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Low Complexity Soft Differential Decoding of QPSK for Forward Error Correction in Coherent Optic Receivers

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Abstract Coherent systems based on QPSK rely on differential encoding to avoid catastrophic error propagation. A simple solution for soft differential decoding is presented that limits the penalty after FEC to 0.75dB, similar to pre-FEC binary differential decoding.

Introduction

First generation coherent systems are mostly based on quadrature phase shift keying (QPSK)\(^{1}\). Differential encoding can be used in order to avoid catastrophic error propagation in presence of cycle slips, e.g. caused by the carrier recovery due to the relatively large laser phase noise or a low signal-to-noise ratio. Binary differential decoding of QPSK leads to a penalty of 0.75dB at a bit-error-rate (BER) of 4e-3 as shown in Fig. 1. The signal can then be processed in the FEC, albeit only using hard-decisions. On the other hand, differential detection of QPSK as used in incoherent receivers (soft DQPSK), computes the differential soft phase between two subsequent symbols as

\[
z_k = r_k r_{k-1}^* e^{j\pi/4}.\]

At 2.45dB, the loss is naturally much higher than for binary differential decoding due to noise enhancement, as shown in Fig. 1.

Differential Encoding Loss

In differentially encoded QPSK, the information is encoded onto the phase difference between two subsequent symbols. After the equalization and carrier recovery of the signal, the information can be recovered using binary differential decoding of QPSK (binary DQPSK) of input \(r_k\), \(k\in\{0,1,...\}\), e.g. given by

\[
z_k = \text{sgn}(r_k)\text{sgn}(r_{k-1}^*)e^{j\pi/4}.\]  

Every bit error in the coherent domain is translated to two bit errors after differential decoding, leading to a 0.75dB penalty at a bit error rate (BER) of 4e-3 as shown in Fig. 1. The signal can then be processed in the FEC, albeit only using hard-decisions. On the other hand, differential detection of QPSK as used in incoherent receivers (soft DQPSK), computes the differential soft phase between two subsequent symbols as

\[
z_k = r_k r_{k-1}^* e^{j\pi/4}.\]

Iterative Decoding

The differential encoding loss can be fully compensated using iterative decoding\(^{7,8}\). Here, error correction codes are used iteratively in concatenation with differential maximum a posteriori (MAP) decoding, as shown in Fig. 2. The optimum MAP soft values DQPSK can be derived from\(^9\). For the Gray-coded input signal constellation \(X\in\{e^{j\pi/4}, e^{j3\pi/4}, e^{j5\pi/4}, e^{j7\pi/4}\}\),

![Fig. 1: Performance of coherent QPSK in comparison with binary and soft DQPSK.](image-url)
and the receive complex signal $y$, the log-likelihood ratio (LLR) for bit $b$ is defined as

$$ \Lambda_b(y) = \log \frac{\Pr \{ b = 0 | y \}}{\Pr \{ b = 1 | y \}} $$

with the exact LLRs for two subsequent symbols $y_0, y_1$ given by

$$ \Lambda_0(y) = \log \frac{\sum_{s \in X} \left( \exp \frac{2}{N_0} A_s + \exp \frac{2}{N_0} C_s \right)}{\sum_{s \in X} \left( \exp \frac{2}{N_0} B_s + \exp \frac{2}{N_0} D_s \right)} $$

$$ \Lambda_1(y) = \log \frac{\sum_{s \in X} \left( \exp \frac{2}{N_0} A_s + \exp \frac{2}{N_0} D_s \right)}{\sum_{s \in X} \left( \exp \frac{2}{N_0} B_s + \exp \frac{2}{N_0} C_s \right)} $$

$$ A_s = \Re \{ s(y_0 + y_1) \}, B_s = \Re \{ s(y_0 - y_1) \}, $$

$$ C_s = \Re \{ s(y_0 + jy_1) \}, D_s = \Re \{ s(y_0 - jy_1) \}. $$

Typically, convolutional codes with a low rate ($r=1/2$, overhead=100%) are used and decoded using either maximum likelihood sequence estimation (MLSE) or maximum a posteriori (MAP) decoding. In principle, it is possible to replace the convolutional code by a soft output low density parity check (LDPC) or turbo code with a low overhead, as they are typically used in fiber optics, in order to compensate for the differential penalty. However, this has neither been demonstrated in fiber optic literature, nor has an assessment of the possible complexity increase taken place.

**Quantized soft DQPSK (Q-DQPSK)**

In this paper, we propose the use of a low complexity quantized differential decoding of QPSK in modular combination with FEC. The block diagram for the proposed algorithm is shown in Fig. 3. First, the signal is quantized independently in the I and Q branches, with the quantization consisting of a compressor, clipping with magnitude $\kappa$, quantization with $q$ bits, and an expander. The compressor has a response curve that is given by

$$ y = \text{sgn}(x) \cdot |x|^\lambda, \quad \lambda < 1, $$

with the expander being the inverse of the compressor. Then, the signal is differentially soft decoded similar to (2). The quantized soft output is fed into the FEC decoder. As it will be shown in the following, the independent quantization of I and Q leads to a noise reduction and an improved performance.

As an example, the code I.5 from the ITU-T recommendation G.975.1 was used for reference. This Super-FEC scheme uses a concatenated code consisting of a Reed-Solomon $R(n=1901, k=1855)$ outer code and an Extended Hamming $(n=512, k=502) \times (n=510, k=500)$ product inner code as illustrated in Fig. 4. Since the recommendation does not specify the decoding, the algorithm described by Pyndiah is employed, with 6 soft iterations of block turbo decoding initially optimized for an AWGN channel. The performance is evaluated for the inner code only, as an inner code BER of $\sim 1e^{-6}$ leads to a post-FEC BER of $\sim 1e^{-12}$.

**Performance**

The coding performance is numerically evaluated for differentially encoded QPSK with the transmitted symbols given by $\{ \pm 1 \pm j \}$ in an additive white Gaussian noise (AWGN) channel. The parameters are initially optimized and set to $\lambda = 0.65$, $\kappa = 0.6$ and with 3 bits of quantization. After the quantized soft differential decoding, the FEC module decodes the product code and de-
termines the BER of the inner code. The performance is shown in Fig. 1 comparing quantized soft DQPSK with MAP-DQPSK and soft DQPSK.

Fig. 5: Performance of the inner soft code (Hamming (512,502)x(510,500) product code) for QPSK and the various DQPSK decoding algorithms.

Here, the BER is plotted vs. \( E_b/N_0 \), where \( E_b \) is the energy per information bit, and is computed from the coded symbol energy as \( E_b = r E_{cs}/2 \) using the code rate \( r = k/n \) of the used code. Using the proposed I/Q quantization, the pre-FEC BER is in fact reduced to the level of binary differential decoding. For the optimum clipping magnitude, the post-FEC BER of Q-DQPSK is within 0.75dB of coherent QPSK, each with 3 bits of quantization, which is the identical penalty to the pre-FEC BER. Non-quantized MAP-DQPSK minimally outperforms Q-DQPSK and achieves identical performance for the same number of quantization bits. Q-DQPSK thus reaches MAP with a much lower complexity and outperforms soft DQPSK by 1.7dB. Reducing the quantization from 3 to 2 bits leads to a penalty of \(< 0.15dB\).

Capacity
In order to further demonstrate the improvement, the mutual information of the proposed quantized soft differential decoding is compared to the channel capacity. The mutual information for an input signal \( X \) and output \( Y \) is given by

\[
I(X;Y) = \sum_i p_i \int_0^\infty p(y|x_i) \log_2 \left( \frac{p(y|x_i)}{\sum_j p_j p(y|x_j)} \right) dy.
\]

For QPSK, the signal is identically distributed with \( p_i = 1/4 \). Fig. 6 shows the numerically computed mutual information of the proposed decoding compared to the ultimate boundaries. For low \( E_{cs}/N_0 \), differentially soft decoded QPSK slightly outperforms quantized decoding. However, in the high \( E_{cs}/N_0 \) region, quantized soft decoding has an edge of \( \geq 1.5dB \).

Fig. 6: Mutual information of quantized differential soft decoded QPSK compared to incoherent differential soft decoding, QPSK and the channel capacity.

Conclusions
We have presented a simple soft differential decoding scheme for QPSK in combination with block turbo coding that limits the performance penalty to 0.75dB compared to coherent QPSK. The scheme outperforms soft DQPSK decoding by 1.7dB after soft FEC decoding, and by 1.5dB with respect to the mutual information for high \( E_{cs}/N_0 \). The gap should be closed completely by applying iterative decoding in future research.

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