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Analytical Calculation of the Force Between a Rectangular Coil and a Cuboidal Permanent Magnet

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This paper concerns the analysis of the force between a rectangular coil and a cuboidal permanent magnet. The magnetic flux density distribution due to the permanent magnet is determined using the surface charge method, and an analytical equation is obtained for the Lorentz force on a cuboidal current carrying volume. These analytical results are used to calculate the Lorentz force on a rectangular coil modeled using four current carrying volumes. The calculations are verified both by measurements and numerical integration. These results can be used in the design and real-time control of planar actuators for industrial levitation/positioning platforms or other (ironless) actuators.

Index Terms—Coil, Lorentz force, permanent magnet, planar actuator.

I. INTRODUCTION

SINGLE-STAGE magnetically levitated planar actuators [1]–[5] are being researched for use in the lithographic industry (e.g., wafer stages). This requires a positioning accuracy in the nanometer (nm) range. These ironless actuators consist of a stator with coils and a plate with a Halbach array of permanent magnets mounted on it (the translator). To achieve such accuracies, the deformation of the translator plate has to be limited. Therefore, expressions describing the force acting on each PM in the array are needed, since these forces cause the deformation. In previous research, the static forces among the permanent magnets in a Halbach array were determined [6]. A next step is to determine the force resulting from the coils in the stator. For design optimization and real-time implementation, this model has to be accurate and fast.

In previous research on magnetically levitated planar actuators using permanent magnets (i.e., [7]), the force between a cuboidal permanent magnet and a rectangular coil is obtained by numerical integration of the magnetic field of the magnet over the coil volume. A disadvantage of this method is that it is very time-consuming, making it unsuitable for design synthesis. A faster method, which has been used for real-time implementation, makes use of the periodicity of the magnetic field of the array to describe the magnet field using harmonic models [7]. The force acting on the magnet plate is then determined using the Lorentz force equations. Therefore, the model only calculates a force (and torque) vector acting on the magnet plate as a whole and not on individual magnets.

This paper presents analytic solutions to the Lorentz force acting on a current carrying cuboidal volume due to a cuboidal permanent magnet, which are used to calculate the force between a rectangular coil and a permanent magnet. Firstly, the magnetic flux density of the permanent magnet is determined

Fig. 1. Permanent magnet modeled using magnetic nodes, magnetization direction indicated by M .

using the magnetic surface charge method. Secondly, these expressions are integrated over a cuboidal volume to determine the Lorentz force on this volume and the results are verified by the finite element method (FEM). Thirdly, the expressions are then used to determine the force on a rectangular coil. Lastly, these force calculations are verified both by numerical integration over the coil volume and by measurements.

II. MODELING OF THE PERMANENT MAGNET

The force between a permanent magnet and a coil can be determined by calculating the Lorentz force on the coil due to the magnetic field of the magnet. The force acting on the magnet is then equal to this force but opposite of sign. The flux density of the magnet can be modeled using the magnetic nodes method [8], which is derived from the magnetic surface charge method [9]. These methods assume a relative permeability (μ_r) of the PM material equal to unity. For a cuboidal permanent magnet as shown in Fig. 1, the magnet can be modeled using eight magnetic nodes at its corners, denoted with the index $k \in \{1, \dots, 8\}$.

The magnetic flux density $\vec{B}(x, y, z)$ due to the permanent magnet magnetized in the positive z -direction as shown in Fig. 1 is given by [8]

$$B_x(x, y, z) = \frac{B_r}{4\pi} \sum_k \log(-y_r + r) \quad (1)$$

$$B_y(x, y, z) = \frac{B_r}{4\pi} \sum_k \log(-x_r + r) \quad (2)$$

$$B_z(x, y, z) = \frac{B_r}{4\pi} \sum_k \tan^{-1} \left(\frac{x_r y_r}{z_r r} \right) \quad (3)$$

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Fig. 2. Permanent magnet and current carrying volume, dimensions in millimeters.

where B_r is the remanent flux density of the permanent magnet material (in Tesla), (x_r, y_r, z_r) is equal to $(x - x_k, y - y_k, z - z_k)$, and

$$r = \sqrt{x_r^2 + y_r^2 + z_r^2}. \quad (4)$$

III. LORENTZ FORCE ON A CUBOIDAL CURRENT CARRYING VOLUME

Using the expressions for the magnetic flux density due to a cuboidal permanent magnet, the Lorentz force on a cuboidal current carrying volume can be obtained. The permanent magnet from Fig. 1 together with a wire segment is shown in Fig. 2. Assume the wire segment has a constant and uniform volume current distribution J equal to

$$\vec{J}_w = \frac{I}{4w_w w_h} \hat{u}_x \quad (5)$$

where I is the current in the wire segment and $\hat{u}_x$ is the unit vector in the x -direction.

The Lorentz force equation is

$$\vec{F}_w = \int_{V_w} \vec{J}_w \times \vec{B} dV_w \quad (6)$$

where V_w is the volume of the wire segment, $\vec{J}_w$ is the current density in the wire segment, and $\vec{B}$ is the external magnetic field. Applying this integral to the current carrying volume and permanent magnet in Fig. 2 results in

$$\vec{F}_w = \vec{J}_w \times \sum_k \int_{z'_r - w_h}^{z'_r + w_h} \int_{y'_r - w_w}^{y'_r + w_w} \int_{x'_r - w_l}^{x'_r + w_l} \vec{B}_k(x, y, z) dx dy dz \quad (7)$$

where $\vec{B}_k(x, y, z)$ is the magnetic flux density due to node k and $(x'_r, y'_r, z'_r) = (x_w - x_k, y_w - y_k, z_w - z_k)$. Therefore, the force on the volume is determined by the volumetric integral of the magnetic flux density over the volume of the wire segment. The solutions to the indefinite volumetric integral of the flux density $\vec{B}_k(x, y, z)$ are given in

$$\begin{aligned} & \iiint B_x(x, y, z) dx dy dz \\ &= \frac{1}{18} \left(-z^3 + 6xzr + 9y(x^2 + z^2) \tan^{-1} \left(\frac{x}{z} \right) \right. \end{aligned}$$

Fig. 3. F_y obtained from analytic integral (blue solid line), F_y obtained from FEM with $\mu_r = 1$ (green $\times$), F_y obtained from FEM with $\mu_r = 1.05$ (red circle), F_z obtained from analytic integral (dashed line), F_z obtained from FEM with $\mu_r = 1$ (magenta $+$), F_z obtained from FEM with $\mu_r = 1.05$ (black triangle).

$$\begin{aligned} & + 3y^3 \left(-4 \tan^{-1} \left(\frac{z}{y} \right) + \tan^{-1} \left(\frac{xz}{yr} \right) \right) \\ & + x^3 \log(8) + 3z^3 \log(x + r) \\ & + 9y \left(z \left(-3x + z \tan^{-1} \left(\frac{xy}{zr} \right) \right) \right. \\ & \quad \left. + x \left(2x \tan^{-1} \left(\frac{z}{x} \right) + x \tan^{-1} \left(\frac{yz}{xr} \right) \right. \right. \\ & \quad \left. \left. + 2z \log(-y + r) \right) \right) \\ & + 3x^3 \log(z + r) \\ & - 3y^2 (-4z + 9x \log(2) + 3z \log(x + r) \\ & \quad + 3x \log(z + r))) \end{aligned} \quad (8)$$

$$\begin{aligned} & \iiint B_y(x, y, z) dx dy dz \\ &= \frac{1}{18} \left(-z^3 + 6yzt + 9x(y^2 + z^2) \tan^{-1} \left(\frac{y}{z} \right) \right. \\ & \quad + 3x^3 \left(-4 \tan^{-1} \left(\frac{z}{x} \right) + \tan^{-1} \left(\frac{yz}{xr} \right) \right) \\ & \quad + y^3 \log(8) + 3z^3 \log(y + r) \\ & \quad + 9x \left(z \left(-3y + z \tan^{-1} \left(\frac{yx}{zr} \right) \right) \right. \\ & \quad \left. + y \left(2y \tan^{-1} \left(\frac{z}{y} \right) + y \tan^{-1} \left(\frac{xz}{yr} \right) \right. \right. \\ & \quad \left. \left. + 2z \log(-x + r) \right) \right) \\ & \quad + 3y^3 \log(z + r) \\ & - 3x^2 (-4z + 9y \log(2) + 3z \log(y + r) \\ & \quad + 3y \log(z + r))) \end{aligned} \quad (9)$$

$$\begin{aligned} & \iiint B_z(x, y, z) dx dy dz \\ &= \frac{1}{6} \left((x^2 + y^2 - 2z^2)r + 3x(-y^2 + z^2) \tanh^{-1} \left(\frac{x}{r} \right) \right. \\ & \quad \left. + 3y(-x^2 + z^2) \tanh^{-1} \left(\frac{y}{r} \right) + 6xyz \tanh^{-1} \left(\frac{xy}{zr} \right) \right) \end{aligned} \quad (10)$$

where r is given by (4).

To verify the analytic solution to the Lorentz integral, the Lorentz force acting on the wire segment was also determined using FEM (Flux 3-D [10]). The force components F_y and F_z are obtained from (8)–(10) and FEM. The results are shown

Fig. 8. Difference in results obtained from analytic and numeric integral in force component (F_x , F_y , F_z).

acting on the coil obtained from the analytic method and from measurements for variation of the coil position y_c and $z_c - z_m = 12.35$ mm, $x_c - x_m = 0$. The results show good agreement with a maximum error of approximately 4.5%, which is partly due to the fact that the analytic method assumes $\mu_r = 1$.

The results obtained from the analytic method are also compared with results from numeric integration over the full coil volume. For variation of the position (x_c, y_c) , the force components $F_{(x,y,z)}$ (in N/At) acting on the coil as obtained from the analytic models are shown in Fig. 7, with the dimensions of the magnet and the coil as shown in Fig. 4. For the force components

$F_{(x,y,z)}$ the error with respect to values obtained from numerical integration (mesh element size equal to 1 mm^3) over the coil volume [7] is shown in Fig. 8. This shows that the error is 1.7% for force component F_x , 6.3% for F_y and 1.4% for F_z . In terms of calculation times, the analytic model is over 600 times faster than numerical integration (0.014 s for the analytic model against 8.6 s for numerical integration per grid point).

VI. CONCLUSIONS

This paper presents analytic equations to calculate the force between a cuboidal permanent magnet and a cuboidal current carrying volume. These equations are used to determine the force between a cuboidal permanent magnet and a rectangular coil. It is verified, both by measurements and numerical integration, that this method is accurate and, compared with numerical integration, the computational load of this analytic method is limited. The method will be used in the design and real-time control of a magnetically levitated planar actuator and is also suitable for the design of other ironless actuators.

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