The benefit of VMI strategies in a stochastic multiproduct serial two echelon system

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GP Kiesmüller, RACM Broekmeulen

Faculty of Technology Management,

Technische Universiteit Eindhoven,

P.O. Box 513,

5600 MB Eindhoven,

The Netherlands

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GP Kiesmüller*, RACM Broekmeulen

Faculty of Technology Management,

Technische Universiteit Eindhoven,

P.O. Box 513,

5600 MB Eindhoven, The Netherlands

e-mail: g.p.kiesmueller@tm.tue.nl

Abstract

We consider a multi-product serial two echelon inventory system with stochastic demand. Inventories at the downstream location are replenished periodically using an automatic ordering system. Under vendor managed inventory strategies the upstream stage is allowed to adapt these orders in order to benefit from economies of scale. We propose three different VMI strategies, aiming to reduce the order picking cost at the upstream location and the transportation costs resulting in reduced total supply chain costs. In a detailed numerical study the VMI strategies are compared with a retailer managed inventory strategy for two different demand models suitable for slow moving products. It is shown that if inventory holding costs are low, compared to handling and transportation costs, efficiencies at the warehouse are improved and total supply chain costs are reduced.

keywords: vendor managed inventory, coordinated replenishment, order-picking

*corresponding author: GP Kiesmüller, fax: + 31 40 2464531, e-mail: g.p.kiesmueller@tm.tue.nl

1 Introduction

The efficient management of a supply chain requires close coordination of different parties and functions in order to maximize total supply chain profit. Since different stages in a supply chain may have different owners it is very likely that they optimize their own profits without taking into account the impact of their decisions on other parties in the supply chain. But also under the same ownership different stages may have conflicting objectives, resulting in decisions which are only locally optimal. Therefore, it is a very challenging task to achieve coordination in a supply chain.

Different programs have been developed and implemented in industry in order to improve the performance of the entire supply chain. Vendor managed inventories (VMI) is one of these initiatives, which is pioneered in the early 1980s by Wal-Mart and since then has gained popularity in many industries as well as in research. With VMI the supplier is authorized to manage the inventory of the retailer and replenishment decisions about the timing and quantity move from the downstream supply chain actor to an upstream stage. As a consequence the supplier must have access to information about inventory levels at the retailers, which requires trust to build up a supply chain relationship, and information technology such as electronic data interchange (EDI) or radio frequency identification (RFID). For more information about the factors which might effect the effectiveness of VMI see the empirical studies of Dong et al. [1] or Kuk [2].

Several advantages and opportunities of VMI are already discussed in the literature (see for example Waller et al. [3]) and different models are developed to analyze the improvements. These models usually assume deterministic and known demand, and focus on production, inventory and/or transportation cost. We refer to Dong et Xu [4], Braglia and Zavanella [5], Valentini and Zavanella [6], and Bertazzi et al. [7]. All these models do not explicitly model the handling operations at the warehouse and the related costs in detail. However, in a recent study at a Dutch retail chain, Broekmeulen et al. [8] have observed that these costs are a very important component of total costs, at least three times higher than inventory costs. Therefore, in this paper we investigate improvement opportunities in vendor managed inventory systems by reducing handling costs at the supplier. We will do this by modeling the warehouse operations in detail.

We study a two echelon supply chain consisting of one vendor and one retailer. At the vendor as well as at the retailer N different products are kept on stock. Stock at the retailer is used to satisfy stochastic customer demand, while the stock at the vendor is used to replenish the inventory of the retailer. We assume that the supplier has ample stock and also at the retailer there is excess shelf space available (Broekmeulen et al. [8]). In contrast to the problem environment studied by Hassini [9], each product has dedicated excess shelf space and therefore we are not interested in the storage space allocation at the retailer. Unsatisfied demand at the retailer is back-ordered. Retailer managed inventory (RMI) as well as vendor managed inventory (VMI) replenishment strategies are considered and compared. In both situations the inventory is reviewed periodically and single item order-up-to policies, widespread at retailers, are used to determine initial order quantities. Under VMI the vendor is allowed to adapt the initial order in order to benefit from economies of scale. We investigate three different VMI policies resulting in different order-picking strategies at the warehouse.

Order-picking activities are, according to Petersen II [10] and de Koster et al. [11], the most costly and labor intensive activities for almost all warehouses. Although other activities contribute to the order-picking time, the travel distance of the picking tour is dominant (see [11]). In our cost model we include linear costs in the traveling distance as well as fixed ordering costs for each product ordered. The order picking strategy as well as the layout of the warehouse have a large influence on these costs. In this paper we consider a single picker – single order strategy. A lot of research is done about optimal layout and optimal routes and order picking strategies, where it is usually assumed that orders from the retailer are given. In contrast to this we allow adapting the initial orders in order to make use of economies of scale and reduce costs. The three policies we propose can be applied in a VMI concept where the vendor has information about the retailers inventory levels through technologies such as EDI or RFID .

Our work is also related to the stochastic demand joint replenishment problem. For an overview of the literature in this field we refer to Aksoy and Erenguc [12] and Khouja and Goyal [13]. In the classical joint replenishment problem besides inventory and backorder costs major (independent from the number of different products per order) and minor ordering costs (for each product included in an order) are considered (see for example,

Chen and Chen [14]). In this paper we relate these cost components to the order picking process in a warehouse and by modeling this activity in detail we can also influence and change these cost components.

The remainder of the paper is organized as follows. In the following section we describe the problem characteristics in more detail. In section 3 the retailer managed inventory situation is described and for special cases analytical models are derived. Three different replenishment strategies under vendor managed inventory are introduced in section 4. A numerical study in order to compare both concepts is presented in section 5. The paper ends with a short summary and conclusions.

2 Problem characteristics

We consider a retailer keeping N different products on stock in order to satisfy stochastic customer demand where unsatisfied demand is backordered. The retailer is resupplied by a single vendor with ample supply. In the RMI as well as in the VMI situation inventory is reviewed periodically and orders can be placed at each review instant where a customer order consists of OL order lines, each line for a unique product, in a certain quantity. For each product the order picker has to stop in order to pick the order and therefore each order line induces a cost of c_{OL} at the warehouse. We suppose the review period to be given and without loss of generality we set the review period equal to one time unit. The replenishment lead time L is assumed to be constant and includes transportation time as well as time necessary for order picking and handling.

In this paper we restrict ourselves to slow moving items and we will use discrete distributions for the demand per period at the retailer.

We consider the most simple warehouse layout where all products are stored in one aisle with length A (see Figure 1). Moreover, we number the products in the aisle such that product 1 is closest to the starting point of the order picker.

The supplier is applying a picker to parts system where order batching is not allowed and the order picker is always picking exactly one order. Costs originated through traveling

Figure 1: Locations of products in the warehouse

of the order-picker are linear in the distance and for each distance unit c_W is charged. The picker is using a (cardboard) container or a box with a finite capacity V to put all products of an order in it. (In the sequel we will only use the word container.) This container is also used for transporting the products to the retailer. There is no restriction on the number of containers available but for each container a fixed cost c_M is charged.

We are interested in the long run average costs of the entire supply chain compound of inventory holding and backorder costs at the retailer as well as the handling and transportation costs of the warehouse. Handling costs consists of two parts: the major ordering costs, linear in the traveling distance of the order picker, and the minor ordering costs for each order line. Inventory and backorder costs at the retailer are charged at the end of a review period and are assumed to be linear. In total the average costs per period given as

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \left(c_W E[W_t] + c_{OL} E[OL_t] + c_M E[M_t] + \sum_{i=1}^N (c_{H,i} E[I_{i,t}^+] + c_{B,i} E[I_{i,t}^-]) \right) \quad (1)$$

where $I_{i,t}^+$ denotes the stock on hand of item i at the end of period t , $I_{i,t}^-$ denotes the backorders of item i at the end of period t and W_t denotes the traveling distance of the order picker in period t , M_t denotes the number of containers used in period t and OL_t the number of order lines of the order placed in period t .

Below we summarize the used notation:

N :	Number of different products
L :	Replenishment lead time
D_i :	Demand of product i in one period
$D_i(T)$:	Demand of product i during T periods
μ_i :	Mean period demand of product i
$I_{i,t}$:	Net-stock (Stock on-hand minus backorders) of item i at the end of review period t
$Y_{i,t}$:	Inventory position of product i just before the review point t , before the decision is made
$Q_{i,t}$:	Order quantity of product i in the review period t
S_i :	Order-up-to level for product i
A :	Length of the aisle in the warehouse
W_t :	Traveling distance of the order-picker in period t
OL_t :	Number of order lines of an order in period t
V :	Capacity of a container in units
M_t :	Number of containers used in period t
c_W :	Handling cost parameter related to traveling distance of the order picker
c_{OL} :	Handling cost per order line
c_M :	Transportation cost parameter per container
$c_{H,i}$:	Holding cost parameter for product i
$c_{B,i}$:	Backorder cost parameter for product i
$E[X]$:	Expectation of a random variable X
F_X :	Cumulative distribution function of a random variable X
X^+ :	$\max(0, X)$
X^- :	$\max(0, -X)$

3 Retailer managed inventory

As a benchmark we analyze the supply chain in the current structure where the retailer is responsible for placing orders at the warehouse.

3.1 The model at the retailer

According to Angerer [15], many retailers are using automatic ordering systems which are build on single item inventory models, neglecting any possible benefits by coordinating orders. We assume that the logic of a periodic order-up-to policy is used to determine

order quantities $Q_{i,t}$, given as the difference between the actual inventory position $Y_{i,t}$ of item i and its order-up-to level S_i .

$$Q_{i,t} := S_i - Y_{i,t} \quad (2)$$

Since we assume that the warehouse has ample supply all quantities ordered are shipped to the retailer. In Cachon [16] this policy is called a full service policy, a policy often used by retailers.

In case of an RMI system the retailer is interested in minimizing his costs composed of inventory holding costs and backorder costs for each product, and he is neglecting the impact of his decisions on the vendor's warehouse operations. The following objective function is minimized:

$$\begin{aligned} G(S_1, S_2, \dots, S_N) \\ = \sum_{i=1}^N c_{H,i} E[(S_i - D_i(L+1))^+] + c_{B,i} E[(D_i(L+1) - S_i)^+] \end{aligned} \quad (3)$$

It is well known that the optimal values S_i^* are given as a solution of the following newsboy inequalities:

$$F_{D_i(L+1)}(S_i^*) \geq \frac{c_{B,i}}{c_{B,i} + c_{H,i}} \quad (4)$$

The resulting expected inventory costs for product i can be computed as

$$E[I_{i,t}^+] = c_{H,i} \sum_{j=0}^{S_i^*} (S_i^* - j) \cdot P(D_i(L+1) = j) \quad (5)$$

and the average costs for backorders are given as

$$E[I_{i,t}^-] = c_{B,i} \sum_{j=S_i^*}^{\infty} (j - S_i^*) \cdot P(D_i(L+1) = j) \quad (6)$$

3.2 The model at the warehouse

When the warehouse receives an order from the retailer, the order is released to the floor and the picker starts his activities, walking or driving through the aisle and picking the products in the ordered quantities. In this situation it is not allowed to pick more products

or other quantities than ordered. The traveling distance of the order picker is determined by the largest index i in an order. It can be computed as follows:

$$W_t = 2A \cdot \frac{1}{N} \cdot \max\{i \mid S_i - Y_{i,t} > 0\} \quad (7)$$

If all the items are fast movers and $P(D_i = 0) \approx 0$ then the order picker always has to travel to the end of the aisle and the traveled distance is $2A$. But in case of slow movers there is a positive probability for having no demand of product i during a period and then the distance the order picker has to travel can be less than $2A$. Expected traveling costs of the order picker can be computed as follows:

$$c_W \cdot E[W] = c_W \cdot \frac{2A}{N} \left(\sum_{j=1}^{N-1} j \cdot P(D_j > 0) \cdot \prod_{k=j+1}^N P(D_k = 0) + N \cdot P(D_N > 0) \right) \quad (8)$$

It can be seen that in this situation the handling costs can only be influenced by changing the layout of the warehouse or the order picking strategy. Moreover, the order-up-to levels at the retailers do not have any influence on the handling costs.

The average number of containers to be used in a period can be computed as

$$E[M] = \sum_{j=0}^{\infty} j \cdot P\left((j-1)V < \sum_{i=1}^N D_i w_i \leq jV\right) \quad (9)$$

where w_i is factor to be used to transform units in capacity (volume and/or weight). In the sequel we assume $w_i = 1$ ($i = 1, 2, \dots, n-1, N$).

Simple closed form expressions of (8) and (9) can easily be derived when all products are identically distributed. We consider two discrete demand distributions: the Bernoulli distribution and the Poisson distribution.

3.2.1 Model 1: Bernoulli distribution

For the Bernoulli distribution with parameter p , we have:

$$D_i := \begin{cases} 1 & : \text{ with probability } p_i \\ 0 & : \text{ with probability } 1 - p_i \end{cases} \quad (10)$$

This results for stochastically identical products in the following closed form expression for $E[W]$, $E[M]$, and $E[OL]$, based on (8) and (9).

$$E[W] = 2A \cdot \left\{ 1 + \frac{1-p}{Np} \left((1-p)^N - 1 \right) \right\} \quad (11)$$

$$E[M] = \sum_{j=0}^{\lceil \frac{N}{V} \rceil} j \sum_{k=(j-1)V+1}^{jV} \binom{N}{k} p^k (1-p)^{N-k} \quad (12)$$

$$E[OL] = N \cdot p \quad (13)$$

3.2.2 Model 2: Poisson distribution

For the Poisson distribution with parameter λ , we have:

$$P(D_i = j) = \frac{\lambda_i^j}{j!} e^{-\lambda_i}, j = 0, 1, 2, \dots \quad (14)$$

This results for stochastically identical products in the following closed form expression for $E[W]$, $E[M]$, and $E[OL]$.

$$E[W] = 2A \left\{ 1 + \frac{e^{-\lambda}}{N(1 - e^{-\lambda})} \left((e^{-\lambda N}) - 1 \right) \right\} \quad (15)$$

$$E[M] = \sum_{j=0}^{\infty} j e^{-\lambda N} \sum_{k=(j-1)V+1}^{jV} \frac{(\lambda N)^k}{k!} \quad (16)$$

$$E[OL] = N \cdot (1 - e^{-\lambda}) \quad (17)$$

Although we consider RMI we are interested in the average total costs of the entire supply chain and we will use (1) as a performance measure.

4 Vendor managed inventory

While RMI as described above is optimal for the retailer it is not necessarily the best solution for the vendor. First, it can happen that the utilization of a container is low resulting in relatively high transportation costs. Second, it may happen that an order-picker has to walk through the whole aisle for picking only a few number of products, resulting in a low utilization of the order-picker, and relatively high handling costs. In the following we will investigate some opportunities arising from VMI to overcome the

problems mentioned above. However, a reduction of the costs at the vendor may lead to an increase of the costs at the retailer. Therefore, it is important to have insights in this trade-off in order to find the optimal balance to minimize total supply chain costs.

In this paper we suggest a hierarchical approach where the vendor is also using the automatic ordering system of the retailer for creating orders but additionally has some flexibility to change order quantities. In the first step the initial order quantities for each product are determined according to (2) and in the second step, the vendor is allowed to enlarge the initial order quantities in order to benefit from economies of scale. Additional backorder costs will always decrease while inventory costs will increase. Please note that we only allow for enlargements, because we do not want the service level to decrease.

Moreover, there are some additional restrictions with respect to the changes in the orders. Since the retailer does not have ample shelf capacity for stocking products it is only allowed to enlarge the order size in such a way that the inventory position of product i after placing the order is equal to $S_i^* + 1$, which means that at most one unit more can be replenished compared to RMI. We further only allow enlargements of order quantities as long as the number of used containers is not changed, which means adaption of orders is restricted by container capacity.

In this paper we investigate three different strategies. For the first strategy, which we call Pick More Units (PMU), we do not allow for the current order to enlarge the traveling distance of the order picker as well as picking more products which would result in more stopping moments. While keeping W_t and OL_t constant for the current period t we expect the average traveling distance and the average number of order lines of an order to decrease in the long-run when more units of a product are picked than actually needed. This means that if in the initial order the largest index of the product ordered is denoted with K , i.e.,

$$K := \max\{i \mid S_i - Y_{i,t} > 0\} \quad (18)$$

then we only allow to change the order quantities of products with $i \leq K$ and $Q_{i,t} > 0$.

The algorithm to compute the new order quantities for the PMU strategy is given as follows:

```

(A-PMU)
 $i := K$ 
while  $i > 0$  and  $V - \sum_{j=1}^K Q_{j,t} > 0$ 
begin
  if  $Y_{i,t} = S_i^*$  and  $Q_{i,t} > 0$  then  $Q_{i,t} = Q_{i,t} + 1$ 
   $i := i - 1$ 
end

```

The second strategy also does not allow an enlargement of the traveling distance of the order picker for the current order, but it allows for picking more units as well as more products than in the initial order. Therefore, this policy is called Pick More Products (PMP). As a consequence, the order picking costs $c_L \cdot OL$ for the current order will increase, but we expect cost savings in coming periods due to less orders and less order lines per order in the future. The algorithm to compute the new order quantities for the PMP strategy is given as follows:

```

(A-PMP)
 $i := K$ 
while  $i > 0$  and  $V - \sum_{j=1}^K Q_{j,t} > 0$ 
begin
  if  $Y_{i,t} = S_i^*$  then  $Q_{i,t} = Q_{i,t} + 1$ 
   $i := i - 1$ 
end

```

In contrast to the PMU and the PMP strategy, enlargement of the traveling distance for the current order is allowed in case of the looking One Product Ahead (OPA) strategy. This may be beneficial in a situation where the picker already has traveled almost to the end of the aisle. In order to avoid traveling to the end of the aisle in the near future he can also pick products close to him in this period, if there is still capacity left, at the retailer's shelf as well as in the container. But the order picker will not always travel to the end of the aisle. He is using a myopic strategy and is only considering the next product when deciding about traveling further or turning around and traveling back to the starting point. If the current position of the order picker is at product $i > K$ then he is only checking if enlargement of the order quantity of product $i + 1$ is possible. If not,

he will turn around, otherwise he will travel to position $i + 1$ and will pick the product and continues in a similar way. A formal description of the algorithm is given as follows:

```

(A-OPA)
 $i := K$ 
while  $i < N + 1$  and  $V - \sum_{j=1}^i Q_{j,t} > 0$ 
begin
    if  $Y_{i,t} = S_i^*$  then  $Q_{i,t} = Q_{i,t} + 1$  and  $i := i + 1$ 
    else  $i := N + 1$ 
end

```

If there is still capacity left then again (A-PMP) will be applied.

In order to be able to analyze the three strategies described above it is necessary to have expressions for the expectations in (1). For the analysis it is in principle possible to use a multi dimensional Markov model, where the state vector $(x_1, \dots, x_N)$ describes the inventory position before an order is placed. But the dimension of the state space is exploding for reasonable numerical values for N and numerical computations are getting intractable. Therefore, we will use simulation to obtain estimates for the expectations in (1).

5 Numerical Study

The input parameters for our numerical study were motivated by empirical data that Huntjens [17] gathered at one of the largest drugstore retailers in the Netherlands. A drugstore retailer is interesting in our setting since it sells predominately slow moving products. Each drugstore places once a week an order at the central warehouse. The sales value of the items in the assortment at the central warehouse varies between 1 and 5 euros. Based on a methods-time measurement study in the order picking area of the central warehouse, we assume that the average walking speed is 1 [m/s] and the average stopping time per order line is 6 seconds. Based on the average wage of the order pickers in the warehouse of 18 [€/h], this results in an average traveling cost c_W of 0.005 [€/m] and an average order line cost c_{OL} of 0.03 [€/stop]. Note that the average order line cost

is six times higher than the traveling cost. Each container could hold on average 78 units and the transportation cost of a container is €3.5.

We did a full factorial experiment in which we tested several levels for each of the input parameters for two demand distributions: Bernoulli and Poisson. In the case of the Bernoulli demand distribution, we have $p_i = \mu_i$ and in the case of the Poisson demand distribution we have $\lambda_i = \mu_i$. We further distinguish between identically distributed product demand, with $\mu_i = \mu$, and different distribution parameters for different products. In the latter case we have set the parameters of the probability distributions in such a way that the mean demand μ_i for product i is decreasing with decreasing product number. We consider

$$\mu_i = e^{-\beta(\frac{i-1}{N})}, \quad \forall i = 1, 2, \dots, N-1, N \quad (19)$$

setting β in such a way that $\mu = \frac{1}{N} \sum_{i=1}^N \mu_i$. This means that in total we have four different demand models.

Throughout the numerical study we consider $N = 100$. The average walking speed is kept at 1 [m/s], but we varied the length of the aisle A to account for different storage densities in the picking area. Varying A has the same effect as varying c_W (see equation (8)) and therefore we fix $c_W = 0.005$ [€/m]. We assume in each experiment that all products have the same product value v and inventory carrying charge $r = 0.1$, resulting in the same holding cost $c_H = vr/50$ per week. Based on the required fill rate P_2 , we derive the back order cost c_B from the holding cost through the newsboy equation.

Further the following numerical values are chosen for the parameters:

L	$= \{0, 1, 2\}$	P_2	$= \{0.90, 0.95, 0.98, 0.99\}$
μ	$= \{0.2, 0.4, 0.6, 0.8\}$	c_H [€/week]	$= \{0.002, 0.005, 0.01\}$
A	$= \{25, 50, 100\}$	c_{OL} [€/orderline]	$= \{0.01, 0.03, 0.05\}$
V	$= \{25, 50, 75, 100, \infty\}$	c_M [€/container]	$= \{1, 3, 5\}$

Following Law and Kelton [18], the reported values for the simulation are the averages from at least 10 replications. In each replication, the first 250 periods were the warming-up periods and statistics are recorded for the last 5000 periods. We replicated until we reached an absolute precision for the relative expected distance $\frac{E[W]}{2A} \pm 0.05$ with 95% confidence.

For the comparison of the two situations we denote the average costs under the uncoordinated replenishment policy by C_{RMI} and the average costs under the coordinated replenishment policy by C_{VMI} . Although it is possible to compute the costs C_{RMI} exactly, we use the value obtained by simulation when computing the relative deviations of the costs defined as

$$\Delta := \frac{C_{RMI} - C_{VMI}}{C_{RMI}} \cdot 100\% \quad (20)$$

5.1 Model 1: Bernoulli demand

Using the Bernoulli demand distribution, the cost improvement through VMI is on average 11.65% for PMU, 16.74% for PMP, and 17.68% for the OPA strategy. Overall, the PMP strategy outperforms the PMU strategy. Compared with the PMP strategy, the OPA strategy gives no significant improvement and will therefore not be discussed further.

From Table 1 we see that there is hardly any difference in performance between the situations with identical products and with non-identical products. The same is true for the lead-time and the fill rate parameters. The negative savings in the table are mainly caused by high inventory holding cost. With low inventory holding costs, we always have positive savings. When the inventory holding cost increases, which is the case for more expensive products, the savings in handling and transportation cost are not always sufficient to cover the increased inventory costs at the retailer.

For the PMU strategy, we expected that the walking distance would remain almost the same, but the number of order lines would reduce significantly. In Table 1 we see that the aisle length has no significant effect on the savings, but the savings increase strongly with the order line cost. The potential savings in the PMU strategy are constrained in the first place by the order lines already in the retailers order and to a lesser degree by the container capacity. Therefore, the savings become higher with higher demand rates and the highest demand parameter results in the highest savings for the PMU strategy in Table 1. The number of additional units, which can be added to the initial order, depends on the container capacity. In the case of infinite container capacity, the PMU strategy does not save on transportation cost compared to the RMI scenario, which results in a saving of just 8.71%. With smaller containers, the savings become larger. The required container

capacity, based on the retailers order, depends on the number of products and the demand rate. The relationship between the container capacity and the savings is therefore not straightforward. This explains also the increased savings from lower container costs, since lower container cost put less emphasis on the savings in transportation by the PMU strategy compared to the order line costs.

For the PMP strategy, we expected almost the opposite from the PMU strategy: the walking distance would reduce significantly and the number of order lines would remain the same or reduce only slightly. Including a product that was not on the original retailers order increases the number of order lines in the short run but could decrease the number of order lines in the long run. In Table 1 we see through the effect on the aisle length that the walking distance is indeed significantly reduced by the PMP strategy. Since the order line cost are on average three times higher than the walking cost in our experimental setup, we see an increase of the savings with higher order line cost that is comparable with the effect of the aisle length. The potential savings in the PMP strategy are first of all constrained by the container capacity and secondly order lines already in the retailers order, since these order lines determine the length of the pick route. Therefore, the savings become higher with larger container capacities and the highest container capacity parameter results in the highest savings for the PMP strategy in Table 1. Compared with the PMU strategy, the PMP strategy also results in relative high savings for low demand rates, because the retailers order can be enlarged with order lines not already on that list. Order enlargement through the PMP strategy reduces in the long run the number of order lines and the number of required containers more than the PMU strategy. Since the PMP strategy has a substantial impact on the number of containers, we also see increased savings with higher container costs.

5.2 Model 2: Poisson demand

Using the Poisson demand distribution, the cost improvement through VMI is on average 8.94% for PMU, 3.54% for PMP, and 3.52% for the OPA strategy. In contrast with the Bernoulli demand model, the PMU strategy clearly outperforms the PMP strategy. Again, the OPA strategy does not differ significantly from the PMP strategy.

Table 1: Cost comparison of the RMI and the VMI scenario using the PMU or the PMP strategy under Bernoulli demand.

Parameter	Level	PMU			PMP		
		Min	Avg	Max	Min	Avg	Max
Lead-time L	0	-18.69	11.58	37.84	-25.44	16.60	40.31
	1	-13.80	11.67	37.70	-20.57	16.78	40.70
	3	-12.80	11.68	37.54	-17.71	16.82	39.85
Fill rate P_2	0.90	-18.69	11.85	37.84	-25.44	17.14	40.31
	0.95	-18.69	11.75	37.84	-25.44	16.96	40.31
	0.98	-18.69	11.48	37.84	-25.44	16.42	40.31
	0.99	-18.69	11.49	37.84	-25.44	16.44	40.31
Mean demand μ	0.2	-18.69	3.60	21.28	-24.10	16.60	33.59
	0.4	-14.94	9.20	27.48	-25.44	13.73	33.50
	0.6	-9.30	15.50	36.82	-19.17	16.60	36.36
	0.8	-5.69	18.46	37.84	-11.00	20.30	40.31
Identical products	Yes	-18.69	11.50	37.84	-25.44	16.55	40.31
	No	-16.28	11.80	37.72	-25.92	16.92	38.68
Container capacity V	25	-14.50	13.41	22.96	-24.10	12.30	19.28
	50	-18.69	14.75	36.82	-25.44	16.26	36.36
	75	-18.69	12.65	37.84	-19.17	17.00	38.68
	100	-18.69	8.71	35.96	-19.30	15.60	33.59
	∞	-18.69	8.71	36.00	-19.30	22.80	40.31
Holding cost c_H	0.002	0.22	15.80	37.84	3.60	22.30	40.31
	0.005	-8.51	12.60	36.85	-9.36	17.35	37.52
	0.010	-18.69	7.79	35.25	-25.44	10.84	35.60
Aisle length A	25	-18.69	11.94	37.84	-25.44	15.65	39.90
	50	-16.86	11.71	36.99	-20.71	16.55	40.60
	100	-12.16	11.27	35.39	-12.64	18.30	40.31
Order line cost c_{OL}	0.01	-18.69	6.19	37.84	-25.44	15.93	40.31
	0.03	-8.87	12.34	36.36	-19.44	16.89	39.29
	0.05	0.16	16.39	36.00	-15.42	17.39	38.61
Container cost c_M	1	-18.69	12.72	36.00	-25.44	12.30	35.89
	3	-10.85	11.48	36.35	-10.50	17.91	39.20
	5	-7.79	10.74	37.84	-5.70	20.28	40.31

We see in Table 2 that the savings are almost the same for the demand model with identical products and the demand model with non-identical products. The lead-time and fill rate parameters result only in a small change in savings, which we already saw in the experiments with the Bernoulli demand model.

The Poisson distribution differs from the Bernoulli distribution in several ways. First, with Bernoulli demand we have the highest variance at $p = 0.5$, which is 0.25 ($Var(X) = p(1 - p)$). The variance of the Poisson distribution is equal to λ , which is much higher at the same demand rate. Secondly, with Bernoulli demand the demand per period is limited to one item, while Poisson demand can be much larger. Therefore, the excess inventory that our proposed strategies create at the retailer do not necessarily reduce the number of order lines in the long run. This reduction in order lines is needed to achieve savings in transport and handling costs. Under low inventory holding cost, the negative savings only disappear for the PMU strategy, which makes the VMI scenarios less interesting for a broad range of possible parameter settings.

Under both strategies, smaller container capacities increase the savings. The PMP strategy ships too many units to the retailer with larger containers due to the potential higher number of order lines. These products increase the inventory holding cost without significantly lowering the handling cost. The effects of the other parameters on the savings are in line with the findings under Bernoulli demand.

6 Summary, conclusions and future research

We have shown that using vendor managed inventory scenarios that focus on the reduction of order lines can reduce handling cost at the warehouse and transportation cost between the warehouse and the stores. The amount of savings require that the inventory holding costs are relatively low compared to the handling and transportation cost. The proposed Pick More Units strategy has modest savings, but performs well over a large range of parameters and under both the Bernoulli and the Poisson demand distribution. The Pick More Products strategy is more interesting under a Bernoulli demand distribution than under a Poisson demand distribution, since the potential savings can be much higher.

Table 2: Cost comparison of the RMI and the VMI scenario using the PMU or the PMP strategy under Poisson demand.

Parameter	Level	PMU			PMP		
		Min	Avg	Max	Min	Avg	Max
Lead-time L	0	-14.14	9.70	32.45	-26.28	3.20	33.40
	1	-11.31	9.80	32.17	-20.17	3.85	32.64
	2	-10.18	8.66	31.91	-19.69	3.56	32.43
Fill rate P_2	0.90	-14.14	9.30	32.45	-26.28	3.47	33.40
	0.95	-11.31	9.21	32.43	-24.62	3.75	32.70
	0.98	-9.50	8.60	31.85	-19.41	3.29	32.42
	0.99	-11.70	8.64	31.83	-21.78	3.66	32.20
Mean demand μ	0.2	-14.14	3.54	19.94	-26.28	1.14	14.92
	0.4	-10.74	8.80	23.10	-24.62	1.37	20.21
	0.6	-8.57	11.50	32.45	-22.17	4.69	33.40
	0.8	-4.35	12.63	31.92	-15.91	6.95	32.13
Identical products	Yes	-14.14	9.30	32.45	-26.28	3.72	32.70
	No	-12.25	8.83	32.43	-22.10	3.37	33.40
Container capacity V	25	-8.19	12.13	22.00	-21.80	9.98	20.21
	50	-14.14	12.60	32.45	-26.28	8.18	33.40
	75	-14.14	9.40	31.92	-26.28	3.56	32.13
	100	-14.14	5.61	24.32	-26.28	-2.60	12.16
	∞	-14.14	5.49	24.23	-26.28	-3.91	12.16
Holding cost c_H	0.002	0.10	11.94	32.45	-7.15	6.67	33.40
	0.005	-7.86	9.80	30.51	-16.72	3.72	31.10
	0.010	-14.14	5.78	27.66	-26.28	0.23	28.10
Aisle length A	25	-14.14	9.90	32.45	-26.28	2.83	33.40
	50	-13.85	8.97	31.76	-22.73	3.41	32.45
	100	-10.50	8.74	30.48	-18.19	4.37	31.33
Order line cost c_{OL}	0.01	-14.14	5.50	32.45	-26.28	3.37	33.40
	0.03	-6.17	9.24	32.40	-22.50	3.56	30.91
	0.05	-1.51	12.60	31.71	-19.73	3.68	29.80
Container cost c_M	1	-14.14	9.90	26.53	-26.28	0.88	23.85
	3	-8.42	8.93	31.30	-15.40	4.16	31.50
	5	-6.79	8.77	32.45	-11.39	5.58	33.40

As future research, we intend to look at different, more complex warehouse layouts to see if our findings still hold. Another way to reduce the number of order lines at the warehouse could be the introduction of assortment boxes and/or order batching. Both approaches have the disadvantage that they create additional handling. With assortment boxes, we need to assemble these boxes before we can start picking them. Batch picking requires sorting after the picking route.

References

- [1] Dong Y, Xu K, Dresner M. Environmental determinants of VMI adoption: an explorative analysis. *Transportation Research Part E* 2007; 43: 355–369.
- [2] Kuk G. Effectiveness of vendor-managed inventory in the electronics industry: determinants and outcomes. *Information and Management* 2004; 41: 645–654.
- [3] Waller M, Johnson ME, Davis T. Vendor managed inventory in the retail supply chain. *Journal of Business Logistics* 1999; 20(1): 183–203.
- [4] Dong Y, Xu K. A supply chain model of vendor managed inventory. *Transportation Research Part E* 2002; 38: 75–95.
- [5] Braglia M, Zavanella L. Modelling an industrial strategy for inventory management in supply chains: the 'Consignment Stock' Case. *International Journal of Production Research* 2003; 41: 3793–3808.
- [6] Valentini G, Zavanella L. The consignment stock of inventories: industrial case and performance analysis. *International Journal of Production Economics* 2003; 81–82: 215–224.
- [7] Bertazzi L, Paletta G, Speranza MG. Minimizing the Total Cost in an Integrated Vendor-Managed Inventory System. *Journal of Heuristics* 2005; 11: 393–419.
- [8] Broekmeulen RACM, van Donselaar KH, Fransoo JC, van Woensel T. Excess shelf space in retail stores: An analytical model and empirical assessment. *BETA Working Paper* 2004; 109.
- [9] Hassini E. Storage space allocation to maximize inter-replenishment times. *Computers and Operations Research* 2005; 32: 3191–3207.
- [10] Petersen II CG. The impact of routing and storage policies on warehouse efficiency. *International Journal of Operations and Production Management* 1999; 19(10): 1053–1064.
- [11] de Koster R, Le-Duc T, Roodbergen KJ. Design and control of warehouse order picking. *European Journal of Operational Research* 2007; 182: 481–501.

- [12] Aksoy Y, Erenguc SS. Multi-item inventory models with coordinated replenishments: A survey. *International Journal of Operations and Productions Management* 1988; 8: 63–73.
- [13] Khouja M, Goyal S. A review of the joint replenishment problem literature: 1989-2005. *European Journal of Operational Research* 2008; 186: 1–16.
- [14] Chen J, Chen T. The multi-item replenishment problem in a two-echelon supply chain: the effect of centralization versus decentralization. *Computers and Operations Research* 2005; 32: 3191–3207.
- [15] Angerer A. The impact of Automatic Store Replenishment Systems on retail. Dissertation, University of St. Gallen, Graduate School of Business Administration, Economics, Law and Social Sciences (HSG), 2005.
- [16] Cachon G. Managing a Retailer’s Shelf Space, Inventory, and Transportation, *Manufacturing and Service Operations* 2001; 3(3): 211–229.
- [17] Huntjens RMG. Tandpasta bij shampoo ... of toch maar apart (in Dutch). MSC thesis, Technische Universiteit Eindhoven, Eindhoven, 2008.
- [18] Law AM, Kelton WD. *Simulation modeling and analysis* (third ed.). McGraw-Hill, Boston, 2000.