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Off-resonance atomic Bragg scattering

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We performed extensive measurements on the scattering efficiency of atomic Bragg scattering on a standing light wave up to sixth order and developed a two-state Demkov model to explain the results. Using an effective 2N-photon coupling between the original and the reflected momentum states and the assumption of a mostly adiabatic transfer we derived the scattering probability. This probability has two factors: an oscillating part which represents the well-known Pendelösung oscillation and a part that is peaked in the energy difference between the two momentum states. This last part describes the tolerance on the Bragg condition for angular misalignment. Comparison of the measurements and numerical simulations with this model shows good agreement.

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Atomic Bragg scattering on a standing light wave ([1], Fig. 1) has become a basic tool in atom interferometry [2], providing a relatively simple way to coherently split an atom beam into two well-defined paths. The largest advantage of Bragg scattering over other schemes is that it can produce relatively large splitting angles allowing for large area atom interferometers with a high sensitivity to, e.g., rotation. With ongoing search for interferometers with higher contrast and sensitivity [3–5], a thorough investigation of these beam-splitters is indispensable.

Atomic Bragg scattering has been studied extensively, both experimentally [6–11] and in theory [12–14]. However, it turns out to be very difficult to find general analytic expressions for the scattering efficiency that allow for simple design criteria. Expressions exist for the case of low laser intensity [10] and for first-order Bragg scattering [8]; however, these fail at higher order scattering and the associated high laser intensity. There are two aspects to the diffraction efficiency: the well-known Pendellösung oscillation [9,10] and the dependence on the angular misalignment, which has been studied in detail for first order only [8]. In this paper we present an alternative model. We will first show that this model describes the Pendellösung oscillations. The largest gain of the model, however, is the ability to predict the acceptance angle for angular misalignment at higher diffraction order with a simple formula. This allows for a straightforward optimization of the total number of atoms in the split beams by matching the divergence of the atomic beam to this angle.

In ideal Bragg scattering the probability for scattering is determined by the Bragg condition, which requires the initial momentum \(p_{i,j}\) along the laser to be equal to an integer number times the laser photon momentum:

\[
p_{i,j} = \pm N \hbar k,
\]

where \(k\) is the laser wave number. The reflected beam then has momentum \(p_{r,i} = \pm N \hbar k\), where \(N\) is called the diffraction order. A large number of diffraction patterns, such as Fig. 1(b), integrated over \(y\), are shown in Fig. 2. A beam of metastable helium is scattered from a standing light wave that is red detuned to the \(2s\,^3S_1 \rightarrow 2p\,^3P_2\) transition (\(\lambda = 1083\) nm) with a detuning \(\Delta = -1.2 \pi\) GHz. The atom beam is collimated to a transverse velocity spread of 0.1 photon recoil (\(v_{\text{rec}} = 0.092\) m/s) and has a longitudinal velocity of \(250 \pm 1\) m/s [6]. At the top of the graph the laser is exactly perpendicular to the atom beam, \(N=0\). From the top down, the laser angle is rotated over an increasing angle. In the scanned range we clearly see the first six Bragg resonances. The atoms that arrive at 14.7 mm are the remains of the original undiffracted beam. The spatial separation with the diffracted beam is caused by the \(2Nv_{\text{rec}}\) velocity difference and the 2 m distance between the interaction region and the detector.

Because the laser is far detuned from resonance with the excited level(s), the effect of the light can be described by the ac Stark shift of the (metastable) lower level [7]. If we assume that these energy shifts are much smaller than the total kinetic energy, we can describe the interaction with the time-dependent Hamiltonian

\[
H(t) = \frac{p^2}{2m} + \frac{\hbar \Omega_2(v, t)}{2} \cos^2(kx),
\]

where \(\Omega_2\) is the effective two-photon Rabi frequency, \(x\) is the coordinate along the laser wave vector, and \(z\) is the transverse coordinate [see Fig. 1(a)]. In terms of the single photon Rabi frequency, \(\Omega_2 = \Omega_1^2 / 2\Delta\). This Hamiltonian couples momentum states that are separated by \(2\hbar k\) in momentum. Starting with momentum \(p_0\) the atom can end up with any momentum \(p_0 \pm 2N \hbar k\). An accurate description of the diffraction profile can be obtained by taking a large number of these momentum states as a (partial) basis and integrate the associated Schrödinger equation numerically. However, although

![Figure 1](image_url)

**FIG. 1.** (a) Schematic representation of the experimental setup. A highly collimated atom beam is diffracted on a standing light wave formed by retroreflection of a laser beam. (b) Example of the resulting image on the two-dimensional detector.
FIG. 2. One-dimensional detector image (integrated over $y$) as a function of the laser angle for fixed laser power. The first six resonances of the Bragg condition are clearly visible.

accurate [6], these fully numerical calculations provide little insight into the diffraction process.

In the case of Bragg scattering we see that the atoms are distributed over the two momentum states $|p_0\rangle$ and $|p_0+2N\hbar k\rangle$. In that case the process can be simplified to a two-level system. In the case of perfect Bragg scattering these two momentum states are completely degenerate. We will, however, consider off-resonant Bragg scattering, where there is a (small) energy difference. With first-order Bragg scattering and a simple omission of higher momentum states, the scattering probability can be solved analytically for a square [8] or a hyperbolic secant laser profile [12]. In this work we expand on these results to higher order scattering using a Demkov model [15].

The Demkov model describes two states that have constant energy difference $\Delta E$ and a peaked interaction strength. Nonadiabatic transitions take place at the points $\pm t_0$ where the interaction strength is equal to the energy difference. If the interaction strength is (locally) exponential with time constant $\gamma$ at the points $\pm t_0$ the transition probability is equal to [15]

$$P_{1\rightarrow2} = \sin^2\left(\frac{\phi_0}{2}\right) \operatorname{sech}^2\left(\frac{\pi \Delta E}{\hbar \gamma}\right).$$

Here, $\Omega(t)=A \exp(\gamma t)$ or the local fit of this form around $\pm t_0$. In Bragg scattering we take the two relevant (momentum) states with constant (kinetic) energy. Because we will consider only the transition probability between two momentum states that are close to the Bragg condition, the energy difference is very small such that $t_0$ lies in the tails of the Gaussian profile. In that case the interaction strength can indeed be approximated by an exponential making the Demkov model ideal for this process.

We can see the qualitative agreement of the Demkov transition probability [Eq. (3)] with Bragg diffraction. There is a resonance peak at $\Delta E=0$, the resonance of the Bragg condition. Furthermore, the first part indicates the Pendellösung oscillation [9] which can be explained by the acquired phases during the adiabatic evolution [16]. The phase $\phi_0$ in Eq. (3) is the total phase area between the coupled states during the entire pulse, with $\Delta E$ set to zero,

$$\phi_0 = \int_{-\infty}^{\infty} \Delta E_H(t) dt / \hbar,$$

where $\Delta E_H$ is the difference between the two eigenvalues of the Hamiltonian. In Fig. 3 the smallest few instantaneous eigenvalues of the full Hamiltonian (in the discrete momentum basis) are plotted versus the transverse position in the standing light wave for typical parameters. The Pendellösung phase $\phi_0$ for fifth-order Bragg scattering is given by the shaded area.

We measured the Pendellösung oscillation by scanning the total laser power when the laser angle was set for fifth-order Bragg scattering. The total fraction of atoms with momentum $4.5 \hbar k < |p| < 5.5 \hbar k$ is plotted in Fig. 4 as a function of the running beam laser power, with the overall maximum scaled to 1. The dashed line in this graph is the result from the phase area calculation as sketched in Fig. 3. The phase of the measured first five oscillations agrees very well with the calculations. At high laser power the amplitude of the oscillation becomes too low by losses to off-resonant momentum states. It should be noted that there are no free parameters in the calculations (except for a 5% correction to the laser power).

A notable feature in this graph is the maximum in the frequency of the oscillation at approximately 20 mW. This effect can readily be understood by considering the two limits in laser power. At low laser intensity the difference between the relevant eigenvalues and thus the phase area $\phi_0$ as depicted in Fig. 3 grows approximately with $P^N$ [cf. Eq. (6)]. At very high laser power the energy levels are approximately equidistant with a difference that is proportional to the laser power resulting in a phase area that is proportional to the laser power. The complete function that describes the phase...
area versus laser power then (provided \( N > 1 \)) necessarily has a maximum slope somewhere between these limits that corresponds to the maximum frequency in Fig. 4.

The acceptance angle for Bragg scattering is determined by the hyperbolic secant part of the Demkov transition probability [Eq. (3)]. The energy difference \( \Delta E \) is the difference in kinetic energy when the laser is slightly misaligned from the Bragg condition and the incoming atoms have a momentum mismatch \( \Delta N h k \). In that case,

\[
\Delta E = 4N\Delta N h \omega_{\text{rec}}.
\]

The time factor \( 1/\gamma \) is obtained from a local expansion of the effective 2N-photon coupling strength at the point \( \ell_0 \) where this effective interaction strength \( \Omega_{2N} \) is equal to \( 2\Delta E/h \). We use the approximation \[10\]

\[
\Omega_{2N} = \frac{\Omega_2^N}{2^{3N-3}[(N-1)!]^2\omega_{\text{rec}}^{N-1}}.
\]

This approximation is only valid for small laser intensity. It is difficult to find a strict validity range, but comparison with numerical calculations gives \( \Omega_2 < 10\omega_{\text{rec}} \) for a maximum error of 10%. Although the maximum value of \( \Omega_2 \) is typically 60\( \omega_{\text{rec}} \), the nonadiabatic transitions occur in the tails of the laser profile and the accuracy should be sufficient. A Gaussian profile of the laser intensity gives

\[
\gamma = \frac{1}{\tau} \sqrt{8N \ln \left( \frac{\Omega_{2N,\text{max}}^N}{8N[(N-1)!]^2\omega_{\text{rec}}^N N}\Delta N \right)},
\]

where \( \tau = W_l/v_z \) and \( W_l \) the \( 1/e^2 \) laser waist radius.

To check this Demkov model, we performed a large number of measurements in which we scanned the laser angle at various settings of the laser waist and laser power. An example for one value of the laser power is given in Fig. 2. The width of a diffraction peak in this plot in terms of laser angle indicates the acceptance angle. We also performed a full se-

![FIG. 4. Pendellösung oscillation as a function of the total laser power for fifth-order Bragg scattering. The points indicate the measured fraction of atoms in the fifth diffraction order, with the overall maximum scaled to 1. The dashed line is the result from the phase area calculation. \( W_l = 1.6 \text{ mm}, v_z = 250 \text{ m/s}. \)](image)

![FIG. 5. FWHM acceptance angle for Bragg scattering as a function of the Bragg order \( [W_l = 1.6 \text{ mm}, \Delta _l = 1(2\pi) \text{ GHz}, P = 21 \text{ mW}]. \) The percentages next to the measurements indicate the maximum scattered fraction of atoms.](image)

![FIG. 6. FWHM acceptance angle for Bragg scattering as a function of the single beam laser power \( [W_l = 1.6 \text{ mm}, \Delta _l = 1(2\pi) \text{ GHz}, N = 2]. \) The three measurements were taken at a maximum of the Pendellösung oscillation. The circles, connected by the dashed lines, give the results from numerical simulations. The size of these circles indicates the maximum transfer at that laser power. The jumps in these acceptance angles occur at the minima of the Pendellösung oscillations.](image)
FIG. 7. FWHM acceptance angle for Bragg scattering as a function of the transit time through the laser beam. The symbols are the result from the Demkov model. The lines depict the simulations. The result from a transit-time broadening model is indicated by the line, where the transit time was taken equal to \( \tau \).

The dependence of power around the maximum. This is most likely caused by the strong, locally dispersive dependence on the laser power. As a result, the acceptance angle is set by the momenta at which the laser waist. Its effect on the acceptance angle is plotted in Fig. 7. Again, there is a strong similarity between the model and the simulations. The results for first-order scattering (\( N=1 \)) differ the most, because the energy difference between low momentum states is small. In that case, the interaction with other off-resonance states is stronger and the system cannot accurately be described as a two level system. For short interaction time, the acceptance angle becomes larger than \( 1/\hbar \). At that point the atoms are scattered to multiple orders [11].

One might be tempted to use a simple transit-time broadening model to explain the acceptance angle. In that case the acceptance angle is set by the transverse momentum at which the energy difference between the original and the reflected beam exceeds the energy uncertainty \( \hbar/\tau \) associated with the interaction time. The results from this model are plotted in both Figs. 5 and 7 where the interaction time was taken to be equal to the laser waist. The plots show that this model is too naive and clearly underestimates the acceptance angle.

In conclusion, we have shown that a two level Demkov model can accurately describe high-order Bragg scattering. It allows for a straightforward optimization for the use in atom interferometers. To be specific, the waist of the standing light wave can now easily be set to create an acceptance angle of \( 1/\hbar \) that allows for a “clean” two-path interferometer with maximum signal at minimum requirements for the collimation of the atomic beam. Because this model holds well even at large diffraction angle it will be very useful for the design of interferometers with large separation and large area.

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