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Analysis of a True Time Delay Photonic Beamformer for Transmission of a Linear Frequency-Modulated Waveform

Ruth Rotman, Oded Raz, *Student Member, IEEE*, and Moshe Tur, *Fellow, IEEE*

Abstract—A generalized conversion matrix (GCM) and numerical analysis are used to study the distortions suffered by a linear frequency-modulated radio frequency (RF) pulse while propagating through photonic links to be used in wideband phased arrays. The analysis shows the effects of dispersion of all orders, coherent crosstalk and nonlinearity of the optical components on the RF pulse, and the high performance needed to achieve acceptable RF performance of the temporal (impulse) response. The effects of the electrical-to-optical (E/O) and optical-to-electrical (O/E) conversions are also considered. Using the GCM, the optical amplitude and phase fluctuations are converted into their RF counterparts, thereby reducing the optical problem into the well-understood RF domain. A photonic wavelength-controlled true delay device is experimentally shown to achieve good RF performance over a 4-GHz bandwidth, with predicted sidelobe levels below 30 dB.

Index Terms—Microwave communication, optical communications, optical devices, optical distortion, optical modulation, phase modulation.

I. INTRODUCTION

PHOTONIC beamformers [1], comprising optical transmitters and receivers, together with some optical circuitry to optically process radio frequency (RF) signals, are candidates for inclusion within wideband RF radar systems [2], which need true time delay (TTD). These radars have increasingly stringent requirements as to their performance within the RF bandwidth. In receive, where many signals arrive at the beamformer simultaneously, the RF power into the optical modulator is, at present, inherently limited by intermodulations caused by the modulator, conflicting with the system need for large dynamic range [3]. However, for a transmitting (Tx) photonic beamformer, which provides TTD to a set of antenna subarrays [2] and uses RF signals of the form $\cos(\vartheta(t))$, with $\vartheta(t)$ an arbitrary function of time, mapping exists between a single instantaneous frequency and every instant of time in the pulse [4]. As a result, nonlinearities in the modulator give rise only to harmonics, rather than to intermodulations. Therefore, the requirements of linearity and dynamic range are far less rigorous and the transmit link can work at high modulation indices (nonlinear regime of the modulator), allowing high S/N

and reasonably low loss, making them candidates for inclusion within present day radar systems and necessitating the need for more in-depth analysis.

In general, for RF wideband pulse transmission [4], RF components within the transmit radar chain are treated as a cascade of RF filters. Ideally, one would like the overall transfer function to have a uniform amplitude and linear phase over the RF band [4]. In reality, each component, starting from the exciter and ending at the antenna element, has its own transfer function with its own deviations from the ideal, resulting in a distorted pulse. When dealing with an optical beamformer, the optical transfer function of its components is not ideal and may eventually limit the RF performance of the system. One would like to find directly an equivalent RF transfer function and apply the well-known theory found in [4] for RF filters. Recently [5], a relationship between an arbitrary optical transfer function and the RF transfer function for a linear time-invariant optical device, such as a Bragg grating or dispersive fiber, has been derived using a generalized conversion matrix (GCM) [5], which relates the input and output complex envelopes of the RF signal (for small-signal modulation). It should be noted that both the magnitude and phase of the resulting RF transfer function are of equal relevance for determining the distortion of wideband RF pulses [5].

In an attempt to provide guidelines for the successful design of photonic beamformers, this paper analyzes the impacts of photonic circuitry on the performance of wideband RF systems, with emphasis on linear frequency modulation (LFM) pulses (Section II). The GCM is presented in Section III and is then used (Section IV) to demonstrate the deleterious effects of various deviations of the optical transfer function from pure delay. Since the conversion matrix is strictly a small-signal modulation formulation, its regime of validity is also examined (Section IV) using numerical techniques. The impact of optical reflections and crosstalk is also studied in this section. Section V discusses the importance of the transfer functions of the electrical-to-optical (E/O) and optical-to-electrical (O/E) converters and describes an experimental study of wideband (4 GHz) RF transmission through a wavelength-division multiplexer (WDM)–demultiplexer-based TTD. Since for high enough optical powers, the RF performance of photonic beamformers may also depend on optical nonlinearities, Section VI extends the conversion matrix formulation to include the effects of self-phase modulation (SPM).

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II. LFM TRANSMISSION THROUGH RF FILTERS

A. Introduction

An ideal RF LFM square pulse at the input of a transmission system is defined by

$$V_{\text{in}}(t) = \cos \left[2\pi f_0 t + \frac{B}{2T} t^2 \right] \text{rect} \left[\frac{t}{T} \right] \quad (1)$$

where f_0 is the pulse center frequency, B is the total bandwidth of the signal, and T is the pulse length. During the pulse life ($-T/2 \leq t \leq T/2$), the instantaneous frequency $f(t) = f_0 + (B/T)t$ linearly spans the frequency range $f_0 - B/2 \leq f \leq f_0 + B/2$. There is thus a mapping [4] between the instantaneous frequency and time in the pulse, so that when the LFM pulse goes through a nonlinear device, it will produce harmonics but not intermodulations, which limit S/N and dynamic range. The Fourier transform of $V_{\text{in}}(t)$ is approximately given for $BT > 100$ by

$$\mathfrak{F}[V_{\text{in}}](f) \propto \exp \left[-j \frac{T}{2B} (f - f_0)^2 \right] \text{rect} \left[\frac{(f - f_0)}{B} \right]. \quad (2)$$

If the RF transfer characteristics between the input and output of the transmission link can be described by the linear RF filter $H_{\text{RF}}(f) = A_{\text{RF}}(f) \exp[-j\varphi_{\text{RF}}(f)]$, then the received signal is the inverse Fourier transform of the product of $H_{\text{RF}}(f)$ and the spectrum of the input LFM signal, given as

$$V_{\text{rec}}(t) \propto \mathfrak{F}^{-1} [\mathfrak{F}[E_{\text{in}}](f) H_{\text{RF}}(f)]. \quad (3)$$

The signal then undergoes correlation with the initial transmitted pulse (pulse compression), either via weighted matched filter processing or dechirp processing [4]. This process removes the parabolic phase and takes an inverse Fourier transform of the resulting pulse, returning back to the time domain producing a relatively narrow output pulse, the impulse response, whose width is of order $1/B$. For a pure time delay of t_d ($H_{\text{RF}}(f) \propto \exp[-j2\pi f t_d]$) and a matched filter with uniform magnitude, the resulting impulse response would be proportional to $\text{sinc}(\pi B(t - t_d))/(\pi B(t - t_d))$, with the maximum possible resolution per bandwidth but with 13-dB sidelobes. Therefore, in many systems, a weighting filter is used to substantially lower the level of the time sidelobes at the expense of a broader main lobe. In the real world, the system RF transfer function $H_{\text{RF}}(f)$ deviates from a pure time delay, and the actual frequency dependence of its magnitude and phase, i.e., $A_{\text{RF}}(f)$ and $\varphi_{\text{RF}}(f)$, define which time resolution and sidelobe level are achievable by the system [4]. To summarize, the quality of the LFM link is measured by the width of the main lobe (at the 3-dB points) and by the ratio of the peak sidelobe level to the main lobe (PSL), with a minimum possible value determined by the weighting window (e.g., -42.4 dB for a Hamming window). Ripples in the magnitude and/or phase of the RF transfer function of the link may broaden the main lobe and increase the PSL.

B. Role of Distortions in $H_{\text{RF}}(f)$

To avoid deterioration of the impulse response, the transfer function describing the transmission system from Tx to receiving (Rx), $H_{\text{RF}}(f) = A_{\text{RF}}(f) \exp[-j\varphi_{\text{RF}}(f)]$, should have flat magnitude ($A_{\text{RF}}(f) = \text{Constant}$) and linear phase ($\varphi_{\text{RF}}(f) \propto f$) in $[f_0 - B/2 < f < f_0 + B/2]$. In practice, this is not the case, and it is customary to express the frequency dependence (excluding the pure delay) of both $A_{\text{RF}}(f)$ and $\varphi_{\text{RF}}(f)$ as a Fourier series of cosines and sines [4]. A periodic modulation imposed on either the magnitude or phase of the spectrum of the LFM pulse (2) will generate, upon the inverse Fourier transformation, a pair of time pulses, symmetrically located a distance of $1/P_f$ [P_f is the period in the frequency domain of the sinusoidal disturbance of either $A_{\text{RF}}(f)$ or $\varphi_{\text{RF}}(f)$] around the main lobe of the impulse response, duly called “paired echos” [4]. The corrupted impulse response will exhibit a complex combination of these paired echos, not necessarily of equal height. Importantly, when $H_{\text{RF}}(f) \propto (1 + r \cos(2\pi f/P_f)) \exp(jr \sin(2\pi f/P_f))$, the particular combination of equal phase-sine and magnitude-cosine modulations result in a single sidelobe. Physically, this occurs if the transmission involves a point reflection, which is weak. The corresponding transfer function is

$$H_{\text{RF_weak_reflection}}(f) \propto \left(1 + r \exp \left(\pm \frac{j2\pi f}{P_f} \right) \right) \approx |r| \ll 1 \left(1 + r \cos \left(\frac{2\pi f}{P_f} \right) \right) \exp \left(jr \sin \left(\pm \frac{2\pi f}{P_f} \right) \right) \quad (4)$$

where the choice of the sign in (4) depends on whether the reflected signal precedes or is delayed in time relative to the primary signal at the detector.

Each of the Fourier components has its own frequency P_f . We will distinguish between cases, where either the period P_f of the ripple is much longer than the pulse bandwidth B , or the opposite $P_f \ll B$. In the first case, the peaks of the paired echos (at $\pm 1/P_f$ away from the time location of the main lobe) fall within the main lobe and may effectively broaden it, thereby deteriorating the radar range resolution. In the second case, the peaks of the paired echos will appear as two sidelobes, $\pm 1/P_f$ away from the main lobe, with a peak sidelobe level of

$$\text{PSL (decibel)} = 20 \log_{10} \left(\frac{x}{2} \right) \quad (5)$$

where x measures the linear amplitude of the corresponding phase or magnitude periodic ripple. The existence of many terms with $P_f/B \ll 1$ will create a floor of sidelobes, which will affect the contrast of the received signal [6]. The spectrum of the phase and amplitude errors and their relative magnitude and phase determine the level and the width of this floor. As described in [4], phase errors up to 20° can usually be tolerated for $P_f/B \gg 1$ or corrected within the processing. However, the demands on terms with $P_f/B \ll 1$ tend to be stricter: Phase and magnitude errors need to be less than 1° and 0.1 dB in order to maintain sidelobes 40 dB below the level of the main lobe.

More rigorous statistical analysis can be found in [6] and more insight into the general phased array problem in [7].

As the bandwidths of today's radars are rapidly growing, and with the interest in ultrawideband systems, terms, which previously were "swallowed up" in the main lobe, are appearing as sidelobes in the impulse response, threatening system performance.

In the next few sections, we analyze LFM transmission through an optical link, having a nonideal optical transfer function, by converting the optical distortions to their RF equivalents. Once the effect of the optical link on the RF transfer function is known, all traditional results for the RF domain, as described above and as found in the RF literature [6], [7], can be applied.

III. GENERALIZED CONVERSION MATRIX

Under the assumption of small-signal modulation, a modulated electromagnetic field at the input of an optical device can be expressed as [8]

$$E_{\text{in}}(t) = \sqrt{\langle I_{\text{in}} \rangle} \left[1 + \frac{\Delta I_{\text{in}}(t)}{(2\langle I_{\text{in}} \rangle)} + j\phi_{\text{in}}(t) \right] \exp[j\omega_c t] \quad (6)$$

where $I_{\text{in}}(t)$ is the optical intensity, $\langle I_{\text{in}} \rangle$ is the average intensity, $\phi_{\text{in}}(t)$ is the optical phase, and ω_c is the radial frequency of the optical carrier. The propagation of this electromagnetic field through a linear optical medium will be investigated using the transfer function $H_{\text{opt}}(\omega) = E_{\text{out}}(\omega)/E_{\text{in}}(\omega) = A_{\text{opt}}(\omega) \exp[-j\varphi_{\text{opt}}(\omega)]$. For weak intensity and phase modulation, it has been shown [5] that the Fourier transforms of the analytic signals, associated with the output fractional intensity $\Delta I(t)/(2\langle I \rangle)$ and the output optical phase variations $\phi(t)$ at the RF f , are related to their input values by

$$\begin{aligned} E_{\text{out}}(t) &= \sqrt{\langle I_{\text{in}} \rangle} A(\omega_c) \cdot \left[1 + \frac{\Delta I_{\text{out}}(t)}{(2\langle I_{\text{in}} \rangle)} + j\phi_{\text{out}}(t) \right] \\ &\quad \cdot \exp[j(\omega_c t - \varphi_{\text{opt}}(\omega_c))] \\ \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right] \\ \phi(f) \end{bmatrix}_{\text{out}} &= \mathbf{T}(f) \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right] \\ \phi(f) \end{bmatrix}_{\text{in}} \\ \mathbf{T} &= \exp[j\Phi_{\text{phase}}(f)] \cdot \begin{bmatrix} A_1(f) & A_2(f) \\ -A_2(f) & A_1(f) \end{bmatrix} \\ A_1(f) &= \Gamma_+(f) \cos[\Phi_{\text{amp}}(f)] - j\Gamma_-(f) \sin[\Phi_{\text{amp}}(f)] \\ A_2(f) &= \Gamma_+(f) \sin[\Phi_{\text{amp}}(f)] + j\Gamma_-(f) \cos[\Phi_{\text{amp}}(f)] \end{aligned} \quad (7)$$

where

$$\begin{aligned} \Gamma_{\pm}(f) &= \frac{[\exp(-\gamma_+(f)) \pm \exp(-\gamma_-(f))]}{2} \\ \exp(-\gamma_{\pm}) &= \frac{A_{\text{opt}}(\omega_c \pm 2\pi f)}{A_{\text{opt}}(\omega_c)} \\ \Delta\varphi^+(f) &= \varphi_{\text{opt}}(\omega_c + 2\pi f) - \varphi_{\text{opt}}(\omega_c) \\ \Delta\varphi^-(f) &= \varphi_{\text{opt}}(\omega_c) - \varphi_{\text{opt}}(\omega_c - 2\pi f) \\ \Phi_{\text{amp}}(f) &= 0.5 [\varphi(\omega_c + 2\pi f) + \varphi(\omega_c - 2\pi f) - 2\varphi(\omega_c)] \\ \Phi_{\text{phase}}(f) &= -0.5 [\varphi(\omega_c + 2\pi f) - \varphi(\omega_c - 2\pi f)]. \end{aligned} \quad (8)$$

For a phase-only filter, $A_{\text{opt}}(\omega) = \text{Constant}$, (7) reduces to

$$\begin{aligned} \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{out}} \\ \phi_{\text{out}}(f) \end{bmatrix} &= \exp[j\Phi_{\text{phase}}(f)] \\ &\quad \cdot \begin{bmatrix} \cos[\Phi_{\text{amp}}(f)] & \sin[\Phi_{\text{amp}}(f)] \\ -\sin[\Phi_{\text{amp}}(f)] & \cos[\Phi_{\text{amp}}(f)] \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{in}} \\ \phi_{\text{in}}(f) \end{bmatrix} \\ &= \mathbf{T} \cdot \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{in}} \\ \phi_{\text{in}}(f) \end{bmatrix}. \end{aligned} \quad (9)$$

If we further assume that $\varphi_{\text{opt}}(\omega)$ can be expanded in a Taylor series

$$\begin{aligned} \varphi(\omega_c + 2\pi f) &= \varphi(\omega_c) + \left. \frac{d\varphi}{d\omega} \right|_{\omega_c} (2\pi f) + \frac{1}{2} \left. \frac{d^2\varphi}{d\omega^2} \right|_{\omega_c} (2\pi f)^2 \\ &\quad + \frac{1}{6} \left. \frac{d^3\varphi}{d\omega^3} \right|_{\omega_c} (2\pi f)^3 + K \end{aligned} \quad (10)$$

it is immediately obvious that when only the first and second derivatives of $\varphi_{\text{opt}}(\omega)$ are nonzero (i.e., constant dispersion), (9) reduces to the results of [8] and to that of [9] when the third-order derivative cannot be neglected. However, for complicated devices with sinusoidal-type ripples and wideband applications, the approximation of (10) breaks down relatively quickly, and the full conversion matrix should be used [(9) for a phase-only transfer function, and (7) otherwise].

In systems where direct optical detection is used, only the complex RF envelope is detected. Thus, for double sideband (DSB) intensity modulation, $\phi_{\text{in}}(f) = 0$, only the top left-hand term of the matrix (7) is relevant to the calculation and the output RF pulse is given by (3) and (7)

$$V_{\text{out}}(t) \propto \Im^{-1} [\Im[V_{\text{in}}(f)] \cdot \exp(j\Phi_{\text{phase}}(f)) \cdot A_1(f)]. \quad (11)$$

When $A_{\text{opt}}(\omega) = \text{Constant}$, $A_1(f)$ reduces to $\cos(\Phi_{\text{amp}}(f))$, and RF signal fading occurs for those RF frequencies, for which the cosine approaches zero.

For single sideband (SSB) modulation, $\phi_{\text{in}}(f) = \pm j[\Delta I_{\text{in}}(f)/2\langle I \rangle]$ and (9) predicts

$$\begin{aligned} \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{out}} \\ \phi_{\text{out}}(f) \end{bmatrix} &= \exp(-\gamma_{\mp}) \exp[j[\Phi_{\text{phase}}(f) \pm \Phi_{\text{amp}}(f)]] \\ &\quad \cdot \begin{bmatrix} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{in}} \\ \phi_{\text{in}}(f) \end{bmatrix}. \end{aligned} \quad (12)$$

 Fig. 1. Photonic RF link having an optical transfer function $H_{\text{opt}}(\omega)$.

For example, when $\phi_{\text{in}}(f) = -j[\Delta I_{\text{in}}(f)/2\langle I \rangle]$ and $[\Delta I_{\text{in}}(t)/2\langle I \rangle] = a \cos(2\pi ft)$, then $\phi_{\text{in}}(f) = a \sin(2\pi ft)$, and (12) at the output leads to

$$\begin{aligned} \left[\frac{\Delta I_{\text{out}}(t)}{2\langle I \rangle} \right] &= \frac{a A_{\text{opt}}(\omega_c + 2\pi f)}{A_{\text{opt}}(\omega_c)} \\ &\cdot \cos(2\pi ft - (\varphi_{\text{opt}}(\omega_c + 2\pi f) - \varphi_{\text{opt}}(\omega_c))) \\ \phi_{\text{out}}(t) &= \frac{a A_{\text{opt}}(\omega_c + 2\pi f)}{A_{\text{opt}}(\omega_c)} \\ &\cdot \sin(2\pi ft - (\varphi_{\text{opt}}(\omega_c + 2\pi f) - \varphi_{\text{opt}}(\omega_c))). \end{aligned} \quad (13)$$

While for SSB modulation, there is no RF magnitude fading; both the magnitude and phase of the RF transfer function do see the ripples and other distortions in the optical transfer function.

An optical beamformer is usually comprised of many components, each with their own conversion matrix. These transfer matrices can be cascaded together to give one total conversion matrix, which expresses the conversion matrix of the entire beamformer.

Deviations of the optical transfer $H_{\text{opt}}(\omega)$ function from flat amplitude pure delay characteristics can occur due to chromatic dispersion of various orders in dispersive optical TTD elements, crosstalk within optical components, etc. Finally, imperfect response of the E/O and O/E converters will also limit the performance of wideband photonic circuits. The following section analyzes some examples.

IV. EFFECT OF ARBITRARY DISPERSION

A. Introduction

Signal propagation through the schematics of Fig. 1 was evaluated using primarily the conversion matrix and also numerically for a monochromatic input optical field of the form $E_{\text{in}}(t) \propto \exp[j\omega_c t]$. Optical power was assumed to be weak enough so that optical nonlinear effects can be ignored. When both the Mach-Zehnder intensity modulator (properly biased for maximum linearity with a positive slope) and the phase modulator are excited by a single-tone RF signal of frequency f (m and s are, respectively, the intensity and phase modulation

indices and ϑ_{RF} and ϑ'_{RF} are the corresponding RF phases), we obtain the modulated optical field at the filter input, given as

$$E_{\text{in}}(t) \propto \cos\left(\frac{m}{2} \cos(2\pi ft + \vartheta_{\text{RF}}) + \frac{3\pi}{4}\right) \cdot \exp\left[j\left(\omega_c t + \frac{s}{2} \cos(2\pi f + \vartheta'_{\text{RF}})\right)\right]. \quad (14)$$

Its optical intensity is proportional to $(1 + \sin(m \cos(2\pi ft + \vartheta_{\text{RF}})))$, reducing to $(1 + m \cos(2\pi ft + \vartheta_{\text{RF}}))$ for small values of the modulation index m . For the exact numerical calculation of the effect of the optical filter on $E_{\text{in}}(t)$ (valid for arbitrary values of m and s), the Fourier transform of $E_{\text{in}}(t)$ was multiplied by the filter transfer function $H_{\text{opt}}(\omega)$ (we always assume that $H_{\text{opt}}(-\omega) = H_{\text{opt}}^*(\omega)$ [4]). Finally, using inverse Fourier transformation, the output optical field $E_{\text{out}}(t)$ is evaluated, from which we numerically deduce all relevant parameters: the deviation of the optical phase from a pure delay as well as the magnitude and phase of the RF output modulation at f . For small values of m and s , (14) reduces to the form of (6), and the GCM can be directly applied. As shown below, the predictions of (7) compare favorably with the numerical results for small but practical values of m and s .

An interesting example of an optical filter, possessing many dispersion orders, is offered by

$$H(\omega_{\text{opt}}) = \left(1 + A \cos\left[\frac{2\pi}{P_{\omega}^{\text{opt}}}(\omega_{\text{opt}} - \omega_c) + \theta_A\right]\right) \cdot \exp\left[jC \cos\left[\frac{2\pi}{P_{\omega}^{\text{opt}}}(\omega_{\text{opt}} - \omega_c) + \theta_C\right]\right] \quad (15)$$

exhibiting periodic ripples in both its amplitude and phase. We will now address a few special cases of the modulation signal and the optical filter function. In all cases, the detected optical signal after the filter was correlated with the initial LFM pulse, and the resulting impulse response is compared to the ideal one, assuming a Taylor weighting window [10] designed to give a PSL of -47 dB.

B. DSB Modulation [$s = 0$ in (14)]

We start with a phase-only filter [$A = 0$ (15)], having a period twice that of the LFM pulse ($P_{\omega}^{\text{opt}}/(2\pi B) = 2$). Using the GCM formulation, it is easily shown that

$$\begin{aligned} \Phi_{\text{amp}}(f) &= C \cos(\theta_C) \left[\cos\left(\frac{2\pi}{P_f} f\right) - 1 \right] \\ \Phi_{\text{phase}}(f) &= -C \sin(\theta_C) \sin\left(\frac{2\pi}{P_f} f\right) \end{aligned} \quad \left. \vphantom{\begin{aligned} \Phi_{\text{amp}}(f) \\ \Phi_{\text{phase}}(f) \end{aligned}} \right\} P_f = \frac{P_{\omega}^{\text{opt}}}{2\pi}. \quad (16)$$

Clearly, from (7), both the magnitude and phase of the equivalent RF transfer function will be periodic in f with a period P_f . As described in Section III, when P_f exceeds the LFM bandwidth, the deterioration of the impulse response occurs mostly in regard to the target resolution, i.e., a wider main lobe is expected. Therefore, the percentage increase of the 3-dB width of the main lobe was checked as a function of the amplitude of the phase ripple (Fig. 2) for various values of θ_C .

Fig. 2. Percentage decrease in resolution as a function of the optical phase ripple strength C for DSB modulation, with $P_{\omega}^{\text{opt}}/(2\pi B) = 2$. Solid line: $\theta_C = 0$; dashed line: $\theta_C = \pi/4$; dash-dotted line: $\theta_C = \pi/2$ (results obtained using GCM).

Fig. 3. PSL in the impulse response, as a function of optical phase ripple (C) for $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$ with DSB modulation using GCM for $A = 0$. Solid line: $\theta_C = \pi/2$; dashed line: $\theta_C = \pi/4$; dash-dotted line: $\theta_C = 0$.

As the ripple (C) increases, there is a larger dependence on the position of the optical carrier with respect to the filter shape (as measured by θ_C), and more resolution is lost when the optical carrier is at the peak of the cosinusoid ($\theta_C = 0$). At this carrier position, $\Phi_{\text{phase}}(f) = 0$ (16), and the phase error of the optical transfer function translates to an RF amplitude error, having nulls at the zeros of $\cos(\Phi_{\text{amp}}(2\pi f))$. This amplitude fading leads to severe pulse distortion. When $\theta_C = \pi/2$, the optical carrier is at the zero crossing of the phase cosinusoid of (14), the magnitude of the equivalent RF transfer function is flat ($\Phi_{\text{amp}}(f) = 0$), while the RF phase ripple is at its maximum. While this phase ripple decreases the resolution of the pulse, its effect is not as prominent as nulling out frequencies within the bandwidth (cf. $\theta_C = 0$). When the carrier position is between these two states, both the magnitude and phase of the RF transfer function are affected.

For a decreased period of the filter, $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$, the resolution remains unaffected, but the peak sidelobe level increases (Fig. 3).

When the optical carrier is at the peak of the cosine ($\theta_C = 0$), we have $\Phi_{\text{phase}}(f) = 0$ (16), and the optical phase ripple

Fig. 4. PSL of the impulse response, as a function of the optical phase ripple C for DSB modulation with $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$, $\theta_C = \pi/4$, $A = 0$ using numerical simulation. Solid line: $m = 0.3$; dashed line: $m = 0.7$; dash-dotted line: $m = 1.0$; dotted line: GCM.

directly translates into an RF magnitude ripple proportional to $[1 - \cos(\Phi_{\text{amp}}(f))] \propto [\Phi_{\text{amp}}(f)]^2$, which represents a periodic ripple with an amplitude of C^2 [for $C \ll 1$ (14)], resulting in $\text{PSL} = 20 \log_{10}(C^2/2)$ dB (5). When the carrier is $\pi/2$ from the peak, $\Phi_{\text{amp}}(f) = 0$, the optical phase ripple directly translates to an RF phase ripple of amplitude C , resulting in $\text{PSL} = 20 \log_{10}(C/2)$ dB. Here, the RF phase ripple is more influential than its magnitude counterpart.

The mathematics leading to the conversion matrix assumes that the modulation of the optical carrier creates just one lower and one upper sidebands at $\omega_c \pm 2\pi f$. Unless the modulation index is really weak, a Mach-Zehnder intensity modulator, even when biased at quadrature, generates a multiplicity of optical sidebands, spaced $2\pi f$ apart, which mix together at the detector, such that the magnitude and phase of the RF transfer function of the whole system at f are affected by the characteristics of the optical filters at $\omega = \omega_c \pm n(2\pi f)$, where the maximum size of the integer n grows with the modulation index. To test the regime of validity of the small-signal analysis, we compared the case of $P_{\omega}/B = 0.25$, $\theta_C = \pi/4$, $A = 0$, and $C = 0.15$ rad for modulation indices of 0.3, 0.7, and 1 using the numerical simulation described above (Fig. 4).

For m as high as 0.3, the GCM approximation (which implicitly, rather than explicitly, assumes small m) and the numerical simulation show very good agreement. For modulation indices higher than 0.3, the accuracy of the approximation deteriorates. However, the agreement still remains within 2 dB even until a modulation depth of 1. Therefore, the GCM approach can offer results with reasonable approximation for many practical cases. While we have checked a particular example, the same conclusions apply to all cases dealt here in this section.

Finally, Fig. 5 shows an example where both A and C are not 0. We took as parameters $C = 0.05$ rad, $\theta_C = \pi/4$, as a function of the optical amplitude ripple A , where $\theta_A = 0$, $\pi/4$, and $\pi/2$. The RF phase and magnitude of the transfer function are affected by both the optical phase and amplitude of the device, and the full set of (7) needs to be applied.

Fig. 5. PSL of the impulse response with DSB modulation for $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$, $C = 0.05$ rad, and $\theta_C = \pi/4$, as a function of the optical amplitude ripple A . Solid line: $\theta_A = 0$; dashed line: $\theta_A = \pi/4$; dash-dotted line: $\theta_A = \pi/2$.

C. SSB Modulation

As mentioned in Section III, SSB modulation, defined as $\phi_{\text{in}}(f) = \pm j[\Delta I_{\text{in}}(f)/2\langle I \rangle]$, will mitigate the effects of RF magnitude fading for wavelength-independent dispersion. Here, we consider optical phase fluctuations, representing higher order dispersion ($\beta_i \neq 0$, $i > 2$), as well as optical amplitude fluctuations and examine their effect on the performance of SSB modulation in LFM photonic systems.

For the case of upper sideband SSB ($\phi_{\text{in}}(f) = -j[\Delta I_{\text{in}}(f)/2\langle I \rangle]$), we found in (13) that the magnitude and phase of the RF transfer function are given by

$$H_{\text{RF}}(f) = \frac{A_{\text{opt}}(\omega_0 + 2\pi f)}{A_{\text{opt}}(\omega_0)} \cdot \exp[-j(\varphi_{\text{opt}}(\omega_0 + 2\pi f) - \varphi_{\text{opt}}(\omega_0))]. \quad (17)$$

While RF fading is clearly avoided [no $\cos(\Phi_{\text{amp}}(f))$ term], RF phase errors are still present, as well as magnitude ripples.

Using the optical and phase fluctuations of (15), we note that (17) predicts RF magnitude and phase fluctuations with amplitudes that do not depend on either θ_A or θ_C . Therefore, the resulting sidelobes of the impulse response should not vary with these parameters. Comparing (16) and (17), we note that when $A = 0$ in (15), the amplitudes of the sinusoidal RF magnitude and phase ripples for the SSB case are identical to those of the DSB case with $\theta_C = \pi/2$. Thus, for example, the effect of the optical ripples on the sidelobes of the impulse response for SSB for the case $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$ can be taken from Fig. 3 of the DSB case with $\theta_C = \pi/2$ (cf. Fig. 3, $\theta_C = \pi/2$, with Fig. 6). Accuracy is pretty good, since in this case, the bandwidth B spans a few cycles of the optical phase fluctuations. In the case of $P_{\omega}^{\text{opt}}/(2\pi B) = 2$, the resolution of the SSB system can be approximately deduced (Fig. 2), with $\theta_C = \pi/2$. Here, however, B does not see a full cycle of the optical phase variations and the exact shape of the impulse response will depend on θ_C . Similarly, for an optical transfer function with only amplitude ripples ($C = 0$), the amplitudes

Fig. 6. PSL in the impulse response, as a function of optical phase ripple (C) for $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$ with SSB modulation using GCM for $A = 0$ (solid line) and DSB modulation with $\theta_C = \pi/4$ (dashed line).

Fig. 7. PSL of the impulse response with SSB modulation for $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$ and $C = 0.05$ rad, as a function of the optical amplitude ripple A .

of the sinusoidal RF magnitude and phase ripples for the SSB case are very close to those of the DSB case.

Fig. 6 compares DSB ($\theta_C = \pi/4$) and SSB for the case when $A = 0$, $P_{\omega}^{\text{opt}}/(2\pi B) = 0.25$, as a function of C . SSB results are slightly worse than DSB results as the entire optical phase error translates to RF phase error.

Fig. 7 shows peak sidelobe levels for SSB when both A and C differ from zero. The PSL is not better than any of the results shown in Fig. 5.

We conclude this section by noting that while SSB modulation can solve the problem of RF magnitude fading, RF phase response may be severely affected by optical phase ripples, and it does not evade distortions associated with optical amplitude ripples.

D. Parasitic Reflections

The main signal route in an optical system is always accompanied by parasitic satellites, originating from multiple reflections in the system components, optical switch leakage, crosstalk in WDMs, or modal dispersion in higher order mode

(HOM) fibers [11]. Due to the relatively long coherence length of modern lasers, these parasitic signals interfere with the main signal and may thus deteriorate system performance. Here, we analyze the effect of a single weak optical satellite on LFM performance, using the GCM.

Assume a main optical signal of unity strength and a single satellite, delayed by τ , resulting in an optical transfer function of the form

$$H_{\text{opt}}^{\text{sat}}(\omega) = 1 + \sqrt{\varepsilon} \exp(-j\varphi_\varepsilon) \exp(-j\omega\tau) \quad (18)$$

where ε is the relative strength of the satellite (powerwise) and φ_ε is its optical phase. In high-quality modern optical components and well-designed systems, $\varepsilon \ll 1$ and (18) can be approximated by

$$H_{\text{opt}}^{\text{sat}}(\omega) \approx (1 + \sqrt{\varepsilon} \cos(\varphi_\varepsilon + \omega\tau)) \cdot \exp(-j\sqrt{\varepsilon} \sin(\varphi_\varepsilon + \omega\tau)). \quad (19)$$

Clearly, this optical transfer function is characterized by its amplitude and phase ripples being in-quadrature. Using the GCM (7) and keeping terms to first order in $\sqrt{\varepsilon}$, we obtain

$$\begin{aligned} \Phi_{\text{amp}}(f) &= \sqrt{\varepsilon} \sin(\omega_c\tau + \varphi_\varepsilon) [\cos(2\pi f\tau) - 1] \\ \Phi_{\text{phase}}(f) &= -\sqrt{\varepsilon} \cos(\omega_c\tau + \varphi_\varepsilon) \sin(2\pi f\tau) \\ \Gamma_+(f) &= 1 + \sqrt{\varepsilon} \cos(\omega_c\tau + \varphi_\varepsilon) [\cos(2\pi f\tau) - 1] \\ \Gamma_-(f) &= -\sqrt{\varepsilon} [\sin(\omega_c\tau + \varphi_\varepsilon) \sin(2\pi f\tau) \\ &\quad + \cos(\omega_c\tau + \varphi_\varepsilon)] \\ A_1(f) &= 1 + \sqrt{\varepsilon} \cos(\omega_c\tau + \varphi_\varepsilon) [\cos(2\pi f\tau) - 1] \\ A_2(f) &= \sqrt{\varepsilon} \sin(\omega_c\tau + \varphi_\varepsilon) [\cos(2\pi f\tau) - 1] \\ &\quad - j\sqrt{\varepsilon} [\sin(\omega_c\tau + \varphi_\varepsilon) \sin(2\pi f\tau) \\ &\quad + \cos(\omega_c\tau + \varphi_\varepsilon)] \\ &= \sqrt{\varepsilon} \sin(\omega_c\tau + \varphi_\varepsilon) \exp[-j2\pi f\tau] \\ &\quad - j\sqrt{\varepsilon} \exp[j(\omega_c\tau + \varphi_\varepsilon)]. \end{aligned} \quad (20)$$

It appears that the GCM $T(f)$ (7) very sensitively depends on $\omega_c\tau$. Given that $\omega_c \approx 10^{15}$ rad/s, even minute temperature changes will affect τ to the extent that $\cos(\omega_c\tau + \varphi_\varepsilon)$ will span the whole $[-1, 1]$ range. Due to its statistical nature, $\cos(\omega_c\tau + \varphi_\varepsilon)$ can neither be controlled nor corrected for, and it is advisable for each application of (20) to pick the worst case scenario.

For DSB modulation $\phi_{\text{in}} = 0$, and combining (7) and (20), we get

$$\begin{aligned} \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{out}} &= A_1(f) \exp[j\Phi_{\text{phase}}(f)] \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{in}} \\ &= (1 + \sqrt{\varepsilon} \cos(\omega_c\tau + \varphi_\varepsilon) [\cos(2\pi f\tau) - 1]) \\ &\quad \cdot \exp[-j\sqrt{\varepsilon} \cos(\omega_c\tau + \varphi_\varepsilon) \sin(2\pi f\tau)] \cdot \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{in}} \end{aligned} \quad (21)$$

when $\cos(\omega_c\tau + \varphi_\varepsilon) = 0$, and the extraneous interfering signal has no effect on the RF transmission. When the optical interfering signal is in phase with the main signal $\cos(\omega_c\tau + \varphi_\varepsilon) = 1$, (21) reduces to

$$\left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{out}} = ((1 - \sqrt{\varepsilon}) + \sqrt{\varepsilon} \cos(2\pi f\tau)) \cdot \exp[-j\sqrt{\varepsilon} \sin(2\pi f\tau)] \left[\frac{\Delta I(f)}{2\langle I \rangle} \right]_{\text{in}}. \quad (22)$$

The equations show that the magnitude and phase ripples as a function of frequency are sinusoidal but 90° out of phase with each other (see also [11]). Being identical in form to (4), we are led to the intuitive result that under LFM DSB modulation, the resulting impulse response comprises a main lobe and a smaller one [down $10 \log_{10}(\varepsilon)$ dB] delayed in time by τ . Depending on its RF phase, it will add algebraically to the original sidelobes of the unperturbed impulse response. Comparing with (4) ($r \equiv \sqrt{\varepsilon}$), we note that a very weak reflection (crosstalk) of $\varepsilon = 10^{-4}$ ($= -40$ dB) will generate $\text{PSL} = 20 \log_{10}(r) = 10 \log_{10}(\varepsilon)$, which is -40 dB. This implies an important design guideline: Reflections and crosstalk within the optical system (measured in the optical power regime) must be kept at a level as low as the desired peak sidelobe level of the system impulse response.

The term $A_2(f)$ in (7) and (20) has to do with the optical phase. Here, the extraneous optical signal will give rise to the conversion of optical phase variations at the input to variations of the RF complex amplitude at the output and vice versa. It is of interest in the case of SSB modulation, or when the laser is afflicted with phase noise, where the resulting RF noise is referred to as multiple path interference (MPI) noise [11]. If the input to the system had been pure phase modulation, the parasitic signal generates intensity modulation at the O/E converter, which generates a response both at time 0 and time τ .

A parasitic reflection with $\varepsilon = 0.05$ ($= -26$ dB) at a time delay of 6.25 ns was added to the simulation (Section IV-A) for a DSB modulation with a modulation index of 0.3 and 1. The sidelobe level is -26 dB, as predicted by the theory. The effect of the higher modulation index is relatively negligible, except for showing a weak sidelobe at $\tau = -6.25$ ns (Fig. 8).

V. ROLE OF DISTORTIONS IN THE E/O AND O/E CONVERTERS

The previous analysis concentrated on distortions contributed by optical components within the optical link. However, both the E/O and O/E converters of Fig. 1 must also be included in the analysis, since they produce RF phase and magnitude ripples. While the behavior of these devices can be theoretically analyzed [12], it is also possible to actually measure the performance of the modulator (E/O) and receiver (O/E). For a high-quality symmetrically driven lithium-niobate integrated-optics Mach-Zehnder intensity modulator, one can assume that an input RF sinusoidal signal will give rise to chirpless ($[\phi(f)]_{\text{mod_out}} = 0$) DSB modulation, where the resulting optical modulation ($[\Delta I(f)/2\langle I \rangle]_{\text{mod_out}}$) can be

Fig. 8. Impulse response in the presence of a small reflection with time delay $\tau = 6.25$ ns and $\varepsilon = 0.05$ (-26 dB). Dashed line: $m = 1$ (numerical analysis); solid line: $m = 0.3$ (both GCM and numerical analysis).

Fig. 9. Experimental setup for the characterization of the RF transfer function of a wavelength-controlled TTD.

experimentally determined using a lightwave component analyzer [12]. Interferometric or equivalent techniques must be employed if $\phi(f)|_{\text{mod_out}} \neq 0$ needs to be measured as well. Assuming that the modulator is well isolated, $[\Delta I(f)/2\langle I \rangle \phi(f)]_{\text{mod_out}}^T$ is the initial input into the conversion matrix formulation of (7) and (9) (the superscript T stands for transpose). Similarly, for direct detection, the receiver complex responsivity $R(f)$ (i.e., both magnitude and phase) can also be measured using a lightwave component analyzer [12] so that the receiver output voltage is given by $R(f)[\Delta I(f)/2\langle I \rangle]_{\text{out}}(2\langle I \rangle)$.

The optical link, shown in Fig. 9, comprises a tunable laser, a lithium-niobate Mach-Zehnder integrated-optics modulator (E/O), an optical receiver (O/E), a WDM [13], [14], and an optical circulator. The electronic amplifier is used to control the modulation index of modulator, while the erbium-doped fiber amplifier compensates for optical loss in the link. Each fiber leg of the demultiplexer is cut to a prescribed length and silver coated. The result is a precise eight-channel wavelength-controlled TTD link, where the only active switching is done at a single point, namely, the tunable laser. A commercial thin-film WDM was chosen, rather than dispersive lines (RF magnitude fading) or chirped Bragg gratings (optical amplitude and phase

Fig. 10. RF phase and amplitude errors over a 4-GHz bandwidth for the photonic link of Fig. 9 at $\lambda = 1558.98$ nm.

Fig. 11. Calculated impulse response for RF bandwidth of 1 (dashed line) and 4 GHz (solid line).

distortions and sensitivity to laser jitter). Care was taken to minimize all parasitic reflections within the system.

Fig. 10 depicts the RF transfer function of the photonic link over the RF band of 3–7 GHz (measured every 5 MHz) at $\lambda = 1558.98$ nm (the curves of Fig. 10 repeat themselves for the other seven wavelengths, indicating that the distortions originate from the E/O and O/E conversions [13]). Using a Taylor window (with a PSL of -47 dB for the ideal case), we used the data of Fig. 10 to calculate the impulse response for two RF bandwidth, namely, 1 and 4 GHz (Fig. 11).

As the bandwidth increases, the effect of the optical link is more detrimental as the period of the ripple becomes substantially smaller than the RF bandwidth. The PSL of the impulse response remains under -30 dB for the 4-GHz bandwidth and under -37 dB for the 1-GHz bandwidth. The RF transfer function of Fig. 10 insignificantly changes when the optical system is bypassed by connecting the E/O output to the O/E input. This shows that by proper choice of the time delay mechanism, and by keeping crosstalk and optical reflections to a minimum, a much better performance could have been achieved, had smoother E/O and O/E devices been available. The results for an actual LFM experiment with $B = 600$ MHz

were reported in [13] with PSL values below -37 dB for all eight wavelengths.

VI. DELAY LINES AND NONLINEARITY

A. GCM for the Nonlinear Case

LFM, or any other radar signal, can be advantageously remoted via an optical fiber for a variety of operational reasons, including target simulation and calibration purposes. If the input optical power into the fiber is high enough and the line is long enough, the optical intensity modulation due to the RF signal will be translated into optical phase modulation, via the nonlinear SPM effect [15]. Propagating through the fiber, the induced nonlinear phase modulation will be converted by the fiber dispersion back to intensity modulation, thereby affecting the overall RF transfer function of the system. This situation has been analyzed [16] and a closed-form solution was found [17] for small-signal propagation, assuming constant dispersion. Below, we extend the conversion matrix of Section III to include optical nonlinear interactions for arbitrary dispersion.

Starting with a short section of fiber of length Δz and ignoring dispersion, the Kerr effect [15] introduces a nonlinear optical phase to the incident optical signal, according to

$$\begin{aligned} E_{\text{out}}(t) &= E_{\text{in}}(t) \\ &\cdot \exp \left[-j \left(n_0 + n_2 \omega_0 \frac{\langle I(z) \rangle + \Delta I(t)}{c A_{\text{eff}}} \right) \Delta z \right] \\ &= E_{\text{in}}(t) \\ &\cdot \exp \left[-j \left(n_0 + n_2 \omega_0 \frac{\langle I(z) \rangle}{c A_{\text{eff}}} \right) \Delta z \right] \\ &\cdot \exp \left[-j(2\gamma) \left[\frac{\Delta I(t)}{2 \langle I(z) \rangle} \right] \langle I(z) \rangle \Delta z \right] \end{aligned} \quad (23)$$

where n_0 is the refractive index of the mode, n_2 is the nonlinear refractive index, c is the speed of light, A_{eff} is the effective area of the propagating mode [15], $\langle I(z) \rangle$ is the average optical intensity, which is range dependent due to fiber loss, and $\gamma = (n_2 \omega_0)/(c A_{\text{eff}})$ is the nonlinearity coefficient, responsible for SPM. Small-signal modulation does not affect $\langle I(z) \rangle$, and the first exponential term on the second row of (23) represents a time-independent phase and, therefore, will be ignored. Thus, the short fiber section only adds a small optical phase $2\gamma[\Delta I(t)/(2\langle I(z) \rangle)]\langle I(z) \rangle \Delta z$ to the subcarrier-modulated optical wave, and the effect of the short section can be described by a matrix, similar to that of the conversion matrix [(7) and (9)], given as

$$\begin{aligned} \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_{z+\Delta z} &= \mathbf{L}(z, \Delta z) \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_z \\ \mathbf{L}(z, \Delta z) &= \begin{bmatrix} 1 & 0 \\ -K \Delta z & 1 \end{bmatrix} \\ K &= 2\gamma \langle I(z) \rangle. \end{aligned} \quad (24)$$

Subject to the approximations behind the split-step algorithm [15] (mainly small Δz), the combined effect of $\mathbf{L}$ and $\mathbf{T}$ can be obtained by multiplying them as

$$\begin{aligned} \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_{z+\Delta z} &\approx \mathbf{L}(z, \Delta z) \mathbf{T}(f) \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_z \\ &\approx \mathbf{T}(f) \mathbf{L}(z, \Delta z) \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_z \end{aligned} \quad (25)$$

and the whole link can be described by a simple concatenation of the local $\mathbf{L}$ and $\mathbf{T}$ matrices. To first order in Δz , and assuming frequency-independent optical attenuation (within the RF bandwidth around the optical carrier), (25) reduces to [see (9)]

$$\begin{aligned} \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_{z+\delta z} &= (1 + j\bar{\Phi}_{\text{phase}}(f) \cdot \delta z) \\ &\cdot \begin{bmatrix} 1 & \bar{\Phi}_{\text{amp}}(f) \cdot \delta z \\ - (K + \bar{\Phi}_{\text{amp}}(f)) \cdot \delta z & 1 \end{bmatrix} \begin{bmatrix} \frac{\Delta I(f)}{2 \langle I \rangle} \\ \phi(f) \end{bmatrix}_z' \end{aligned} \quad (26)$$

where $\bar{\Phi}_{\text{amp}}(f)$ and $\bar{\Phi}_{\text{phase}}(f)$ are the phases of (8) per unit length. Equation (26) can now be converted to a set of two linear differential equations

$$\begin{aligned} \frac{\partial \bar{p}_N(z, f)}{\partial z} &= [2 \langle I \rangle_{z=0} \bar{\Phi}_{\text{amp}}(f) \cdot \phi(z, f) \\ &\quad + j \bar{\Phi}_{\text{phase}}(f) \bar{p}_N(z, f)] \\ \frac{\partial \phi(z, f)}{\partial z} &= - \left[\gamma \exp(-\alpha z) + \frac{\bar{\Phi}_{\text{amp}}(f)}{2 \langle I \rangle_{z=0}} \right] \bar{p}_N(z, f) \\ &\quad + j \bar{\Phi}_{\text{phase}}(f) \cdot \phi(z, f) \end{aligned} \quad (27)$$

where, following [16] and [17], $\bar{p}_N(z, f) = \Delta I(z, f)/\exp(-\alpha z)$, where α is the (intensity) attenuation coefficient. Equation (27) is valid for arbitrary dispersion and reduces to (1) and (2) of [17] for the case of first-order dispersion.

B. Discussion

The Kerr effect is a relatively weak one, and even for reasonably strong input optical powers (~ 10 – 20 mW), its manifestation in microwave photonics is generally observed in long fiber links. As implied by the term $(K + \bar{\Phi}_{\text{amp}}(f))$ of (26), both the Kerr effect (K) and the dispersive properties of the fiber ($\bar{\Phi}_{\text{amp}}(f)$) modify the optical phase, which is eventually responsible to the detrimental RF magnitude fading discussed above. Neglecting high-order terms, the dispersion of an optical fiber is described by the wavelength derivative of the propagation delay per unit length D [$= 17$ ps/(nm $\cdot$ km) for single mode fiber (SMF) at 1550 nm]. D is related to the second-order radial frequency derivative of the optical phase $\varphi(\omega)$ through $\beta_2 = \partial^2 \varphi(\omega)/\partial \omega^2|_{\omega_c} = -\lambda^2/(2\pi c)D$ (λ is the optical wavelength corresponding to ω_c , and c is light velocity). For such a fiber, $(K + \bar{\Phi}_{\text{amp}}(f)) = K + 2\pi^2 \beta_2 f^2$. However, since $K > 0$ but $\beta_2 < 0$, the nonlinear Kerr effect will postpone (in the RF domain) the occurrence of the null. Moreover, as the RF increases, the role of the Kerr effect diminishes. Fig. 12 shows the conversion of the nonlinearly induced optical phase

Fig. 12. Magnitude of the RF transfer function of a 100-km SMF fiber ($D = 17$ ps/(nm · km), 0.2 dB/km of attenuation, $n_2 = 3.2 \cdot 10^{-20}$ m/W, and optical input power $\langle I(z=0) \rangle = 10$ mW), as a function of the normalized RF. Solid lines represent the small-signal approximation, while the dashed lines show solution of the nonlinear Schrödinger equation for initial modulation indexes of 0.8 ($\gamma = 0$) and 0.5 ($\gamma \neq 0$).

modulation to RF magnitude modulation. Power is transferred from the optical carrier to the RF component, and the detected RF power can actually increase substantially for the nonlinear case, although as noted [16], [17], other factors such as Raman scattering will come into play. The two solid lines, obtained using the conversion matrix [(9) for the linear case ($\gamma = 0$), and (25) for the nonlinear case ($\gamma \neq 0$)] exactly match the results of [17]. As the RF increases, the nulls of the $\gamma \neq 0$ case monotonically approach those of the linear case. To investigate the validity of the small-signal approximation, we used the VPItransmissionMaker software [18] to calculate the magnitude of the RF transfer function for modulation indices as high as 0.8 for the linear case and 0.5 for the nonlinear case. In the linear case, the null locations remain the same, while other portions of the curve show very slight changes. In the nonlinear case, the null locations somewhat deviate from the predictions of the small modulation approximation, but the deviations decrease as the RF increases.

When several different types of fibers and/or in-line optical elements are joined together, the conversion matrix approach is easily applied by multiplying together the matrices corresponding to the various fibers and optical elements. As an example, we analyze a photonic link made of 100 km of standard SMF [$D = 17$ ps/(nm · km), $A_{\text{eff}} = 70 \mu\text{m}^2$, and $\alpha = 0.2$ dB/km], followed by 17 km of dispersion-compensating fiber [$D = -100$ ps/(nm · km), $A_{\text{eff}} = 20 \mu\text{m}^2$, and $\alpha = 0.5$ dB/km]. In the linear case, since the total accumulated dispersion is zero, no RF magnitude fading is observed (Fig. 13). In the nonlinear case and for a reasonably moderate optical input power of 10 mW, SPM helps the dispersion compensation process to the extent that the link as a whole is overcompensated and some fading is observed at around 8 GHz. Results of numerical simulation of the same link using VPI are indistinguishable from those of the predictions of the conversion matrix formulation (25).

Small-signal modulation studies of nonlinear effects have dealt so far only in low-order dispersion ($\beta_n = 0$, $n > 2$) (10). Our technique (25) easily tackles dispersion of arbitrary order. We analyze here the case of $\beta_4 \neq 0$. In normal fibers, β_4 is insignificantly small. However, some of the recently introduced holey and photonic-crystal fibers have the most peculiar disper-

Fig. 13. Magnitude of the RF transfer function of a link comprising a 100-km SMF fiber ($D = 17$ ps/(nm · km), $A_{\text{eff}} = 70 \mu\text{m}^2$, and $\alpha = 0.2$ dB/km) and a dispersion-compensating fiber ($D = -100$ ps/(nm · km), $A_{\text{eff}} = 20 \mu\text{m}^2$, and $\alpha = 0.5$ dB/km), $n_2 = 3.2 \cdot 10^{-20}$ m/W, as a function of the normalized RF, obtained using proper concatenation of (24). $\gamma \neq 0$ represents the nonlinear case with optical input power $\langle I(z=0) \rangle$ of 10 mW.

Fig. 14. Magnitude of the RF transfer function of a 50-km “special” fiber ($\beta_2 = \beta_3 = 0$; $\beta_4 = 2 \cdot 10^{-50}$ s⁻⁴/m, 0.2 dB/km of attenuation, and $n_2 = 3.2 \cdot 10^{-20}$ m/W) as a function of normalized RF. The solid line represents the nonlinear case with an optical input power of $\langle I(z=0) \rangle = 20$ mW, while the dash-dotted line describes the linear case ($\gamma = 0$).

sion properties [19]. In Fig. 14, we show the effect of $\beta_4 > 0$ on the magnitude of the RF transfer function. Since β_4 and K of (24) have the same sign, nonlinearities push the nulls to lower frequencies (cf. Fig. 12). While not shown, VPI-based numerical simulations accurately substantiate the accuracy of the GCM results for small to moderate (0.2) modulation indices.

The GCM, extended to include SPM, is a convenient and useful tool to analyze analog fiber architectures. It is much faster than commonly available split-step solutions of the nonlinear Schrödinger equation, and while strictly valid for small-signal modulation, the results predict with practical accuracy the behavior for modulation indices as high as 0.5.

VII. CONCLUSION AND GUIDELINES

This paper has analyzed the effects of amplitude and phase ripples of the optical transfer function of an optical beamformer on the performance of LFM transmission. A generalized conversion matrix (GCM) formulation was employed, which can handle arbitrary optical transfer functions. While this method is strictly valid only for small-signal modulation, numerical

analysis has been employed to show that results obtained using the conversion matrix are quite accurate up to useful values of the modulation index (~ 0.3). Using the GCM, we could convert the optical amplitude and phase fluctuations into their radio frequency (RF) counterparts. Thus, we have reduced the optical problem into the well-understood RF domain. The results can be summarized in the form of a few conclusions.

- 1) The amplitude and phase fluctuations of the optical transfer function must be kept small to avoid loss of resolution (for slowly varying fluctuations) or increased sidelobe levels for fast varying fluctuations (all with respect to the LFM bandwidth). How small depends on the specifications of the RF system.
- 2) While single sideband (SSB) has many positive benefits for long-distance transmission, where RF magnitude fading is avoided, it does not have any particular benefit for RF transmission with a large instantaneous bandwidth through systems with optical components with rapid amplitude and phase fluctuations.
- 3) The resulting RF transfer function may depend on the exact position of the optical carrier relative to the optical phase and amplitude fluctuations, requiring prohibitive stabilization measures.
- 4) While some distortions can be corrected for by suitable calibration procedures, other perturbations, e.g., those depending on optical interferometric processes, cannot.
- 5) Modern electrooptic technology can offer devices that can satisfy quite stringent requirements, although the weak links are the electrical-to-optical (E/O) and optical-to-electrical (O/E) converters [13].

When optical nonlinearities are of concern, the GCM has been extended to treat self-phase modulation (SPM) effect in a computationally efficient way.

In conclusion, it appears that when properly designed, photonic beamformers have ripened to the point, where they can replace classical electronic beamformers, offering extremely wide bandwidth and large scanning angles.

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