Excited states of the $\Omega^-$

Aerts, A.T.M.; Heller, L.

Published in:
Physics Letters B

DOI:
10.1016/0370-2693(89)91497-4

Published: 01/01/1989

Citation for published version (APA):
EXCITED STATES OF THE $\Omega^-$

A.T.M. AERTS
Department of Mathematics and Computing Science, Eindhoven University of Technology, Eindhoven, The Netherlands

and

L. HELLER
Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Received 7 November 1988

The hyperfine structure of the excitation spectrum of the $\Omega^-$ baryon is studied in a theoretical model. An interpretation for the excited $\Omega^-$ state at 2251 $\pm$ 12 MeV is given.

In a recent measurement Biagi et al. [1] obtained values for the masses of two excited states of the $\Omega^-$ baryon. One excitation was observed to have a mass of 2251 $\pm$ 12 MeV; the other one was reported to have a mass of 2384 $\pm$ 12 MeV. No information about the spin-parity of the states was presented.

Several years ago we published a calculation of the excitation spectrum of the $\Omega^-$ baryon [2,3] using a spin-independent potential which was derived using a Born–Oppenheimer approximation to the MIT bag model [4]. The mass spectrum we calculated covers the mass range of the lower observed state, but not that of the higher one. On the basis of the calculated spectrum the state at 2251 MeV could be interpreted as either an S ($L=0$) state with $J=\frac{1}{2}$ or a D ($L=2$) state with $J=\frac{3}{2}$.

This ambiguity and the accuracy of the mass value have prompted us to investigate the influence of the spin-dependent parts of the potential on the mass spectrum. In this short note we report on the results of this investigation.

The dominant spin-dependent interaction within the framework of the MIT bag model description of hadrons is the hyperfine interaction term associated with the one-gluon-exchange interaction. It consists of two terms: a spin–spin and a tensor term. The spin–spin term is short-ranged but the precise space dependence is unknown. Nonrelativistically, this term is a contact term, but relativistic effects are expected to spread it out. Recent work on the hyperfine shift in quark–antiquark systems containing at least one heavy quark [5] suggests that a reasonable approximation is given by

$$V_{\text{hyp}} = \frac{3}{2} \alpha_s m_\pi \frac{N}{\zeta r} \exp[-(r/r_0)^2/\zeta^2],$$

where the scale $\zeta$ lies somewhere between 0.15 and 0.25 fm. The factor $N$ is a normalisation constant which has to be introduced to keep the strength of this term correct. The tensor term is given by

$$V_{\text{hyp},ij} = \frac{\alpha_s}{6m_\pi^2 r_0} (\vec{r}_i \cdot \vec{r}_j - \vec{r}_i \cdot \vec{r}_j),$$

where $\vec{r}_i$ is the unit vector pointing from quark $i$ to quark $j$, and $r_{ij}$ is their separation.

We take the hyperfine potential term to be the sum over three two-body terms,

$$V_{\text{hyp}} = \sum_{i>j} V_{\text{hyp},ij} + V_{\text{hyp},ij}.$$

The energy shifts due to the hyperfine interaction are evaluated by taking the expectation value of $V_{\text{hyp}}$ in the various states of the $\Omega^-$ spectrum. We used the wave functions of ref. [3], which were obtained from
a hyperspherical expansion. The mass of the strange quark then is 571 MeV. The strong coupling constant was taken to be 0.37 [5].

The precise value for $\alpha$, as defined in eq. (1), for a system containing strange quarks is not known. Such a system was not studied in ref. [5]. The value for $\alpha$ needed in ref. [5] to reproduce the $J/\psi-\eta$ splitting was $\alpha=0.15$ fm; the one needed to reproduce the $D-D^*$ splitting was $\alpha=0.25$ fm. Using a simple gaussian wave function for the quark–antiquark groundstate wave function and evaluating the hyperfine shift for the potential of eq. (1) one finds that the $F-F^*$ splitting can be reproduced with values for $\alpha$ which interpolate between those needed to reproduce the $J/\psi-\eta$ and the $D-D^*$ splitting. (We used $\alpha_s=0.37$, $m_s=350$ MeV and $m_q=1364$ MeV in these calculations.) It is therefore not unreasonable to choose a value for $\alpha$ in the range between 0.15 and 0.25 fm.

We have calculated the hyperfine shift for the six $s^3$ levels we also considered in ref. [3], for a number of values of $\alpha$ in the above cited range. The hyperfine shifts of these states are somewhat sensitive to the precise value of $\alpha$; the deviations from the average value found are on average at the 10% level, but at most at the 20% level.

As is clear from eq. (3), there are two contributions from the hyperfine interaction to the level shifts: a spin–spin and a tensor part. Our results for the spin–spin part of the level shifts of five excited states minus that of the ground state are presented in table 1. These differences are found to be much less sensitive to the precise value of $\alpha$.

The last column of table 1 shows the excitation spectrum without any hyperfine interaction #1 and we see that the spectrum of the $s^3$ system has come down a little upon inclusion of the hyperfine splitting. This is a consequence of the fact that the mass shift of the ground state $s^3$ state due to the hyperfine splitting is the largest (about 25 MeV upwards), that of its radial excitation is slightly smaller and that of the other states is some 15 MeV smaller.

Most of the levels in table 1 will be shifted somewhat further, as a function of their spin, by the tensor part of the hyperfine interaction. The tensor term has a direct effect only in the $D$ states, and it splits them by only a few MeV. Our results for the full hyperfine interaction of eq. (3) are given in table 2, but only for $\alpha=0.200$ fm. Table 2 also includes the effect of off-diagonal tensor matrix elements in states of the same total angular momentum $J$ and parity. Omitting the off-diagonal matrix elements would change the numbers in table 2 by at most 2 MeV.

Table 2 also shows the results of Chao et al. [6] for the same states. Our results for the hyperfine splittings are a (state dependent) factor of 4 or more smaller for both the spin–spin and tensor terms. The differences arise from the use of different coupling constants and masses, different wave functions, and also from our use of a spin–spin term that is spread out. Our wave functions were obtained from a potential model which incorporates two-body Coulomb

---

# Table 1

Excitation spectrum of the $s^3$ system including the spin–spin part of the hyperfine shift. Splittings are in MeV, $\alpha_s=0.37$ and $m_s=571$ MeV. Values of $\alpha$ are given in units of fm. The last column shows the energy levels without any hyperfine interaction.

<table>
<thead>
<tr>
<th>State</th>
<th>$\alpha$</th>
<th>0.150</th>
<th>0.175</th>
<th>0.200</th>
<th>0.225</th>
<th>0.250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\bar{s}}$ ($\Omega^+$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P_M$</td>
<td>0.150</td>
<td>304</td>
<td>304</td>
<td>304</td>
<td>304</td>
<td>304</td>
</tr>
<tr>
<td>$S_S$</td>
<td>307</td>
<td>506</td>
<td>506</td>
<td>506</td>
<td>505</td>
<td>509</td>
</tr>
<tr>
<td>$S_M$</td>
<td>308</td>
<td>568</td>
<td>568</td>
<td>569</td>
<td>569</td>
<td>585</td>
</tr>
<tr>
<td>$D_S$</td>
<td>307</td>
<td>574</td>
<td>574</td>
<td>575</td>
<td>576</td>
<td>577</td>
</tr>
<tr>
<td>$D_M$</td>
<td>308</td>
<td>581</td>
<td>582</td>
<td>582</td>
<td>583</td>
<td>583</td>
</tr>
</tbody>
</table>

---

# Table 2

Excitation spectrum of the $s^3$ system including the full hyperfine splitting. Splittings are in MeV, $\alpha_s=0.37$, $m_s=571$ MeV and $\alpha=0.200$ fm.

<table>
<thead>
<tr>
<th>State</th>
<th>$J^P$</th>
<th>This work</th>
<th>Chao et al. [6]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\bar{s}}$ ($\Omega^+$)</td>
<td>$\frac{3}{2}^+$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P_M$</td>
<td>$\frac{1}{2}^-$</td>
<td>304</td>
<td>345</td>
</tr>
<tr>
<td>$S_S$</td>
<td>$\frac{1}{2}^-$</td>
<td>305</td>
<td>345</td>
</tr>
<tr>
<td>$S_M$</td>
<td>$\frac{1}{2}^+$</td>
<td>567</td>
<td>515</td>
</tr>
<tr>
<td>$D_S$</td>
<td>$\frac{3}{2}^+$</td>
<td>574</td>
<td>535</td>
</tr>
<tr>
<td>$D_M$</td>
<td>$\frac{3}{2}^+$</td>
<td>578</td>
<td>550</td>
</tr>
<tr>
<td>$\bar{P}_M$</td>
<td>$\frac{1}{2}^-$</td>
<td>575</td>
<td>540</td>
</tr>
<tr>
<td>$\bar{S}_S$</td>
<td>$\frac{3}{2}^+$</td>
<td>577</td>
<td>535</td>
</tr>
<tr>
<td>$\bar{S}_M$</td>
<td>$\frac{3}{2}^+$</td>
<td>583</td>
<td>590</td>
</tr>
<tr>
<td>$\bar{D}_S$</td>
<td>$\frac{3}{2}^+$</td>
<td>583</td>
<td>590</td>
</tr>
</tbody>
</table>
terms and a three-body confining term, whereas the model of ref. [6] uses exclusively two-body harmonic oscillator potentials. The result of these effects is a hyperfine interaction which is a rather smaller perturbation than the corresponding interaction of ref. [6].

Fig. 1 shows the mass spectrum that we have calculated, and the range covered by the experimental level at 2251 ± 12 MeV. It is seen that seven of our levels fall within this range: \( S_M \left( \frac{1}{2}^+ \right), D_s \left( \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+ \right), \) and \( D_M \left( \frac{3}{2}^+, \frac{5}{2}^+ \right) \). From the spectrum of Chao et al. the \( D_M \) levels at 2265 MeV emerge as their best candidates. This implies that should further measurements favor a high \( J \) value of \( \frac{3}{2} \) or a low \( J \) value of \( \frac{1}{2} \) our model is favored over the model of ref. [6]. A \( J \) value of \( \frac{3}{2} \) or \( \frac{1}{2} \) would not allow a distinction between our model and that of ref. [6].

The model of ref. [6] has been “relativized” by Capstick and Isgur [7]. In ref. [7] an harmonic oscillator expansion is again used for the wave function but the confining potential is now taken to be a string potential. Our three-body confining potential (ref. [2]) is different from this string potential except at very large distances, which are not important in the calculation. After giving the actual predictions of their model, the authors show in their fig. 9 their “best” predictions which include an additional correction. According to this figure there are two candidates for the experimental level, a \( \frac{5}{2}^+ \) and \( \frac{7}{2}^+ \), which we estimate are located at approximately 2240 MeV and 2260 MeV, respectively. Obviously, experimental evidence for low \( J \) values of \( \frac{3}{2} \) or \( \frac{1}{2} \) would favor our model in comparison with the model of ref. [7]. Any indication of substructure of the level at 2251 MeV could serve as additional evidence for distinguishing between the various models.

We would like to thank A. Martin for informing us of the experimental result contained in ref. [1]. A.T.M.A. would like to thank the Medium Energy Physics Theory Group at Los Alamos for their hospitality during a visit when part of this work was done. This research was supported by the US Department of Energy.

References