Computing Electromagnetic Fields in Engineering Applications: a Diakoptic Approach

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Abstract—The trends and needs in the development of technologies like antennas and microwave circuits have clearly indicated a constantly increasing level of complexity of these structures, as well as the need of efficient and accurate analysis and synthesis tools. Such tools should provide an efficient yet accurate design approach, avoiding or minimizing the time consuming experimental optimization phase. In this paper, we present an overview of electromagnetic modeling techniques, responding to these trends, developed in the authors’ research groups.

I. INTRODUCTION

In several fields like astronomy, radar, electromagnetic effects, space technology, optical fibres, lithography, portable and wearable devices, RF design, and medical applications, there is a continuous request for applied electromagnetic research. The following main trends can be clearly identified:

- The increasing complexity and high speed of devices in electrical engineering make the electromagnetic wave character of the underlying physical phenomena increasingly more important. Neither the interconnects in integrated electronics, nor the packaging of integrated circuits can be described solely in terms of straightforward circuit theory, moreover the design of printed-circuit boards for high-speed electronics must proactively account for EMC/EMI aspects.
- In telecommunications, radar as well as astronomy, there is a common need for integrated antennas. These antennas have often to meet very demanding requirements not only from the electromagnetic point of view, but also in terms of structural integration with the supporting platforms (with their mechanical and thermodynamic requirements) and integration with the electronics. These trends and the never ending request of cost reduction, require the development of novel concepts and new manufacturing techniques.

These trends go hand in hand with an increasing demand for the “exact” numerical analysis of electromagnetic fields in large, complicated configurations, in particular for synthesis problems. For the foreseeable future, even the more and more powerful computer platforms that are becoming available are, in the authors’ opinion, unable to carry out such computations “brute force”, within an acceptable time frame. Fortunately, there are two circumstances that point us in the right direction. First, the users are typically not interested in the electromagnetic field as such, but in some derived “observable”. Second, the synthesis need not be carried out for the entire configuration at once, but for a bounded subdomain.

II. ENGINEERING ELECTROMAGNETIC

One of the most expensive and time consuming phases in the development of microwave and antenna systems is the design fine tuning of the structure. This is often based on experimental techniques or on trial-and-error processes, based on the use of general-purpose CAD tools. In both stages, human intelligence is needed to steer the process, in particular with respect to the choice of the design criteria and the choice of the direction in which the improvement or “update” is sought. This intelligence is usually obtained from a few experienced engineers, and this makes the design process vulnerable and expensive.

Engineering Electromagnetic has reached the stage where commercial software seems capable of replacing some or all of the prototyping steps. Nevertheless, although these general-purpose tools are extremely powerful and user-friendly, and can simulate very complex structures, they do not always provide an easy physical interpretation of the electromagnetic behaviour, and furthermore they often require very long computation times. Thus, like in prototyping, only a few geometries are analyzed before the design is finalized. Stochastic techniques like genetic or particle-swarm optimization are capable of searching an optimum in a large parameter space with multiple local optima. However, they might have to analyze the electromagnetic behaviour of many thousands of “candidates”. Using state-of-the-art computational electromagnetic for this purpose does not seem realistic for the foreseeable future.

An efficient computer aided design/synthesis process requires, therefore, a two step strategy. At first, a stochastic optimization can be applied to approximate models to allow a fast evaluation of the cost and fitness function. Equivalent circuits or extracted parameters, as they are widely used in engineering practice, may be suitable for this purpose. An alternative approach is reduced-order modelling [1], [2], which is capable of reducing the size of a linear system of equations for an unknown field quantity to a much smaller system, in which the behaviour of relevant physical parameters is preserved.
In a second stage, we assume that a suitable initial estimate is available, either from the first stage or from engineering experience. Thus, local optima are avoided, and deterministic algorithms based on a local linearization may be employed for the optimization. Even this requires the evaluation of the electromagnetic behaviour of tens of candidates, therefore the computation time must still be reduced “from hours to minutes” for this procedure to be realistic.

In the remainder of this paper, we concentrate on the second stage, where an initial estimate is available. In that stage, full-wave modeling must be combined with line-search optimization, which amounts to successive sweeps with respect to a line-search parameter. Two concepts have been developed in our groups to reduce the duration of a single field computation, so that it may be repeated tens of times in the optimization. The first one can be generically classified as diakoptic approach, which basically consists in separating, in smart ways, large and complex structures into smaller sub-domains. The second concept, named “marching-on-in-anything”, falls in the category of in-line search optimization techniques.

### III. A DIAKOPTIC APPROACH

The first basic idea, in order to substantially reduce the computation time for the analysis of complex structures, is to separate large, complicated configurations into smaller sub-domains. The electromagnetic field in these sub-domains is then computed locally, for a simpler environment. Subsequently, the thus obtained field distributions are used as basis functions in a global version of the Method of Moments. This may be regarded as an application of Gabriel Kron’s concept of diakoptics in electromagnetic field analysis.

This approach is increasingly used in modelling large, finite antenna arrays consisting of metallic patches. The currents on these patches are determined for an isolated patch and/or for a patch in an infinite array. In the synthetic function (SFX, [3]) and characteristic basis function (CBF, [4]) approaches, this is achieved by moving an elementary source in the vicinity of the patch, and using the singular-value decomposition to extract independent distributions. In the eigen-function approach [5], the eigen-currents of the integral operator for the isolated patch are used as basis functions. Thus, the choice of the varying excitation and the subsequent Singular Value Decomposition (SVD) are avoided. These methods have already been applied successfully to structures consisting of electrically isolated domains, while overlapping basis functions for electrically connected domains are also emerging [3]. In all cases, the metallic surface, on which the induced electric surface currents are computed, is subdivided.

An alternative is to subdivide the entire three-dimensional space. Typically, we consider an observation domain and an environment. The environment is supposed to be fixed, while the constitution of the observation domain is varied and optimized. Again, different possibilities exist to realize this concept. For closed or fully periodic structures, decomposition into modes has led to the integral equation multimode equivalent network approach [6]-[9].

As an example, Fig. 1 shows a multilayer Frequency Selective Surface based structure, consisting of three planar dielectric layers loaded by patch- and slot-based resonant elements, cascaded to an infinite periodic array of open-ended waveguides. For this kind of structure, either the scattering problem is tackled simultaneously at all layers, by solving the corresponding system of coupled integral equations [7], or it is treated separately at each layer by resorting to a modular methodology.

Fig. 1. Example of multilayer antenna structure: an iris-loaded waveguide array integrated with a generic FSS structure, comprising a slot-based FSS and a patch-based FSS sandwiched between different dielectric layers.

The latter approach is computationally convenient, especially when the optimization procedure requires the modification of only one part of the overall structure (e.g. thickness of dielectric layers, FSS geometry, waveguide tuning elements etc.). In this case, in fact, only the equivalent network representation of this part is recomputed. Moreover, it can be implemented in a general-purpose software tool that allows analyzing any arbitrary number of layers. Techniques based on the derivation of equivalent microwave networks can be used for this purpose.

Starting from a modal expansion of the fields in each layer, these techniques lead to a representation of the layer and of the transition between two adjacent layers in terms of an equivalent network; the different networks are then cascaded to represent the entire structure. The form of the matrix describing this network depends on the type of parameter used to characterize the structure performances: for example the generalized scattering matrix (GSM) [10], the generalized impedance matrix (GIM) and the generalized admittance matrix (GAM) [11]. In many cases, the calculation of the GIM or GAM equivalent representation is the goal of the analysis. An appealing characteristic of these impedance/admittance representations is that they can be directly interpreted in terms of physical parameters of the represented structure. On the other hand, as already pointed out in [12], the GAM and GIM representations encounter a stability problem when matrices with a large number of entries are cascaded. To prevent this problem, additional operations should be undertaken to derive
from the complete matrix a reduced form, corresponding only to the interacting modes between adjacent discontinuities. This is achieved with the introduction of the concept of accessible and localized modes. Accessible are the modes that contribute to the interaction between adjacent discontinuities, they include not only the propagating ones, but also modes below cut-off, still responsible for a sensible interaction with the adjacent discontinuity. All remaining higher order modes are indicated as localized. Fig. 2 shows the junction between two uniform waveguide regions, indicated as regions 1 and 2, and gives an intuitive representation of the accessible and localized modes. This approach restricts the multi-mode equivalent network representation to those modes that are observed in a homogeneous region between sharp interfaces, while the localized modes contribution is kept in the kernel of the integral representation. This allows a significant increase in the computational efficiency.

In LEGO (Linear Embedding via Green's Operators, [13], [14]), the equivalence principle is used to model the interaction between different sub-domains. The advantage is that this method is applicable to boundaries of arbitrary shape. To explain the basic principle, let us consider the two-dimensional multiple-scattering problem shown in Fig. 3. An electrically polarized line source is exciting two identical scattering objects, and the aim is to evaluate the electromagnetic field in an efficient manner. In LEGO, we proceed as follows.

First, the scattering problem is solved for an isolated object in domain $D_1$ with a homogeneous environment in $D_2$ excited by a line source on the boundary $C$. The equivalence principle is then used to translate the resulting scattered field in $D_1$ into one originating from an equivalent current on $C$, e.g., by solving an Electric Field Integral Equation (EFIE). This equivalent current distribution is used to define a “scattering operator” for a single object.

Second, we place a single object in $D_1$ and choose the location of a line source on $C_2$ the boundary of an adjoining domain. With the aid of the known two-dimensional Green's function for a homogeneous space, we are then able to translate the “scattering operator” for domain $D_1$ into a “reflection operator” for domain $D_2$.

Finally, since both domains are identical, we may now combine the scattering and reflection operators for both domains into an integral equation for equivalent currents on $C_1$ and $C_2$. The resulting currents would produce the correct fields inside $D_1$ and $D_2$ in a homogeneous environment. Since the elementary solutions are known from the analysis of a single object, the superposition principle may be invoked to evaluate the field in $D_1 \cup D_2$.

The same procedure may be repeated to combine multiple domains. It should be remarked that sub-domains need not to be identical, and that the order of their combination may be chosen for convenience. This enables us to combine the known portions of a complicated geometry into a fixed environment, for which a reflection operator is determined for an empty observation domain. By combining this operator with the scattering operator for an object in an empty environment, we obtain the full electromagnetic response. This enables us to optimize an object in the observation domain without re-evaluating the response of the complicated environment.

As an illustration, Fig. 4 shows the field in an optimized power splitter in a finite EBG structure of 17 x 17 cells. In the optimization step, only the radius of a cavity at the junction between the wave-guiding channels was varied. More details can be found in [14].
where \( u(p) \) is a discretized field and \( f(p) \) corresponds to the excitation. We are interested in the situation where this problem must be solved for a large number of sample values of the parameter \( p \), e.g., \( p_m = p_0 + m\Delta p \), with \( m=0, 1, \ldots, M \). To this end, we minimize the squared error:

\[
\text{ERR}^{(n)} = \| u^{(n)} - f^{(n)} \|^2 = \langle u^{(n)} - f^{(n)} \rangle
\]

with \( r^{(n)} = Lu^{(n)} - f \) with the aid of a standard conjugate gradient method. This procedure is accelerated significantly when the initial estimate for \( p = p_0 \) is generated from a few previous “final” results, according to:

\[
u^{(0)}(p_m) = \sum_{k=1}^{K} \gamma_k u(p_{m-k})
\]

where the \( \{ \gamma_k \mid k=1, \ldots, K \} \) are found by minimizing the squared error (2). The value of the coefficients \( \{ \gamma_k \} \) can be found from the system of linear equations:

\[
\begin{align*}
\sum_{k=1}^{K} < L(p_{m-1})^{-1} L(p_m) u(p_{m-k}) > \gamma_k = & \\
< L(p_m) u(p_{m-1}) | f(p_m) >
\end{align*}
\]

with \( l=1, \ldots, K \). The procedure has been demonstrated for boundary and domain integral equations in two and three dimensions. Typically \( K = 2 \) or \( K = 3 \), i.e., storing two or three previous final results suffices, and the acceleration rate varies between 10 and 50.

For “finite” (difference or element) methods, the solution of the discretized equation (1) is as such is less efficient because of the poor convergence of the conjugate gradient method. This can be explained from the structure of the adjoint operator which is employed to generate the update directions. This operator now links a few local field values, so that the update remains local as well. This problem can be remedied by applying a pre-conditioner based on a spectral decomposition of the discretized field. This opens up the possibility to combine the resulting pre-conditioned scheme with the extrapolation procedure outlined above.

V. CONCLUSIONS

In this paper, we have presented a short overview of electromagnetic modelling techniques, responding to the trends of technologies like integrated antennas and microwave circuits. A more detailed description of EM tools developed in the authors’ research groups has been presented and a two-stage approach has been proposed for using computational electromagnetic in antenna engineering. This approach consists of stochastic optimization techniques used in combination with approximate models, for the first stage, and in line-search techniques combined with full-wave modelling for the second stage.

REFERENCES
