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Experiments and simulations on self-organization of confined quasi-two-dimensional turbulent flows with discontinuous topography

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Decaying, quasi-two-dimensional turbulent flows in a rotating rectangular domain with a step-like topography are investigated by means of laboratory experiments and numerical simulations. The aim is to describe the role of a discontinuous topography on the evolution and organization of the vortices. Initially, vortex interactions lead to the self-organization of the flow, as in two-dimensional turbulence. Afterwards, the interaction of vortices with the step leads to a flow along the topography that always maintains the shallow region on the right. The simulations have revealed the existence of a critical value determined by the strength of the flow and the step height, after which structures are not able to cross the topography. As a result, the flow evolves almost independently at the shallow and deep regions affecting the efficiency of the organization and therefore the final distribution of vorticity. The existence of a preferential distribution of vorticity due to the step for long times (several rotation periods) is discussed. Different distributions are found when using slightly different flow parameters, and therefore the existence of such a preferential final state is analyzed by using statistical methods. © 2010 American Institute of Physics.

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I. INTRODUCTION

A geophysical flow system can be characterized by the existence of a great number of scales of motion that may vary from planetary to microscales. Such a wide range of scales of motion defines the turbulent character of most of the oceanic and atmospheric phenomena. For example, the kinetic energy transfer from smaller to bigger spatial scales characterizes the two-dimensional (2D) turbulence decay, which results in the self-organization of the flow.^{1,2} This effect is known as the inverse energy cascade, which is not observed in three-dimensional (3D) flows. In contrast, both cases (2D and 3D) present an enstrophy transfer to small scales, being one of the main characteristics of decaying turbulent flows. Large scale systems (small wave numbers) tend to be conservative, i.e., the kinetic energy dissipation is weak, resulting in the persistence of structures such as eddies and currents. This kind of phenomena has been studied with increasing interest during the past decades by means of laboratory experiments, numerical simulations, and theoretical considerations.

The evolution of 2D turbulent flow in the presence of lateral walls has been a research topic in a number of recent investigations. Clercx *et al.*³ described the inverse energy cascade on decaying 2D turbulence in a bounded domain, and compared the results using no-slip and stress-free boundary conditions. They found that the self-organization of the

flow in a square box shows a relaxation toward the so-called Stokes fundamental eigenmode, which consists of a single vortex with size comparable to the domain. Maassen *et al.*⁴ studied the self-organization of decaying, quasi-2D turbulence in a stratified fluid within rectangular containers by means of laboratory experiments and numerical simulations. Their results were focused on the final states of the flow as a function of the domain geometry and found a clear difference with the prediction of quasistationary final states from statistical-mechanical theories. The authors found that the cell patterns are not merely determined by the shape of the container, but also depend significantly on the formation and detachment of viscous boundary layers. These studies showed the role of no-slip boundaries as sources of vorticity and net angular momentum in 2D turbulence in bounded domains, and demonstrated that these effects play a crucial role in the establishment of these quasistationary final states of the flow (see also Refs. 5–7).

In contrast with these works, the evolution of 2D turbulent flow over topography has been less studied, especially by means of laboratory experiments. The study of decaying turbulent flows with rotation and topography in terms of the formation of coherent vortices leads to remarkable results with great applications, since the configuration of such structures is an important factor responsible for the transport of physical, chemical, and biological properties in the oceans and in the atmosphere. In one of the pioneering works using topography, Bretherton and Haidvogel⁸ describe numerically the inverse energy cascade of a quasigeostrophic decaying turbulent flow over random topography. They found that the flow tends toward a stationary state aligned with the topog-

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raphy, with cyclonic (anticyclonic) circulation around a depression (bump) (see also Refs. 9 and 10). This behavior has been widely studied for different types of flows (vortices and currents) over a number of different bottom topographies (slopes, seamounts, and ridges). For a recent review, see van Heijst and Clercx.¹¹

A particular idealized topography is a step-like bottom, dividing the flow domain in a shallow and a deep region. The flow evolution is then strongly influenced by changes in height of fluid columns, as they move from one region to another. Studies on barotropic currents interacting with step topographies are reported by Spitz and Nof¹² and Stern and Austin.¹³ The reflection of cyclonic (anticyclonic) vortices from step-down (up) topographies is described in Zavala Sansón *et al.*¹⁴ For the same kind of topography, Zavala Sansón *et al.*¹⁵ found a flow along the topography that always keeps the shallow region on the right when a coastal current reaches a step-up or down. Tenreiro *et al.*¹⁶ investigated the interaction of dipolar structures with a step-like topography and found an equivalent result: a persistent flow along the topography with the shallow region on its right. Konijnenberg *et al.*¹⁷ describes a spin-up problem using this same configuration and finds always a cyclone (anticyclone) in the deep (shallow) region.

The occurrence of some geophysical processes such as mesoscale currents and vortices is persistent in time and space due to different factors associated with bottom topography and basin geometry. A good example is the Gulf of California, which is an extraordinary natural laboratory, where a seasonal circulation of mesoscale geostrophic eddies along the main axis is observed. Hydrographic historical data prove the existence of these eddies that can reach up to 1000 m depth, with horizontal dimensions (<200 km) comparable with that of the basins that characterize the topography of the region.¹⁸ Another case is the sequence of altimetry data showing the presence of cyclonic and anticyclonic eddies inside and outside the Gulf of Aden, aligned along the main Gulf axis (Ref. 19). The authors report that the presence of cyclones and anticyclones in the vicinity of the Gulf of Aden are due to westward propagating Rossby waves generated in the interior of the Arabian Sea. These two examples show how the circulation pattern seems to depend on the topography and geometry of the basin. In the present work, where laboratory experiments and numerical simulations are done using an idealized geometry and topography, we seek to contribute for a better understanding on how these flow patterns are generated.

This paper addresses the organization of decaying quasi-2D turbulence in rectangular containers with discontinuous topography. The objective is to describe the process in this system and to determine the preferential final state of the flow field due to the presence of the step. The study is carried out by means of laboratory experiments in a rotating tank and by numerical simulations. The laboratory experiments provide physical evidence of the main features expected on 2D decaying turbulence with topography. The numerical simulations, based on a barotropic quasi-2D model with topography, will help to gain a better understanding of the processes involved. Most of these are related with the

FIG. 1. Schematic picture of the experimental setup. The initial flow is generated by moving the set of vertical bars from right to left.

dynamics of dipolar structures generated during the flow evolution. The numerical results show that the flow organization depends crucially on the step height. An important result is that the flow along the step forces the existence of a preferential solution for long times (several rotation periods).

The paper is organized in four main sections. In Sec. II, the experimental setup and two particular experiments are discussed in terms of self-organization and topography signal. Numerical simulations with a similar arrangement are presented in Sec. III, where the main features on the flow organization are described. Section IV is reserved for the discussion of the results and to present the conclusions.

II. LABORATORY EXPERIMENTS

A. Experimental setup

The laboratory experiments were performed in a rectangular, rotating tank with horizontal dimensions 150×75 cm² filled with fresh water. The aspect ratio of the tank is $\delta=2$, defined as the ratio between length and width. The bottom of the container was divided in two square regions, deep and shallow, by means of a 3 cm step. The height of the water column at the deepest part was $H_0=20$ cm (Fig. 1 shows a schematic picture of the experimental setup).

The rotation rate of the tank around the vertical axis was fixed at $\Omega=0.5$ rad s⁻¹, which corresponds to a Coriolis parameter $f=2\Omega=1$ s⁻¹. The decay induced by bottom friction is associated with the Ekman period, $T_E=H_0/(\nu\Omega)^{1/2} \approx 280$ s, for $\nu=0.01$ cm² s⁻¹ (kinematic viscosity of water at 20 °C) which is roughly the duration of a typical experiment. The time scale associated to the Ekman decay is much longer than the rotation period, $T=2\pi/\Omega \approx 12$ s. The experimental procedure consisted of setting the tank in rotation at a constant angular speed for about 30 min before starting an experiment, in order to ensure that the fluid has reached a state of solid body rotation. The parabolic free-surface (1–3 mm) effects are ignored, assuming that the change in depth due to the step (3 cm) in the bottom topography is more important.

A disordered small-scale initial flow field is generated by passing a grid of vertical bars (15 bars with diameter $d = 6$ mm and 4 cm spacing between each one) through the fluid, parallel to the longer sides of the container. The grid is moved with constant speed by an electric motor mounted on the table (a similar configuration is used in Ref. 4). When the bars arrive at the other side of the container, the grid is removed by vertically lifting it out of the fluid (Fig. 1). The initial characteristic vorticity ω_0 is about 0.5 s^{-1} , which corresponds to a Rossby number $R_o = \omega_0/f$ always smaller than 1, ensuring the two-dimensionality of the flow.

In the presence of variable topography there are always 3D effects. Nevertheless, for a rotating fluid system with small to moderate Rossby number and small Ekman number, the flow presents a strong columnar motion, which is modulated by depth changes.

For qualitative experiments, the vortices are visualized by adding fluorescent dye to the fluid. Quantitative experiments are performed using passive tracers ($\sim 250 \text{ }\mu\text{m}$) floating on the surface. The flow field evolution was recorded with a corotating camera mounted at some distance above the tank. Particle image velocimetry (PIV) was used for quantitative experiments. The main results were clearly reproducible in all experiments.

B. Results

Several experiments were performed in order to observe the different processes involved in the self-organization of the flow field. Two typical experiments are discussed, one being qualitative and the other leading to quantitative information about the flow evolution.

Figure 2 shows a sequence of photographs from a typical qualitative experiment (a dye visualization of the flow is used). Hereafter, in all figures the lower (upper) side of the domain corresponds with the deep (shallow) region, the horizontal black line represents the step position and time is made dimensionless as $t = t^*/T$, where t^* is time and T the rotation period of the tank. Initially ($t=3$), the small-scale motions, characteristic of the flow field originated by the passage of the rake through the fluid, are observed. At $t=6$ it is already possible to identify small coherent structures such as dipoles in both regions of the domain. For $t=10$ new structures grow and the effect of the step results in a weak flow along the topography, which is sketched with an arrow. For longer times, the flow organizes into larger structures in both regions. Note, for instance, the large dipole at the shallow side ($t=15-25$) and the smaller dipole near the left wall at the deep region ($t=15-20$). The important point to remark here is that the flow generated along the step effectively separates the shallow and deep regions as larger vortices are formed.

Figure 3 presents the velocity and vorticity fields from a quantitative experiment after several rotation periods. It shows the advection toward the left wall of a cyclonic (anticyclonic) structure at the deep (shallow) region, near the step. The presence of the step generates a flow toward the negative x -direction maintaining the shallow region on its right, which results in the absence of structures above the

FIG. 2. (Color online) Sequence of top view photographs showing the evolution of an experimental 2D decaying turbulent flow with rotation and discontinuous topography. The black line divides the domain into a deep (lower) and shallow (upper) regions. This experiment is visualized with dye distributed randomly inside the domain before passing the grid.

step. For these times ($t=20-35$) the flow in both regions behaves almost independently. Note that the dipole moves along the step with each one of its parts at a different region: the cyclone (anticyclone) is always at the deep (shallow) part of the domain.

The two experiments shown above are representative of the observed interactions, among which the most remarkable are the flow along the step and the formation of fewer and larger vortices at both sides of the discontinuity. However, after repeating several experiments, it was not possible to determine a clear trend toward a preferred configuration of the flow for long times. Thus, the final states were configured by one or two vortices at each side of the step, either cy-

FIG. 3. (Color online) Top view data from PIV showing the step region of an experimental flow field at two different times. The small arrows represent the velocity field $\mathbf{u} = (u, v)$. Dark regions represent strong positive (negative) vorticity center at the deep (shallow) side of the step. The black line indicates the position of the step. The big black arrows schematize the flow behavior. The area is a rectangle with dimensions $\sim [75 \text{ cm}, 80 \text{ cm}]$.

TABLE I. Summary of the laboratory results in terms of number, sign of vorticity, and region of the vortices for $t=25$. +D (+S) denotes cyclones in the deep (shallow) region, while D- (S-) indicates anticyclones in the deep (shallow) region.

Expt.	+D-	+S-
1	1-1	1-1
2	1-0	1-1
3	1-2	2-1
4	0-1	0-1
5	1-1	1-1
6	1-0	1-1
7	1-0	0-1
8	1-0	2-2
9	0-1	1-1
10	1-1	0-1
No. of vortices	15	20
%	53-47	45-55

clonic, anticyclonic, or both. This is shown in Table I, which contains a list of the number of vortices, sign of vorticity, and domain region, where they were found for $t=25$ in ten experiments. The vortices are counted by identifying the strongest vortical structures for each experiment. The percentage of vortices in both regions indicates a slightly higher probability to find cyclones (anticyclones) in the deep (shallow) part of the domain. This result, however, is not conclusive at all. One important factor in the experiments is the presence of Ekman friction. Indeed, bottom friction effects for long times strongly reduce the energy of the flow, halting the organization process, and therefore inhibiting the formation of even larger vortices (recall that $t=25 \approx 0.9T_E$). In order to avoid this problem, numerical simulations without Ekman effects are carried out. The results are shown in Sec. III.

III. NUMERICAL SIMULATIONS

In this section, numerical simulations of decaying quasi-2D turbulence with discontinuous topography are presented. The flow is represented by means of a barotropic, shallow-water model. The equations in the $\omega-\psi$ formulation solved with a finite differences code (see, e.g., Ref. 20) are

$$\frac{\partial \omega}{\partial t} + J(q, \psi) = \nu \nabla^2 \omega, \quad (1)$$

$$\omega = -\frac{1}{h} \nabla^2 \psi + \frac{1}{h^2} \nabla h \cdot \nabla \psi, \quad (2)$$

where $\omega = \partial v / \partial x - \partial u / \partial y$ is the relative vorticity with (u, v) the horizontal, depth-independent velocity, $q = (\omega + f/h)$ is the potential vorticity, $h(x, y)$ is the fluid depth, ψ is the transport function, and J is the Jacobian operator. Note that the local depth h is time independent, according with the rigid lid approximation, and therefore only depends on the local depth of the fluid. This formulation can be reduced to the quasigeostrophic approximation when the step height h_B is much smaller than the maximum fluid depth, $h_B/H \ll 1$. The shallow-water model is adopted here, however, since it

TABLE II. Characteristic parameters of the numerical simulations.

Domain	$L \times \delta L$	0.5×1
Maximum depth	H_0	0.2
Kinematic viscosity	ν	10^{-6}
Rotation period	$T = 2\pi/\Omega$	12.5
Vortex diameter	a	0.05
Initial rms velocity	U_{rms}	0.005
Step height	Δh	Low (high) 0.01 (0.05)

is a more appropriate approximation for high steps. A comparison of the two models in the presence of abrupt topography is reported in Zavala Sansón *et al.*²¹

The simulations represent a rectangular domain with horizontal dimensions $L \times \delta L$ with $L=0.5$, and $\delta=2$ being the aspect ratio of the tank. A step-like topography divides the flow domain in two geometric squares with aspect ratio 1. In all simulations the height of the water column in the deep region is $H_0=0.2$. The rotation rate around the vertical axis is fixed at $\Omega=0.5$, corresponding to a Coriolis parameter $f=2\Omega=1$. The rotation period is $T=4\pi/f$. The flow decay is induced by lateral friction effects, where viscosity is $\nu=10^{-6}$. No-slip conditions are imposed at the side walls. Ekman friction effects are not considered. The discretization consists of 257×257 grid points and the dimensionless time step is always $\Delta t=10^{-3}$ for all simulations. Due to the discretization, the step-like topography in the numerical simulations is actually a very narrow slope, whose width is much smaller than the size of the vortices.

The initial condition (IC) used in all simulations is an 8×16 array of cyclonic and anticyclonic Gaussian vortices with maximum vorticity $|\omega|=1$ and diameter $a=0.05$. Similar initial distributions are used in several laboratory experiments, where vortices are forced electromagnetically (Refs. 22 and 23) or by dragging a rake of vertical bars through the surface (as in this study; Ref. 4). Twelve different ICs are used, with the relative positions between adjacent vortices slightly and randomly changed. The random perturbations of the positions are about 15% of the vortex diameter. The Reynolds number $\text{Re}=2500$ is defined as $\text{Re}=LU_{\text{rms}}/\nu$, with $U_{\text{rms}}=0.005$ being the characteristic velocity of the initial flow field.

Two step heights are used, $\Delta h=1/20H_0$ and $\Delta h=1/4H_0$. Since these values determine substantial differences in the flow evolution, they will be referred to as *low* and *high step*, respectively. Table II shows a summary of the numerical parameters used for the simulations.

A. General features

Figure 4 shows the evolution of the relative vorticity field for a low step using lateral no-slip boundary conditions in a particular simulation. The black line at $y=0.5$ represents the position of the step that divides the domain in deep and shallow regions. Three main processes can be observed: (1) fast flow auto-organization [panels (a) and (b)]; (2) strong interaction of the vortices with the lateral boundaries and the topography, with the formation of coherent structures with

FIG. 4. Vorticity contours from a simulation with a low step and no-slip boundary conditions. Dashed contours represent negative values of vorticity, and solid contours represent positive values. The contour level increment is the following: [(a) and (b)] 0.02, (c) 0.01, and (d) 0.002.

sizes comparable with the square regions [panel (c)]; and (3) final configuration of two coherent structures more or less placed at the geometrical center of the square deep and shallow regions [panel (d)].

When the simulation starts, vortices with the same sign of vorticity merge and generate larger vortices. Vortices with different sign form self-propagating dipolar structures, which in turn will interact with other vortices. Initially, the flow is dominated by these dipolar structures. When reaching the step topography, some of them are able to cross it, while others are reflected, depending on the step height and the strength and size of the structures.¹⁶ The effect of the boundaries when a vortex approaches a wall is the generation of thin filaments with opposite sign vorticity. These intense filaments are injected into the flow interior, sometimes forming new dipolar structures.^{5,24} At the step region, a flow along

FIG. 5. As Fig. 4, but now for a high step.

the topography is generated. This flow, which maintains the shallow region at the right, forces the formation of two structures, a cyclone at the deep and an anticyclone at the shallow part of the domain [panel (c)]. Due to the interaction with the left wall the anticyclone generates at the shallow region a cyclonic structure which eventually prevails in this area. The flow field reaches a final pattern consisting of two large positive structures surrounded by negative relative vorticity.

In order to see the effect of the step height on the flow evolution, a numerical run with the same IC but now using a high step topography is performed (see Fig. 5). Initially, the flow behavior is very similar to the low step case [compare panel (a) in Figs. 4 and 5]. However, after 15 rotation periods it can be noticed that there are no structures above the step [panel (b)], which indicates a clear separation between both regions. The flow along the step is generated and intense interactions with the left wall are observed [panel (c)]. In this

FIG. 6. Normalized mean velocity components along six grid lines adjacent and parallel to the step in the deep region for the simulations shown in Figs. 4 and 5. The velocity is normalized with the maximum value for each transect. The time step is $\Delta t=1$. Dashed contours represent negative values and solid contours represent positive values. Dashed-point contours represent zero. The contour level interval is 0.2.

case, the final pattern consists of two large scale structures: an anticyclone at the deep and a cyclone at the shallow region [panel (d)]. It is important to remark that the main effect of the high step is the earlier separation of the shallow and deep regions. A more precise time scale of this process in terms of the step height is given Sec. III E. Note also that the final distribution has a very different pattern compared to the low step simulation.

B. Flow at the step region

In order to get more information about the step influence on the flow evolution and organization, the velocity field along six parallel and consecutive grid lines adjacent to the step in the deep region is analyzed. In Fig. 6, the u and v components are examined independently for the same simulations shown in Figs. 4 and 5. The plots present these com-

FIG. 7. Zoom of Figs. 6(a) and 6(c). The line indicates the separation time between regions and the circle the initial position of the jet for a low (a) and high (b) steps.

ponents (horizontal axis) along the step during the whole simulation (vertical axis). The flow along the low step is clearly shown by the negative values of the u -component from $t \sim 40$ [panel (a)]. Before $t \sim 40$, the existence of small-scale structures associated with the vortices crossing above the step is observed. Note also that near the right wall there are some periodic inversions of the u -component, which are also correlated with the v -component. This behavior is associated with the existence of topographic waves along the step. Panel (c) shows the u -component of the velocity field,

in this case, along a high step. It can be seen that negative values of this component appear from $t \sim 5$ due to the step influence. Before $t \sim 5$, the existence of small structures is related with the IC. For $t \sim 25$, a positive flow near the central point of the step is formed and grows in time. This positive flow at the step is directly associated with the presence of an anticyclone at the deep region (see Fig. 5).

The main point to emphasize here is the time at which the negative flow along the step is established: $t \sim 35$ for the low and $t \sim 5$ for the high step case (see Fig. 7). This time is

FIG. 8. (Color online) Scatter plots at two different times of simulations shown in Figs. 4 and 5. Panels (a) and (b): low step; panels (c) and (d): high step. The values are normalized with maximum absolute values. Dark (gray) points are located in the deep (shallow) region.

a good estimation of the moment at which the flows in the shallow and deep regions begin to evolve almost independently, which mainly depends on the step height.

C. Final configurations

The final flow configuration is well-represented by means of scatter plots. The relation between relative vorticity (ω) and transport function (ψ) is shown in Fig. 8 for a low [panels (a) and (b)] and a high [panels (c) and (d)] step. Note that points in the deep and shallow regions are marked with different colors. For $t=15$ the disordered character of the flow field can be noticed by the absence of a clear relationship between both quantities. However, this dispersion is smaller for the high step case, which indicates a faster organization of the flow field. For later times, $t=200$, a nearly linear relationship is found separately for the two resulting coherent structures. The important point to remark is the different distribution of large vortices at the end of the simulation: two cyclones for the low step, and for the high step one cyclone in the deep region and an anticyclone in the shallow region.

However, the final configurations shown above are not always obtained when slightly varying the ICs. To illustrate this assertion, Fig. 9 shows the relative vorticity contour lines and the corresponding scatter plots for three different numerical simulations at very long times using a low step. Recall that the simulations differ in the position of the vortices of the IC, which are weakly and randomly perturbed. It can be seen that all the simulations present a different final cell pattern distribution. Panel (a) shows the formation of cyclonic vortices at each side of the step. For panels (c) and (e), intermediate configurations of different structures are

FIG. 9. (Color online) Vorticity contours and scatter plots of three different simulations with a low step-like topography. Dashed contours represent negative values of vorticity, and solid contours represent positive values. The contour level increment is 0.001. Black (gray) dots in scatter plots are located in the deep (shallow) region.

found. The dominant vortices are easily identified with the corresponding scatter plots. Similar results for different numerical simulations using a high step were obtained: simulations with small variations in the ICs also presented different final cell pattern distributions.

Given the sensitivity of the final flow pattern for slightly different ICs, an ensemble of 12 simulations was carried out for each topography. In Table III, a summary of the numerical results in terms of the final cell pattern distribution is presented. This table shows that the flow evolves toward a preferential final distribution of vorticity given by one coherent structure in each region; the correlation between the sign of these structures and the topography (deep versus shallow) appears to be weak.

TABLE III. Summary of the numerical results for low and high steps in terms of number, sign of vorticity, and region of the vortices at time $t = 400$. +D (+S) cyclone at the deep (shallow) region; D- (S-) anticyclone at the deep (shallow) region. The symbol 0 denotes cases in which no structures were found.

IC	Low step		High step	
	+D-	+S-	+D-	+S-
1	1-0	0-1	1-1	0-1
2	1-0	2-2	0-1	0-1
3	1-1	0-1	0-1	0-1
4	1-0	1-0	0-1	1-0
5	0-1	1-1	1-0	1-0
6	1-0	0-1	0-1	0-1
7	1-0	1-0	0-1	1-0
8	0-1	1-0	0-1	0-1
9	1-0	0-1	0-1	0-0
10	1-1	1-0	1-0	1-0
11	0-1	1-0	1-0	1-0
12	1-0	1-0	0-1	1-0
No. of vortices	14	16	13	11
%	64-36	56-44	31-69	55-45

D. Generalization based on ensemble averages

In order to get more general information about the processes involved in the turbulent decay in the presence of a step, the time evolution of two global quantities, the kinetic energy (E), and the enstrophy (Z) are investigated. These functionals are defined as

$$E = \frac{1}{2} \int (u^2 + v^2) dx dy, \quad (3)$$

$$Z = \frac{1}{2} \int \omega^2 dx dy. \quad (4)$$

The ratio Z/E can be interpreted as $1/l^2$, where l is the mean scale of the structures of the turbulent flow field. The quantity l must grow in time due to the inverse energy cascade and it is indicative of the efficiency of the self-organization process.

The ensemble average is based on 12 runs using 12 slightly different ICs, as mentioned above. Figure 10 shows the decay of global quantities for a low and high step. In panel (a) the normalized kinetic energy and enstrophy decay is presented. As can be seen, the decays are very similar in both quantities. From the ratio $Z(t)/E(t)$ [panel (b)] different features can be noticed. The total algebraic decay rate of Z/E between $4 \leq t \leq 400$ for a low step is $\sim t^{-0.58}$, while for a high step is $\sim t^{-0.63}$. These values are quite similar. Nevertheless, they are somewhat larger than the one reported by Maassen²⁵ for a rectangular container with the same aspect ratio ($\delta=2$) but without topography ($t^{-0.48}$). In other words, the presence of the step increases the decay of this quantity or, equivalently, the efficiency of the organization of larger structures with respect to the strictly 2D case. Besides, the value measured by Clercx *et al.*³ for a square container ($\delta=1$) without bottom topography is $t^{-0.63}$. Therefore, the exponents found

FIG. 10. Time evolution of global quantities with low (solid) and high (dashed) steps: (a) Normalized kinetic energy ($E/E_{(t=0)}$) and normalized enstrophy ($Z/Z_{(t=0)}$); (b) ratio of Z/E ; (c) mean scale structures; (d) number of vortices. The data are computed from ensemble averages of 12 runs with slightly different ICs.

here suggest that the flow in each region tends to behave independently, as in two separated square boxes. Panel (c) shows the mean scale of the structures. For $t > 40$ slightly larger structures are found for the high step case.

Another analyzed quantity is the number of vortices as a function of time [panel (d)]. The vortex count is done by making use of the Okubo-Weiss function, $Q = s_1^2 + s_2^2 - \omega^2$, where the strain components are defined as $s_1 = u_x - v_y$ and $s_2 = u_y + v_x$. $Q(x, y, t)$ allows to distinguish between rotation ($Q < 0$) and strain ($Q > 0$) dominated regions.²⁶ A vortex is counted in closed regions, where $Q_{\min} \leq Q \leq Q_{\min}/10$. As the flow organizes, it is expected that the number of vortices decreases. It can be seen that the number of vortices in both regions is reduced at almost the same rate. Nevertheless, the high step case presents always fewer vortices indicating a more effective self-organization of the flow.

E. Step limit and step signal

The separation of the flow in two different regions due to the step presence was found to occur at $t \sim 5$ ($t \sim 35$) for a high (low) step. The physical mechanism associated with this behavior consists of the cease of vorticity exchanges between regions as flow structures are less able to cross the step from one side to the other. This process depends on the flow properties such as circulation and vorticity and on the step height. In order to relate the separation time T^* , the step height Δh and the characteristic strength of the flow ω^* at this time, we define the latter as the integrated relative vorticity in the deep region

FIG. 11. Time evolution of integrated relative vorticity in different regions due to different step heights. (a) Schematic representation of the integration regions. Vorticity adjustment in (b) deep, (c) R1, (d) R4, (e) R2, and (f) R3 regions.

$$\omega^* = \omega_{\text{deep}} = \int_0^{0.5} \int_0^{0.5} \omega dx dy. \quad (5)$$

For this analysis, we could also use ω_{shallow} , which is equal to $-\omega_{\text{deep}}$ because the total circulation is zero $\int_0^{0.5} \int_0^{1.0} \omega dx dy = 0$. Note that $\omega^*(0) \sim 0$ due to the ICs. For latter times we expect an increase in ω^* as fluid columns crossing toward the deep region are stretched, gaining positive relative vorticity.

Figure 11 shows the ensemble average of ω^* from 12 runs for each step height. An additional set of simulations with a very low step ($\Delta h = 0.005$) is also included. The time

evolution of ω^* for the different steps is shown in panel (b). It can be noticed that, as expected, ω^* increases in time and reaches a maximum. Afterwards it decays without strong oscillations for each step height. The existence of this maximum suggests the time after which there are no more structures crossing the step. Indeed, this time is $t \sim 10$ for the high step and $t \sim 40$ for the low step; these values correspond well with those previously found with independent measures. The magnitude of the maximum ω^* depends on the size of the step: larger steps induce stronger exchanges of vorticity across the topography. Panels (c)–(f) show the integrated vorticity on four subregions [see panel (a) for a schematic representation of the different integration areas]. It is clear that the increment of ω^* is mainly associated with the R1 and R4 regions, where a stronger signal is measured compared to the other two placed away from the step. Another interesting feature is that R2 and R3 regions present a strong correlation at early times. This means that the step signal takes several rotation periods to be noticed at the southern wall.

F. Statistical analysis of the final configuration

In Secs. II B and III C, it was found that small differences in the ICs lead to significant differences in the final vorticity distribution. Figure 12 shows the final mean vorticity distribution ($t=400$) calculated from ensembles of 12 simulations for the three steps discussed above. As expected, few large scale structures remain at each side of the topography. However, this average does not show the strong variability of the final configurations observed from individual simulations. In order to quantify these differences and to find spatial structures that explain them an empirical orthogonal function (EOF) technique is used.^{27–29}

The EOF analysis seeks structures that explain the maximum amount of variance in a 2D data matrix. A space-realization array at a single time ($t=400$) is used where col-

FIG. 12. Mean spatial vorticity distribution at $t=400$ for (a) very-low step, (b) low step, and (c) high step. Dashed contours represent negative values of vorticity, dot-dashed contours represent zero values, and solid contours represent positive values. The contour level increment is: (a) 0.001 and [(b) and (c)] 0.0008.

FIG. 13. Mean spatial vorticity distribution and respective principal components at $t=400$ for the very-low step EOF analysis: [(a) and (b)] first, [(c) and (d)] second, and [(e) and (f)] third modes.

umns are the space vorticity vectors ($m=1,2,\dots,257^2$) and rows are the results of each IC used ($n=1,2,\dots,12$)

Realization $\rightarrow$

$$\mathbf{X} = \begin{bmatrix} \omega_{1,1} & \omega_{1,2} & \cdots & \omega_{1,N} \\ \omega_{2,1} & \omega_{2,2} & & \\ \vdots & & \ddots & \\ \omega_{M,1} & \cdots & & \omega_{M,N} \end{bmatrix} \downarrow \text{Space},$$

where $\omega_{m,n}$ is the vorticity at the grid position m for the simulation n . The M dimension represents the data structure and N the realization.

By using the singular value decomposition for $M > N$, matrix $\mathbf{X}$ can be written as

$$\mathbf{X} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T, \quad (6)$$

where the columns of $\mathbf{U}_{(M \times M)}$ and $\mathbf{V}_{(N \times N)}$ are the eigenvectors (singular vectors) of the covariance matrices $\mathbf{X}\mathbf{X}^T$ and $\mathbf{X}^T\mathbf{X}$, respectively. The diagonal elements of $\mathbf{\Sigma}_{(M \times N)}$ are the corresponding eigenvalues (singular values). $\mathbf{U}$ and $\mathbf{V}$ are orthogonal and $\mathbf{\Sigma}$ is diagonal. The singular values $\sigma_{m,m}$ contain the amplitude information of the data set in descending order of magnitude in the first N positions of the matrix.

Figure 13 shows the first three modes for the smaller step case. The first mode [$\mathbf{U}_{m,1}$, panel (a)] explains 60% of the variance to a preferential pattern ($\sigma_{1,1}^2 / \sum \sigma^2 \times 100$) and is spatially composed by two structures with the same sign covering the deep and shallow regions. In panel (b), the principal components times the single value of this mode

($\sigma_{1,1} \mathbf{V}_{n,1}^T$) shows that half of the simulations are composed by two cyclones and the other half by two anticyclones (this product gives the sign and the amplitude of the spatial distribution). The second mode [panels (c)–(d)] explains 27% of the variance and represents a preferential distribution composed by a cyclone at the deep and an anticyclone at the shallow region. Only in three cases the inverse solution was found. The third mode [panels (e)–(f)] explains 6% of the variance and it is composed by two different signed structures at each region, with a cyclone and an anticyclone near the step at the deep and shallow regions, respectively.

Figure 14 shows the three modes of the intermediate step height case. The first mode [panels (a) and (b)] explains half of the variance (50%) and its spatial structure is composed by two different signed structures. The principal components show a cyclone (anticyclone) at the deep (shallow) region for seven simulations and the inverse distribution for the other five. The second mode (33%) shows a distribution, where there is a tendency to have two positive structures at each region. The third mode, which explains 9% of the variance, shows a strong step signal with the structures slightly rotated when compared to the previous case.

For the higher step case, Fig. 15 shows two structures for the first mode, with half of them being cyclones and the other half anticyclones [panels (a) and (b)]. Nevertheless, a clear asymmetry between the structures is noticed. The second mode explains 38% of the variance and the solution is composed by an anticyclone at the deep region and a very weak distribution of positive vorticity at the shallow one.

FIG. 14. As Fig. 13, but now for the low-step case.

The third mode presents more structures near the step and only 5% of the variance is explained. In particular, the third mode distribution indicates a strong step signal on the separation of the domain in two independent regions.

From this analysis, several features appear in order to quantify the step signal on the flow organization. The smaller step is the only case in which more than 50% of the variance (60%) is explained by the first mode, where a symmetric

FIG. 15. As Fig. 13, but now for the high-step case.

FIG. 16. (Color online) Summary of the EOF analysis results for different step heights.

distribution of vorticity between regions is obtained. For the intermediate and high steps the variance explained by the first mode is 50% and the spatial structure in the shallow region is stronger and more coherent than the one in the deep side. For both intermediate and high steps, the spatial distribution of the second mode coincides with the statistical result shown in Table III. This means that this mode represents the vorticity distribution in terms of the number of vortices. Third modes explain less than 10% of the variance in all cases but shows a possible mechanism for a step induced solution: In all cases a flow along the step with the shallow region on its right is maintained by the presence of structures near the step.

Figure 16 summarizes the EOF analysis results in terms of percentage explained by each one of the first three modes and step height. It can be seen that the first two modes explain a high percentage of the variance [panels (a) and (b)]. Nevertheless, the existence of a preferential solution in terms of the vorticity distribution due to step effects remains unknown. From the percentage of variance explained by the third modes, a weaker role on the existence of a preferential solution by the presence of a high step is shown [panel (c)]. This supports the idea that there are critical, intermediate steps inducing stronger signals on the organization of the flow into a preferential solution.

IV. DISCUSSION AND CONCLUSIONS

The self-organization of confined 2D flows in rectangular containers with a step topography dividing the domain in two square regions with different depths, strongly depends on the height of the discontinuity. The conclusions are supported by laboratory experiments and numerical simulations on decaying quasi-2D turbulence.

The laboratory experiments were performed in a rectangular container with aspect ratio $\delta=2$, where a discontinuous topography divides the domain in deep and shallow regions, each with aspect ratio $\delta=1$. The experiments showed the self-organization of an initial turbulent flow field into a well-organized flow pattern that almost fills up the entire domain. The final number of vortices tends to be $\eta=2$, one in each region. The final distribution depends on the formation of one or two strong vortices that dominate the flow organization, where the step-like topography plays a fundamental role. These strong structures are forced by the flow along the

step that, when reaching the lateral boundary, interact continuously with it and injects vorticity into the flow interior. This intense step-wall-flow interaction seems to dominate the cell formation, where a cyclone (anticyclone) is formed at the deep (shallow) region. Nevertheless, different distributions were also found.

Laboratory experiments are affected by the presence of Ekman damping, which eventually drains most of the energy of the system and halts the self-organization process. Thus, numerical simulations with no Ekman friction allow to study the flow behavior for much longer times. Numerical simulations using no-slip boundary conditions show the same principal features observed in the laboratory experiments. The organization of the flow field into two structures placed near the geometrical center of each region, and a jet along the step which maintains always the shallow region on its right (for $f>0$), are systematically obtained. The mass transfer from one side to the other across the step makes fluid columns to gain or lose relative vorticity due to potential vorticity conservation: columns traveling from shallow to deep regions acquire positive vorticity, and traveling from deep to shallow get negative vorticity. This explains the positive (negative) vorticity patches along the deep (shallow) part for intermediate times [see Fig. 4(c)]. As the flow decays, rotation effects and pressure gradients arising when the step is present are balanced and originate the flow along the step that always maintains the shallow region on the right. One remarkable feature of the presence of a discontinuous topography on a bounded domain is that, for intermediate times, there are always a cyclone and an anticyclone on the deep and shallow regions, respectively, near the left wall. This is due to the flow along the step

It was found that a higher step induces a faster flow organization. This is related with the existence of a critical time T^* determined by the strength of the flow and the step height, after which structures are not able to cross the topography. Afterwards, the flow evolves almost independently in each region. Such a time scale is longer for lower steps.

Nevertheless, this effect is not related with the existence of a preferential solution for long times. A slight tendency toward the existence of a preferential vorticity distribution due to the step was observed for lower steps with a cyclone (anticyclone) at the deep (shallow) region. From the EOF analysis, the existence of a preferential distribution of vortic-

ity associated with the geometry induced by the discontinuity seems to depend on longer flow-step interactions. This supports the idea that such distribution is favored for intermediate steps, over which this interaction has a longer duration than over a high step.

Finally, it must be pointed out that despite the simplicity of the system, the difficulty to predict the long-term configuration of the flow is remarkable. Further research is in progress in order to analyze the relevance of different aspect ratios of the container, as well as different geometries.

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