The total energy $W$ radiated into the vacuum can be calculated by integrating the Poynting vector over a surface in the far field, and over time

$$W = \frac{1}{\mu_0} \int \int \mathbf{E}^h(r, t) \times \mathbf{B}^h(r, t) \cdot d\mathbf{A} dt,$$  \hspace{1cm} (A-31)

where $d\mathbf{A}$ designates a surface element in the far field. If we substitute the Fourier expansions of $\mathbf{E}^h(r, t)$ and $\mathbf{B}^h(r, t)$ into Eq. (A-31), and use the fact that only Fourier components of equal frequency give a contribution to the integration over time, we obtain

$$W = \frac{2\pi}{\mu_0} \int \int \mathbf{E}^h(r, \omega) \times \mathbf{B}^{h*}(r, \omega) \cdot d\mathbf{A} d\omega.$$  \hspace{1cm} (A-32)

If we furthermore use $\mathbf{E}^h(r, \omega) \times \mathbf{B}^{h*}(r, \omega) = \sqrt{\varepsilon_0 \mu_0} |\mathbf{E}_h^\theta(r, \omega)|^2 \mathbf{e}_r$, with $\mathbf{e}_r$ the unit vector in spherical radial direction, this can be written as

$$W = 4\pi \sqrt{\varepsilon_0 \mu_0} \int_0^\infty \int |\mathbf{E}_h^\theta(r, \omega)|^2 r^2 d\Omega d\omega,$$  \hspace{1cm} (A-33)

with $d\Omega$ unit solid angle. Note that we have used the symmetry of the integrand to change the integration interval. The energy radiated per unit frequency and unit solid angle into the vacuum can thus be written as

$$\frac{\partial^2 W}{\partial \omega \partial \Omega} = 4\pi r^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} |\mathbf{E}_h^\theta(r, \omega)|^2 = 16\pi^3 \sqrt{\frac{\varepsilon_0 \omega^2}{\mu_0 c^2}} |\mathbf{E}_{k_r}^h(\frac{\omega}{c r}, \omega)|^2.$$  \hspace{1cm} (A-34)
Bibliography


Appendix B

B-1 Form factor for ellipsoidal electron density

We start with

\[ \int h(\sqrt{x^2 + y^2 + \alpha^2 z^2}) e^{-ik_\rho \rho - i \frac{\omega}{c} z} dx dy dz. \]  

(B-1)

If we substitute \( x = R \cos \varphi \sin \psi, \ y = R \sin \varphi \sin \psi \) and \( z = \frac{R}{\alpha} \cos \psi \) for the spatial coordinates, and \( k_x = k_\rho \cos k_\varphi \) and \( k_y = k_\rho \sin k_\varphi \) for the transverse wave numbers in Eq. (B-1) we obtain

\[ \frac{1}{\alpha} \int_0^\infty \int_0^{2\pi} \int_0^\pi h(R)e^{-ik_\rho R \sin \psi \cos(k_\varphi - \varphi) - i \frac{\omega}{c} R \cos \psi R^2 \sin \psi} dR d\varphi d\psi. \]  

(B-2)

This can be written as

\[ \frac{2\pi}{\alpha} \int_0^\infty \int_0^\pi h(R)J_0(k_\rho R \sin \psi)e^{-i \frac{\omega}{c} R \cos \psi R^2 \sin \psi} dR d\psi, \]  

(B-3)

in which \( J_0 \) is the zeroth order Bessel function of the first kind [1]. Next we substitute \( \xi = \cos \psi \) and use the fact that resulting integrand is even, to change the interval of integration [2]

\[ \frac{4\pi}{\alpha} \int_0^\infty \int_0^{1} h(R)J_0(k_\rho R \sqrt{1 - \xi^2}) \cos(\frac{\omega R}{c} \xi) R^2 dR d\xi = \]  

\[ \frac{4\pi}{\alpha} \int_0^\infty h(R) \sqrt{\frac{2}{\pi}} \frac{J_1(R \sqrt{k_\rho^2 + \omega^2/c^2})}{(R \sqrt{k_\rho^2 + \omega^2/c^2})^2} R^2 dR = \]  

\[ \frac{4\pi}{\alpha} \int_0^\infty h(R) \sin(R \sqrt{k_\rho^2 + \omega^2/c^2}) \sqrt{k_\rho^2 + \omega^2/c^2} R dR, \]  

(B-4)

where we used \( \sqrt{\frac{\pi}{2}} x^{-\frac{1}{2}} J_{\frac{1}{2}}(x) = x^{-1} \sin x \) (see Ref. [3]) in the last step. If we substitute \( k_\rho = \sin \theta c \) into Eq. (B-4) we obtain

\[ \frac{4\pi}{\alpha} \int_0^\infty h(R) \sin(R \Theta \omega/c) \frac{R}{\Theta \omega/c} dR, \]  

(B-5)

with \( \Theta = (\alpha \beta)^{-1} \sqrt{\alpha^2 \beta^2 \sin^2 \theta + 1} \).
Bibliography


Appendix C

C-1 Derivation of Eq. (4.2) and discussion of measured emittance

If the trace-space \(x-x' (x' = p_x/p_z)\) volume of an electron bunch is a uniformly filled ellipsoid its transport through an linear optical system can be described using Twiss parameters [1]. This theory can be applied if the particle motions are paraxial and all particles have the same axial velocity. Furthermore it is assumed that space-charge forces are negligible and that transverse focusing forces vary linearly with displacement from the main axis, independent of the transverse velocity.

Because the equidensity contours of a Gaussian trace-space distribution form ellipsoids, it is possible to use the same Twiss parameters to calculate the transport of a Gaussian distribution through a linear optical system. Instead of using the half-width of an ellipsoid one can use the RMS values of the Gaussian distribution in \(x\) and \(x'\) direction. This is what we did for the system depicted in Fig. C-1. We assumed that the beam is non-skewed in trace-space at a distance \(l_1\) before the lens with \(\sigma_v\) the RMS size in \(x\)-direction. If we then use the Twiss parameters to calculate the RMS size \(\sigma_x\) at a position of \(l_2\) after the lens, Eq. 4.2 is the result.

To determine the influence of experimental errors on the measured emittance, \(\sigma_x\) is expanded in a power series around the smallest value \(\sigma_0\) in terms of the relative change in the focal strength \(\Delta f_x = f - f_0\), with \(f_0\) the focal strength at \(\sigma_x = \sigma_0\). If \(\Delta f_x / f_0 \ll 1\) then \(\sigma_x\) can be written as

\[
\sigma_x = \sigma_0 + \frac{\epsilon_x^2 l_2^2}{2 \sigma_0^2 f_0^2} (\frac{\Delta f_x^2}{f_0^2} - 2 \frac{\Delta f_x^3}{f_0^3}).
\]  

(C-1)

The relative change \(\Delta f_x / f_0\) in focal strength is known very accurately since it is equal to the relative change in current through the quadrupole. The uncertainty is therefore mainly determined by the pre-factor.

The value of \(\sigma_0\) is measured during the quadrupole scan and is therefore known accurately. For the value of \(l_2\) we took the distance from the middle of the quadrupole to the phosphor screen, which is a good approximation for the quadrupole we used [2]. The remaining parameter \(f_0\) is based on field map measurements in combination with a particle tracking simulation using TRANSPORT [3]. Since the field map has been measured accurately the value of \(f_0\) is known with a few percent accuracy, mainly limited by the absolute error in the used current supply.
A large contribution to the error of the measured emittance comes from the deviation of the actual trace-space distribution from a Gaussian distribution. To estimate this we determined the values of $\sigma_x$ shown in Fig. 4.12 using a different method. We took the pixel row containing the most energy and subtracted the background signal from it. The row was subsequently truncated such that it contained 95% of the integral in the entire row. Finally the standard deviation of the truncated row was determined. To determine $\epsilon$ the measured $\sigma_x$ versus $f_x$ data points were fitted with Eq. 4.2, resulting in a $\epsilon_x^n$ of 0.9 mm mrad. Based on this we estimate an error of approximately 10% in the measured emittances both at high and low charge.
Bibliography


**Appendix D**

### D-1 Calculation of focus of CTR pulses created by ellipsoidal mirror

In order to compare the measured free-space CTR (Ch. 5) with theory (Ch. 2), we need to calculate the field distribution of the focused CTR, created by the ellipsoidal mirror. For this purpose we use the physical theory of diffraction (PTD) [1], which assumes that the surface current density $\mathbf{K}$ on the metal surface is given by,

$$\mathbf{K} = 2\mu_0^{-1} \mathbf{n} \times \mathbf{B}_{\text{in}}, \quad (D-1)$$

where $\mathbf{B}_{\text{in}}$ is the incident magnetic field, $\mu_0$ is the permeability of free space, and $\mathbf{n}$ denotes the outward normal vector at the metal surface, see Fig. D-1. In addition we assume that the surface charge density $\sigma$ is given by,

$$\sigma = 2\varepsilon_0 \mathbf{n} \cdot \mathbf{E}_{\text{in}}, \quad (D-2)$$

with $\mathbf{E}_{\text{in}}$ the incident electric field, and $\varepsilon_0$ the permittivity of free space. Using $\sigma$ and $\mathbf{K}$ we calculate the scattered fields via the vector potential $\mathbf{A}$,

$$\mathbf{A}(\mathbf{r}, t) = \int_S \frac{\mathbf{K}(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})}{4\pi\mu_0^{-1} |\mathbf{r} - \mathbf{r}'|} d^2 \mathbf{r}', \quad (D-3)$$

![Figure D-1: Geometry used in calculation of CTR focus.](image)
and the scalar potential $\phi$, 

$$
\phi(r, t) = \int_S \frac{\sigma(r', t - \frac{|r-r'|}{c})}{4\pi\varepsilon_0 |r-r'|} d^2r',
$$

(D-4)

where $S$ is the surface of the ellipsoidal mirror. The scattered electric field $E$ is given by 

$$
E(r, t) = -\nabla \phi(r, t) - \frac{\partial A(r, t)}{\partial t},
$$

(D-5)

and the scattered electric field $B$ by 

$$
B(r, t) = \nabla \times A(r, t).
$$

(D-6)

## D-2 Normal vector and surface area on ellipsoidal mirror

We evaluate Eqs. D-3 and D-4 assuming that the CTR is emitted at one of the foci of the ellipsoidal mirror, which we will call $F_1$. We define a coordinate system $(x, y, z)$ with the origin located in $F_1$, as illustrated in Fig. D-1. We need to find an expression for $n$ and $d^2r$ on the surface of the ellipsoidal mirror. In addition, we need an expression for the distance from a point on the ellipsoidal mirror to the origin, which we call $r_e$. The surface of the mirror is described by

$$
\frac{x^2}{b^2} + \frac{y^2}{b^2} + \frac{(z - f_c)^2}{a^2} = 1,
$$

(D-7)

where $f_c = \sqrt{a^2 - b^2}$, $a$ is the semimajor axis, and $b$ is the semiminor axis. If we write Eq. (D-7) in spherical coordinates of $(r, \varphi, \theta)$, with $r$ the distance from a point to the origin, $\varphi$ the azimuthal angle in the x-y plane from the x-axis, and $\theta$ the polar angle from the z-axis, we can find an expression for $r_e$

$$
r_e(\theta) = \frac{b^2}{a(1 - e_c \cos \theta)},
$$

(D-8)

with $e_c = \sqrt{1 - \frac{b^2}{a^2}}$ the eccentricity. The normal vector $n$ can be calculated by taking the gradient of Eq. (D-7) and rewriting the result into 

$$
n = -\frac{1}{\sqrt{1 + e_c^2 - 2e_c \cos \theta}} (\sin \theta e_\rho + (\cos \theta - e_c) e_z),
$$

(D-9)

with $e_z$ the unit vector in z-direction, and $e_\rho = \cos \varphi e_x + \sin \varphi e_y$. The surface area $d^2r$ on the ellipsoidal mirror is given by 

$$
d^2r = r_e \sin \theta \sqrt{\left(\frac{\partial r_e}{\partial \theta}\right)^2 + r_e^2 d\varphi} d\theta.
$$

(D-10)
In order to determine $\sigma$ and $K$ we need to specify the incident electromagnetic field. We use the expression for CTR assuming that the electron bunch traveled from vacuum into metal under $45^\circ$ incidence. We furthermore assume that the transverse dimension of the electron bunch is negligible compared to the longitudinal dimension $\sigma_l$. Using these assumptions the incident electrical field is given by

$$E_{in}(r, t) = \frac{eN\beta}{8\pi^2 \varepsilon_0 c^2} \sqrt{2\pi} (\mathcal{E_\varphi} e_\varphi + \mathcal{E_\theta} e_\theta) \exp\left[-\frac{(t - \frac{r}{c})^2}{2\tau_g^2}\right], \tag{D-11}$$

with $\tau_g = \sigma_l/c/\beta$ the RMS bunch duration, $c$ the speed of light, $\beta = v/c$ with $v$ the speed of the electrons, and $N$ the number of electrons in a bunch. The components $\mathcal{E_\varphi}$ and $\mathcal{E_\theta}$ are given by

$$\mathcal{E_\varphi} = \frac{\sin \varphi}{1 - \beta \cos \varphi \sin \theta}, \tag{D-12}$$

$$\mathcal{E_\theta} = \frac{(\beta - \cos \theta) \cos \varphi - \sin \theta}{(1 - \beta \cos \theta)(1 - \beta \cos \varphi \sin \theta)}. \tag{D-13}$$

The incident magnetic field is given by

$$B_{in}(r, t) = \frac{eN\beta}{4\pi^2 \varepsilon_0 c^2} \sqrt{2\pi} \tau_g r (-\mathcal{E_\varphi} e_\theta - \mathcal{E_\theta} e_\varphi) \exp\left[-\frac{(t - \frac{r}{c})^2}{2\tau_g^2}\right], \tag{D-14}$$

The surface charge density can be calculated by substituting Eq. (D-11) into Eq. (D-2), resulting in

$$\sigma(r, t) = \frac{eN\beta}{4\pi^2 \mu_0 \varepsilon_0 c^2} \tau_g r (n_\rho \cos \theta - n_z \sin \theta) \mathcal{E_\theta} \exp\left[-\frac{(t - \frac{r}{c})^2}{2\tau_g^2}\right], \tag{D-15}$$

where $n_\rho$ is the $\rho$-component of $n$, and $n_z$ the $z$-component, see Eq. (D-9). Substituting Eq. (D-14) into Eq. (D-1) leads to

$$K(r, t) = \frac{eN\beta}{4\pi^2 \mu_0 \varepsilon_0 c^2} \sqrt{2\pi} \tau_g r (-n_z \mathcal{E_\theta} e_\rho - (n_\rho \sin \theta + n_z \cos \theta) \mathcal{E_\varphi} e_\rho + n_\rho \mathcal{E_\theta} e_z) \exp\left[-\frac{(t - \frac{r}{c})^2}{2\tau_g^2}\right]. \tag{D-16}$$

### D-4 Expression for scalar and vector potential

We will evaluate $A$ and $\phi$ in the rectangular coordinates $(x, y, z)$. If we use Eqs. (D-3), (D-4), (D-8), (D-10), (D-15), and (D-16) we can write

$$\begin{pmatrix}
  cA_x(r, t) \\
  cA_y(r, t) \\
  cA_z(r, t) \\
  \phi(r, t)
\end{pmatrix} = \frac{eN\beta \sqrt{2\pi}}{4\pi^2 \varepsilon_0 c^2 \tau_g} \int_0^{2\pi} \int_{\theta_{min}}^{\theta_{max}} \exp\left[-\frac{(t - c^{-1}(|r - r_e| + r_e))^2}{2\tau_g^2}ight] \frac{I_x}{4\pi |r - r_e|} \sin \theta \sqrt{\left(\frac{\partial r_e}{\partial \theta}\right)^2 + r_e^2 d\varphi d\theta}, \tag{D-17}
$$

$$\begin{pmatrix}
  I_x \\
  I_y \\
  I_z \\
  I_\sigma
\end{pmatrix}$$
where
\[
\begin{pmatrix}
I_x \\
I_y \\
I_z \\
I_\sigma
\end{pmatrix} = \begin{pmatrix}
-n_z \mathcal{E}_\phi \cos \varphi + (n_\rho \sin \theta + n_z \cos \theta) \mathcal{E}_\phi \sin \varphi \\
n_z \mathcal{E}_\theta \sin \varphi - (n_\rho \sin \theta + n_z \cos \theta) \mathcal{E}_\phi \cos \varphi \\
n_\rho \mathcal{E}_\theta \\
(n_\rho \cos \theta - n_z \sin \theta) \mathcal{E}_\phi
\end{pmatrix},
\]
(D-18)
and \( \mathbf{r}_e = \mathbf{r}_e(\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta) \), and \( \theta_{\text{min}} = 0.05 \) and \( \theta_{\text{max}} = 0.25 \) determined by the ellipsoidal mirror.

### D-5 Evaluation of Electric field

The electric field can be calculated using Eqs. (D-5) and (D-17), resulting in

\[
\mathbf{E}(\mathbf{r}, t) = -\frac{eN\beta}{4\pi^2\varepsilon_0 c\tau_g} \int_0^{2\pi} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \frac{\mathbf{r} - \mathbf{r}_e}{|\mathbf{r} - \mathbf{r}_e|} I_\sigma \exp\left[-\frac{(t-c^{-1}(|\mathbf{r} - \mathbf{r}_e|+r_e)^2)}{2\tau_g^2}\right] \times
\]
\[
\times \left(1 - \frac{|\mathbf{r} - \mathbf{r}_e| + r_e}{c}\right) \frac{1}{|\mathbf{r} - \mathbf{r}_e|} \sin \theta \sqrt{\left(\frac{\partial \mathbf{r}_e}{\partial \theta}\right)^2 + r_e^2 d\varphi d\theta}
\]
\[
+ \frac{eN\beta}{4\pi^2\varepsilon_0 c\tau_g} \int_0^{2\pi} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \left(\begin{array}{c}
I_x \\
I_y \\
I_z
\end{array}\right) \exp\left[-\frac{(t-c^{-1}(|\mathbf{r} - \mathbf{r}_e|+r_e)^2)}{2\tau_g^2}\right] \times
\]
\[
\times \frac{1}{c\tau_g^2} \left(t - \frac{|\mathbf{r} - \mathbf{r}_e| + r_e}{c}\right) \sin \theta \sqrt{\left(\frac{\partial \mathbf{r}_e}{\partial \theta}\right)^2 + r_e^2 d\varphi d\theta},
\]
(D-19)

and

\[
\mathbf{B}(\mathbf{r}, t) = \frac{eN\beta\sqrt{2\pi}}{4\pi^2\varepsilon_0 c^2\tau_g} \int_0^{2\pi} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \frac{\mathbf{r} - \mathbf{r}_e}{|\mathbf{r} - \mathbf{r}_e|} \times \left(\begin{array}{c}
I_x \\
I_y \\
I_z
\end{array}\right) \exp\left[-\frac{(t-c^{-1}(|\mathbf{r} - \mathbf{r}_e|+r_e)^2)}{2\tau_g^2}\right] \times
\]
\[
\times \left(\frac{1}{c\tau_g^2} \left(t - \frac{|\mathbf{r} - \mathbf{r}_e| + r_e}{c}\right) - \frac{1}{|\mathbf{r} - \mathbf{r}_e|}\right) \sin \theta \sqrt{\left(\frac{\partial \mathbf{r}_e}{\partial \theta}\right)^2 + r_e^2 d\varphi d\theta}.
\]
(D-20)

The numerical evaluation of the integrals for \( \mathbf{E} \) and \( \mathbf{B} \) are performed using Mathematica.
Bibliography

Summary

For a long time it has been recognized that electromagnetic radiation in the terahertz (1 THz = $10^{12}$ Hz) frequency range holds many promises. However, only in the last two decades THz science started living up to its potential. To push the limits of THz science even further the need exists for a broadband THz source, capable of delivering powerful pulses which are either propagating in free-space, or coupled onto a metal wire.

We combine a small electron accelerator, with a footprint of an optical table, and a femtosecond laser to create THz radiation. Single-cycle, high-field THz pulses are created using the coherent transition radiation (CTR) process. Because the used relativistic electrons are concentrated into a volume having dimensions smaller than the corresponding wavelength ($\lambda \approx 0.3$ mm), they emit the radiation coherently at THz frequencies. This leads to powerful THz pulses, either propagating in free-space or coupled onto a metal wire, because the energy collectively released is proportional to the number of electrons ($\sim 10^9 = 160$ pC) squared. This thesis describes an experimental study into the use of CTR to produce intense single-cycle THz pulses, either propagating in free-space or coupled onto a metal wire.

In Ch. 2 the theory of the generation of free-space THz CTR is treated, appropriate for our experimental situation.

In Ch. 3 we present the theory of CTR generated by ellipsoidal electron bunches, which can be used for diagnostics of ellipsoidal electron bunches. Realization of such bunches would solve the problem of space-charge induced emittance degradation.

The production of few-MeV, 100 pC electron bunches, which can be focused into a sufficiently small volume, requires state-of-the-art radiofrequency (RF) photogun technology. A major part of the experimental effort has therefore gone into the development of such a RF photogun. In Ch. 4 we discuss the 1.5 cell RF photogun, and the quality of the produced electron bunches, which is comparable to the present state of the art.

In Ch. 5 we present the measurements of single-cycle free-space THz pulses, generated using the CTR process. To increase the signal-to-noise ratio we focussed the CTR THz pulses into a small spot. We show that we understand the electric field profile in the focal spot, both the qualitative behavior and the quantitative field strengths. This enables us to make a realistic prediction of the performance of the 1.5 cell RF photogun as a source of free-space THz CTR radiation. We show that is is possible to create THz pulses with peak-electric fields of 100 MV/m, having energies per pulse up to 10 $\mu$J, and bandwidths
of 0.1 - 10 THz, using the CTR process in a compact manner. However, this requires more than a single RF photogun because the electron bunches need to be compressed in the longitudinal dimension to \( \sim 100 \) fs. We also find, however, that by focusing a broadband THz pulse one inevitably throws away a substantial part of the pulse energy, because the low frequency components cannot be focused as well as the high frequency components. This problem can be resolved by coupling a THz CTR pulse onto a metal wire.

In Ch. 6 we present measurements of intense THz surface plasmon polaritons (SPPs) on a metal wire, generated by launching electron bunches onto a tapered end of the wire. The SPPs were generated on a metal wire of 1.5 mm thickness and had a peak electric field of 0.5 MV/m and a full-width-half-maximum (FWHM) pulse length of \( \sim 6 \) ps. We have compared the measured properties of the SPPs with a newly developed theory. The theory predicts the bandwidth correctly, but the measured spectral amplitude was typically a factor of 5 less than the calculated amplitude. Probable causes for the discrepancy are the tip-to-wire transition and electron scattering in the metal tip, which are both not modeled in the theory. By optimizing the electron beamline and focusing the SPPs to a dimension below the wavelength, by tapering the metal wire into a tip, electric field strengths in excess of \( \sim 100 \) MV/m are possible.
Samenvatting

Elektromagnetische straling in het terahertz (1 THz = 10^{12} Hz) frequentiegebied biedt vele fascinerende toepassingen, zowel praktisch als in de fundamentele wetenschap. Pas in de laatste 20 jaar is het onderzoek in het THz gebied echt tot bloei gekomen, voornamelijk door de opkomst van femtoseconde lasertechniek, die het mogelijk maakt zeer breedbandige THz pulsen te maken: door het gelijkrichten van een femtoseconde laserpuls wordt een stralingspuls opgewekt die gedurende 1 picoseconde slechts enkele trillingen uitvoert, en dus met een bandbreedte van de orde van 1 THz. Om de grenzen nog verder te verleggen is er grote behoefte aan een gepulste THz bron, die niet alleen zeer breedbandig is (bandbreedte groter dan 1 THz), maar ook zeer krachtige pulsen levert (veldsterktes van 10-100 MV/m). Tot voor kort werd het onderzoek aan THz bronnen vooral gericht op het genereren van vrij propagerende THz pulsen. Recentelijk is er ook grote belangstelling ontstaan voor de mogelijkheid om zogenaamde 'oppervlakteplasmonen' op te wekken in het THz gebied. Gekoppeld aan het oppervlak van een metalen draad kunnen dergelijke elektromagnetisch oppervlaktegolven niet alleen efficiënt getransporteerd worden, maar ook gefocuseerd tot ver beneden de diffractielimiet. Hiermee zouden geheel nieuwe onderzoeksmogelijkheden gecreëerd worden.

Om dergelijke THz pulsen op te kunnen wekken combineren wij een femtosecond laser met een elektronenversneller, die voldoende compact is dat hij gemakkelijk past op een standaard optische tafel. Met behulp van het zogenaamde 'coherente transitiestraling'-proces worden intense THz pulsen gemaakt die slechts een enkele trilling uitvoeren. Relativistische elektronen zenden transitiestraling uit als ze, bijvoorbeeld, door een metalen folie vliegen. De relativistische elektronenpulsen, die door de versneller geproduceerd worden, stralen coherend bij THz frequenties, omdat ze zijn geconcentreerd in een volume kleiner dan de corresponderende golflengte (typisch 0.3 mm). Omdat de collectief uitgestraalde energie schaalt met het aantal elektronen in een puls in het kwadraat (de pulslading is typisch 160 pC, overeenkomend met ongeveer 10^{9} elektronen), leidt dit tot zeer krachtige THz pulsen. Dit proefschrift beschrijft een experimentele studie naar het gebruik van coherente transitiestraling om intense THz pulsen te genereren die slecht een enkele trilling uitvoeren. Het onderzoek richt zich zowel op het genereren van vrij propagerende THz pulsen als op het opwekken van THz oppervlakteplasmonen gekoppeld aan een metalen draad.

In Hfst. 2 wordt de theorie behandeld van het opwekken van vrij propagerende THz pulsen door middel van coherente transitiestraling. In Hfst. 3 wordt de theorie van Hfst. 2
toegepast op ellipsvormige elektronenpulsen met een homogene ladingsverdeling. Dergelijke
elektronenpulsen, die pas sinds kort experimenteel gerealiseerd kunnen worden, bieden de
mogelijkheid om de helderheid te behouden van intense elektronenbundels waarvan de
dynamiek geheel beheerst wordt door ruimteladingskrachten. Coherente transitiestraling
maakt diagnostiek mogelijk van dergelijke elektronenpulsen.

Voor het opwekken van coherente transitiestraling moeten picoseconde elektronenpulsen
met een energie van enkele MeV en pulsladingen van ongeveer 100 pC gefocuuseerd worden
in een brandpunt van enkele honderden micron. Dit vereist een zogenaamd ‘radiofrequent
fotokanon’, waarin foto-emitie van elektronenpulsen door een femtoseconde laser gecombi-
neerd wordt met ultramoderne radiofrequente (RF) versnellertechnologie. Een groot deel
van het experimentele werk is besteed aan het ontwikkelen van een RF fotokanon van zeer
hoge kwaliteit, wat beschreven wordt in Hfst. 4. We laten zien dat het door ons gebouwde
1.5 cel RF fotokanon een elektronenbundel levert die in helderheid kan concurreren met de
allerbeste fotokanonnen die wereldwijd nu in bedrijf zijn.

In Hfst. 5 presenteren we de metingen van vrij propagerende, breedbandige THz pulsen,
gegenereerd door middel van coherente transitiestraling. De elektrische veldsterkte van de
THz pulsen wordt direct gemeten als functie van de tijd met behulp van de zogenaamde
‘elektro-optische’ meettechniek. Voldoende signaalsterkte wordt gerealiseerd door de THz
pulsen scherp te focuusen. We laten zien dat we het gedrag van het elektrische veld in het
brandpunt zowel als functie van de tijd als van positie nauwkeurig kwantitatief begrijpen.
Op basis hiervan kunnen we een realistische voorspelling maken hoe goed een RF fotokanon
in principe kan fungeren als bron van vrij propagerende THz pulsen. We laten zien dat
het mogelijk is om met een compacte opstelling coherente transitiestraling THz pulsen te
creëren met een elektrische veldsterkte van 100 MV/m, 10 µJ energie per puls, en band-
breedtes van 0.1 - 10 THz. Dit vereist echter meer dan alleen een RF fotokanon, omdat
de elektronenpulsen gecompresseerd moeten worden in de longitudinale richting tot ~ 100
fs. Daarnaast volgt uit onze analyse dat een groot deel van de pulsenergie onvermijdelijk
verloren gaat als een breedbandige THz puls gefocuuseerd wordt. De reden hiervoor is dat
straling bij relatief lage frequenties, en dus relatief lange golflengtes, niet zo goed gefocu-
seerd kunnen worden als de hogere frequenties in een breedbandige puls. Dit probleem kan
opgelost worden door THz pulsen te genereren in de vorm van oppervlakteplasmonen op
een metalen draad.

In Hfst. 6 presenteren we metingen van intense THz oppervlakteplasmonen gekoppeld
aan een metalen draad, die gegenereerd zijn door elektronenpulsen op de punt van een
metalen draad te schieten. Op een metaaldraad van 1.5 mm dik worden oppervlakteplas-
monen opgewekt met een elektrisch veldsterkte van 0.5 MV/m en een puls lengte van ~ 6
ps. De gemeten bandbreedte van de oppervlakteplasmonen is in overeenstemming met een
recent ontwikkelde theorie. De theorie voorspelt echter elektrische veldsterktes die typisch
een factor 5 hoger dan de gemeten waarden. Mogelijke redenen voor de discrepantie zijn
effecten die niet gemodelleerd worden in de theorie, zoals elektronenverstrooiing in de punt
van de draad of de overgang van kegelvormige punt naar een evenwijdig draad. Door op-
timalisatie van de bundellijn gecombineerd met focuusen van de oppervlakteplasmonen
door de metaaldraad toe te laten lopen in een spitse punt, is het mogelijk om elektrische
veldsterktes van meer dan $\sim 100$ MV/m te bereiken.
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Curriculum vitae

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