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Noninertial lateral migration of vesicles in bounded Poiseuille flow

Gwennou Coupier,1,a) Badr Kaoui,1,2 Thomas Podgorski,1 and Chaouqi Misbah1
1Laboratoire de Spectrométrie Physique, CNRS–UMR 5588, Université Grenoble I, B.P. 87, 38402 St. Martin d’Hères Cedex, France
2Faculté des Sciences Ben M’sik, Laboratoire de Physique de la Matière Condensée, Université Hassan II–Mohammedia, BP 7955 Casablanca, Morocco

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Cross-streamline noninertial migration of a vesicle in a bounded Poiseuille flow is investigated experimentally and numerically. The combined effects of the walls and of the curvature of the velocity profile induce a movement toward the center of the channel. A migration law (as a function of relevant structural and flow parameters) is proposed that is consistent with experimental and numerical results. This similarity law markedly differs from its analog in unbounded geometry. The dependency on the reduced volume $\nu$ and viscosity ratio $\lambda$ is also discussed. In particular, the migration velocity becomes nonmonotonous as a function of $\nu$ beyond a certain $\lambda$. © 2008 American Institute of Physics. [DOI: 10.1063/1.3023159]

Flow of confined soft entities, such as vesicles (closed quasi-inextensible lipid membranes) or blood cells in the circulatory system and in microfluidic devices, is a problem of paramount importance with both fundamental and practical interests. While inertial effects can induce lateral migration of any flowing body in a channel,1–3 the ability of these soft entities to adapt their shapes under nonequilibrium conditions gives them the possibility to migrate transversally even at low Reynolds number. Transverse migrations induce non-uniform lateral distributions of the suspended entities, which has important consequences on the rheology of a confined suspension (e.g., the Fahraeus–Lindquist effect in blood vessels4), or should impact on transport efficiency in the various sorting microfluidic devices that are now being developed.5

Despite the considerable interest for transverse migration in many circumstances, there is, to our knowledge, yet no quantitative law that would allow one to relate the lateral migration velocity of a deformable entity flowing in a channel, even isolated, with its position, its mechanical properties, and the flow parameters. We propose a law for the case of a single vesicle placed in a bounded Poiseuille flow, from experiments in microfluidic devices as well as simulations based on the boundary integral method.

The behavior of vesicles under unbounded shear flow has been the subject of several theoretical6–10 and experimental11–13 studies. When the viscosity ratio between the inner and the outer fluids is small, vesicles perform a tank-treading dynamics where the orientation of the main axis of the vesicle is constant and the membrane undergoes a tank-treading motion. In a bounded Poiseuille flow, tank-treading vesicles experience a transverse force and reach the center where they assume a steady shape. The latter stage has been described in several papers.14–16 Migration in Poiseuille flows has been also reported on capsules,17–19 red blood cells,18,19 and drops.20,21 The latter seem to have a very different behavior: Depending on the viscosity ratio and the confinement, the reported equilibrium positions are not always on the centerline. In addition, a drop interface and a vesicle membrane are mechanically different: A vesicle has a constant surface area and its equilibrium shape is not a sphere in general. As a consequence of these intrinsic differences the vesicle shape under flow is described (in the comoving frame), to leading order, by a nonlinear shape equation while a drop obeys a linear equation.8

Lateral migration has two distinct sources. (i) A wall-induced lift force.22–25 In agreement with the numerical and theoretical studies for simple shear flows,22,24 we showed recently that the migration velocity decreases like $1/y^2$, where $y$ is the distance to the wall of the center of mass of the vesicle.26 (ii) The nonconstant shear rate in a parabolic velocity profile (even unbounded) leads to a subtle interplay between the gradient of shear and the shape,27 resulting in migration toward the center with a constant drift velocity except near the centerline. In a realistic channel, both effects coexist and we shall see that this leads to a new and non-trivial noninertial migration law.

The considered microfluidic channel is straight and has a rectangular cross section. The flow direction is $Ox$, and we investigate lateral migration along $Oy$. Let $2w$ denote the channel width in the $y$-direction, and $v_0$ the imposed flow velocity at the center of the channel in the absence of vesicle. The two walls are located at $y=0$ and $y=2w$.

A vesicle is characterized by two geometrical parameters: Its effective radius $R_0$, determined from its constant volume $V$ by $R_0=(3V/4\pi)^{1/3}$, and its reduced volume $\nu=V/[4\pi(S/4\pi)^{3/2}/3]$ ($S$ is the constant area of the vesicle) characterizing vesicle deflation. Volumes are calculated at lift-off (see below) by assuming axisymmetric shape about the vesicle’s main axis. The viscosity ratio is defined as $\lambda=\eta_i/\eta_o$, where $\eta_i$, $\eta_o$ denote the inner and the outer viscosities. Relevant space and time scales are the vesicle radius $R_0$ and the characteristic time needed by the vesicle to relax to its equilibrium shape (in the absence of imposed flow),

a)Electronic mail: gcoupier@spectro.ujf-grenoble.fr.
which is given by $\tau = \eta_{\text{out}} R_0^2 / \kappa$, where $\kappa \sim 20k_B T$ is the membrane’s bending rigidity (typically, $\tau \sim 10^{-3}$ s).

Therefore dynamics depends \textit{a priori} on the four dimensionless parameters $(\tilde{w} = w/R_0, \tilde{v}_0 = v_0 \nu \tau / R_0, \nu, \lambda)$. We consider first the case $\lambda = 1$. The strategy is to vary $\tilde{w}$ and the imposed velocity $\tilde{v}_0$ and investigate the migration law $\tilde{y}(\tilde{t}) = y(t/\tau) / R_0$ for each value of $\nu$. Then we discuss the influence of $\nu$ and $\lambda$.

In the experiments, we used straight channels of height $h_0 = 66.6$ μm (in the direction of gravity $z$) and width $2w$ (rectangular cross section) between 70 and 140 μm. The walls of the channels are made of polydimethylsiloxane glued to a glass slide. The flow is induced by gravity, by connecting the inlet and the outlet to reservoirs at different heights. Vesicles are prepared following the electroformation method. They are made of a dioleoylphosphatidylcholine lipid bilayer enclosing an inner solution of sugar (sucrose or glucose) in water or in a 1:4 glycerol-water ($w:w$) mixture. Samples are diluted in a slightly hyperosmotic outer solution of the same type, in order to deflate them by osmosis. Dextran can be added to one of the solutions to modify the viscosity ratio $\lambda$. Vesicle size $R_0$ lies in the range of $7-37$ μm while $v_0$ varies between 200 and 1100 μm s$^{-1}$. Note that for our solutions of viscosity and density close to the one of water, the Reynolds number $R_e = \rho v_0 R_0 / \eta_{\text{heat}}$ is always lower than $4 \times 10^{-2}$.

A particular design of the upstream channel creates an initial condition where incoming vesicles touch the $y=0$ wall in the observation area and start to be lifted away from it. In particular, they have already developed a nearly ellipsoidal shape tilted with respect to the wall.23–25 The two-dimensional (2D) fluid velocity profile in the $xy$ plane where their center of mass lies is nearly parabolic, since the rectangular cross section of the three-dimensional (3D) channel obeys $2w / R_0 \ll 3$.28 Moreover, we wait for the flow to be established for a long time, resulting in a parabolic profile of the vesicles in the $z$ direction. The imposed velocity profile is thus written as $v'_y(r) = c(y = y^*)/2$, where $c = 2v_0/\nu^2$ is the curvature. A vesicle is tracked along its trajectory with a phase contrast microscope, and the position $y$ of its center of mass is determined by image processing, starting with $y(t = 0) = y_0$, where $y_0$ is the position just before lift-off, which is close to $R_0$.

The evolution with time of the $y$ position of two vesicles with $\lambda = 1.1$ is shown in Fig. 1(a). The vesicles quickly move away from the wall, then the migration velocity decreases to zero as they approach the centerline. Along their trajectory, they continuously deform from a tilted ellipsoid to a symmetric bulletlike shape. For a given $\nu$, the function $\tilde{y}(\tilde{t})$ depends \textit{a priori} on both parameters $\tilde{w}$ and $\tilde{v}_0$. In order to determine this functional dependence, which is not known \textit{a priori}, we rescale the time variable. The choice of a relevant time scale is not obvious, however. Indeed, while the inverse of the shear rate yields a natural scale, this is not an adequate choice since the shear rate is not constant along the trajectory. The trick is to rescale each infinitesimal time step $dt$ around the time $t$ by the local shear rate $\gamma(y) = d\tilde{v}/dy = c(w - y)$ of the unperturbed flow at the position $\tilde{y}(t)$. The new dimensionless timelike parameter is then obtained by integrating the rescaled time steps: $\tilde{t} = \int_0^t y'(\tilde{y}) d\tilde{t}' = \int_0^y d\tilde{t}' = \tilde{y}(\tilde{t}) - \tilde{y}_0$. The corresponding curves clearly collapse on a single one. For clarity, the sets of curves corresponding to different $\nu$ intervals are switched vertically with an increment of 0.5.

This law constitutes the central result of our finding. In the range $0.970 < \nu < 0.975$ we find for instance $\xi = \alpha \beta^l / a = 1.2 \times 10^{-2} \pm 0.2 \times 10^{-2}$ and $\delta = 1/\alpha - 1 \pm 0.1$. The error bars for these coefficients are mainly due to the uncertainties on the measure of $\nu$, of order $\pm 0.005$, since, as we shall see, the velocity depends on the reduced volume. Note that the differential equation (1) has no analytical solution but can be easily solved numerically and the result used to fit the raw data $y(t)$ without rescaling procedure [Fig. 1(a)].

In the simulations, we studied 2D neutrally buoyant vesicles (the 2D geometry captures the essential features)
having no viscosity contrast ($\lambda=1$). The fluid flows inside and outside the vesicle are governed by the Stokes equations ($r_m \in \partial\Omega$ is a membrane point)

$$-\nabla p(r) + \eta \nabla^2 \mathbf{v}(r) = -\delta(r-r_m)\mathbf{f}(r), \quad \nabla \cdot \mathbf{v}(r) = 0,$$

where $p$ is the pressure, $\mathbf{v}$ is the velocity, and $\mathbf{f}$ the membrane force, given by Eq. (8) of Ref. 27. Thanks to the linearity of Eq. (2), we solve them using a boundary integral method adapted to vesicle problems.\textsuperscript{27,29,30} The membrane velocity is then given by the following integral equation, which we solve numerically:

$$v_i(r_m) = \frac{1}{4\pi \eta} \oint_{\partial \Omega} G_{ij}^W(r_m,r') f_j(r') ds(r') + \nu^v_i(r_m),$$

where $G_{ij}^W$ is the Green’s function for a fluid bounded by a steady infinite plane wall located at $y=0$,

$$G_{ij}^W(r,r') = G_{ij}(r-r') - G_{ij}(r-r_j') + 2r_j^2 G_{ij}^{SD}(r-r_j') - 2r_j^2 G_{ij}^{SD}(r-r_j').$$

This Green’s function for an unbounded fluid, or Stokeslet, $r_j'=(r_j',-r_j')$ is the image of $r'$ with respect to the wall. The function

$$G_{ij}^{SD}(r) = (\delta_{ij} - \delta_{ij}) \left( \frac{\delta_{ij}}{r_j^2} - 2 \frac{r_i r_j}{r^2} \right)$$

is the Stokeslet doublet, and the source doublet is

$$G_{ij}^{SD}(r) = r_i G_{ij}^{SD}(r) + (\delta_{ij} - \delta_{ij}) \frac{\delta_{ij} r_i - \delta_{ij} r_j}{r^2}.$$
vesicle in the case of a simple shear flow bounded by a wall, although the scaling is different.

In conclusion, our experiments and simulations yield a similarity law for the lateral migration velocity of a vesicle in a bounded Poiseuille flow as a function of its distance to the walls and to the centerline, its effective radius, the channel’s width, and the flow velocity. We showed that the effects of the walls and of the curvature of the velocity field are coupled in a nonlinear manner: Curvature not only induces migration but also affects the shape and orientation, which affects the lift force. The law $v_m \sim \dot{\gamma}(y)/y$ markedly differs from what the naive extrapolation of the results for a vesicle near a wall and in a linear shear flow would give $v_m \sim \dot{\gamma}(y)/y^2$.

Deflating a spherical vesicle increases its deformability, thus its asymmetry under shear, and leads to higher migration velocities. However, beyond a given viscosity ratio, the tank treading to tumbling transition is approached when the deflation increases, and the migration velocity undergoes a decline which can be understood on the ground of general symmetry considerations.

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