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Almost Decentralized Lyapunov-based Nonlinear Model Predictive Control

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Abstract—This paper proposes an almost decentralized solution to the problem of stabilizing a network of discrete-time nonlinear systems with coupled dynamics that are subject to local state/input constraints. By “almost decentralized” we mean that each local controller is allowed to use the states of neighboring systems for feedback, whereas it is not permitted to employ iterations between the systems in the network to compute the control action. The controller synthesis method used in this work is Lyapunov-based model predictive control (MPC). The stabilization conditions are decentralized via a set of structured control Lyapunov functions (CLFs) for which the maximum over all the functions in the set is a CLF for the global network of systems. However, this does not necessarily imply that each function is a CLF for its corresponding subsystem. Additionally, we provide a solution for relaxing the temporal monotonicity of the CLF for the overall network. For structured CLFs defined using the infinity norm, we show that the decentralized MPC algorithm can be implemented by solving a single linear program in each network node. A non-trivial example illustrates the effectiveness of the developed theory and shows that the proposed method can perform as well as more complex distributed, iteration-based MPC algorithms.

I. INTRODUCTION

Over the last few years control of networks of interacting dynamical systems has gained a continuously increasing attention from the systems and control community. Examples of such networked dynamical systems (NDS) are electrical power networks (see e.g., [1], [2]), automated highways with formation control of autonomous vehicles (see for instance [3]) and urban water supply networks (see e.g., [4]), to name just a few. The large size and complexity of networked dynamical systems generally hamper the application of centralized control laws, which is the main reason for which the non-centralized implementation of controllers for NDS has become a major concern.

The design of non-centralized control laws for NDS is not straightforward, as these systems are often subject to strong coupling between local dynamics, hard and possibly coupled constraints on the control actions and states, and communication constraints. These characteristics are generally reflected in the structure of the control law applied to stabilize a particular type of NDS. Roughly speaking, we can divide non-centralized control schemes into two categories: *decentralized* techniques, in which local controllers operate

without mutual exchange of information, and *distributed* methods that exploit (iterative) communication over a usually predefined sparsely structured communication network to compute the control action. Although solutions to specific varieties of structured control problems exist, a general theory for synthesizing stabilizing control laws under arbitrary system and information constraints is still lacking.

Recently, a lot of research has been dedicated to model predictive control (MPC) as a tool for setting up non-centralized control algorithms. Interesting examples of non-centralized MPC can be found in [1]–[11]. When stability is the main focus, a successful technique within MPC is the so-called Lyapunov-based MPC (L-MPC) approach, see, e.g., [12]–[14]. L-MPC, which makes use of an explicit control Lyapunov function (CLF) to achieve stability, was already successfully applied to networked control systems, see [15], [16]. Therein the focus is more on communication network effects such as time delays and packet dropouts, rather than on decentralized stabilization of large-scale NDS.

In this paper we propose a non-centralized L-MPC scheme for discrete-time nonlinear NDS that are subject to coupled local dynamics and separable constraints. The key ingredient of the proposed approach is a set of structured CLFs with a particular type of convergence conditions. While these conditions do not impose that each of the structured functions should decrease monotonously, as typically required for a CLF, they provide a standard CLF for the overall network. Still, a demand for monotonous convergence of the overall CLF candidate might be too conservative in practice. Therefore, we provide a solution for relaxing the temporal monotonicity of the global CLF based on an adaptation of the Lyapunov-Razumikhin technique (see for instance [17], [18]), which was originally developed for systems with time delays. The proposed L-MPC scheme needs no global coordination and can be implemented in an almost decentralized fashion. By this we mean that the controller only requires one run of information exchange between direct neighbors per sampling instant. This is in contrast to many of the existing non-centralized MPC schemes, which either require iterative computations or global information, see e.g., [1], [2], or, employ contractive constraints or small gain conditions, see e.g., [9], [10], to guarantee closed-loop stability.

For systems that are affine in the control input, we show that by employing infinity-norm based structured CLFs, the proposed L-MPC setup can be implemented by solving a single linear problem per sampling instant and node. The

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effectiveness and computational complexity of the proposed scheme is assessed on a non-trivial example.

II. PRELIMINARIES

A. Basic Notions and Definitions

Let $\mathbb{R}, \mathbb{R}_+, \mathbb{Z}$ and $\mathbb{Z}_+$ denote the field of real numbers, the set of non-negative reals, the set of integer numbers and the set of non-negative integers, respectively. For every $c \in \mathbb{R}$ and $\Pi \subseteq \mathbb{R}$ we define $\Pi_{\geq c(\leq c)} := \{k \in \Pi \mid k \geq c(k \leq c)\}$ and $\mathbb{Z}_\Pi := \{k \in \mathbb{Z} \mid k \in \Pi\}$. For a finite set of vectors $\{x_i\}_{i \in \mathbb{Z}_{[1, N]}}$, $x_i \in \mathbb{R}^{n_i}$, $N \in \mathbb{Z}_+$, we use $\text{col}(\{x_i\}_{i \in \mathbb{Z}_{[1, N]}})$, and equivalently $\text{col}(x_1, \dots, x_N)$, to denote the column vector $(x_1^\top, \dots, x_N^\top)^\top$. Let $\mathbf{0}_n$ denote the zero vector in $\mathbb{R}^n$. For a set $\mathcal{S} \subseteq \mathbb{R}^n$, we denote by $\text{int}(\mathcal{S})$ the interior of $\mathcal{S}$. For a vector $x \in \mathbb{R}^n$, let $\|x\|$ denote an arbitrary p -norm and let $[x]_i$, $i \in \mathbb{Z}_{[1, n]}$ be the i -th component of x . The ∞ -norm of a vector $x \in \mathbb{R}^n$ is defined as $\|x\|_\infty := \max_{i=1, \dots, n} |[x]_i|$, where $|\cdot|$ denotes the absolute value. For a matrix $M \in \mathbb{R}^{m \times n}$, let $\|M\| := \max_{x \neq \mathbf{0}_n} \frac{\|Mx\|}{\|x\|}$ denote its corresponding induced matrix norm. Let $\mathbf{z} := \{z(l)\}_{l \in \mathbb{Z}_+}$ with $z(l) \in \mathbb{R}^n$ for all $l \in \mathbb{Z}_+$ denote an arbitrary sequence. Define $\|\mathbf{z}\| := \sup\{\|z(l)\| \mid l \in \mathbb{Z}_+\}$ and $\mathbf{z}_{[0, k]} := \{z(l)\}_{l \in \mathbb{Z}_{[0, k]}}$. For some $s \in \mathbb{R}$, let $\lfloor s \rfloor := \max\{n \in \mathbb{Z} \mid n \leq s\}$ be the floor function. A function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\varphi(0) = 0$. A function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to class $\mathcal{K}_\infty$ if $\varphi \in \mathcal{K}$ and it is radially unbounded, i.e., $\lim_{s \rightarrow \infty} \varphi(s) = \infty$.

B. Lyapunov Stability

Consider the discrete-time, autonomous nonlinear system

$$x(k+1) \in \Phi(x(k)), \quad k \in \mathbb{Z}_+, \quad (1)$$

where $x(k) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state at the discrete-time instant $k \in \mathbb{Z}_+$. The (possibly nonlinear) set-valued mapping $\Phi : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is such that $\Phi(x)$ is compact and nonempty for all $x \in \mathbb{X}$. We assume that the origin is an equilibrium of (1), i.e., $\Phi(\mathbf{0}_n) = \{\mathbf{0}_n\}$.

Definition II.1 A set $\mathcal{P} \subseteq \mathbb{R}^n$ is *Positively Invariant (PI)* for system (1) if $\forall x \in \mathcal{P}$ it holds that $\Phi(x) \subseteq \mathcal{P}$.

Definition II.2 (i) System (1) is *Lyapunov stable* if $\forall \varepsilon > 0$, $\exists \delta(\varepsilon) > 0$ such that for all state trajectories of (1) it holds that $\|x(0)\| \leq \delta(\varepsilon) \Rightarrow \|x(k)\| \leq \varepsilon$ for all $k \in \mathbb{Z}_+$. (ii) Let $\mathbb{X} \subseteq \mathbb{R}^n$ and $\mathbf{0}_n \in \text{int}(\mathbb{X})$. The origin of (1) is *attractive* in $\mathbb{X}$ if for any $x(0) \in \mathbb{X}$ it holds that all corresponding trajectories of (1) satisfy $\lim_{k \rightarrow \infty} \|x(k)\| = 0$. (iii) System (1) is *asymptotically stable* in $\mathbb{X}$ if it is Lyapunov stable and attractive in $\mathbb{X}$.

Theorem II.3 Let $\mathbb{X}$ be a PI set for system (1) and let $\mathbf{0}_n \in \text{int}(\mathbb{X})$. Furthermore, let $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, $\rho \in \mathbb{R}_{[0, 1]}$ and let $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ be a function such that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|) \quad (2a)$$

$$V(x^+) \leq \rho V(x) \quad (2b)$$

for all $x \in \mathbb{X}$ and all $x^+ \in \Phi(x)$. Then system (1) is asymptotically stable in $\mathbb{X}$.

A function V that satisfies the conditions of Theorem II.3 is called a *Lyapunov function*. The proof of Theorem II.3 can be obtained from [19], Theorem 2.8. Note that in [19] continuity of the function V on $\mathbb{X}$, i.e., not solely at the origin as specified by Theorem II.3, is required only to show certain robustness properties. See also [20] for results on stability of discrete-time systems via discontinuous Lyapunov functions.

C. CLFs for discrete-time systems

Consider the discrete-time constrained nonlinear system

$$x(k+1) = \phi(x(k), u(k)), \quad k \in \mathbb{Z}_+, \quad (3)$$

where $x(k) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state and $u(k) \in \mathbb{U} \subseteq \mathbb{R}^m$ is the control input at the discrete-time instant $k \in \mathbb{Z}_+$. The function $\phi : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is nonlinear with $\phi(\mathbf{0}_n, \mathbf{0}_m) = \mathbf{0}_n$. We assume that $\mathbb{X}$ and $\mathbb{U}$ are bounded sets with $\mathbf{0}_n \in \text{int}(\mathbb{X})$ and $\mathbf{0}_m \in \text{int}(\mathbb{U})$. Next, let $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and let $\rho \in \mathbb{R}_{[0, 1]}$.

Definition II.4 A function $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ that satisfies

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathbb{R}^n, \quad (4)$$

and for which there exists a control law, possibly set valued, $\pi : \mathbb{R}^n \rightrightarrows \mathbb{U}$ such that

$$V(\phi(x, u)) \leq \rho V(x), \quad \forall x \in \mathbb{X}, \forall u \in \pi(x),$$

is called a control Lyapunov function (CLF) in $\mathbb{X}$ for (3).

For results on CLFs for discrete-time systems we refer the interested reader to [21] and [22].

III. MAIN RESULTS

In order to set-up the control algorithm, we first introduce a framework for defining a network of systems. Consider a *directed connected graph* $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ with a finite number of vertices $\mathcal{S} = \{\varsigma_1, \dots, \varsigma_N\}$ and a set of directed edges $\mathcal{E} \subseteq \{(\varsigma_i, \varsigma_j) \in \mathcal{S} \times \mathcal{S} \mid i \neq j\}$. In a network of dynamically coupled systems, a dynamical system is assigned to each vertex $\varsigma_i \in \mathcal{S}$, with the dynamics governed by the following difference equation:

$$x_i(k+1) = \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))), \quad k \in \mathbb{Z}_+, \quad (5)$$

for vertex indices $i \in \mathcal{I} := \mathbb{Z}_{[1, N]}$. In (5), $x_i \in \mathbb{X}_i \subseteq \mathbb{R}^{n_i}$ denotes the state and $u_i \in \mathbb{U}_i \subseteq \mathbb{R}^{m_i}$ represents the control input of the i -th system, i.e., the system assigned to vertex ς_i . With each directed edge $(\varsigma_j, \varsigma_i) \in \mathcal{E}$ we associate a function $v_{ij} : \mathbb{R}^{n_j} \rightarrow \mathbb{R}^{n_{v_{ij}}}$ that defines the interconnection signal $v_{ij}(x_j(k)) \in \mathbb{R}^{n_{v_{ij}}}$, $k \in \mathbb{Z}_+$, between system j and system i , i.e., $v_{ij}(x_j(k))$ characterizes how the states of system j influence the dynamics of system i . We use $\mathcal{N}_i := \{j \mid (\varsigma_j, \varsigma_i) \in \mathcal{E}\}$ to denote the set of indices corresponding to the *direct neighbors* of system i . A direct neighbor of system i is any system in the network whose dynamics (e.g., states or outputs) appear explicitly (via the function $v_{ij}(\cdot)$) in the

state equations that govern the dynamics of system i . Clearly, if system j is a direct neighbor of system i , this does not necessarily imply the reverse. Let $\bar{\mathcal{N}}_i := \mathcal{N}_i \cup \{i\}$. We define $x_{\mathcal{N}_i}(k) := \text{col}(\{x_j(k)\}_{j \in \mathcal{N}_i})$ as the vector that collects all the state vectors of the direct neighbors of system i and $v_i(x_{\mathcal{N}_i}(k)) := \text{col}(\{v_{ij}(x_j(k))\}_{j \in \mathcal{N}_i}) \in \mathbb{R}^{n_{v_i}}$ as the vector that collects all the vector valued interconnection signals that “enter” system i . The functions $\phi_i(\cdot, \cdot, \cdot)$ and $v_{ij}(\cdot)$ are arbitrary nonlinear and satisfy $\phi_i(\mathbf{0}_{n_i}, \mathbf{0}_{m_i}, \mathbf{0}_{n_{v_i}}) = \mathbf{0}_{n_i}$ for all $i \in \mathcal{I}$ and $v_{ij}(\mathbf{0}_{n_j}) = \mathbf{0}_{n_{v_{ij}}}$ for all $(i, j) \in \mathcal{I} \times \mathcal{N}_i$. For all $i \in \mathcal{I}$ we assume that $\mathbf{0}_{n_i} \in \text{int}(\mathbb{X}_i)$ and $\mathbf{0}_{m_i} \in \text{int}(\mathbb{U}_i)$.

The following reasonable standing assumption is instrumental for obtaining the results presented in this paper.

Assumption III.1 The value of all the interconnection signals $\{v_{ij}(x_j(k))\}_{j \in \mathcal{N}_i}$ is known at each discrete-time instant $k \in \mathbb{Z}_+$, for any system $i \in \mathcal{I}$.

Notice that Assumption III.1 does not require knowledge of any of the interconnection signals at future time instants. From a technical point of view, Assumption III.1 is satisfied, e.g., if all interconnection signals $v_{ij}(x_j(k))$ are directly measurable¹ at all $k \in \mathbb{Z}_+$. Alternatively, Assumption III.1 is satisfied if all directly neighboring systems $j \in \mathcal{N}_i$ are able to communicate their local measured state $x_j(k)$ to system $i \in \mathcal{I}$.

Finally, let

$$x(k+1) = \phi(x(k), u(k)), \quad k \in \mathbb{Z}_+, \quad (6)$$

denote the dynamics of the overall network of interconnected systems (5), written in a compact form. In (6), $x = \text{col}(\{x_i\}_{i \in \mathcal{I}}) \in \mathbb{R}^n$, $n = \sum_{i \in \mathcal{I}} n_i$, and $u = \text{col}(\{u_i\}_{i \in \mathcal{I}}) \in \mathbb{R}^m$, $m = \sum_{i \in \mathcal{I}} m_i$, are vectors that collect all local states and inputs, respectively.

A. Structured max-CLFs

Next, we introduce the notion of a set of “structured max-CLFs”, which provides an alternative to the structured CLFs defined recently in [23].

Definition III.2 Let $\alpha_1^i, \alpha_2^i \in \mathcal{K}_\infty$ for $i \in \mathcal{I}$ and let $\{V_i\}_{i \in \mathcal{I}}$ be a set of functions $V_i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}_+$ that satisfy

$$\alpha_1^i(\|x_i\|) \leq V_i(x_i) \leq \alpha_2^i(\|x_i\|), \quad \forall x_i \in \mathbb{R}^{n_i}, \quad \forall i \in \mathcal{I}. \quad (7a)$$

Then, given $\rho_i \in \mathbb{R}_{[0,1]}$ for $i \in \mathcal{I}$, if there exists a set of control laws, possibly set-valued, $\pi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_{v_i}} \rightarrow \mathbb{U}_i$ such that

$$V_i(\phi_i(x_i, u_i, v_i(x_{\mathcal{N}_i}))) \leq \rho_i \max_{j \in \bar{\mathcal{N}}_i} V_j(x_j), \quad (7b)$$

$\forall x_i \in \mathbb{X}_i, \forall u_i \in \pi_i(x_i, v_i(x_{\mathcal{N}_i}))$, the set of functions $\{V_i\}_{i \in \mathcal{I}}$ is called a set of “structured max control Lyapunov functions” in $\mathbb{X} := \{\text{col}(\{x_i\}_{i \in \mathcal{I}}) \mid x_i \in \mathbb{X}_i\}$ for system (6).

¹For example, in electrical power systems, where a dynamical system is a power generator, the interconnection signal is the generator bus voltage and line power (or current) flow in the corresponding power line, which can be directly measured.

In the above definition the term *structured* emphasizes the fact that each V_i is a function of x_i only, i.e., the structural decomposition of the dynamics of the overall interconnected system (5) is reflected in the functions $\{V_i\}_{i \in \mathcal{I}}$. Moreover, the term *max* originates from the corresponding convergence condition, i.e., (7b). Next, based on Definition III.2, we formulate the following feasibility problem.

Problem III.3 Let $\rho_i \in \mathbb{R}_{[0,1]}$, $i \in \mathcal{I}$ and a set of “structured max-CLFs” $\{V_i\}_{i \in \mathcal{I}}$ be given. At time $k \in \mathbb{Z}_+$, let the state vector $\{x_i(k)\}_{i \in \mathcal{I}}$, the set of interconnection signals $\{v_i(x_{\mathcal{N}_i}(k))\}_{i \in \mathcal{I}}$ and the values $\{V_i(x_i(k))\}_{i \in \mathcal{I}}$ be known, and calculate a set of control actions $\{u_i(k)\}_{i \in \mathcal{I}}$, such that:

$$u_i(k) \in \mathbb{U}_i, \quad \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))) \in \mathbb{X}_i, \quad (8a)$$

$$V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))) \leq \rho_i \max_{j \in \bar{\mathcal{N}}_i} V_j(x_j(k)), \quad (8b)$$

for all $i \in \mathcal{I}$. □

Let $\pi(x(k)) := \{\text{col}(\{u_i(k)\}_{i \in \mathcal{I}}) \mid (8) \text{ holds}\}$ and let

$$\begin{aligned} x(k+1) &\in \phi_{\text{CL}}(x(k), \pi(x(k))) \\ &:= \{\phi(x(k), u(k)) \mid u(k) \in \pi(x(k))\} \end{aligned} \quad (9)$$

denote the difference inclusion corresponding to system (6) in “closed loop” with the set of feasible solutions obtained by solving Problem III.3 at each discrete-time instant $k \in \mathbb{Z}_+$.

Theorem III.4 Let $\alpha_1^i, \alpha_2^i \in \mathcal{K}_\infty$ and $\rho_i \in \mathbb{R}_{[0,1]}$, $\forall i \in \mathcal{I}$ be given and choose a set of structured max-CLFs $\{V_i\}_{i \in \mathcal{I}}$ in $\mathbb{X} = \{\text{col}(\{x_i\}_{i \in \mathcal{I}}) \mid x_i \in \mathbb{X}_i\}$ for system (6). Suppose that Problem III.3 is feasible for all $x(k) \in \mathbb{X}$ and the corresponding signals $\{v_i(x_{\mathcal{N}_i}(k))\}_{i \in \mathcal{I}}$. Then the difference inclusion (9) is asymptotically stable in $\mathbb{X}$.

Proof: Let $x(k) \in \mathbb{X}$ for some $k \in \mathbb{Z}_+$. Then, feasibility of Problem III.3 ensures that $x(k+1) \in \phi_{\text{CL}}(x(k), \pi(x(k))) \subseteq \mathbb{X}$ due to constraint (8a). Hence, Problem III.3 remains feasible and thus, $\mathbb{X}$ is a PI set for system (9). Now consider the function $V(x) := \max_{i \in \mathcal{I}} V_i(x_i)$. Together with condition (8b) this yields

$$\begin{aligned} V(x(k+1)) &= \max_{i \in \mathcal{I}} V_i(x_i(k+1)) \\ &\leq \rho \max_{i \in \mathcal{I}} \max_{j \in \bar{\mathcal{N}}_i} V_j(x_j(k)) \\ &= \rho \max_{i \in \mathcal{I}} V_i(x_i(k)) = \rho V(x(k)), \end{aligned} \quad (10)$$

for all $x(k) \in \mathbb{X}$, where $\rho := \max_{i \in \mathcal{I}} \rho_i \in \mathbb{R}_{[0,1]}$.

Next, we derive a lower bound for $V(x)$. Observing that the maximum element of a set always equals or exceeds the average value of the elements and using (7a) yields

$$\begin{aligned} V(x) &:= \max_{i \in \mathcal{I}} V_i(x_i) \geq \frac{1}{N} \sum_{i \in \mathcal{I}} V_i(x_i) \\ &\geq \frac{1}{N} \sum_{i \in \mathcal{I}} \alpha_1^i(\|x_i\|). \end{aligned} \quad (11)$$

Next, note that

$$\begin{aligned} \sum_{i \in \mathcal{I}} \alpha_1^i(\|x_i\|) &\geq \sum_{i \in \mathcal{I}} \tilde{\alpha}_1(\|x_i\|) \\ &\geq \tilde{\alpha}_1(\max_{i \in \mathcal{I}} \|x_i\|) \geq \tilde{\alpha}_1\left(\frac{1}{N} \sum_{i \in \mathcal{I}} \|x_i\|\right), \end{aligned} \quad (12)$$

where $\tilde{\alpha}_1(s) := \min_{i \in \mathcal{I}} \alpha_1^i(s) \in \mathcal{K}_\infty$. With $\hat{x}_i := \text{col}(\mathbf{0}_{n_1}, \dots, \mathbf{0}_{n_{i-1}}, x_i, \mathbf{0}_{n_{i+1}}, \dots, \mathbf{0}_{n_N})$ we have that

$$\sum_{i \in \mathcal{I}} \|x_i\| = \sum_{i \in \mathcal{I}} \|\hat{x}_i\| \geq \left\| \sum_{i \in \mathcal{I}} \hat{x}_i \right\| = \|x\|. \quad (13)$$

Using this property, the fact that $\tilde{\alpha}_1 \in \mathcal{K}_\infty$ is strictly increasing and (11) gives the desired lower bound, i.e.,

$$\begin{aligned} V(x) &\geq \frac{1}{N} \sum_{i \in \mathcal{I}} \alpha_1^i(\|x_i\|) \\ &\geq \frac{1}{N} \tilde{\alpha}_1\left(\frac{1}{N} \|x\|\right) =: \alpha_1(\|x\|), \end{aligned} \quad (14)$$

for all $x \in \mathbb{R}^n$ and where $\alpha_1 \in \mathcal{K}_\infty$.

Next, we search for an upper bound on $V(x)$. For this, we first prove that $\|x_i\| \leq \|x\|$, $\forall x = \text{col}(\{x_i\}_{i \in \mathcal{I}}) \in \mathbb{R}^n$, $\forall i \in \mathcal{I}$, and any p -norm. For $1 \leq p < \infty$, the inequality follows from the definition of the p -norm:

$$\|x\|_p^p := \sum_{j=1}^n |[x]_j|^p = \sum_{l \in \mathcal{I}} \sum_{j=1}^{n_l} |[x]_j|^p = \sum_{l \in \mathcal{I}} \|x_l\|_p^p.$$

Hence

$$\|x_i\|_p^p = \|x\|_p^p - \sum_{l \in \mathcal{I} \setminus i} \|x_l\|_p^p \leq \|x\|_p^p, \quad \forall i \in \mathcal{I}. \quad (15)$$

From this and the observation that $f(s) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $f(s) := s^{\frac{1}{p}}$ and $p \geq 1$ is strictly increasing it follows that $\|x_i\|_p \leq \|x\|_p$ for $1 \leq p < \infty$. It is straightforward to see that the inequality holds for the ∞ -norm as well:

$$\begin{aligned} \|x\|_\infty &= \max_{j \in \mathbb{Z}[1, n]} |[x]_j| = \max_{l \in \mathcal{I}} \max_{j \in \mathbb{Z}[1, n_l]} |[x]_j| = \max_{l \in \mathcal{I}} \|x_l\|_\infty \\ &\geq \|x_i\|_\infty, \quad \forall i \in \mathcal{I}. \end{aligned} \quad (16)$$

Next, using (7a), the fact that α_2^i is strictly increasing for all $i \in \mathcal{I}$ and (15), (16), we obtain the desired upper bound, i.e.,

$$\begin{aligned} V(x) &:= \max_{i \in \mathcal{I}} V_i(x_i) \leq \max_{i \in \mathcal{I}} \alpha_2^i(\|x_i\|) \\ &\leq \max_{i \in \mathcal{I}} \alpha_2^i(\|x\|) =: \alpha_2(\|x\|), \end{aligned} \quad (17)$$

for all $x \in \mathbb{R}^n$ and where $\alpha_2 \in \mathcal{K}_\infty$.

The result now follows directly from Theorem II.3, with $V(x) := \max_{i \in \mathcal{I}} V_i(x_i)$ as a CLF for the overall system. ■

Notice that in Problem III.3 the functions V_i do not need to be CLFs (in conformity with Definition II.4) in $\mathbb{X}_i$ for each system $i \in \mathcal{I}$, respectively. Condition (8b) permits a *spatially* non-monotonous evolution of V_i . More precisely, the local functions are allowed to increase, as long as for each system the value of its function V_i at the next time

instant is less than ρ_i times the maximum over the current values of its own function and those of its direct neighbors.

Moreover, observe that Problem III.3 is separable in $\{u_i\}_{i \in \mathcal{I}}$. The set of feasible control inputs is defined by (8), which only contains inequalities at a local level. Therefore, it is possible to solve Problem III.3 by solving N feasibility problems independently, with each problem assigned to one local controller, corresponding to one system $i \in \mathcal{I}$. In order to compute $u_i(k)$, each controller needs to measure or estimate the current state $x_i(k)$ of its system, and have knowledge of the interconnection signals $\{v_i(x_{\mathcal{N}_i})\}_{i \in \mathcal{N}_i}$ and the values $\{V_i(x_i(k))\}_{i \in \mathcal{N}_i}$. Clearly, a single run of information exchange among direct neighbors per sampling instant is sufficient to acquire this knowledge. Therefore, an attractive feature of the control scheme proposed in this work is that it can be implemented in an almost decentralized fashion.

B. Temporal non-monotonicity

In general, it may be difficult to find functions $\{V_i\}_{i \in \mathcal{I}}$ that satisfy (7) for all $x_i \in \mathbb{X}_i$. Systematic methods for synthesizing CLFs for an arbitrary nonlinear system do not exist, although candidate CLFs can often be generated using linearized system dynamics. However, the region of validity for these CLFs is often limited to a neighborhood of the origin. Supposing that we have a set of structured max-CLFs in $\tilde{\mathbb{X}} \subset \mathbb{X}$, we propose a method to relax the conditions on the candidate CLFs, based on an adaptation of the Lyapunov-Razumikhin (LR) technique for time-delay systems [17], [18]. The LR method allows the Lyapunov function to be non-monotonous in order to compensate for the effects of the delay. Next, we show how the LR technique can be applied to discrete-time systems as well, to permit a *temporal* non-monotonous evolution of the candidate CLF for the full network.

Problem III.5 Let $N_\tau \in \mathbb{Z}_{\geq 1}$ be given. Consider Problem III.3 for a set of “structured max-CLFs” $\{V_i\}_{i \in \mathcal{I}}$ in $\tilde{\mathbb{X}} \subset \mathbb{X}$, with (8b) replaced by

$$\begin{aligned} &V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))) \\ &\leq \rho_i \max_{\tau \in \mathbb{Z}[0, N_\tau - 1]} \max_{j \in \mathcal{N}_i} V_j(x_j(k - \tau)), \end{aligned} \quad (18)$$

for all $k \in \mathbb{Z}_{\geq N_\tau - 1}$ and $i \in \mathcal{I}$. □

Let $\bar{\pi}(x(k)) := \{\text{col}(\{u_i(k)\}_{i \in \mathcal{I}}) \mid (8a) \text{ and } (18) \text{ hold}\}$ and let

$$\begin{aligned} &x(k+1) \in \bar{\phi}_{\text{CL}}(x(k), \bar{\pi}(x(k))) \\ &:= \{\phi(x(k), u(k)) \mid u(k) \in \bar{\pi}(x(k))\} \end{aligned} \quad (19)$$

denote the difference inclusion corresponding to system (6) in “closed loop” with the set of feasible solutions obtained by solving Problem III.5 at each time instant $k \in \mathbb{Z}_+$.

Theorem III.6 Let $\alpha_1^i, \alpha_2^i \in \mathcal{K}_\infty$, $N_\tau \in \mathbb{Z}_{\geq 1}$ and $\rho_i \in \mathbb{R}_{[0, 1]}$, $\forall i \in \mathcal{I}$ be given and choose a set of structured max-CLFs $\{V_i\}_{i \in \mathcal{I}}$ in $\tilde{\mathbb{X}} \subseteq \mathbb{X} = \{\text{col}(\{x_i\}_{i \in \mathcal{I}}) \mid x_i \in \mathbb{X}_i\}$

for system (6). Suppose that Problem III.5 is feasible for all $x(k) \in \mathbb{X}$, all $k \in \mathbb{Z}_+$ and the corresponding signals $\{v_i(x_{\mathcal{N}_i}(k))\}_{i \in \mathcal{I}}$. Then the difference inclusion (19) is asymptotically stable in $\mathbb{X}$.

Proof: Let $x(k) \in \mathbb{X}$ for some $k \in \mathbb{Z}_+$. Positive invariance of $\mathbb{X}$ follows from feasibility of (8a), as shown in the proof of Theorem III.4. Now consider the function $V(x) := \max_{i \in \mathcal{I}} V_i(x_i)$. Condition (18) implies that

$$\begin{aligned} V(x(k+1)) &= \max_{i \in \mathcal{I}} V_i(x_i(k+1)) \\ &\leq \rho \max_{i \in \mathcal{I}} \max_{\tau \in \mathbb{Z}_{[0, N_\tau-1]}} \max_{j \in \mathcal{N}_i} V_j(x_j(k-\tau)) \\ &= \rho \max_{\tau \in \mathbb{Z}_{[0, N_\tau-1]}} \max_{i \in \mathcal{I}} V_i(x_i(k-\tau)) \\ &= \rho \max_{\tau \in \mathbb{Z}_{[0, N_\tau-1]}} V(x(k-\tau)), \end{aligned} \quad (20)$$

for all $k \in \mathbb{Z}_{\geq N_\tau-1}$ and with $\rho := \max_{i \in \mathcal{I}} \rho_i \in \mathbb{R}_{[0,1]}$. Recursive application of (20) gives

$$\begin{aligned} V(x(k)) &\leq \rho \max_{l \in \mathbb{Z}_{[0, N_\tau-1]}} V(x(l)), \quad k \in \mathbb{Z}_{[N_\tau, 2N_\tau-1]}, \\ V(x(k)) &\leq \rho \max_{l \in \mathbb{Z}_{[N_\tau, 2N_\tau-1]}} V(x(l)) \\ &\leq \rho^2 \max_{l \in \mathbb{Z}_{[0, N_\tau-1]}} V(x(l)), \quad k \in \mathbb{Z}_{[2N_\tau, 3N_\tau-1]}, \\ &\vdots \\ V(x(k)) &\leq \rho^{\lfloor \frac{k}{N_\tau} \rfloor} \max_{l \in \mathbb{Z}_{[0, N_\tau-1]}} V(x(l)), \quad \forall k \in \mathbb{Z}_+. \end{aligned} \quad (21)$$

Furthermore, we derived in the proof of Theorem III.4 that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathbb{R}^n, \quad (22)$$

with $\alpha_1(s) := \frac{1}{N} \min_{i \in \mathcal{I}} \alpha_1^i(\frac{1}{N}s) \in \mathcal{K}_\infty$ and $\alpha_2(s) := \max_{i \in \mathcal{I}} \alpha_2^i(s) \in \mathcal{K}_\infty$. As $\mathcal{K}_\infty$ -functions are strictly increasing, we know that $\max_{l \in \mathbb{Z}_{[0, N_\tau-1]}} V(x(l)) \leq \max_{l \in \mathbb{Z}_{[0, N_\tau-1]}} \alpha_2(\|x(l)\|) = \alpha_2(\|\mathbf{x}_{[0, N_\tau-1]}\|)$. Combining this bound with (21), (22) gives

$$\|x(k)\| \leq \alpha_1^{-1}(\rho^{\lfloor \frac{k}{N_\tau} \rfloor} \alpha_2(\|\mathbf{x}_{[0, N_\tau-1]}\|)).$$

The fact that

$$\lim_{k \rightarrow \infty} \|x(k)\| \leq \lim_{k \rightarrow \infty} \alpha_1^{-1}(\rho^{\lfloor \frac{k}{N_\tau} \rfloor} \alpha_2(\|\mathbf{x}_{[0, N_\tau-1]}\|)) = 0$$

proves attractivity of the closed-loop system (19). Moreover, Lyapunov stability follows as for every $\varepsilon > 0$ we can find a $\delta(\varepsilon) := \alpha_2^{-1}(\alpha_1(\varepsilon)) > 0$, such that $\|\mathbf{x}_{[0, N_\tau-1]}\| < \delta$ implies that $\|x(k)\| < \alpha_1^{-1}(\rho^{\lfloor \frac{k}{N_\tau} \rfloor} \alpha_2(\delta)) \leq \varepsilon$ for all $k \in \mathbb{Z}_+$. This proves asymptotic stability of (19) in $\mathbb{X}$. ■

The distinctive feature of Problem III.5 is that it allows the trajectories of the local functions $V_i(x_i(k))$ to be non-monotonous, and relaxes the convergence condition on the candidate CLF for the overall network, i.e., $V(x(k))$, as well. The evolution of $V(x(k))$ can be arbitrary, as long as it remains within the asymptotically converging envelope generated by (18) and the first N_τ values of $V(x(k))$.

Note that if we combine Problem III.3 or Problem III.5 with the optimization of a set of local cost functions, the

feasibility-based stability guarantee and the possibility of an almost decentralized implementation still hold. This enables the formulation of a one-step-ahead predictive control algorithm in which stabilization is decoupled from performance, and in which the controllers do not need to attain the global optimum at each sampling instant, as typically required for stability in classical MPC (see [24]).

For the remainder of the article we therefore consider the following almost-decentralized MPC algorithm, supposing that a set of local objective functions $\{J_i(u_i(k), x_i(k))\}_{i \in \mathcal{I}}$ is known.

Algorithm III.7 At each instant $k \in \mathbb{Z}_+$ and node $i \in \mathcal{I}$:

Step 1: Measure or estimate the current local state $x_i(k)$ and transmit $v_{ji}(x_i(k))$ and $V_i(x_i(k))$ to nodes $\{j \in \mathcal{I} \mid i \in \mathcal{N}_j\}$.

Step 2: Specify the set of feasible local control actions $\bar{\pi}_i(x_i(k), v_i(x_{\mathcal{N}_i}(k))) := \{u_i(k) \mid (8a) \text{ and } (18) \text{ hold}\}$. Minimize the cost $J_i(u_i(k), x_i(k))$ over $\bar{\pi}_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))$ and denote the optimizer by $u_i^*(k)$;

Step 3: Use $u_i(k) = u_i^*(k)$ as control action.

Remark III.8 In Algorithm III.7, each controller optimizes its own local objective. However, many distributed MPC schemes (see for instance [1], [2]) optimize a global cost function (e.g., some convex combination of local objectives) and aim for optimization of global performance by employing network-wide information or iterations. Therein, stability is attained by assuming optimality (for example, in [2]) or by imposing a contractive constraint on the norm of the local states (e.g., in [1]). The L-MPC conditions proposed in this paper can be used in those implementations as well, as an alternative way to achieve stability that is less conservative than contractive constraints, while time-consuming iterations would only be used for achieving global optimality. □

C. Implementation Issues

In what follows, we will consider nonlinear systems that are affine in the control input, i.e.,

$$\begin{aligned} x_i(k+1) &= \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))) \\ &= f_i(x_i(k), v_i(x_{\mathcal{N}_i}(k))) \\ &\quad + g_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))u_i(k), \end{aligned} \quad (23)$$

with $f_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_{v_i}} \rightarrow \mathbb{R}^{n_i}$, $f_i(\mathbf{0}_{n_i}, \mathbf{0}_{n_{v_i}}) = \mathbf{0}_{n_i}$, $g_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_{v_i}} \rightarrow \mathbb{R}^{n_i \times m_i}$ and $g_i(\mathbf{0}_{n_i}, \mathbf{0}_{n_{v_i}}) = \mathbf{0}$ for all $i \in \mathcal{I}$. For these systems and polytopic state and input sets $\mathbb{X}_i$ and $\mathbb{U}_i$, respectively, it is possible to implement Step 2 of Algorithm III.7 by solving a single linear program, without introducing conservatism.

For this, we restrict our attention to structured CLFs defined using the infinity-norm, i.e.,

$$V_i(x_i) = \|P_i x_i\|_\infty, \quad (24)$$

where $P_i \in \mathbb{R}^{p_i \times n_i}$ is a full-column rank matrix. Note that this type of structured max-CLF satisfies (7a), for $\alpha_1^i(s) := \frac{\sigma_{P_i}}{\sqrt{p_i}} s$ (where $\sigma_{P_i} > 0$ is the smallest singular value of P_i) and for $\alpha_2^i(s) := \|P_i\|_\infty s$.

By definition of the infinity norm, for $\|x\|_\infty \leq c$ to be satisfied for some vector $x \in \mathbb{R}^n$ and constant $c \in \mathbb{R}$, it is necessary and sufficient to require that $\pm[x]_j \leq c$ for all $j \in \mathbb{Z}_{[1,n]}$. So, for (18) to be satisfied, it is necessary and sufficient to require that

$$\pm[P_i\{g_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))u_i(k)\}]_j \leq \zeta_i(k) \mp [P_i\{f_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))\}]_j, \quad (25)$$

for $j \in \mathbb{Z}_{[1,p_i]}$ and $k \in \mathbb{Z}_{\geq N_\tau-1}$, and where

$$\zeta_i(k) := \rho_i \max_{\tau \in \mathbb{Z}_{[0, N_\tau-1]}} \max_{j \in \mathcal{N}_i} V_j(x_j(k-\tau)) \in \mathbb{R}_+$$

is constant at any $k \in \mathbb{Z}_{\geq N_\tau-1}$. This yields a total of $2p_i$ linear inequalities in the optimization variable u_i .

Moreover, by choosing infinity-norm based local cost functions of the form

$$J_i(x_i(k), u_i(k)) := \|Q_1^i \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))\|_\infty + \|Q_0^i x_i(k)\|_\infty + \|R^i u_i(k)\|_\infty, \quad (26)$$

with full-rank matrices $Q_1^i \in \mathbb{R}^{n_{q1,i} \times n_i}$, $Q_0^i \in \mathbb{R}^{n_{q0,i} \times n_i}$ and $R^i \in \mathbb{R}^{n_{r,i} \times m_i}$, we can reformulate Step 2 of Algorithm III.7 as the linear program

$$\min_{u_i(k), \varepsilon_1, \varepsilon_2} \varepsilon_1 + \varepsilon_2 \quad (27)$$

subject to (8a), (25) and

$$\begin{aligned} \pm[Q_1^i \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))]_j + \|Q_0^i x_i(k)\|_\infty &\leq \varepsilon_1 \\ \pm[R^i u_i(k)]_l &\leq \varepsilon_2, \end{aligned}$$

for $j \in \mathbb{Z}_{[1, n_{q1,i}]}$ and $l \in \mathbb{Z}_{[1, n_{r,i}]}$.

IV. ILLUSTRATIVE EXAMPLE

Consider the nonlinear NDS (5) with $\mathcal{S} = \{\varsigma_1, \varsigma_2\}$, $\mathcal{N}_1 = \{2\}$, $\mathcal{N}_2 = \{1\}$, $\mathbb{X}_1 = \mathbb{X}_2 = \{x \in \mathbb{R}^2 \mid \|x\|_\infty \leq 5\}$ and $\mathbb{U}_1 = \mathbb{U}_2 = \{u \in \mathbb{R} \mid |u| \leq 2\}$. Its dynamics are given by:

$$\begin{aligned} \phi_1(x_1, u_1, v_1(x_{\mathcal{N}_1})) &:= \begin{bmatrix} 1 & 0.7 \\ 0 & 1 \end{bmatrix} x_1 + \begin{bmatrix} \sin([x_1]_2) \\ 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0.245 \\ 0.7 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ ([x_2]_1)^2 \end{bmatrix}, \end{aligned} \quad (28a)$$

$$\begin{aligned} \phi_2(x_2, u_2, v_2(x_{\mathcal{N}_2})) &:= \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} x_2 + \begin{bmatrix} \sin([x_2]_2) \\ 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0.125 \\ 0.5 \end{bmatrix} u_2 + \begin{bmatrix} 0 \\ [x_1]_1 \end{bmatrix}. \end{aligned} \quad (28b)$$

The method of [25] was used to compute the weights $P_1, P_2 \in \mathbb{R}^{2 \times 2}$ of two local infinity-norm based candidate CLFs, i.e., $V_1(x_1) = \|P_1 x_1\|_\infty$ and $V_2(x_2) = \|P_2 x_2\|_\infty$ with $\rho = \rho_1 = \rho_2 = 0.8$ and linearizations of (28a), (28b), respectively, around the origin, in closed-loop with the local state-feedback laws $u_1(k) := K_1 x_1(k)$, $u_2(k) := K_2 x_2(k)$, $K_1, K_2 \in \mathbb{R}^{1 \times 2}$, yielding

$$\begin{aligned} P_1 &= \begin{bmatrix} 2.5598 & 0.3345 \\ 1.8629 & 5.0219 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -0.9715 & -2.1190 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} -0.3898 & -0.3836 \\ 0.2703 & 0.9763 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.3896 & -2.7822 \end{bmatrix}. \end{aligned}$$

Fig. 1. Input trajectories for $\rho = 0.8$ and $N_\tau = 3$.

Fig. 2. Evolution of $V_1(x_1(k))$, $V_2(x_2(k))$, $V(x(k))$, and their upper bounds over time, for $\rho = 0.8$ and $N_\tau = 3$.

Note that the control laws $u_1(k) = K_1 x(k)$ and $u_2(k) = K_2 x_2(k)$ are only employed off-line, to calculate the weight matrices P_1, P_2 and they are not used for controlling the system. For each system $i \in \mathcal{I}$, we employed the following cost functions in Algorithm III.7: $J_i(x_i(k), u_i(k)) := \|Q_1^i \phi_i(x_i, u_i, v_i(x_{\mathcal{N}_i}))\|_\infty + \|Q_0^i x_i(k)\|_\infty + \|R^i u_i(k)\|_\infty$, where $i \in \{1, 2\}$, $Q_1^1 = Q_1^2 = 4I_2$, $Q_0^1 = Q_0^2 = 0.1I_2$ and $R^1 = R^2 = 0.4$.

In the simulation scenario we tested the system response in closed-loop with Algorithm III.7 for $x_1(0) = [3, -1]^\top$, $x_2(0) = [1, -2]^\top$ and $N_\tau = 3$. Fig. 1 shows the control inputs for system 1 and system 2, along with the input constraints that are represented by the dash-dotted lines. Note that these constraints are not violated, although they are active at some time instants. The corresponding evolutions of $V_1(x_1(k))$, $V_2(x_2(k))$, $V(x(k))$ and the upper bounds generated by (18) are shown in Fig. 2. The simulation illustrates that $V(x(k))$ is allowed to vary arbitrarily within the asymptotically converging envelope defined by (18),

resulting in closed-loop stability. Moreover, note that the proposed L-MPC algorithm allows a spatially non-monotonous evolution of the structured max-CLFs (at time instant $k = 2$, $V_2(x_2(k))$ increases although $V(x(k))$ does not), whereas the candidate CLF itself can be non-decreasing as well (which is the case for $k = 4$). The attained performance in terms of convergence speed is similar to the one attained by the method in [23] for the same example and initial conditions. However, the technique in [23] requires global coordination and iterative optimization to guarantee closed-loop stability, whereas the method proposed in this work does not.

V. CONCLUSIONS

This paper proposed an almost decentralized solution to the problem of stabilizing a network of discrete-time nonlinear systems with coupled dynamics that are subject to local state/input constraints. By “almost decentralized” we mean that each local controller is allowed to use the states of neighboring systems for feedback, whereas it is not permitted to employ iterations between the systems in the network to compute the control action. The stabilization conditions were decentralized via a set of structured control Lyapunov functions for which the maximum over all the functions in the set is a CLF for the overall network of systems. However, this does not necessarily imply that each function is a local CLF for its corresponding system. Additionally, we provided a solution for relaxing the temporal monotonicity of the CLF for the overall network. A non-trivial example illustrated the effectiveness of the developed scheme and demonstrated that the proposed L-MPC technique can perform as well as more complex distributed, iteration-based MPC algorithms.

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