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Dynamic inverse modeling and its testing in large-eddy simulations of the mixing layer

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We propose new identities for dynamic subgrid modeling in large-eddy simulation involving an explicit filter and its inverse. Exact defiltering of a class of numerical realizations of the top-hat filter is developed. The approach is applied to large-eddy simulation of the temporal mixing layer. Smagorinsky's model is adopted as base model and the results are compared to the standard dynamic eddy-viscosity model as well as to filtered DNS (direct numerical simulation) results. The difference between the results of the two models for the present application is found to be quite small. This is explained by performing a sensitivity analysis with respect to the dynamic coefficient, which hints towards a "self-restoring" response underlying the observed robustness of the physical predictions. Using DNS data the validity of the assumption that the model coefficients are independent of filter width is tested and found to favor the inverse modeling procedure. The computational effort of the dynamic inverse model is 15% smaller than of the standard dynamic eddy-viscosity model. © 1999 American Institute of Physics. [S1070-6631(99)00212-3]

I. INTRODUCTION

Modeling of transitional and turbulent flow, aimed at reducing the effective number of degrees of freedom of the underlying dynamical system, forms a field of considerable interest, although developments in numerical methods and computer resources enable the use of direct numerical simulation (DNS) for increasingly complex flows. In large-eddy simulations (LES) this modeling process starts with the application of a spatial convolution filter to the Navier–Stokes equations. Filtering of the nonlinear convective terms gives rise to the turbulent stress tensor

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j, \quad i, j = 1, 2, 3, \quad (1)$$

which contains contributions from the filtered ($\bar{u}_i$) and the unfiltered (u_i) velocity components. Since in an LES only the filtered velocity components are calculated, the turbulent stress tensor has to be modelled. Various modeling strategies have been proposed. The dynamic modeling approach² has shown to give rise to quite accurate LES predictions.¹³ This approach involves the introduction of a second "explicit" filter (also called test filter) and is aimed at an appropriate optimization of parameters contained in an assumed base model. The optimization is performed in accordance with an

exact algebraic identity involving τ at different filter levels, and yields dynamically determined solution dependent model parameters.

In large-eddy simulation one concentrates in particular on modeling dissipative and similarity properties of the turbulent stress tensor. Since in geometrically complex transitional and turbulent flow strong spatial and temporal variations in the local structure of the flow are encountered, correspondingly large variations in the above properties of the subgrid model for τ are necessary. In case the well-known Smagorinsky subgrid model is adopted with a constant eddy viscosity, previous experience has shown this model to be too dissipative in smooth (laminar) regions of the flow. A substantial and elegant improvement in subgrid modeling was arrived at with dynamic modeling based on the Germano identity. In this formulation the local and instantaneous structure of the flow determines the local value of the eddy viscosity and eliminates most of the excessive dissipation. In actual simulations several additional assumptions are usually made in dynamic modeling. As an example it is common to assume the model coefficient to be constant over the width of the filter and the test filter. Since the latter can be quite large and even outside an inertial range, a possible conflict with the validity of the assumed base model may arise with *a priori* unknown consequences. Therefore, if dynamic modeling is developed involving mainly length scales comparable to or smaller than the basic filter width such conflicts can be avoided and more accurate, self-consistent modeling may result. This strategy is realized in

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dynamic inverse modeling which is described and applied in this paper.

In particular, we propose to extend the traditional dynamic approach and introduce apart from the explicit filter also its inverse. This implies new algebraic identities which involve the base model at various filter levels. In one of the extensions these filter levels involve only the smallest resolved scales in the LES as well as yet smaller scales. Hence, inaccuracies arising from the use of a larger filter width associated with the explicit filter in the traditional dynamic approach can be partially compensated and basic modeling assumptions can be maintained more strictly. This especially refers to the similarity assumption related to different filter levels in an inertial range of the energy spectrum.

We will apply this dynamic inverse modeling strategy to LES of the temporal mixing layer at low Mach number. As base model we adopt the Smagorinsky eddy-viscosity subgrid model. The results will be compared with filtered DNS results and results from LES using the traditional dynamic eddy-viscosity model. In Sec. II we will briefly recapitulate the dynamic modeling approach and introduce its extension to dynamic inverse modeling. Numerical realizations of the explicit filter and their inverse will be developed in Sec. III. The temporal mixing layer is described in Sec. IV and the results will be presented in Sec. V. Finally, in Sec. VI some conclusions will be drawn.

II. DYNAMIC INVERSE MODELING

In this section we first briefly review the traditional dynamic subgrid modeling using the Smagorinsky model as base model. Subsequently, we formulate the extension to inverse modeling in the dynamic context.

In large-eddy simulation of turbulence the flow quantities are decomposed into a large-scale contribution and a small-scale contribution by a spatial filter. The large-scale contributions are explicitly calculated, whereas only the effects of the small-scale contributions on the large-scale flow are described by a so-called subgrid model. The main term to be modeled is the turbulent stress tensor (1). Since viscous dissipation mainly acts on the smallest scales in a flow, which due to the filtering have been removed from the DNS flow, dissipation should be an important ingredient of the subgrid model. In the widely used Smagorinsky model¹⁰ the turbulent stress tensor is modeled by a viscous term through the introduction of an eddy viscosity:

$$\tau_{ij} = -C_s^2 \Delta^2 |\bar{S}| \bar{S}_{ij},$$

where Δ is the filter width, $\bar{S}_{ij}$ is the strain rate tensor based on the filtered velocity and

$$|\bar{S}|^2 = \frac{1}{2} \bar{S}_{ij} \bar{S}_{ij}.$$

For the Smagorinsky constant C_s various values have been used ranging from $C_s = 0.065$ in turbulent channel flow⁸ to $C_s = 0.17$ in homogeneous turbulence.⁹ A major drawback of this model is the excessive dissipation in laminar regions of the flow,¹² e.g., in boundary layer flow close to solid walls or in regions of the flow domain where transition has not (yet) set in.

In order to overcome this problem Germano *et al.* proposed the dynamic procedure, where C_s^2 is replaced by a coefficient C_d which is dynamically adjusted to the local structure of the flow.² The main assumption in most realizations of the dynamic procedure is that the coefficient C_d is independent of the filter width.¹ To specify C_d the Smagorinsky model is substituted in Germano's identity, which reads

$$T_{ij} - \hat{\tau}_{ij} = \widehat{\bar{u}_i \bar{u}_j} - \hat{\bar{u}}_i \hat{\bar{u}}_j, \quad (2)$$

where the hat ($\hat{\cdot}$) denotes the explicit filter operation and T_{ij} is the turbulent stress tensor corresponding to the consecutive application of the two filters

$$T_{ij} = \widehat{\bar{u}_i \bar{u}_j} - \hat{\bar{u}}_i \hat{\bar{u}}_j. \quad (3)$$

The right-hand side in identity (2) contains only resolved flow quantities and is known in an LES, whereas the terms on the left-hand side involve the turbulent stress tensors at different filter levels. If the base model is substituted for these tensors, the only unknown in the identity is the coefficient C_d , which can, for example, be calculated with a least-squares approach as a function of space and time.⁷

It is clear that the independence of C_d of the filter width can only hold within certain bounds and is approximately valid if the model is adopted at filter levels inside the inertial range. In order to save calculation time the filter width in an LES is usually chosen as large as possible. In this situation, however, the validity of the model on the test filter level may become questionable and it would be preferable if the base model is only applied at smaller scales. This can be achieved by using a generalization of Germano's identity (2) to any combination of filters.

In order to illustrate this generalization it is convenient to introduce a new notation, in which we use the symbol L for a filter operator, and define the product operator S by

$$S(f, g) = fg,$$

for any two functions f and g . Further the commutator of two operators is written in bracket notation, e.g.,

$$[L, S](u_i, u_j) = L(S(u_i, u_j)) - S(L(u_i, u_j)).$$

Using this notation the turbulent stress tensor can be written as

$$\tau_{ij} = [L, S](u_i, u_j).$$

This nonlinear commutator shares a number of properties with the classical Poisson bracket,⁵ such as antisymmetry. Another important property of Poisson brackets is readily verified

$$[\mathcal{L}_1 \mathcal{L}_2, S] = [\mathcal{L}_1, S] \mathcal{L}_2 + \mathcal{L}_1 [\mathcal{L}_2, S], \quad (4)$$

where both $\mathcal{L}_1$ and $\mathcal{L}_2$ denote any filter operator. This identity is a reformulation of the Germano identity (3) which was first established in the context of LES in Ref. 3. Similarly Jacobi's identity holds for the operators S , $\mathcal{L}_1$, and $\mathcal{L}_2$.

In the standard dynamic models $\mathcal{L}_2$ is the common LES filter and $\mathcal{L}_1$ is the test filter. However, as Eq. (4) holds for

any two filters we can also take $\mathcal{L}_2 = \mathcal{H}^{-1}L$ and $\mathcal{L}_1 = \mathcal{H}$, where L is the LES filter, $\mathcal{H}$ is any explicit filter and $\mathcal{H}^{-1}$ its inverse. Here we assume that the test filter $\mathcal{H}$ has an exact inverse $\mathcal{H}^{-1}$, but a mathematically consistent inverse modeling can also be based on approximate inversion.⁴ This choice for the two filters leads to the identity

$$\tau_{ij} - \mathbb{T}_{ij} = \mathbb{L}_{ij}, \quad (5)$$

where

$$\mathbb{T}_{ij} = \mathcal{H}(\mathcal{H}^{-1}(\bar{u}_i \bar{u}_j) - \mathcal{H}^{-1}(\bar{u}_i) \mathcal{H}^{-1}(\bar{u}_j))$$

and

$$\mathbb{L}_{ij} = \mathcal{H}(\mathcal{H}^{-1}(\bar{u}_i) \mathcal{H}^{-1}(\bar{u}_j)) - \bar{u}_i \bar{u}_j.$$

In these and all following formulas the bar denotes the LES-filter L . Note that identities (2) and (5) correspond to Germano's identity applied to different filters. For example, if the hat-filter is replaced by $\mathcal{H}$ and the bar-filter by $\mathcal{H}^{-1}L$, the tensor T_{ij} in Eq. (2) changes into τ_{ij} .

The term $\mathbb{L}_{ij}$ in Eq. (5) is known in an LES, whereas the terms on the left-hand side are (filtered) turbulent stress tensors on a certain filter level and can be modelled. If Smagorinsky's eddy-viscosity model is adopted for the terms on the left-hand side, we find the following relation for the coefficient C_d

$$C_d M_{ij} = \mathbb{L}_{ij}, \quad (6)$$

where

$$M_{ij} = -\Delta^2 |\tilde{S}| \tilde{S}_{ij} + ((\kappa \Delta)^2 |\tilde{S}| \tilde{S}_{ij})^\wedge. \quad (7)$$

Here $\kappa \Delta$ is the effective filter width of $\mathcal{H}^{-1}L$, to which we will return later and $\tilde{S}$ is the strain rate tensor based on the velocity $\mathcal{H}^{-1}L(\mathbf{u})$. The hat after the last term on the right-hand side implies that the whole term between parentheses is filtered with $\mathcal{H}$. In the actual LES shown here the coefficient C_d is calculated with a least-squares approach⁷

$$C_d = \frac{\langle M_{ij} \mathbb{L}_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}. \quad (8)$$

Furthermore, in order to prevent numerical instability caused by negative values of C_d , the numerator and denominator are averaged over homogeneous directions, which is expressed by the symbol $\langle \cdot \rangle$, and the coefficient is artificially set to zero at locations where it would otherwise be negative.

Based on identity (4) also other forms of dynamic inverse modeling can be derived, for example when taking $\mathcal{L}_2 = \mathcal{H}L$ and $\mathcal{L}_1 = \mathcal{H}^{-1}$. However, in Eq. (7) the model is applied at level L and at level $\mathcal{H}^{-1}L$, which thus involves scales smaller than the filter width in the LES. Hence, this form of dynamic inverse modeling provides the best guarantee that the model is applied only in the inertial range. Dynamic inverse modeling can also be applied with other base models. In the present paper we will, however, only consider Smagorinsky's eddy-viscosity model as base model. Other base models, such as the mixed model and the gradient model, will be studied in the future.

The model described above can easily be extended to compressible flow. The most important difference is that

Favre filtering should be used for the velocity field. Hence, for example, $\tilde{S}$ in Eq. (7) should be based on the velocity field $\mathcal{H}^{-1}L(\rho \mathbf{u})/\mathcal{H}^{-1}L(\rho)$, where ρ is the density.

In the next section we will first discuss the explicit filters $\mathcal{H}$ and $\mathcal{H}^{-1}$ which appear in the dynamic inverse model. After that we will apply the model in a large-eddy simulation of the turbulent temporal mixing layer.

III. NUMERICAL FILTERS

In LES the roles of the filters L and $\mathcal{H}$ differ essentially. Whereas L is central in the theoretical development and does not appear explicitly in an LES calculation, the test filter $\mathcal{H}$ and its inverse $\mathcal{H}^{-1}$ are explicitly applied in case dynamic modeling based on the identity derived in the previous section is adopted. Usually, the filter L is defined as a mapping from the space of continuous functions in itself. For example, the well-known top-hat filter is in one spatial dimension defined by

$$\bar{f}(x) = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} f(x + \xi) d\xi. \quad (9)$$

In an LES, however, the resolved fields are known only at grid points $(\{x_m\}_{m=0}^N)$. Therefore, the explicit filter $\mathcal{H}$ and its inverse can only be applied as finite dimensional operators. In this section we will describe a numerical test filter and its exact inverse. This numerical filter is an approximation of a continuous filter.

The numerical approximation of a one-dimensional convolution filter is written as

$$\mathcal{H}(u)(x_m) = \sum_{j \in \mathbb{Z}} \alpha_j u(x_{m+j}),$$

where for consistency $\sum_j \alpha_j = 1$. To satisfy the realizability conditions¹¹ we require the coefficients α_j to be non-negative. As an example consider the numerical filters with $\alpha_0 = 1 - \alpha$, where $0 < \alpha < 1$, $\alpha_1 = \alpha_{-1} = \alpha/2$ and $\alpha_j = 0$ if $|j| \geq 2$. A special case arises with $\alpha = \frac{1}{3}$ which corresponds to Simpson quadrature applied to the top-hat filter with $\delta = 2h$ where h is the grid spacing and δ the width of $\mathcal{H}$. The effect of a numerical filter on a Fourier mode $u = \exp(ikx)$ is given by

$$\begin{aligned} \mathcal{H}(u)(x_m) &= (1 - \alpha + \alpha \cos(kh)) \exp(ikx_m) \\ &= H_\alpha(kh) u(x_m). \end{aligned}$$

This filter can only be inverted if the 'characteristic' function H_α is strictly positive, which holds if $\alpha < \frac{1}{2}$. The inverse is then given by

$$\mathcal{H}^{-1}(\exp(ikx)) = \frac{1}{H_\alpha(kh)} \exp(ikx).$$

The continuous top-hat filter L has a characteristic function

$$F(k\delta) = \sin(k\delta/2)/(k\delta/2),$$

which is closely approximated by $H_{1/3}$ for sufficiently small k .

The application of $\mathcal{H}^{-1}$ to a general solution which is known only by its grid values $\{u(x_m)\}$ can be specified using

discrete Fourier transforms between grid- and wavenumber space. A periodic grid function can be decomposed into N Fourier modes according to

$$u(x_m) = \sum_k a_k \exp(ikx_m),$$

where the sum extends over $k = 2\pi k^*/hN$ with k^* all integers between $-N/2$ and $N/2$. The amplitude of each mode is given by

$$a_k = (1/N) \sum_{j=0}^{N-1} u(x_j) \exp(-ikx_j).$$

Combining these equations and applying the inverse filter to each mode yields

$$\mathcal{H}^{-1}(u)(x_m) = \frac{1}{N} \sum_{j=0}^{N-1} \sum_k H_\alpha^{-1}(kh) \cos(kh(m-j)) u(x_j).$$

In practice the double summation makes application of the inverse filter inefficient if N becomes large. However, for values of N larger than about 10 the result hardly depends on N . Taking the limit $N \rightarrow \infty$ in the summation over k we find

$$\begin{aligned} \mathcal{H}^{-1}(u)(x_m) &= \sum_{j=0}^{N-1} \left(\frac{\alpha - 1 + \sqrt{1 - 2\alpha}}{\alpha} \right)^{|m-j|} \\ &\quad \times (1 - 2\alpha)^{-1/2} u(x_j). \end{aligned} \quad (10)$$

For all positive values of α smaller than $\frac{1}{2}$ the power series in this expression converges and the convergence rate increases if α decreases. Hence, for small values of α an accurate and efficient approximation of the inverse filter can be obtained with only a few terms. Extension of the one-dimensional numerical filter and its inverse to three spatial dimensions is straightforward and boils down to successive application of the filter in all three dimensions. The final result does not depend on the order of application of the individual one-dimensional filter operators.

Before we turn to a description of the temporal mixing layer in the next section we discuss the effective filter width of the numerical filter $\mathcal{H}$ and its inverse. For a one-dimensional continuous filter, such as the top-hat filter the definition of the filter width is obvious. Usually, the filter width in three spatial dimensions is defined as

$$\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3},$$

where Δ_i is the width of the filter in the x_i direction. In dynamic models which use Smagorinsky's model as a base model, also the effective filter width of the composition of two filters is needed, for example in the standard dynamic eddy-viscosity model the width of $\mathcal{H}L$, where $\mathcal{H}$ is the test filter, and in the dynamic inverse model the width of $\mathcal{H}^{-1}L$ is required. If $\mathcal{H}$ is a numerical filter, it can be seen as the discrete approximation of a continuous filter as described above and its filter width can be defined as the width of the corresponding continuous filter. As an example, the numerical filter with $\alpha = \frac{1}{3}$ described above is an approximation of the top-hat filter with filter width equal to $2h$. So, we define the width of the numerical filter as $\delta = 2h$.

Next, we turn to the effective filter width of the composition of two filters. In Ref. 13 it has been shown that the composition of two top-hat filters is not a top-hat filter, and that the top-hat filter which is the best approximation has a width given by

$$\Delta_{\mathcal{H}L}^2 = \Delta_{\mathcal{H}}^2 + \Delta_L^2. \quad (11)$$

This relation is exact for Gaussian filters. A problem still remains in the definition of the effective filter width of $\mathcal{H}^{-1}L$. The composition $\mathcal{H}^{-1}\mathcal{H}$ equals the identity operator, which is the limit of the top-hat filter for $\Delta \rightarrow 0$ and thus has a filter width equal to zero. If we now consider Eq. (11) as a definition of the effective filter width for any filter operator or its inverse, we find

$$\Delta_{\mathcal{H}^{-1}\mathcal{H}}^2 = \Delta_{\mathcal{H}^{-1}}^2 + \Delta_{\mathcal{H}}^2,$$

or

$$\Delta_{\mathcal{H}^{-1}}^2 = -\Delta_{\mathcal{H}}^2.$$

As a final step we can define the effective width of $\mathcal{H}^{-1}L$ by using Eq. (11) again to find

$$\Delta_{\mathcal{H}^{-1}L}^2 = \Delta_L^2 - \Delta_{\mathcal{H}}^2.$$

An interesting case arises when $\mathcal{H}$ is a numerical approximation of L . Then $\mathcal{H}^{-1}L$ is an approximation of the identity operator, and hence, has effective width $\kappa\Delta$ equal to zero. It follows that the second term on the right-hand side of Eq. (7) is absent. This makes this particular dynamic inverse model very cost effective, the explicit filtering operations being necessary in the calculation of the first term in $\mathbb{L}_{ij}$ only. In the sequel we will use this model and refer to it as dynamic inverse model (DIM).

IV. TEMPORAL MIXING LAYER

In this section we will describe the test case of a compressible temporal mixing layer adopted in this paper. The computational domain is a cube. Periodic boundary conditions are imposed in the streamwise and spanwise directions, whereas the boundaries in the normal direction are free-slip walls. As initial condition a hyperbolic tangent velocity profile is taken as the mean streamwise velocity, the other mean velocity components are zero, the mean pressure is uniform and the mean temperature is obtained from the Busemann–Crocco law. In order to initiate turbulence, perturbations consisting of eigenfunctions provided by linear stability theory (LST) are superimposed on the mean profile. The length of the domain equals four times the wavelength of the most unstable mode according to LST. In this way two subsequent pairings of the rollers are allowed. Subharmonic modes are added to initiate the vortex pairings and oblique modes are added to introduce three dimensionality. The Mach number of the simulation is 0.2, which makes the simulation practically incompressible. The Reynolds number is 50, based on the upper velocity and half the initial vorticity thickness.

The initial condition is prepared on a uniform grid with 192^3 grid points. On this grid a direct numerical simulation (DNS) is performed for comparison purposes. The initial

condition is filtered onto a uniform grid with 32^3 grid points, on which the LES is carried out. The filter width equals $D/16$, where D is the length of the computational domain, i.e., $\Delta = 2h$ with h the grid spacing in the LES. The filter used is the top-hat filter. The numerical method for both the DNS and the LES is a fourth-order accurate finite volume method without artificial dissipation for the spatial discretization and a second order four-stage compact-storage Runge–Kutta method for the time integration. More details on the test case and on the numerical method can be found in Ref. 13.

Visualisation of the DNS demonstrates the roll-up of the fundamental instability and successive pairings. After some time four rollers with mainly negative spanwise vorticity are formed. After the first pairing the flow becomes highly three dimensional. After the second pairing the flow exhibits a complex structure with many regions of positive spanwise vorticity (see Fig. 1).

V. RESULTS

In this section we will present LES results for both the traditional dynamic subgrid model (DSM) and the dynamic inverse model (DIM) using Smagorinsky's eddy-viscosity model as base model. We will compare the LES results with the results from the DNS filtered onto the LES grid. First we concentrate on the actual LES predictions which surprisingly show that several physical properties of the flow are approximately independent of the model. The "self-restoring" mechanism leading to this insensitivity is clarified subsequently. Moreover, the DNS results allow a verification of the assumption that the model coefficient is independent of the filter width. The findings favor the incorporation of the small length scale contributions in DIM over the use of scales of the size of the test-filter width as in DSM.

In the DSM the explicit test filter has a width equal to $2\Delta = 4h$ and this is implemented using the trapezoidal rule. In the DIM we use the numerical explicit filter discussed in Sec. III with $\alpha = \frac{1}{3}$. Since the corresponding effective filter width equals the filter width of L this implies that $\kappa\Delta = 0$ in Eq. (7). Note that this implies that the model is only used at the LES-level L and the assumption that the coefficient C_d is independent of the filter width is not actually needed. Equation (10) is adopted for the explicit inverse filter, where the sum is taken over $j = m-5, \dots, m+5$. Since the sum of the coefficients should be equal to 1, the coefficient of the central term ($j=m$) has been slightly changed. It has been checked that $\mathcal{H}\mathcal{H}^{-1}$ is in good approximation equal to the identity operator. Furthermore, the simulation results with the sum in the inverse filter taken over $j = m-10, \dots, m+10$ have also been generated and appear to be indistinguishable. As both the streamwise and the spanwise direction are homogeneous in the temporal mixing layer, averaging over these two directions is carried out in the calculation of C_d according to Eq. (8).

The simulations for both dynamic models reproduce the large-scale roller structures found in the DNS and undergo a transition to turbulence. Figure 2 displays the spanwise vorticity at $t=80$ in a plane in spanwise direction for both dy-

FIG. 1. Contours of spanwise vorticity of the DNS in a plane perpendicular to the spanwise direction at $t=40$ (above) and at $t=80$ (below). Solid and dotted contours indicate negative and positive vorticity, respectively.

namic models and for the filtered DNS results. By comparing the latter to Fig. 1 it can be seen that the filtering efficiently removes the smallest scales. Both dynamic models are qualitatively in good agreement with the filtered DNS results: The peak values of the vorticity are quite well predicted and roughly the correct amount of small-scale structures is present. The differences between the results of the two models are quantitatively very small. This holds for mean quantities, but also for turbulence intensities. As two examples we show the momentum thickness as a function of time in Fig. 3 and the streamwise energy spectrum in the turbulent regime in Fig. 4.

In Fig. 5 the dynamic coefficient C_d is plotted as a func-

FIG. 2. Contours of spanwise vorticity in a plane perpendicular to the spanwise direction at $t=80$. Solid and dotted contours indicate negative and positive vorticity, respectively. (a) DSM, (b) DIM, and (c) filtered DNS.

tion of time at the center-line of the mixing layer. We see that there is a sizeable difference in the value of the dynamic coefficient of 20%–50%, which apparently hardly influences the simulation results. In order to study the sensitivity of the simulation results on the value of C_d we performed another simulation in which we determined the value of C_d in the same way as in the DSM, but multiplied it afterwards by a factor of 2 (DSM2). A similar experiment has been carried out by Jiménez⁶ for isotropic turbulence. In the initial stages

FIG. 3. Momentum thickness as a function of time; solid: DSM, dashed: DIM, and $\circ$: Filtered DNS.

FIG. 4. Streamwise energy spectrum at $t=80$; solid: DSM, dashed: DIM, and $\circ$: Filtered DNS.

of the simulation the resulting subgrid dissipation turns out twice as large as in the DSM. However, this reduces the amount of small scales present, and thus decreases the strain rate tensor. Hence, in the later stages of the simulation the molecular dissipation is smaller and the subgrid dissipation is only roughly 20% larger than in the DSM. As an illustration in Fig. 6 the integral of the subgrid dissipation and molecular dissipation over space is plotted as a function of time for both DSM and DSM2. The effect on mean quantities, such as momentum thickness and kinetic energy, is seen to be very small and even turbulent intensities change by only some 10%. This is consistent with the results for DSM and DIM.

The difference between the two dynamic models can also be investigated by an *a priori* test. To this end we take the DNS results at $t=80$ and filter them to obtain the “exact” LES field at this time. In this way the dynamic coefficient can be calculated for both models. Since the value of the coefficient is averaged over the streamwise and spanwise

FIG. 5. Dynamic coefficient as a function of time at the center-line; solid: DSM, and dashed: DIM.

FIG. 6. Subgrid dissipation and molecular dissipation as function of time; solid: DSM-molecular, dashed: DSM-subgrid, dotted: DSM2-molecular, and dash-dotted: DSM2-subgrid.

directions, C_d is only a function of the normal coordinate x_2 . In Fig. 7 we compare the dynamic coefficients obtained in this way with each other and with the “optimal” value. This optimal value is defined in the following way. From the DNS field the exact turbulent stress tensor τ_{ij} can be calculated. By equating the stress tensor with the model $-C_d \Delta^2 |\bar{S}| \bar{S}_{ij}$, a similar relation for the dynamic coefficient as in Eq. (6) appears, i.e., $C_d M_{ij} = \tau_{ij}$, where $M_{ij} = -\Delta^2 |\bar{S}| \bar{S}_{ij}$, from which the coefficient can be determined in the usual way. Figure 7 shows that both models roughly follow the spatial behavior of the optimal C_d , although the DSM result is somewhat smoothened. The DSM results yield a coefficient which is $\sim 20\%$ higher, whereas the DIM coefficient is about 20% lower. The difference between DSM and DIM is qualitatively in agreement with Fig. 5 at $t=80$.

The availability of the DNS results also enables a verification of the assumption that the dynamic coefficient is independent of the filter width. To this end we calculate the

FIG. 7. Dynamic coefficient as a function of x_2 at $t=80$; solid: optimal-DNS, dashed: DSM, and dash-dotted: DIM.

FIG. 8. Optimal dynamic coefficient as a function of x_2 at $t=80$ for different filter width; solid: $\Delta=2h$, dashed: $\Delta=h$, and dash-dotted: $\Delta=4h$.

optimal C_d value for three different filter widths: $\Delta=h$, $\Delta=2h$, and $\Delta=4h$, keeping h fixed. The results at $t=80$ are displayed in Fig. 8. The qualitative agreement between the results for various filter widths is very good, thus substantiating the dynamic modeling approach. More quantitatively, we see that the larger filter width gives rise to a smoother behavior of C_d as a function of x_2 , which is not surprising. From Fig. 7 we infer that the coefficient of the new DIM follows the optimal C_d profile more closely than the coefficient arising in the DSM. Incorporating the results of Fig. 8 we notice that the improved behavior of DIM coincides with a more strict adherence to the assumption of filter width independence of C_d for the range of length scales incorporated in DIM. This is no longer as accurate for the range of length scales involved in DSM. In this instance the use of the inverse modeling approach clearly enhances the internal consistency of the approach.

Finally, we compare the efficiency of both subgrid models. The only difference between the two models is the way in which the coefficient C_d is calculated. However, as the calculation of C_d involves the filtering of several flow fields, this is a relatively costly part of the program. Since in the DIM the second term in Eq. (7) equals zero, the amount of work is less than in the DSM, resulting in a decrease in calculation time of $\sim 15\%$.

VI. CONCLUSION

We applied dynamic inverse modeling to LES of a temporal mixing layer, using Smagorinsky's model as a base model and compared the results with the standard dynamic eddy-viscosity model. To this end we introduced a numerical filter and its exact inverse. Although the dynamic coefficient calculated with both models differs, the quantitative simulation results are almost the same, both for mean quantities and for turbulent intensities. It is argued that this is caused by the insensitivity of the predictions to relatively small changes in the dynamic coefficient. A sensitivity analysis revealed that even a large change in the value of the dynamic coefficient

hardly influences mean flow quantities. An *a priori* analysis based on accurate DNS results revealed that the assumption that the dynamic coefficient is independent of the filter width holds in good approximation. An advantage of the dynamic inverse modeling is the decrease in calculation time compared to the dynamic eddy-viscosity model. Dynamic inverse modeling can also be applied to other base models and more complex flows. This will be a subject of future research.

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