Low complexity blind estimation of residual carrier offset in orthogonal frequency division multiplexing-based wireless local area network systems

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Abstract: The presence of residual carrier frequency offset (CFO) in an orthogonal frequency division multiplexing (OFDM) system leads to a loss of orthogonality between the subcarriers. This introduces inter-subcarrier interference and degrades the system performance significantly. In the literature, Liu and Tureli proposed a blind CFO estimation method based on the observation that in a typical OFDM system not all the subcarriers are used for data transmission. However, the computational complexity of such a method is very high. Based on practical considerations, we propose an approximate closed-form solution for the blind estimation of CFO that is easily implementable at a very low cost. We also propose a successive CFO estimation and compensation procedure, which reduces the performance degradation of the proposed algorithm as compared with the method of Liu and Tureli when relatively large CFO values are assumed. In addition, a decision-directed extension of the successive algorithm, which further improves the CFO estimation at a slightly higher complexity, is also given.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) enables high data-rate transmission over frequency selective fading channels. It has been adopted as a standard for various wireless applications, such as digital audio and video broadcasting and IEEE 802.11a wireless local area network (WLAN) [1].

A typical OFDM system with frequency synchronisation is depicted in Fig. 1. In such a system, the input data symbols are assigned to different subcarriers according to a subcarrier allocation table. The modulation is then performed by using inverse fast Fourier transform (IFFT). Because of the difference between the reference frequencies of the transmitter and receiver local oscillators (LO) and the possible channel frequency doppler spread, the received signal is subject to certain amount of carrier frequency offset (CFO). The presence of CFO destroys the orthogonality between the subcarriers and introduces inter-carrier interference (ICI), which causes severe degradation in the performance of OFDM systems. Therefore accurate estimation and compensation of CFO is essential in order to ensure a good performance. In OFDM systems, CFO estimation is usually carried out in both analog and digital parts of the system. To ensure that the local oscillator at the analog front end of the receiver is operating with sufficient accuracy, its reference frequency is continuously adjusted by the frequency offset estimated in the analog coarse frequency synchronisation unit [2], normally through the use of a phase-locked loop. The received signal after analog to digital conversion can be expressed as [3]

\[ y(k) = E W \rho H s(k) e^{j \phi_k (k-1) N + N_k} + n(k) \]  

where \( E = \text{diag}(1, e^{j \phi_0}, \ldots, e^{j(N-1)\phi_0}) \) is a diagonal matrix containing the residual CFO \( \phi_k \) after analog coarse frequency synchronisation. This model was derived by using a cyclic prefix that is longer than or equal to the channel memory length.

The CFO can be estimated using training signals known to the receiver. The most popular training-based methods use periodic training sequences [4, 5]. Using the periodicity of the training sequence, the CFO can be estimated by auto-correlating the periodic received training signals. Another class of CFO estimation method is the so-called blind methods. In this case, no transmission of training sequence is required. The CFO estimation is entirely based on the characteristics of the received signal. We are going to focus on a class of blind CFO estimation methods first proposed by Liu and Tureli [3].

In a practical OFDM system, some subcarriers at both ends of the allocated spectrum are left empty to avoid aliasing to the adjacent channels. We will refer to the non-data-carrying subcarriers as null subcarriers, and to the data-carrying subcarriers as simply data subcarriers. Let \( P \) out of \( N \) be data subcarriers, then \( W_P \) is an \( N \times P \) submatrix that is obtained from the \( N \times N \) inverse discrete Fourier transform (IDFT) matrix \( W_N \). \( H \) is a diagonal matrix containing the channel frequency response, \( s(k) \) is a \( P \times 1 \) vector containing the transmitted symbols, \( N_c \) denotes the length of the cyclic prefix and \( n(k) \) is an additive white Gaussian noise (AWGN) vector.
In [3], Liu and Tureli proposed a blind CFO estimation based on null subcarriers. They showed that CFO can be estimated by minimising the following cost function [3]

\[ J(z) = \sum_{l=1}^{L} \sum_{k=1}^{K} || w_l^H Z^{-1} y(k) ||^2 \] (2)

where \([l_1, l_2, \ldots, l_L]\) are the indices of \(L\) null subcarriers, \(K\) is the total number of OFDM symbols used for CFO estimation and \(Z = \text{diag}(1, z, z^2, \ldots, z^{N-1})\). \(w_l^H\) is the \(l\)th row of the DFT matrix. Using (2), it can be shown that \(z = e^{j \phi}\) is a zero of \(J(z)\) in the absence of noise [3]. The identifiability problem of this CFO estimation method was studied in [6, 7].

As shown in [3], the minimisation of (2) can be attained by using either a Music-like search algorithm or a rooting method. This algorithm is shown to have a good performance as compared to Cramer-Rao bound (CRB) [3] and it is not limited to very small CFOs as in [8]. The practical aspects of this algorithm and its experimental implementations are further studied in [9]. However, the cost function \(J(z)\) represents a polynomial in the complex variable \(z\) of order \(2(N-1)\). For a typical application, like wireless LAN (IEEE 802.11a standard), \(N = 64\). Hence, \(J(z)\) order becomes 126, and the computational complexity required to find its zeroes is very high. To reduce this complexity, Tureli et al. proposed an ESPRIT-like method in [10]. However, the computational complexity is still very high as a subspace computation is required for the method.

In [11], we proposed a method to reduce the computational complexity of Liu et al. method. Our method was derived by taking into account the fact that the residual CFO after analog coarse frequency synchronisation tends to be very small in practice. This is due, on one hand, to the use of a coarse synchronisation at the analog part of the receiver [2, 12] and, on the other hand, the stability of currently available oscillators [13] which helps significantly in minimising the drifting. In IEEE Standard 802.11a-1999 [1], coarse and fine CFO estimation are both used, and the stability of the carrier frequency that is required is \(\pm 20\) ppm maximum. This leads to worst case CFO value of \(\pm 0.063\) ( \(\pm 0.64\) subcarrier spacing). Because of the advances in radio frequency system design, the accuracy of the LOs is further improved in recent years. In the comparison criterion document of the IEEE 802.11n high-throughput wireless LAN working group [14], the CFO value between the transmitter and receiver LOs has been relaxed to 13.675 ppm which corresponds to normalised CFO value of 0.023. Therefore the assumption of \(|\phi| \ll 1\) is practically valid. As compared to [10], the method in [11] was shown to lead to a very low cost implementation while maintaining a comparable performance for small CFO when both Gaussian and multipath channels are used. Whereas, as compared to [8], the method in [11] leads to much better results when multipath channels are used. We will not repeat this study, which is already reported in [11].

In the following, however, we will show how to further improve the method in [11]. We propose, in particular, a new factorisation for the CFO matrix that helps approximate \(Z^{-1}\) with a very limited number of Taylor’s series terms. By limiting the number of these terms to 2 (first-order approximation), we derive a closed-form solution for the estimation of CFO. Our proposed factorisation method is based on the assumption that the residual CFO is small in a practical system. However, low order (first and second order) Taylor’s series approximation of the cost function in (2) can lead to some performance degradation. This degradation is more obvious in medium to high signal-to-noise ratio (SNR) region, where an error floor appears. To mitigate this degradation and bridge the performance gap between the proposed method and the method in [3] when low-order approximation is used, we further propose a procedure, which carries out successive CFO estimation and compensation. We will also introduce a convergence monitoring mechanism which ensures the convergence of the algorithm. We will show that the performance of this method can converge to the performance of [3] in two to three iterations even for the worst case CFO values of \([-0.64, 0.64]\) specified by the IEEE 802.11a standard [1]. Moreover, the complexity of the successive method is still significantly lower than that of [10]. We will also present a decision-directed procedure where the performance of CFO estimation can be further improved at a slightly more computational cost.

This paper is organised as follows. The method in [11] is first briefly reviewed in Section 2. In Section 3, we propose the new factorisation method and derive a closed-form solution for CFO estimation. The successive CFO estimation and compensation method and its extension to a decision-directed method are presented in Section 4. In Section 5, we present the simulation results of the proposed methods using a multi-path fading channel, and the concluding remarks are drawn in Section 6.

2 Previous method

In this section, we give a brief review of the method in [11], which will pave the way to our newly proposed method. The inverse diagonal matrix \(Z^{-1}\) in (2) can be re-written as follows

\[ Z^{-1} = \text{diag}(1, e^{-j \phi}, e^{-j 2\phi}, \ldots, e^{-j (N-1) \phi}) \]

\[ = e^{-j \phi (N-1)/2} \text{diag}(e^{j \phi (N-1)/2}, e^{j \phi (N-3)/2}, \ldots, e^{j \phi (1-N)/2}) \] (3)

where \(\phi\) now denotes a real variable. Using Taylor’s series expansion of an exponential function, we can have

\[ Z^{-1} \approx e^{-j \phi (N-1)/2} \sum_{n=0}^{\infty} \frac{(j \phi)^n}{2^n n!} D^n \] (4)

where \(D = \text{diag}(N-1), (N-3), \ldots, (1-N)\) and \(Q\) is a suitable integer (\(Q \ll N\)) satisfying

\[ Q \geq \frac{|\phi (N-1)|}{2} \] (5)

such that the error because of the series truncation is negligible [11]. Substituting (4) into (2) and letting
where the polynomial coefficients \( c_l \) are given by

\[
c_l = \left( \frac{j}{2} \right)^l \sum_{m=0}^l \binom{l}{m} (1)^{l-m} \sum_{k=1}^L a_{l-m}(k)a_{l,m}(k)
\]

with \( a_{l,k}(k) = 0 \) for \( l > Q \). The new cost function (6) is a polynomial of the real variable \( \phi \) of degree 2. In addition, it has been proven in [11] that all the polynomial coefficients are real. The minimisation of (6) is carried out by setting the derivative to 0 and using some standard root finding methods to search for the estimated CFO. As both \( \phi \) and the polynomial coefficients are real, the root finding methods only require real arithmetic operations. This on its own provides large saving in computations. In general, one complex multiplication is equal to four real multiplications. A further reduction in complexity is achieved by limiting the number of terms in the Taylor’s series to a minimum.

3 Proposed approximation method

3.1 Improving the approximation of \( Z^{-1} \)

The factorisation in (3) was aimed at increasing the denominators of Taylor’s series terms (4) by a factor of \( 2^4 \) so that a good approximation to \( Z^{-1} \) can be achieved with a limited number of terms. In the following, we will propose another factorisation which allows to increase this number to \( 2^3 \) and more if needed. Based on (3), we can write

\[
Z^{-1} = e^{-j(d(N-1)/2)}(E_1 + E_2)
\]

Where

\[
E_1 = \text{diag}(e^{j(d(N-1)/2)}, e^{j(d(N-3)/2)}, \ldots, e^{j(d(1/2)}, 0, \ldots, 0)
\]

\[
e^{j(d(N-1)/4)} \text{diag}(e^{j(d(3-N)/4)}, \ldots, e^{j(d(3-N)/4)}, 0, \ldots, 0)
\]

and

\[
E_2 = \text{diag}(0, \ldots, 0, e^{-j(d(1/2)}, \ldots, e^{j(d(1-N)/2))}
\]

\[
e^{j(d(1-N)/4)} \text{diag}(0, \ldots, 0, e^{j(d(3-N)/4)}, \ldots, e^{j(d(3-N)/4))}
\]

Now, using Taylor’s series expansion in (9), we obtain

\[
E_1 = \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (j\phi)^{m+n} \frac{1}{4^{m+n}m!n!} (N-1)^m D_1^m
\]

where

\[
D_1^m = \text{diag}((N-1)^m, (N-5)^m, \ldots, (3-N)^m, 0, \ldots, 0)
\]

Similarly, we can show that

\[
E_2 = \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (-1)^n (j\phi)^{m+n} \frac{1}{4^{m+n}m!n!} (N-1)^m D_2^m
\]

where

\[
D_2^m = \text{diag}(0, \ldots, 0, (N-3)^m, (N-7)^m, \ldots, (1-N)^m)
\]

Substituting (11) and (13) into (8) leads to

\[
Z^{-1} = e^{j(d(N-1)/2}) \frac{1}{2} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (j\phi)^{m+n} \frac{1}{4^{m+n}m!n!} (N-1)^m D_1^m \times (N-1)^n \frac{1}{2} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (j\phi)^{m+n} \frac{1}{4^{m+n}m!n!} (N-1)^n D_2^m
\]

If Taylor’s series of each exponential is truncated to a suitable number of \( O \) terms. Then, (15) can be approximated by the following matrix polynomial

\[
Z^{-1} \approx e^{j(d(N-1)/2}) \sum_{q=0}^{2^O} C_q \phi^q
\]

where

\[
C_q = \frac{j}{4} \sum_{m=0}^{q} \sum_{n=0}^{q-m} (N-1)^m \frac{1}{2} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (j\phi)^{m+n} \frac{1}{4^{m+n}m!n!} (N-1)^m D_1^m \times (N-1)^n \frac{1}{2} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (j\phi)^{m+n} \frac{1}{4^{m+n}m!n!} (N-1)^n D_2^m
\]

Note that \( C_q \) just given above is defined for \( q = 0, 1, \ldots, 2O \) and it is equal to the zero-matrix for \( q > 2O \). To simplify the calculations, let us define the scalar \( b_{q,k} \) as follows

\[
b_{q,k} = w_q^q C_q \phi^q
\]

Now, let us substitute (16) into the cost function (2). This leads to the new approximate cost function

\[
J(\phi) \approx e^{j(d(N-1)/2}) \sum_{q=0}^{2^O} \sum_{i=0}^{q} \sum_{j=0}^{q-i} \sum_{k=1}^{q} b_{q,k} b_{q,k}\]

where the polynomial coefficients are given by

\[
d_q = \sum_{l=0}^{q} \sum_{k=1}^{q} b_{q,l-k} b_{q,k}
\]

We can notice that \( b_{q,k} \neq 0 \) for \( q > 2O \) and \( d_q = d'_q \), that is all the coefficients of the polynomial are real-valued. This method can be reiterated again if needed. However, this also leads to an increase in the polynomial degree. Since our aim is twofold: (1) to reduce the computational complexity of the carrier offset estimation problem while minimising the error because of the truncation of the Taylor’s series, (2) to estimate the carrier offset using a closed-form expression, we will not proceed any further. Let \( n \) be a positive integer. The roots of a polynomial of degree \( n \) can be computed using an algebraic formula if and only if \( n \leq 4 \). If the proposed method is reiterated, then we will obtain a polynomial for the cost function whose degree can be shown to be much larger than 4 even if we set \( O = 1 \).

3.2 Approximate closed-form solution

In practice, the residual CFO can be so small that only a very limited number of Taylor’s series is needed for the approximation. In this case, we can compute directly the carrier offset through a simple formula as follows. For \( O = 1 \), the cost function polynomial is of degree 4 and its derivative with respect to \( \phi \) is a cubic polynomial whose zeros or roots can be computed directly using Cardano’s formula [16]. To this end, we should first rewrite this cubic polynomial (i.e. derivative of the cost function) as follows

\[
\phi^3 + u\phi^2 + v\phi + r = 0
\]
where
\[ u = \frac{3d_1}{4d_3}, \quad v = \frac{d_2}{2d_4}, \quad r = \frac{d_1}{4d_4} \] (22)

Now, let us compute
\[ a = (3v - a')/3 \quad b = (u^3 - 9uv + 27r)/27 \]
\[ S = \left( \frac{b}{2} + \frac{\sqrt[3]{b^2 + 4a^3}}{27} \right)^{3/2} \quad T = \left( \frac{b}{2} - \frac{\sqrt[3]{b^2 + 4a^3}}{27} \right)^{3/2} \] (23)

Finally, the three roots are given by [16]
\[ \phi_1 = (S + T) - \frac{u}{3} \]
\[ \phi_2 = -\frac{1}{2}(S + T) + \frac{\sqrt{3}}{2}(S - T) - \frac{u}{3} \] (24)
\[ \phi_3 = -\frac{1}{2}(S + T) - \frac{\sqrt{3}}{2}(S - T) - \frac{u}{3} \]

The solution sought is the root that once substituted in (19) leads to the minimum. As we are looking for a real solution, we should test only the real roots for the minimum. The complex-valued ones need not be tested. It is worthwhile to mention that for a cubic polynomial with real coefficients as in our case one of the roots will always be real [15]. The summary of this algorithm is given in Table 1.

3.3 Computational complexity of the proposed method

Since \( Q = 1 \), the computational complexity because of (22), (23) and (24) is negligible as compared to the complexity of computing the four polynomial coefficients \( d_1, d_2, d_3 \) and \( d_4 \). Every coefficient \( d_q \) requires the computation of \( b_{i,q} \) in (18), where we can easily notice that \( C_q \) is just a diagonal matrix and both \( w_1^T \) and \( y(k) \) are just vectors. As a result, the computational complexity of the polynomial coefficients can be shown to be about \( O(LKN) \), that is, it is similar to the method in [11] when \( Q = 1 \). While the ESPRIT method in [10] is based on the estimation and construction of a certain matrix \( \mathbf{A} \), its pseudo inverse and the construction of another matrix called \( \Delta \). These operations have at least the following computational complexities \( O(K(N - M)(M + 1)^2) \), \( O(P^3) + O(MP^3) \) and \( O(MP^3) \), where \( M \geq P \) and \( P \) is the number of data subcarriers. In practice, both \( M \) and \( P \) are of comparable size as \( N \). Hence, it should be obvious that the computational complexity of the proposed method is much lower than the methods in [3, 10].

4 Successive blind frequency offset estimation and compensation

The performance of the proposed closed-form solution for CFO estimation depends on the accuracy of Taylor’s series approximation of (2), which is determined by the number of terms used in the summation as well as the residual CFO values. As we only use \( Q = 1 \) (low cost) in the proposed method, the accuracy of the approximation is degraded when the true CFO value is relatively large. As a result, there will be some performance degradation in the mean square error (MSE) of the CFO estimation using the proposed method compared to the method in [3]. An error floor is likely to be visible at high SNR values when the residual CFO is equal to one subcarrier spacing. To further improve the performance of the proposed method, we present, in this section, an effective successive CFO estimation and compensation method.

In the first iteration, we use the proposed low complexity method to find an initial estimate for the CFO \( \phi_1 \). Then, carrier offset compensation can be performed on the received signal \( y(k) \). To this end, let us define the frequency offset compensation matrix in the first iteration as \( C_1 = \text{diag} \left( 1, e^{-j\phi_1}, \ldots, e^{-j(N-1)\phi_1} \right) \). The time domain received signal after CFO compensation can thus be written as
\[ y(k) = C_1 e^{-j\phi_1 k} x(k-N_0) + n(k) \] (25)
where \( \phi_0 = \phi_0(k) \) is the true CFO, \( E_y = \text{diag} \left( 1, e^{j\phi_1}, \ldots, e^{j(N-1)\phi_1} \right) \) denotes the residual CFO matrix, and \( n(k) \) is the residual CFO after compensation in the first iteration. The noise vector \( n(k) = C_1 e^{-j\phi_0(k-1)} n(k-1) \) is still AWGN.

Let us consider \( E_y e^{j\phi_0 k} = E(n(k)) \) in (25). It is obvious that when the CFO estimation is perfect, we should have \( \phi_0 = \phi_0(k) = 0 \) and \( E(n(k)) = I \). In the following, we consider the case where the CFO is not perfectly estimated (\( \phi_0 \neq \phi_0(k) \)) but it is close enough for the following condition to hold
\[ |\phi_0(k)| = |\phi_0 - \phi_0(k)| \leq |\phi_0| \] (26)

Note that the mean of the noise term \( n(k) \) after CFO compensation is \( E(n(k)) = 0 = E(n(k)) \), and its covariance matrix can be calculated as
\[ R_{n(k),n(k)} = E(n(k)) n(k)^T = \sigma_n^2 I = R_{n(k),n(k)} \] (27)
From (27), we can see that \( n(k) \) is statistically the same as \( n(k) \) and both are Gaussian. This means that the noise power remains constant after carrier offset compensation. The estimation and compensation process does not introduce any noise amplification. In the second iteration, since (26) holds, it is obvious that \( e^{-j\phi_0} \) can be represented more accurately using the first-order Taylor’s series approximation as compared to \( e^{-j\phi_0} \) and therefore our estimation method may lead to a better estimate. That is after the second iteration, we get \( \phi_2 \) such that \( |\phi_2| = |\phi_0 - \phi_2| = |\phi_0 - (\phi_0 + \phi_2)| \) is very small. Now if at every iteration, we have \( |\phi_{r+1}| = |\phi_{r} - \phi_{r+1}| < |\phi_0| \), then eventually, we will have
\[ \lim_{r \to \infty} |\phi_{r+1}| = 0 \Rightarrow \lim_{r \to \infty} E_y e^{j\phi_{r+1} k} = I \]

Table 1: Proposed closed-form solution CFO estimation method

<table>
<thead>
<tr>
<th>Initialisation</th>
<th>Start with the received signal vector ( y(0) ), and set ( Q = 1 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>(1) Compute the coefficients ( d_q ) of the cost function polynomial ( J_4(\phi) ) using (20) for ( q = 1, 2, 3, 4 ).</td>
</tr>
<tr>
<td></td>
<td>(2) Compute the coefficients ( u, v ) and ( r ) of ( 1/4d_4 ) ( \frac{\partial J_4(\phi)}{\partial \phi} ) using (22).</td>
</tr>
<tr>
<td></td>
<td>(3) Compute the three roots ( \phi_1, \phi_2 ), and ( \phi_3 ) using (24) and discard those which are complex. At least one root should be real as explained in the text.</td>
</tr>
<tr>
<td></td>
<td>(4) If more than one root is real, then compute ( J_4(\phi) ) for each root and choose the one that leads to the smallest value for ( J_4(\phi) ).</td>
</tr>
</tbody>
</table>

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Table 2: Summary of the successive CFO estimation and compensation algorithm

Initialisation
Set iteration number $i = 1$ and the CFO threshold $\eta$.

Algorithm
(1) Substitute $y(k)$ into cost function (2) and solve for $\phi_i$ that minimises (2) using the proposed method (Table 1).
(2) Perform carrier CFO compensation and update the new $y(k)$ according to (25).
(3) If $|\phi_i| < \eta$, exit iteration, else go to (4).
(4) If $|\phi_i| < |\phi_{i-1}|$ and $i > 1$ or $i = 1$ $i = i + 1$, go back to (1).
else
Go back to the $[i - 1]$th iteration and increase the order of Taylor's series approximation.

The question now is how can we ensure, or at least monitor the algorithm such that the previous condition is maintained so as to guarantee convergence. In the successive algorithm (Table 2), we can monitor the convergence through the amplitude of $|\phi_i|$ from the second iteration onwards. If $|\phi_i| < |\phi_{i-1}|$, then the algorithm is moving towards the right direction. Ideally, we should monitor the amplitude of $|\phi_i|$ for different iterations. If $|\phi_i| < |\phi_{i-1}|$, then the algorithm is converging. However, we do not know $|\phi_i|$ as it requires the knowledge of the true CFO value $\phi_k$ to be estimated. To overcome this, an alternative way is to monitor the amplitude of the CFO estimates $|\phi_i|$ at different iterations.

If the algorithm is converging, then we should expect the amplitude of the CFO estimate $|\phi_i|$ to decrease as the number of iterations increases. We can also use $|\phi_i|$ to stop the algorithm should we find that $|\phi_i|$ is too small and reiterating the estimator one more time will not lead to any significant improvement in CFO estimation.

Our objective is to minimise the computational complexity while improving the CFO estimation. Therefore we need to keep the order of Taylor’s series approximation as small as possible and the number of iterations as low as possible. If $|\phi_i| > |\phi_{i-1}|$, we know that the iteration is likely to diverge. In this case, we need to have a better estimate to ensure convergence. This could be achieved, for example, by increasing the order of Taylor’s series approximation in the $[i - 1]$th iteration. In the simulations, we have found that under practical considerations, the first-order Taylor’s series approximation was fine.

Given that the convergence condition is continuously monitored and enforced through iterations, the proposed successive CFO estimation and compensation method could eventually achieve the same performance as the search method in [3]. The algorithm is summarised in Table 2. Here, $\eta$ is a small threshold value for CFO. When the estimated residual CFO gets smaller than this value, there is no point in going on with the iterations as the improvement will be marginal.

The computational complexity of this method is roughly $n_{d}$ times the complexity of the proposed closed-form solution, where $n_{d}$ is the number of iterations. As we are going to show later in the simulations, the algorithm converges to the performance of the method in [3] in two to three iterations for practical CFO values. Therefore the complexity of the successive algorithm is still much less than the ESPRIT method in [10].

Because of averaging, the MSE of CFO estimation gets smaller as the number of null subcarriers, that is, $L$ used in cost function (2), gets larger. In practice, we cannot afford to have many null subcarriers in one OFDM symbol as the bandwidth efficiency is reduced. On the other hand, if we know all the transmitted signals $s(k)$, we have all the subcarriers available for CFO estimation. It is obviously impossible to know the transmitted signals at the receiver for blind method. If the CFO estimation is accurate in the initial iteration, by performing CFO compensation and OFDM detection on a set of high SNR data subcarriers, we are able to obtain relatively accurate estimates $\hat{s}(k)$ of the transmitted signals $s(k)$ on these subcarriers. Here, in order to keep the computational complexity low, we limit ourselves to the case where only one OFDM symbol is used for CFO estimation, that is, $K = 1$ as in (2). Later in the simulation results section, we will show that $K = 1$ gives us good performance for all the proposed algorithms. In this case, we can drop the OFDM symbol index $k$ in all the following formulations. Let us denote the selected high SNR data subcarriers as $d$ and the set of null subcarriers as $I$. We can thus use a decision-directed method and re-formulate the cost function as

$$J(z) = \sum_{i \in d} \|w_i^H z^{-1} y - h_i \hat{s}_i^* \|^2 + \sum_{i \in I} \|w_i^H z^{-1} y \|^2$$

where $h_i, \hat{s}_i$ are the channel response and detected signal on subcarrier $i$. The two terms of the summation correspond to the cost function on the selected data subcarriers and the null subcarriers. Using Taylor’s series expansion as before, the decision directed cost function is given by

$$J(\phi) = \sum_{i \in d} \|w_i^H \sum_{n=0}^{\infty} \frac{\phi_i^n}{n!} D_i^n y - h_i \hat{s}_i^* \|^2 + J_{d0}(\phi)$$

where $D_i = \text{diag}(0, -1, -2, \ldots, 1 - N)$ and we use subscript $d$ to denote decision directed. $J_{d0}(\phi)$ is the cost function on the null subcarriers given in (19). Comparing this to (3), we can see that for the data subcarriers, we can no longer bring $e^{-J\phi}(N-1)/2$ out of the modulus operation. Therefore in Taylor’s series summation, we do not have the $2^\text{nd}$ term at the denominator. As a result, the amplitudes of higher order terms do not decay as fast as compared to (2) and we need to include more terms in Taylor’s series summation in order to get a good approximation of $Z^1$.

Suppose we use $M$ terms in Taylor’s series approximation and set $a_{i,n} = w_i^H D_i y$, the cost function in (29) could be expanded as

$$J(\phi) = \sum_{i \in d} \left[ \sum_{m=0}^{M} \sum_{n=0}^{M} \frac{(\phi_i)^n (\phi_j)^m}{n! m!} a_{i,n} a_{j,m} - 2\Re\left( \sum_{m=0}^{M} \frac{(\phi_i)^n a_{i,n}(h_i \hat{s}_i)^m}{n!} \right) + |h_i \hat{s}_i|^2 \right]$$

$$+ J_{d0}(\phi)$$

where $\Re(\cdot)$ denotes the real part of a complex number. Here, we assume the channel to be known at the receiver. The channel can also be estimated blindly using blind channel estimation methods such as [17]. In practice, the decision-directed method is only invoked starting from the second iteration onwards as the detected symbol is only available after the first iteration. That means an initial CFO estimate is already obtained in the first iteration. Therefore what the decision-directed method needs to estimate is only the residual CFO, which is much smaller than $\phi_0$. Note that the cost function of data subcarriers is a function of $\phi$ with order $2M$. In practice, we can set $M = 2$ such
that the cost functions of the data subcarriers and the null subcarriers are both of order 4. The overall cost function in (30) is therefore also order 4 and we can use the proposed closed-form solution to find $\phi$ that minimises the total cost. However, the coefficients of the polynomial need to be re-calculated according to (30). Note that to ensure convergence of the algorithm, we again need to monitor the amplitude of the CFO estimate $|d_0|$ at different iterations.

The complexity of the decision-directed blind CFO estimation technique is higher because more subcarriers are used in the cost function calculation. In practice, the size of $d$ should be chosen such that a good trade-off between complexity and performance is achieved.

5 Simulation results

5.1 Simulation results for the proposed closed-form solution

Computer simulations were performed for an OFDM system with 64 subcarriers and length-16 cyclic prefix. According to the specifications given in IEEE 802.11a, the null subcarriers are placed consecutively from subcarriers 27–37 [1]. We define the subcarrier spacing as $\omega = 2\pi/N$. To assess the performance of the proposed method, we define the estimation MSE as [10]

$$\text{MSE} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left( \frac{\hat{\phi} - \phi_0}{\omega} \right)^2$$

(31)

where $\hat{\phi}$ and $\phi_0$ represent the estimated and true CFOs, respectively, and $N_s$ denotes the total number of Monte Carlo trials. In all the simulations, we only use one OFDM symbol to perform CFO estimation, that is $K = 1$ in all the cost functions. We studied the performance of the proposed closed-form approximation method using channel model A of the HiperLan II channel models [18]. It is a multipath Rayleigh fading channel with exponential power delay profile and root mean square (RMS) delay spread equal to one modulation symbol interval.

Fig. 2 shows the MSE performance of the proposed closed-form solution. Here, the true CFO for each OFDM symbol is chosen from a uniformly distributed random variable between $[-0.25\omega, 0.25\omega]$. We compare the performance of the proposed method with the method in [11]. We can see that the proposed method with $Q = 1$ shows a better MSE performance than the method in [11] with both $Q = 1$ and $Q = 2$. Using the proposed method with $Q = 1$, the cost function to solve is a third-order polynomial. From the complexity point of view, this is similar with the method in [11] with $Q = 2$. However, because of the $4^n$ term in the denominator of Taylor’s series expansion, the proposed method achieves better performance. Also shown in the same figure is the MSE performance of the search method in [3]. Only at high SNR regions, the proposed method suffers small degradation as compared to the search method in [3].

It was shown in [7] that the optimal placement of the null subcarriers to minimise the Cramer-Rao bound is to place them with even spacing across the whole OFDM symbol. We adopted this optimal placement of null subcarriers in the subsequent simulations. We used a total of 11 subcarriers (same as the consecutive null subcarrier case) spaced 6 subcarriers apart, that is we placed the null subcarriers at the following locations [1, 7, ..., 55, 61]. If it is necessary to place some null subcarriers consecutively at both ends of the spectrum as guard band, we could still place a few extra null subcarriers evenly across the remaining subcarriers to achieve optimality as in [7].

The MSE performance of the proposed method for $-0.5\omega \leq \phi_0 \leq 0.5\omega$ is shown in Fig. 3. In this case, we purposely set the CFO value larger such that the degradation because of lower order approximation in the cost function is more visible. We can see that the proposed method with $Q = 1$ still performs better than method in [11] with $Q = 1$ and achieves similar performance as method in [11] with $Q = 2$. The performance advantage of the proposed method is more obvious if we compare the symbol
error rate (SER) performance of the three schemes as shown in Fig. 3. In this case, we first use the proposed blind method to obtain the CFO estimate. The estimated offset is then compensated from the received signal and normal OFDM detection is carried out to detect the transmitted data. To separate issues of channel estimation from CFO estimation, we assume that the channel estimation is perfect. We can see that without CFO estimation and compensation, the OFDM system fails. Using the proposed method with $Q = 1$, the performance is about 8 dB better than the method in [11] ($Q = 1$) at SER of $10^{-3}$. The proposed method achieves similar performance as the method in [11] for $Q = 2$.

One major observation from Fig. 3 is the error floor effect at high SNR values because of the first-order approximation of the cost function used in the proposed method. The same observation can be made in the SER performance in Fig. 3. We are going to show later that this error floor can be effectively removed using the proposed successive CFO estimation and compensation procedure.

### 5.2 Simulation results for the successive CFO estimation and compensation

The performance of the successive CFO estimation method is also studied following the same simulation setup as before. Fig. 4 shows the MSE performance of the proposed method with successive CFO estimation and compensation for $-0.7 \omega \leq \phi_0 \leq 0.7 \omega$. Here, we purposely increased the values of the CFO such that the worst case CFO of $\pm 0.64 \omega$ specified by IEEE 802.11a [1] is included. We can see that the performance is improved significantly using the proposed algorithm (Table 2). The MSE of first iteration, which is the same as the proposed blind method without any iteration, has an error floor at MSE around $3 \times 10^{-3}$. With the proposed method, this error floor can be effectively removed after the second iteration. Hence, in practice, the successive method could be stopped after the second iteration and the extra complexity introduced is very low. If we compare the MSE after convergence with the MSE of the method in [3], we can see that after two iterations, the proposed method achieves almost the same MSE performance as the search method in [3]. We have implemented the convergence monitoring mechanism shown in Table 2 in the algorithm. We found that $Q = 1$ is good enough to guarantee convergence in all the SNR values. Fig. 4 also shows the SER performance of the successive CFO method. We can see that the SER performance takes two iterations only to achieve a performance similar to the case where we have a perfect knowledge and compensation of the CFO.

Fig. 4 shows the convergence behaviour of the successive algorithm for a particular channel realisation at SNR of 20 dB. The actual CFO value is fixed at 0.7 subcarrier spacing. From the upper figure in Fig. 5, we can see that the MSE of the CFO estimation converges to the search method in two iterations, which is consistent with the results shown in Fig. 4. The lower figure in Fig. 5 plots the amplitude of the CFO estimates $|\phi_i|$ for different iterations. We can see that the amplitude of $|\phi_i|$ is indeed decreasing as the number of iterations $i$ increases, which is an indication of the convergence of the algorithm as explained earlier.

As the successive CFO estimation and compensation is a generic algorithm, it is applicable to the method in [11] as well. Fig. 6 shows the performance of the proposed method when used with the method in [11] for $Q = 1$. The successive method again significantly improves the performance of blind CFO method and effectively removes the error floor at high SNR values. In this case, the MSE also converges to the same MSE as the method in [3] after three iterations.

Simulations were also carried out to study the performance of the decision directed CFO estimation algorithm. Fig. 7 shows the MSE performance. Here, in the first iteration, we use the proposed closed-form approximation method to get the initial CFO estimate. The estimated CFO is then compensated. We then use least squares detection to obtain the estimate of the transmitted signal on the data subcarriers. From the second iteration onwards, we use both the null subcarriers and the data subcarriers to
perform CFO estimation. As we have more subcarriers available to perform CFO estimation, we expect better performance compared to non-decision-directed method. This is also verified from Fig. 7. In this simulation, we choose the 11 highest-SNR data subcarriers combined with the 11 null subcarriers for CFO estimation. We use $M=2$ and $Q=1$ in the cost function (30). A performance gain of 3 dB can be achieved as compared to non-decision-directed method as we use twice the subcarriers.

6 Conclusion

We proposed an approximate closed-form solution for blind estimation of residual carrier offset in OFDM-based wireless LAN systems. We also proposed a successive CFO estimation and compensation procedure which further improves the performance of the proposed algorithm. Indeed, by reiterating the proposed algorithm, we were able to achieve similar performance to the method in [1] yet at a much lower computational complexity. A decision-directed extension of the proposed method was also given, which achieves even better performance at the cost of slightly higher computational complexity.

7 References

1 IEEE P802.11a Part11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications: high-speed physical layer in the 5GHz band, 1999