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Dependence of the transition from Townsend to glow discharge on secondary emission

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In a recent paper, Šijačić and Ebert [Phys. Rev. E. 66, 006410 (2002)] systematically studied the transition from Townsend to glow discharge, referring to older work by von Engel and M. Steenbeck [Elektrische Gasentladungen. Ihre Physik und Technik (Springer, Berlin 1934), Vol. II] up to Raizer [Gas Discharge Physics (Springer, Berlin, 1991)]. Šijačić and Ebert stated that this transition strongly depends on secondary emission γ from the cathode. We show here that the earlier results of von Engel and Raizer on the small current expansion about the Townsend limit actually are the limit of small γ of the Šijačić and Ebert expression, and that for larger γ the old and the Šijačić and Ebert new results vary by no more than a factor of 2. We discuss the γ dependence of the transition, which is rather strong for short gaps.

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\[ \bar{\alpha}(|E|) = \alpha_0 e^{-|E|/\beta}. \] (4)

(In [1], the generalized case \( \bar{\alpha}(|E|) = \alpha_0 \exp(-E_0/|E|) \) was treated.) Boundary conditions at the anode \( x=0 \) and for secondary emission at the cathode \( x=d \) are

\[ J_e(0) = 0, \quad |J_x(d)| = \gamma |J_x(d)|. \] (5)

The discharge is characterized by the potential \( U \) and total electric current \( J \), as

\[ U = \int_0^d dx \, E(x), \quad J = e(n_+\mu_+ + n_\mu)E. \] (6)

It is useful to introduce dimensionless voltage and current, as

\[ u = \frac{U}{E_0\alpha_0}, \quad \tilde{J} = \frac{J}{e_0\alpha_0E_0} \mu \tilde{E}_0, \] (7)

where \( \tilde{J}/\mu \) with the definition of \( j \) from [1]. It should be noted that only bulk gas parameters have been used as units; therefore, the dimensionless \( u \) and \( \tilde{J} \) are independent of \( \gamma \).

Further dimensional analysis yields that the current-voltage characteristics \( u = u(\tilde{J}) \) can depend on three parameters only: namely, on the dimensionless gap length \( L = \alpha_0 d \), on the coefficient \( \gamma \) of secondary emission, and on the mobility ratio \( \mu = \mu_+/\mu_\mu \). In practice, the dependence on the small parameter \( \mu \) is almost negligibly weak [1]; therefore, \( u = u(\tilde{J}, L, \gamma) \). Here, the dimensionless gap length \( L \) is related to \( pd \) through \( L = Apd \) as long as the coefficient \( \alpha_0 \) is related to pressure as \( \alpha_0 = Ap \).

How strongly does the characteristics \( u = u(\tilde{J}, L, \gamma) \) depend on \( \gamma \)? In [1], Šijačić and Ebert (SE) calculated the whole Townsend-to-glow regime numerically and derived, by expanding systematically in powers of current \( \tilde{J} \) about the Townsend limit, that

\[ u = u_T - A_{SE} \tilde{J}^2 + O(\tilde{J}^3), \] (8)

\[ \bar{\alpha}(|E|) = \alpha_0 e^{-|E|/\beta}. \] (4)
The coefficient of Fig. 1 of \( \left[ \right] \) is reproduced as Eq. (50) in [1] for \( F(\gamma, \mu) \) which was corrected, namely, the missing factor \( 1/(8\pi) \) in (8.8) is substituted by \( \varepsilon_0/2 \) in (16), since we here write the Poisson equation (2) in MKS units rather than in Gaussian units; cf. (8.6) in [2].

In (8.6), another small current expansion was derived from (1)–(3), assuming \( n_s \approx n_e \) and \( n_s(x) = \text{const} \). This approximation was criticized in [1], since it is in contradiction with the boundary condition (5); however, for very small \( \gamma \), it is a good approximation in a large part of the gap. The resulting equations (8.8) and (8.10) from [2] read in the notation of the present paper

\[
U = U_T = \frac{U_T - 1}{48} \frac{2E_T}{J_L^2} \left( \frac{J}{J_L} \right)^2,
\]

(15)

\[
J_L = \frac{\varepsilon_0 \mu L^2}{2 d^3}.
\]

(16)

(Here, a misprint in [2] was corrected, namely, the missing factor \( U_T \) in the coefficient of \( J^2 \) in (15), is now included. Furthermore, the factor \( 1/(8\pi) \) in (8.8) is substituted by \( \varepsilon_0/2 \) in (16), since we here write the Poisson equation (2) in MKS units rather than in Gaussian units; cf. (8.6) in [2].)

In (15), the physical current density \( J \) is compared to \( J_L \). \( J_L \) is the current density at which deviations from the Townsend limit through space charges start to occur; it explicitly depends on \( \gamma \) through \( U_T \) (12).

Comparison of the results of Šijačić and Ebert (8) and of von Engel and Raizer (ER) (15) show that the coefficients \( A_{SE, ER} \) in the expansion (8) are related as

\[
A_{SE} = A_{ER} \frac{12 \frac{F(\gamma, \mu)}{L^3}}{L^3}, \quad A_{ER} = \frac{1 - 2E_T}{2E_T} \frac{L^3}{12 E_T^3}.
\]

(17)

The coefficients \( A_{SE} \) and \( A_{ER} \) depend in the same way on \( L \), and they are essentially independent of \( \mu \) for realistic values of \( \mu \). Therefore, the ratio \( A_{SE}/A_{ER} \) depends only on \( \gamma \) as shown in Fig. 1. For \( \gamma \to 0 \), the ratio tends to unity. For a
large range of $\gamma$ values, the deviation is not too large, approaching a factor 0.44 for $\gamma=10^{-1}$.

Figure 2 shows that the factor $A_{SE}$ indeed strongly depends on $\gamma$ for the given $L$.

The strong dependence of $A_{SE}$ or $A_{ER}$ on $\gamma$ for a given short gap length $L$ means that we can obtain both negative and positive differential resistance $dU/dJ$ close to the Townsend limit for the same gap length. Therefore, the choice of $\gamma$ is important since it can change the differential conductivity and therefore the stability of a Townsend discharge in a short gap.

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